Astrometric Methods

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Amongst others, this talk covers:

- Relative vs. Absolute Astrometry
- Ground vs. Space
- single Telescope vs. Interferometer

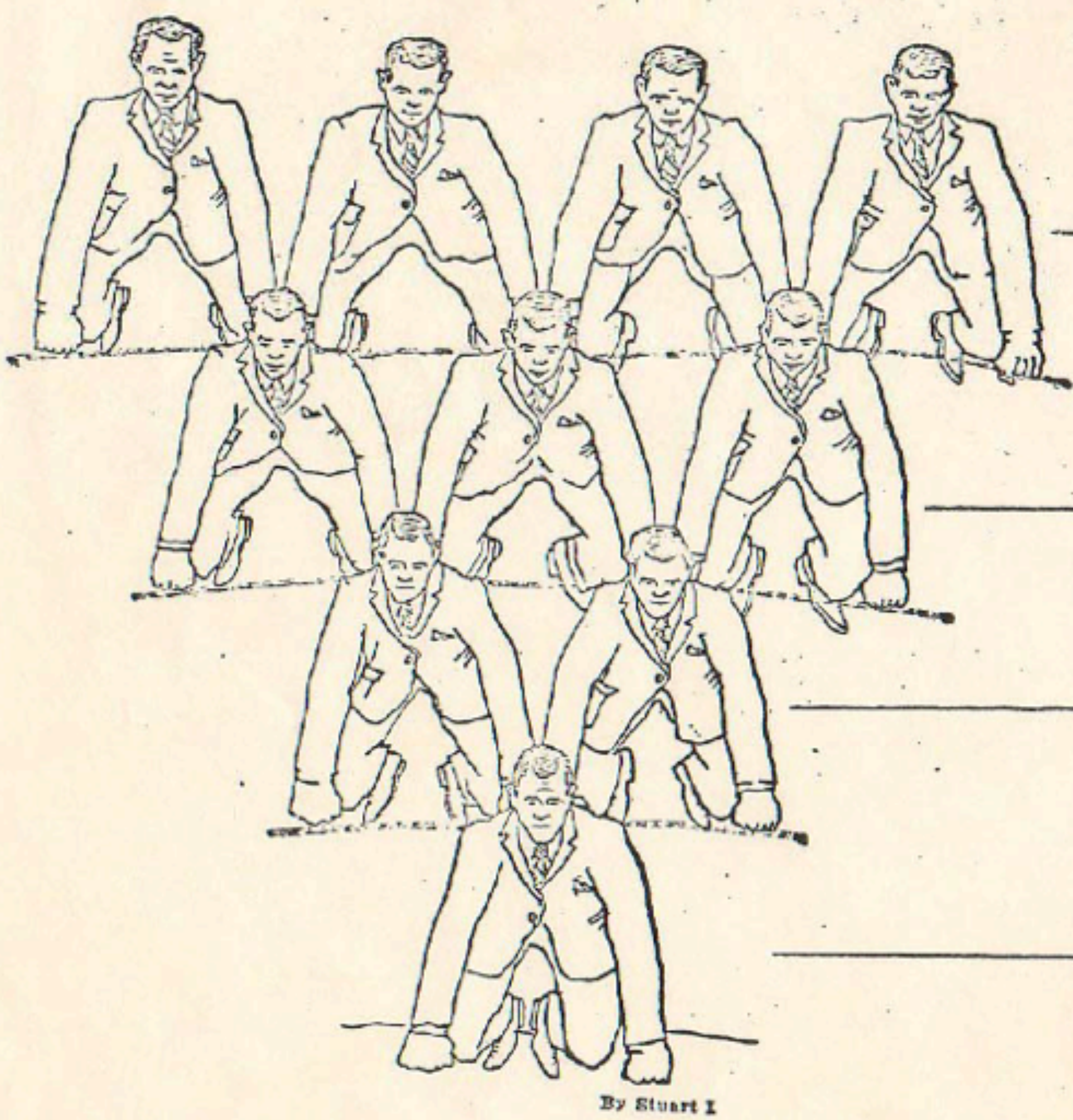
What is Astrometry?

- Oldest subfield of Science
  (note: not just Astronomy!)
- Astrometry is about measuring positions of objects
- ... as a function of time
- ... with ultrahigh precision
- Astrometry is also about establishing a suitable coordinate system for those measurements

Astrometry is the basis of Astronomy

THE ASTRONOMICAL PYRAMID

ILLUSTRATING THE INTERDEPENDENCE OF THE VARIOUS AREAS OF STUDY



COSMOLOGISTS, GENERAL RELATIVISTS, CRANKS, OTHER FUZZY-BRAINED PENCIL-PUSHERS.

ATMOSPHERES, INTERIORS, INTERSTELLAR MED. THEORISTS

SPECTROSCOPISTS PHOTOMETRISTS

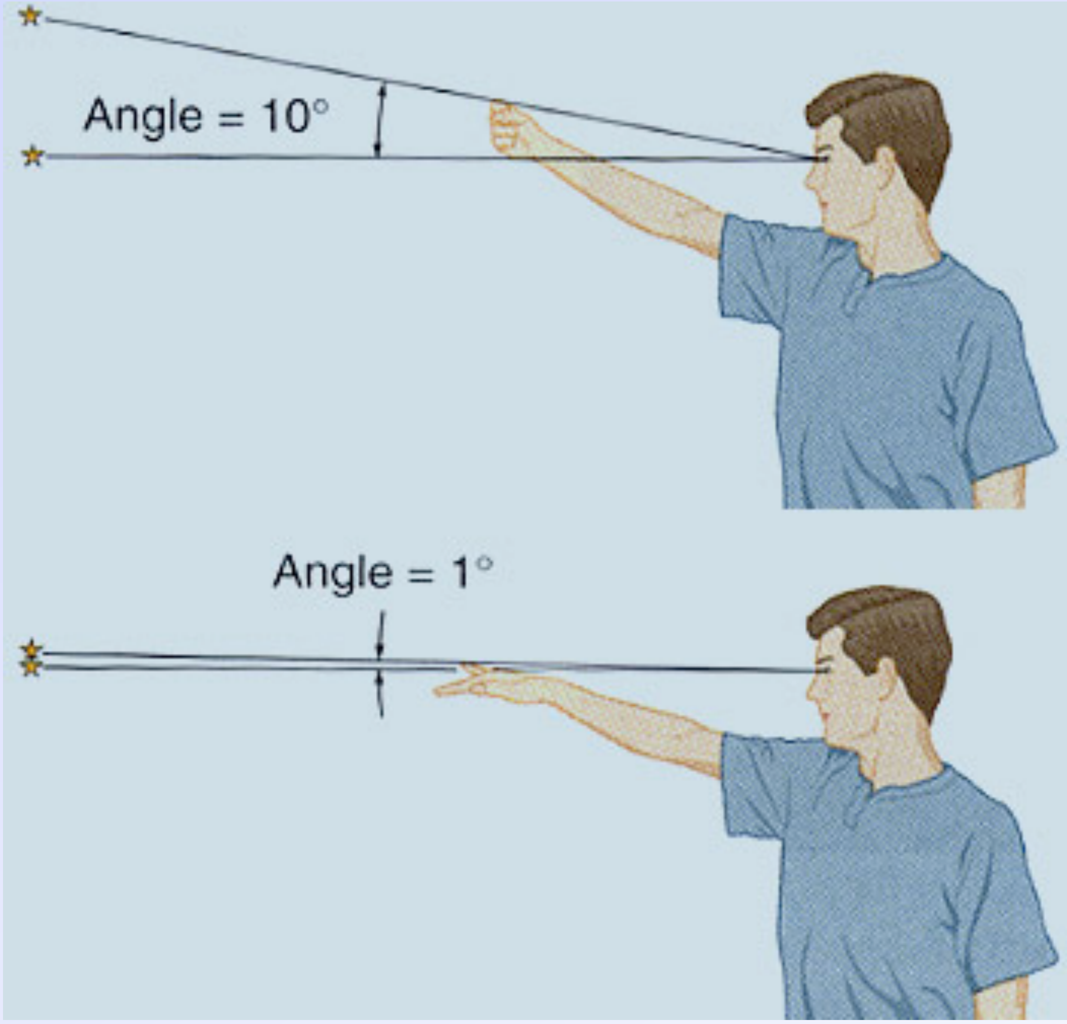
ASTROMETRISTS

GET BACK TO BASICS -- SUPPORT ASTROMETRY

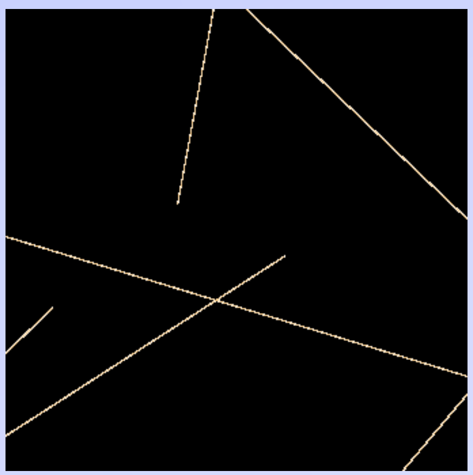
circulated ca. 1974 by Ron Probst at UVa

Why is it so difficult?

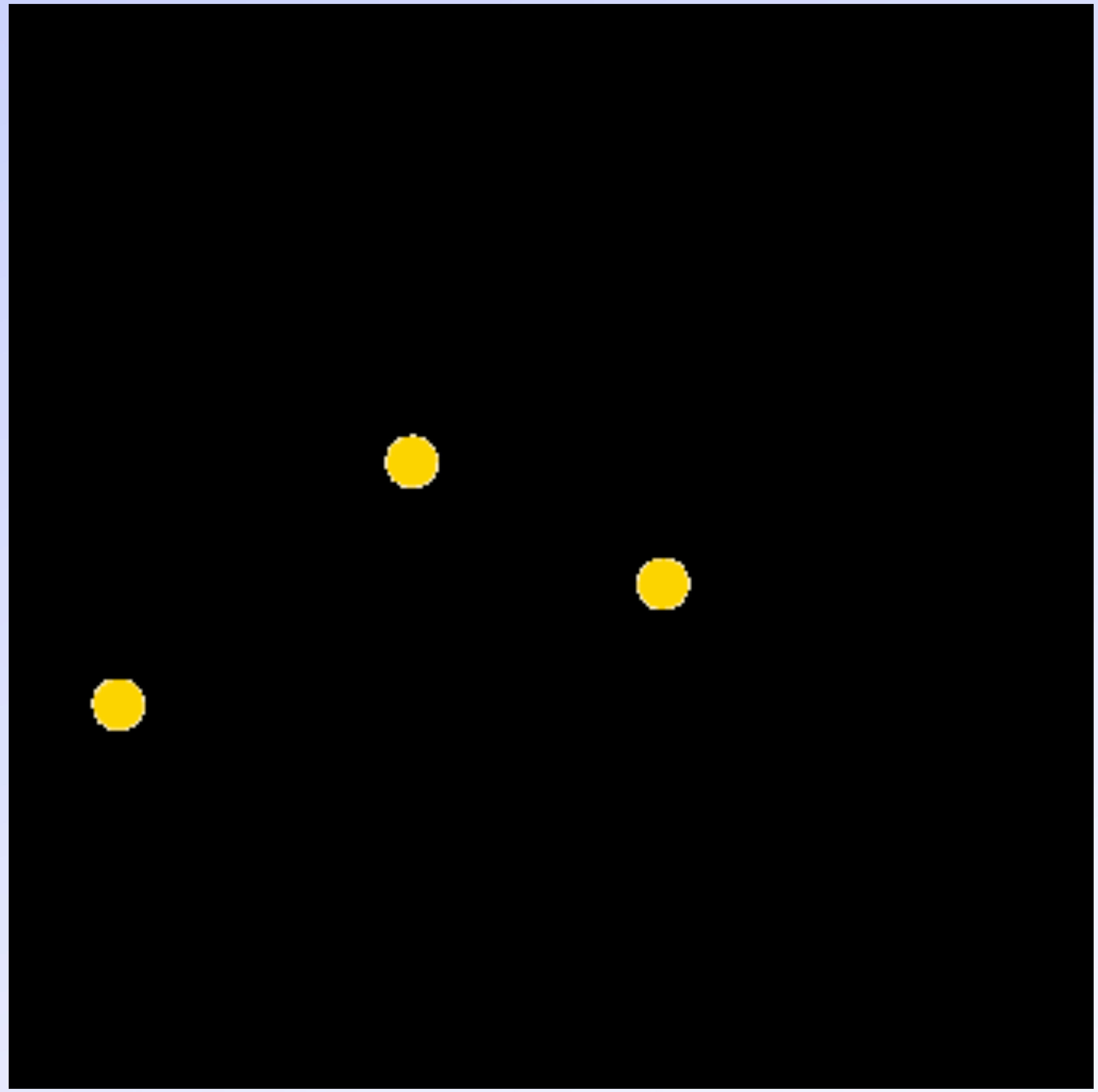
- Measuring positions sounds easy!
- Possible even without instruments:



(1) It's difficult, because...

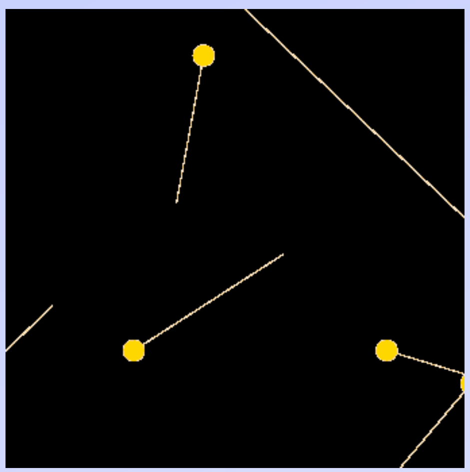


shown: proper motion



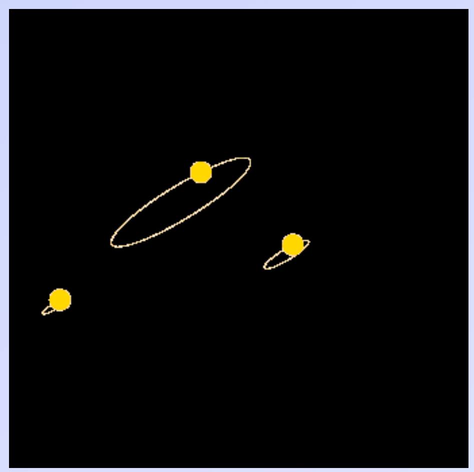
...everything on the sky moves, no fixed reference point!

(2) It's difficult, because...



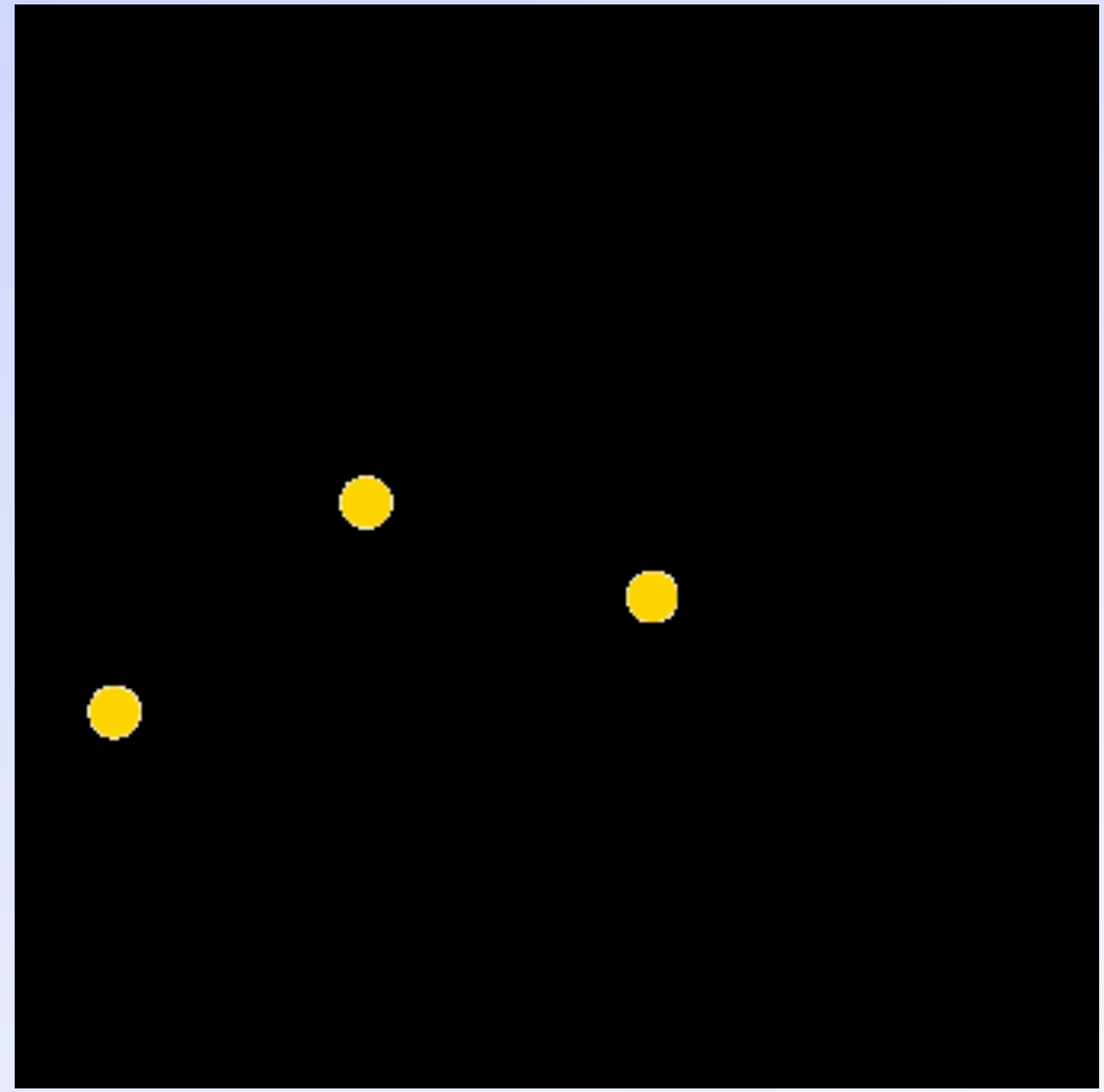
proper motion

+



parallax

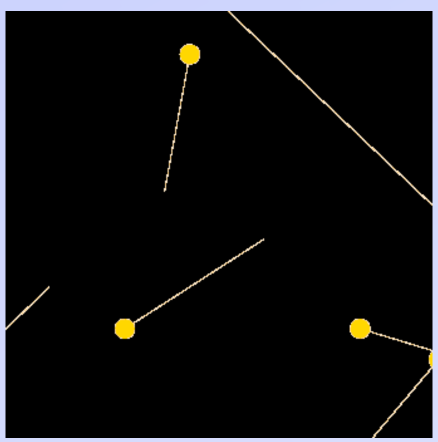
=



... those motions can be quite complicated!

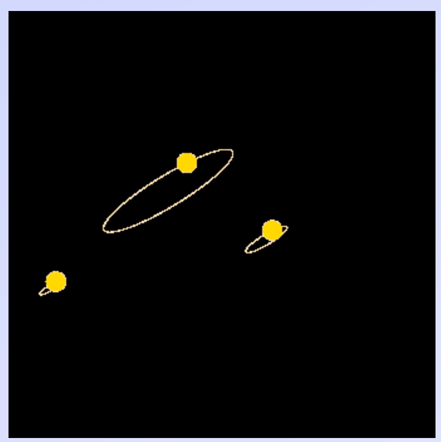
… and that was just the simplest case …

- if it's a binary or a star with a planet, the motion would look like this:



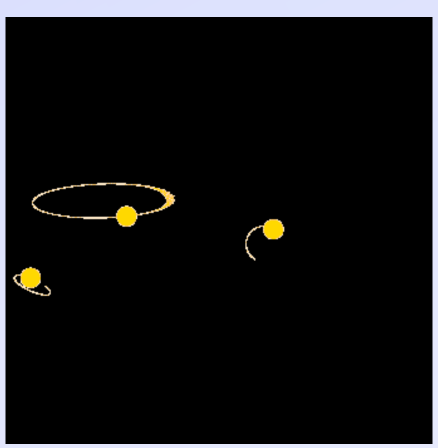
proper motion

+



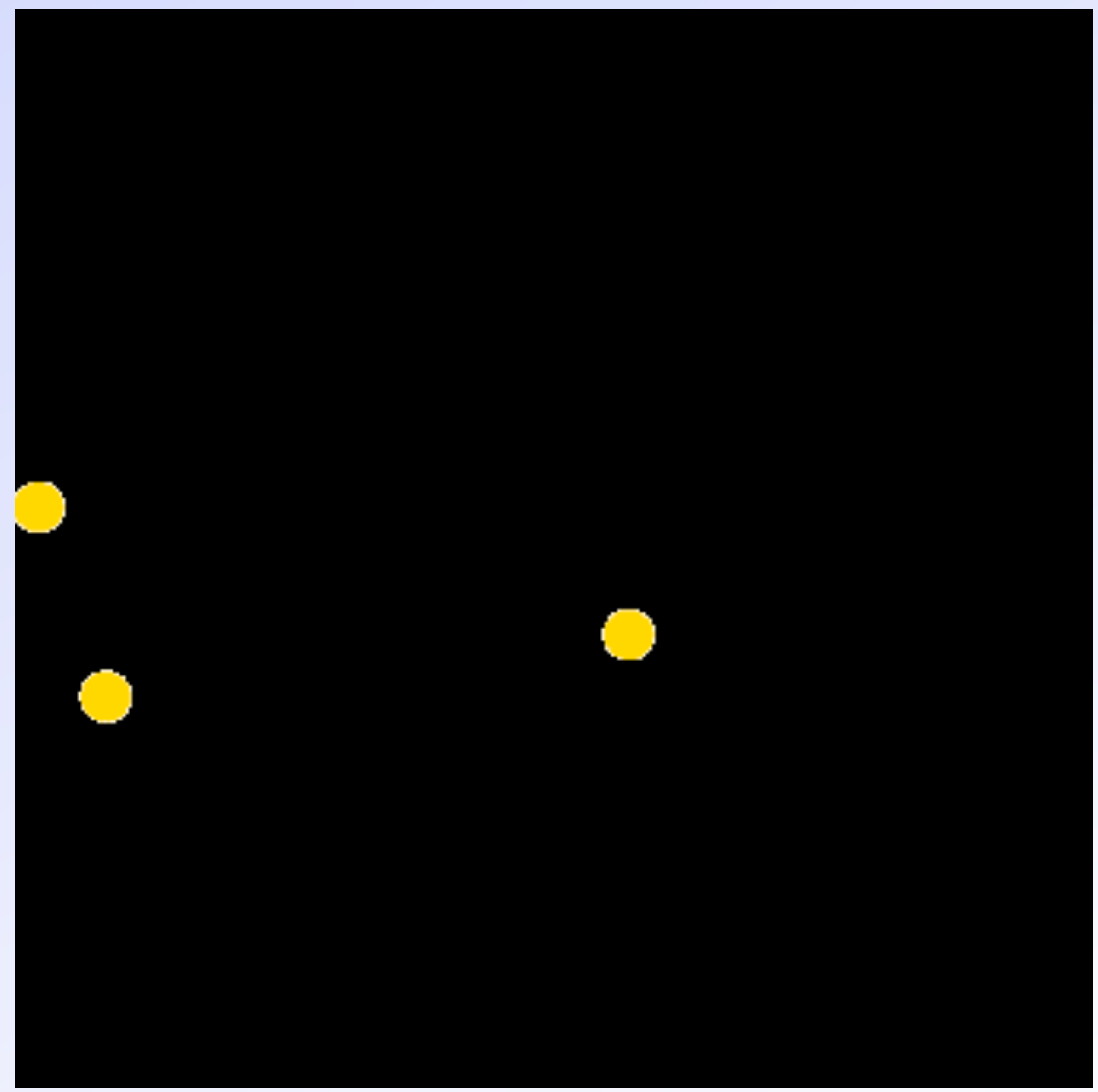
parallax

+



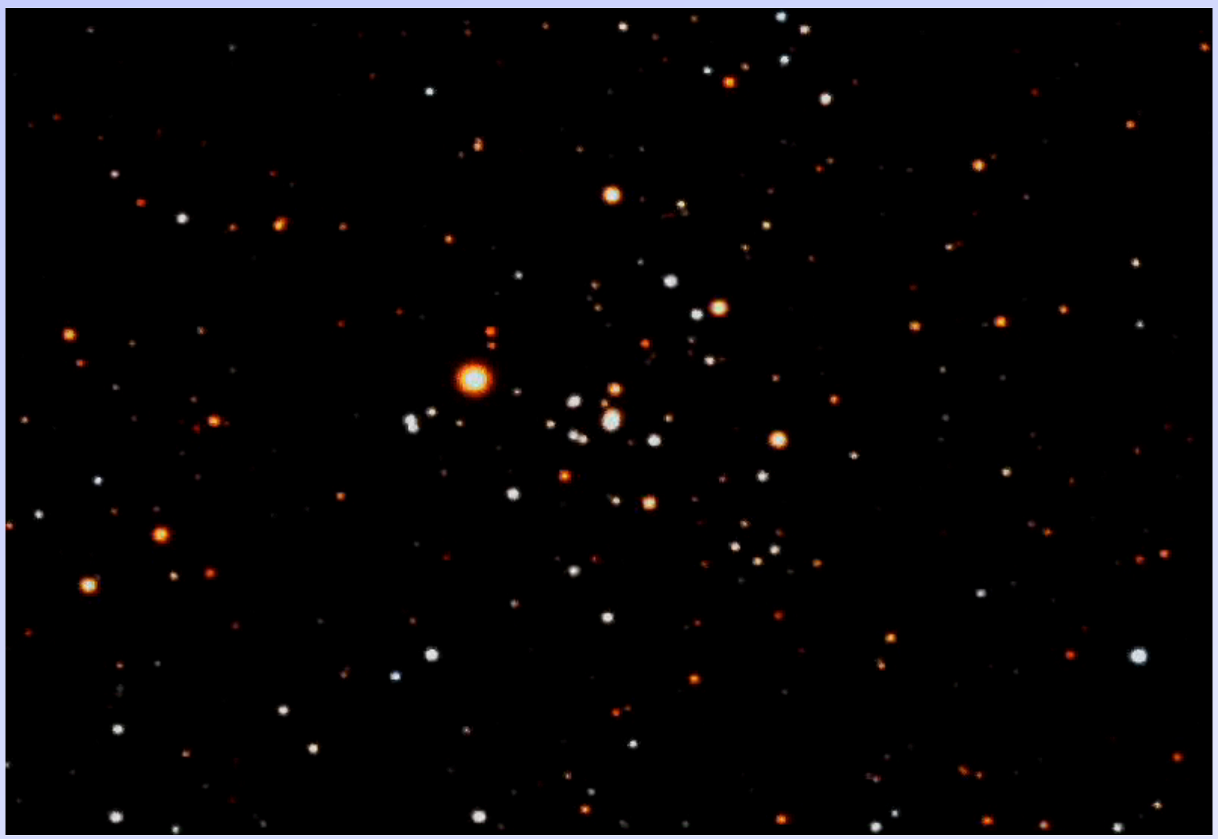
orbital motion

=



(3) It's difficult, because...

motion of the Hyades over 60 000 years



CREDIT: M. PERRYMAN & J. DE BRUIJNE

- required precision: milli- or micro-arcseconds
- 1 mas corresponds to the growth of a human hair in 10 seconds as viewed from a distance of about 10m!

... those motions are tiny!

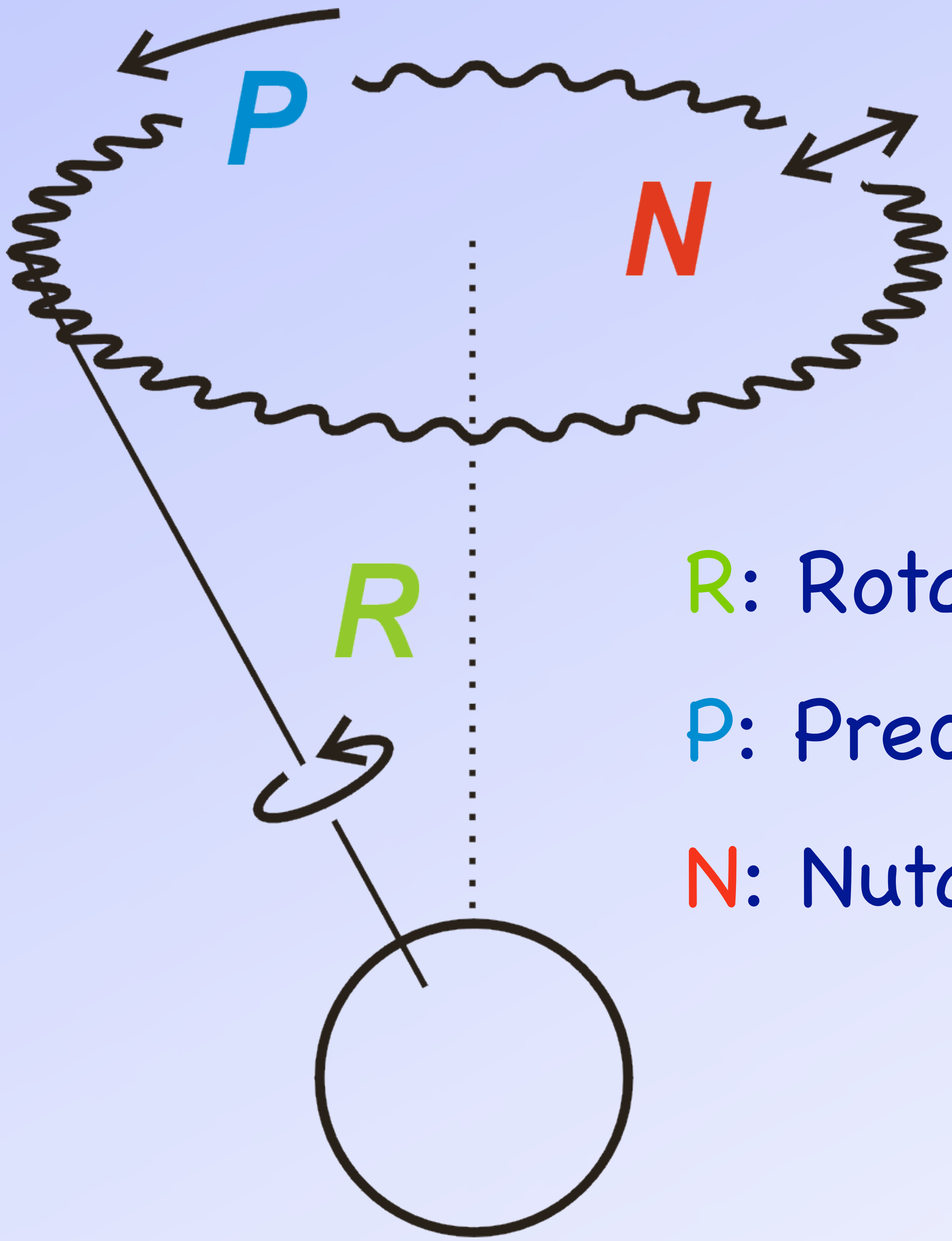
(4) It's difficult, because...

Earth Rotation

CREDIT: C.J.HAMILTON

... the Earth (or a satellite) will move, too!

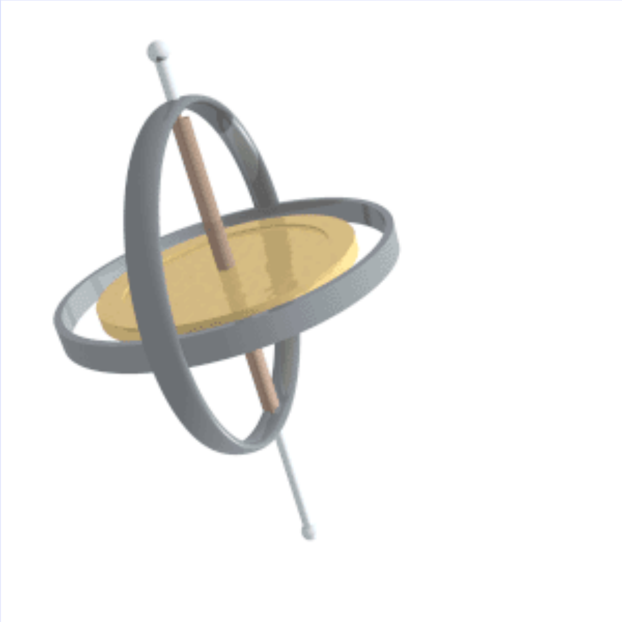
(5) It's difficult, because...



R: Rotation

P: Precession

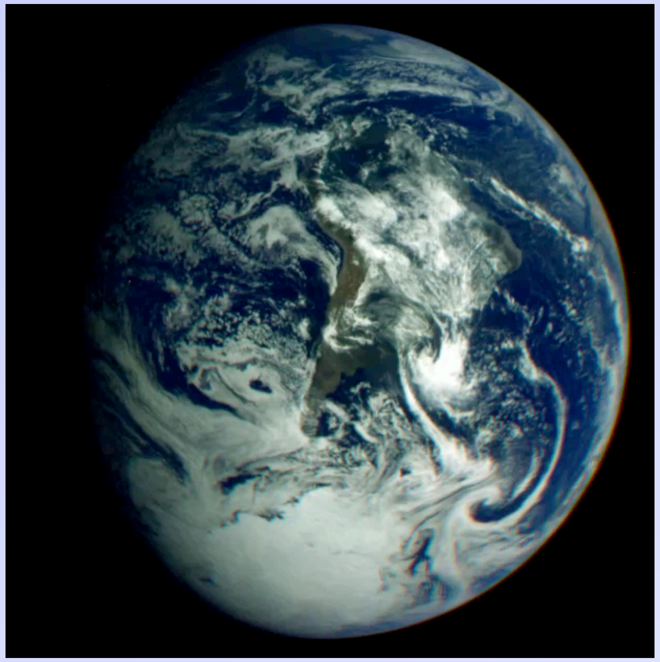
N: Nutation



we are trying to measure tiny angles from the surface of a spinning gyroscope!

... the Earth axis will not be fixed, either!

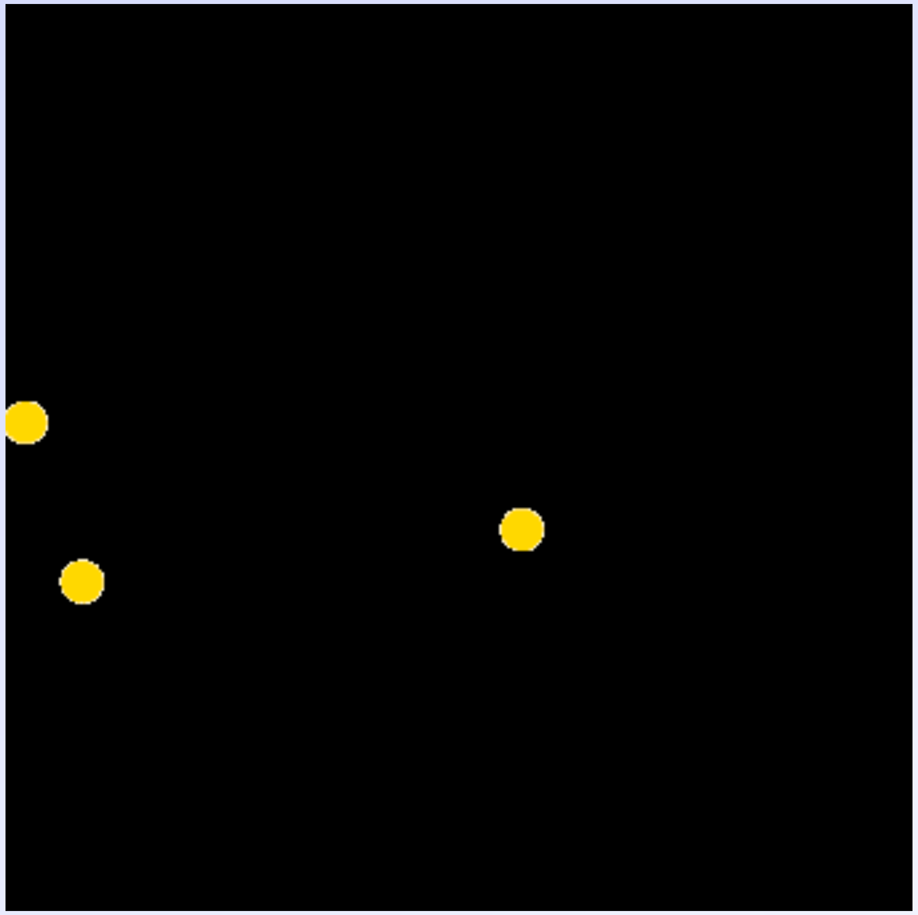
Putting it all together almost makes it look impossible:



credit: D. Ellison, JPL/NASA

from this

observing this



Can it work? – YES!!!

How do we make it work?

1. everything on the sky moves

   --> establish a local or global reference system
   (sometimes, relative astrometry in a small field is enough)

2. motions are tiny

   --> clever measurement techniques
   (e.g. transit circles, space astrometry, interferometry)

3. Earth moves

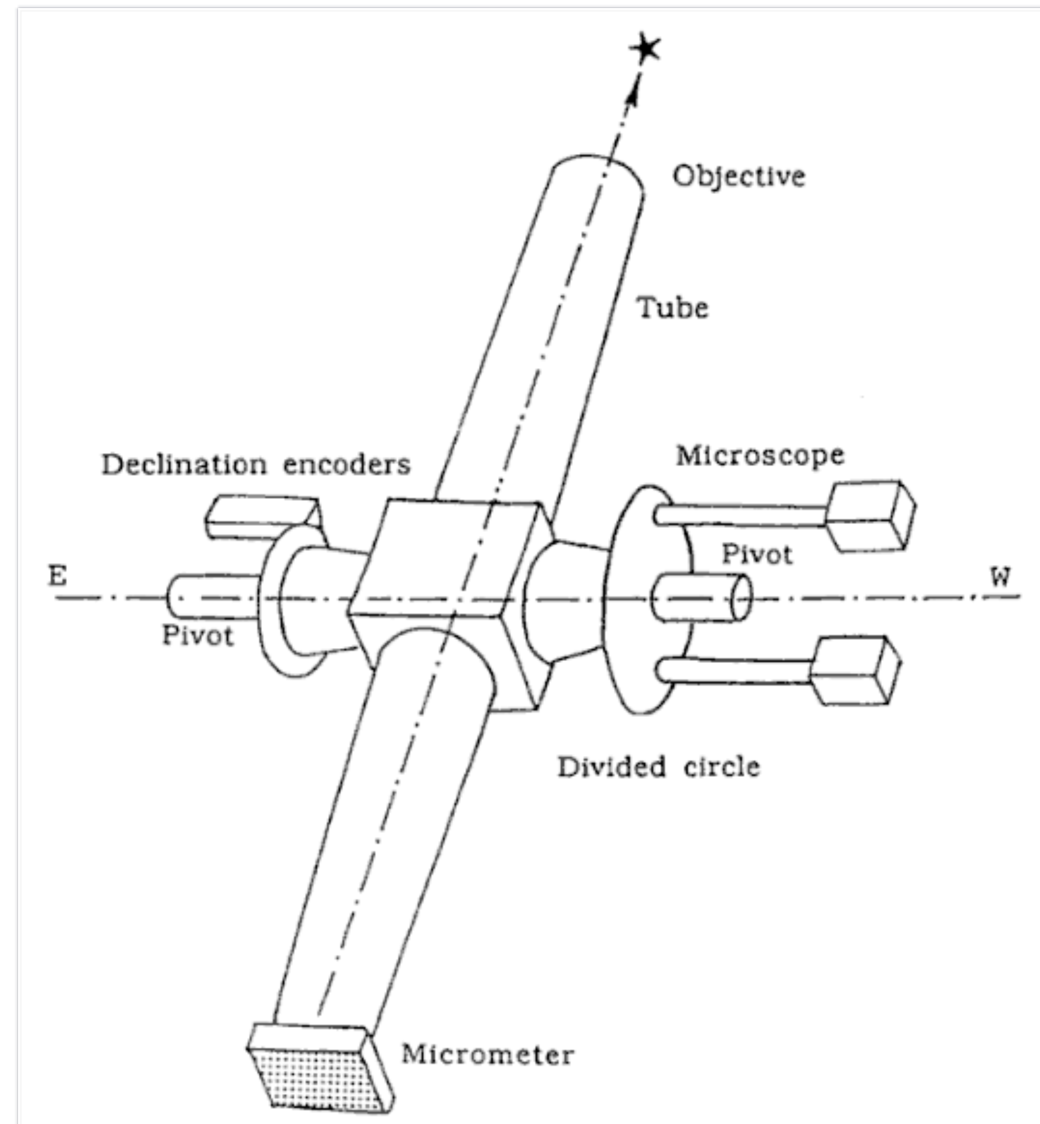
   --> meaure Earth movement with high precision and remove
   (VLBI, lunar laser ranging, etc.)

Transit (Meridian) Circles

- basic astrometric instrument for centuries!
- the transit (upper culmination) of star is measured:

  right ascension is equal to sidereal time during transit of the meridian

  declination is given by latitude of the observer plus/minus the (measured) zenith distance



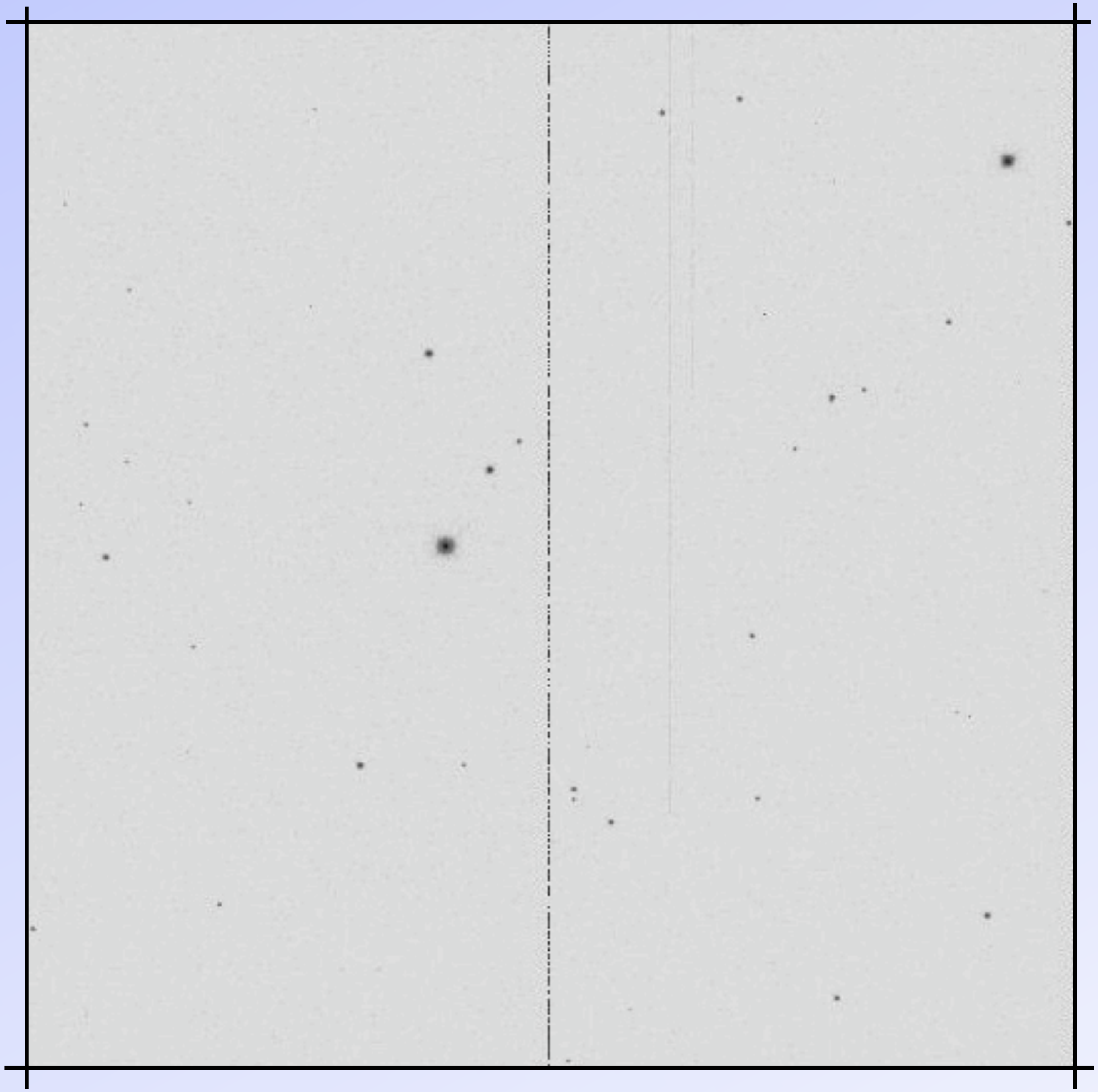
credit: J. Kovalevsky, Modern Astrometry

remember: the local meridian is the great circle passing through north and the celestial pole!

Relative Astrometry with a Single Telescope

- classical approach
- with a single telescope, one repeatedly takes an image of the interesting region over the course of years
- works with photographic plates or CCDs
- measure position of a target star as a function of time in the local reference frame of surrounding (background) stars
- often, target is much closer than reference stars, so proper motion and parallax will be measurable

Data Reduction



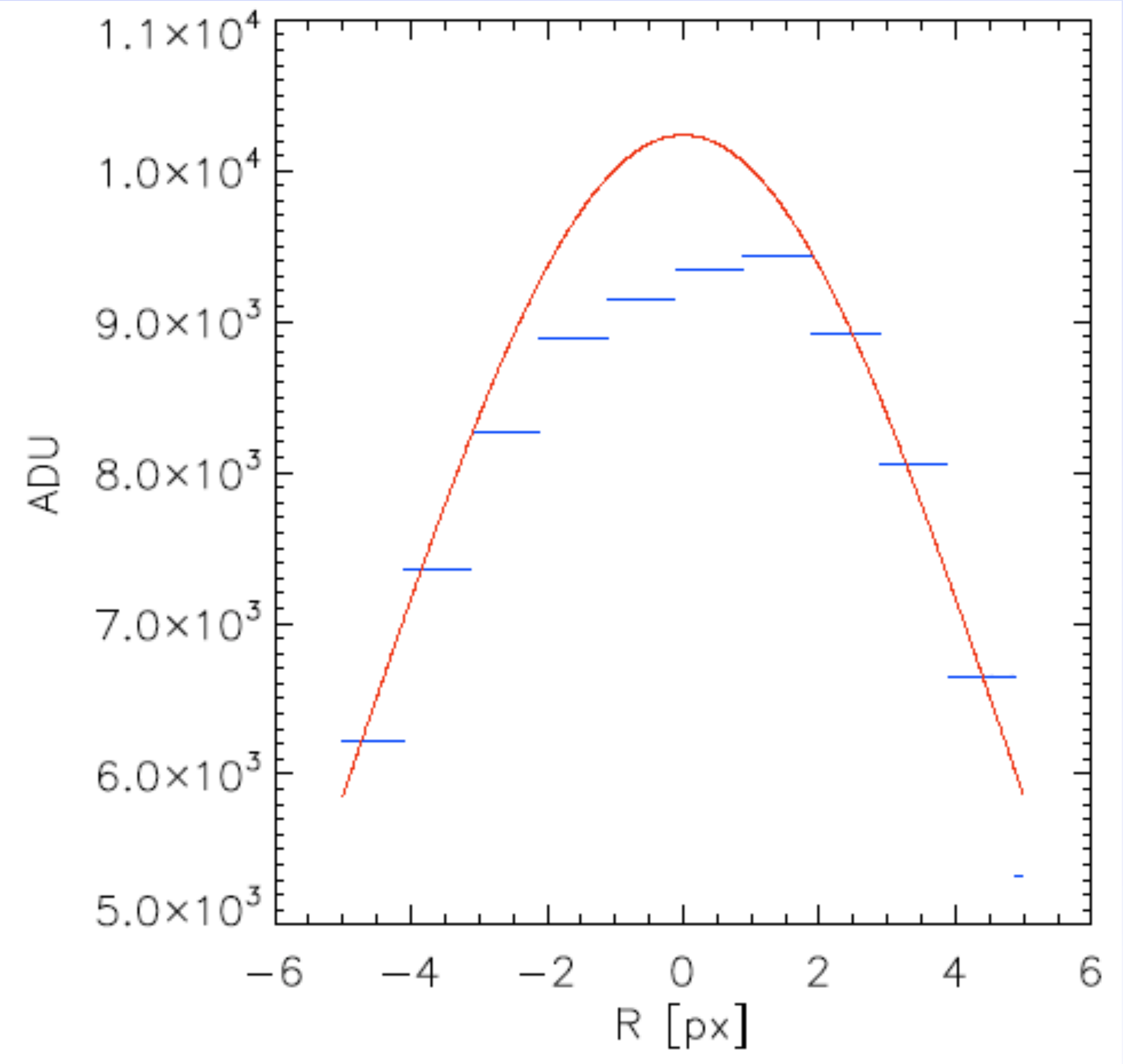
TWA 1 with SUSI2/NTT

credit: B. Sturm

- take image
- flat-field, remove bias and bad pixels
- label your reference stars and measure their positions!

Simple Centroiding

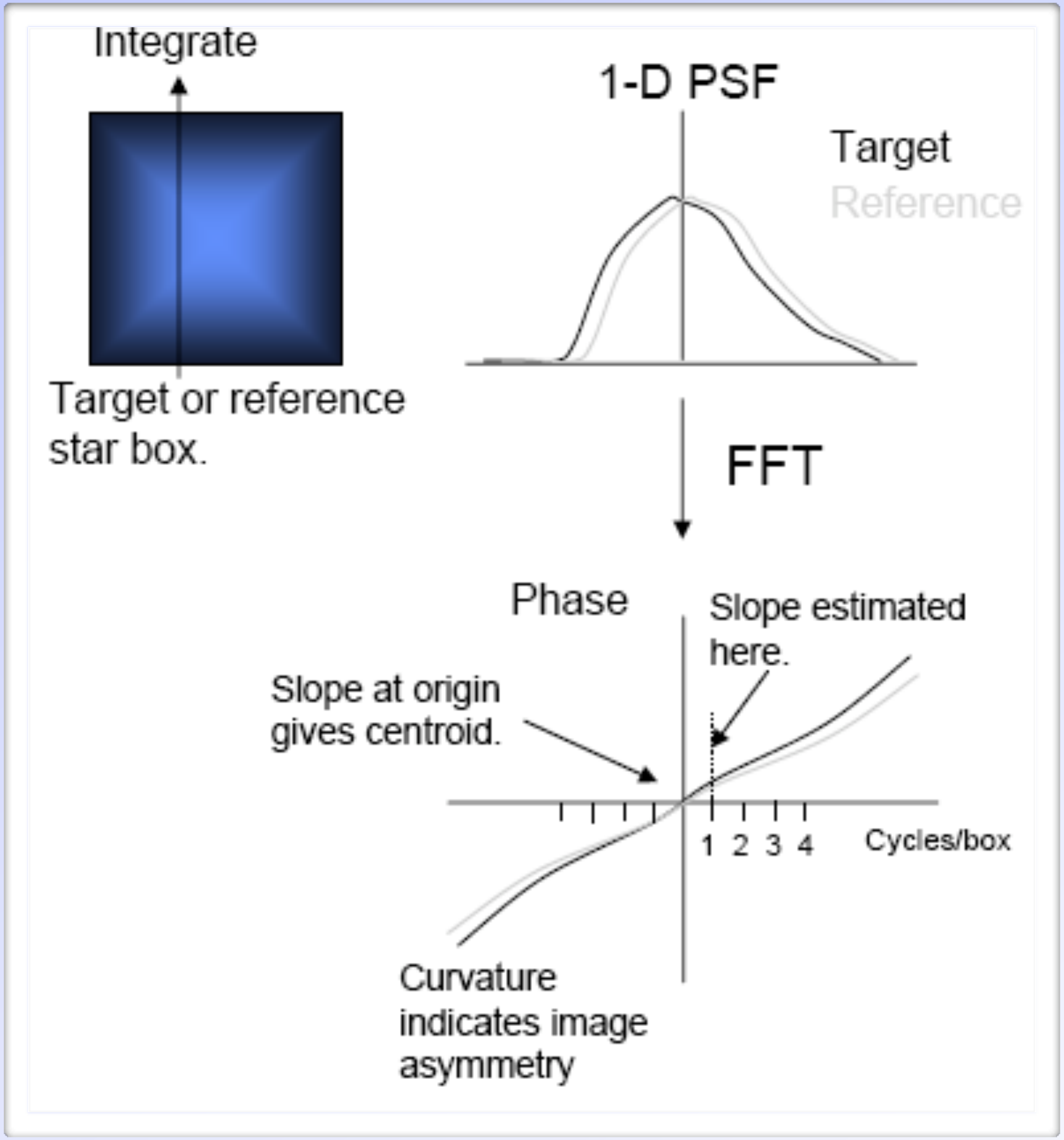
- a critical step in the data reduction is the precise measurement of the position of each object on the frame, i.e. the centroid of the image
- sometimes, if PSF is not symmetric, an elliptical Gaussian fit might not be good enough



intensity distribution

Gaussian Fit

Sophisticated Centroiding



the STEPS Survey does something more sophisticated

Credit: Stuart Shaklan

Pixel Scale and Field of View (FOV)

- pixel scale: usually the precision of a measurement will scale with the pixel scale
- good centroiding algorithm will measure the centroid to about 1/50 of a pixel
- at the same time, the FOV should be large to contain many not too faint reference stars

1/50

CTIO 0.9m (RECONS)	FOV: 8.6′	401 mas/pixel	8.0 mas
FORS UT1 8.2m	FOV: 3.3′	100 mas/pixel	2.0 mas
Palomar 5m (STEPS)	FOV: 2′	61 mas/pixel	1.2 mas

roughly achievable single measurement precision

Atmospheric Image Motion

- biggest complication on the ground: atmospheric image motion (turbulence etc.)
- typical atmospheric noise: 1 mas
- correlation timescale of order only 1 second

→ can be overcome by taking many observations in a row and averaging
if more than 1 second in between images, atmospheric noise is uncorrelated and improves as sqrt(t)

1 image - 1 mas accuracy

100 images - 0.1 mas accuracy

Differential Chromatic Refraction (DCR)

- refraction itself is not a problem, as long as it's the same for all stars (remember, we only do relative astrometry)
- but if it's different for individual stars, it matters!
- differential chromatic refraction depends on the color of the star -> need to correct for that
- need temperature, pressure, humidity, and star color
- easier to correct for smaller bandpass
  -> use narrow filter, if possible
- also, size of DCR wavelength dependent
  (smaller in red than in the blue)
- depending on the particulars of the observing program, DCR is often the limiting factor in for ground-based astrometry!

Plate Solutions

- next, we want to combine all our measurements over several epochs of observations
- in order to do that, we have to correct for possible variations in the focus and the CCD orientation (translation, rotation etc.)
- sometimes, coordinates will also depend on magnitude (magnitude equation)
- this is achieved with a plate solution, where certain parameters depending on the relative CCD orientation are solved in a large global solution of all frames

Example for a Plate Solution

$$\xi = Ax + By + C$$

$$\eta = Dx + Ey + F$$

ξ, η: coordinates after transformation

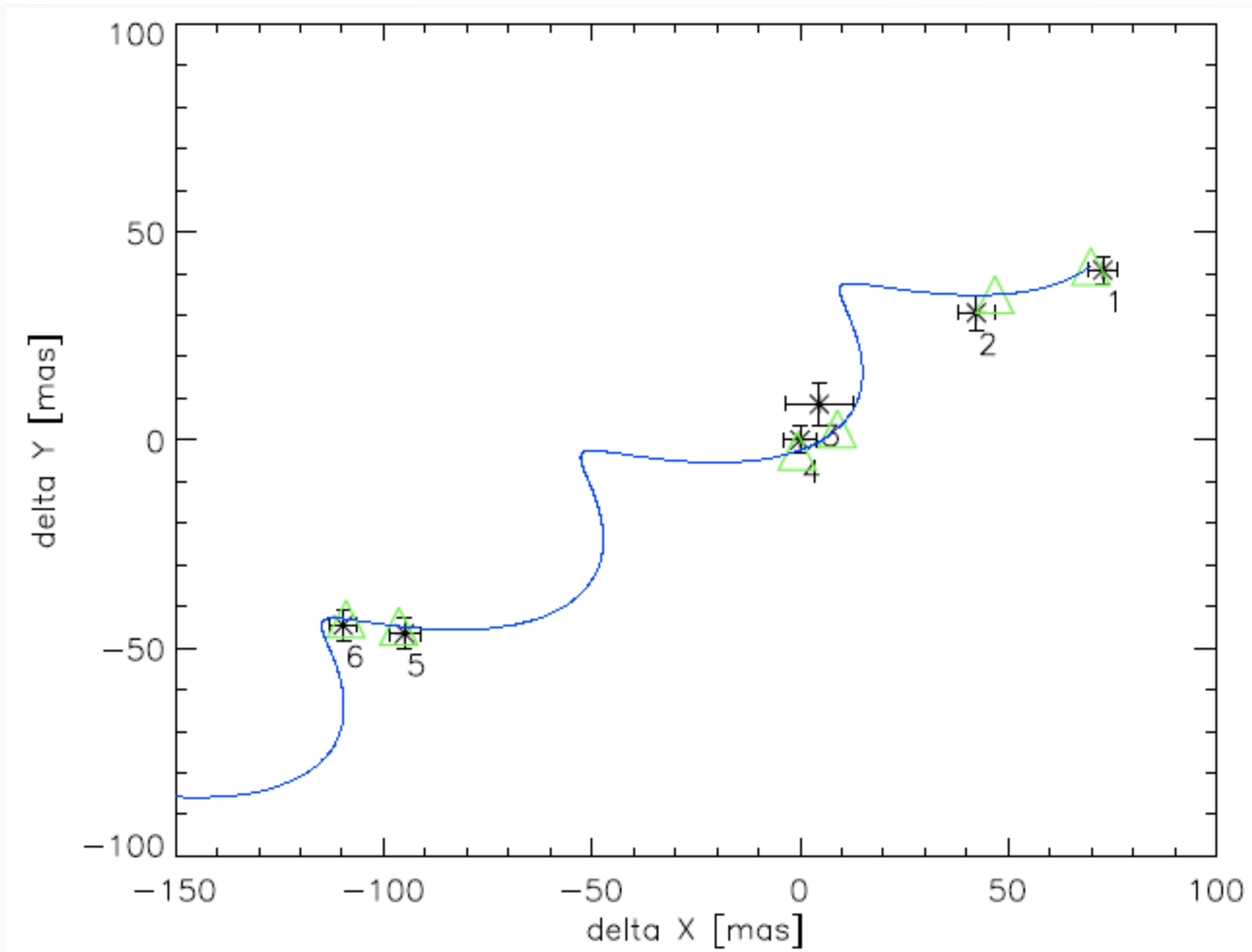
A, B, C, D, E, F: plate constants

A, E: scale (focus)
B, D: field rotation
C, F: translation

Note: this model does not include higher order terms dependent e.g. on distance from center of field (radial distortion), magnitude etc.

Finally: Relative Astrometry

- now, we have fully reduced everything on a common reference frame and can combine all measurements:



credit: B. Sturm

x measurements

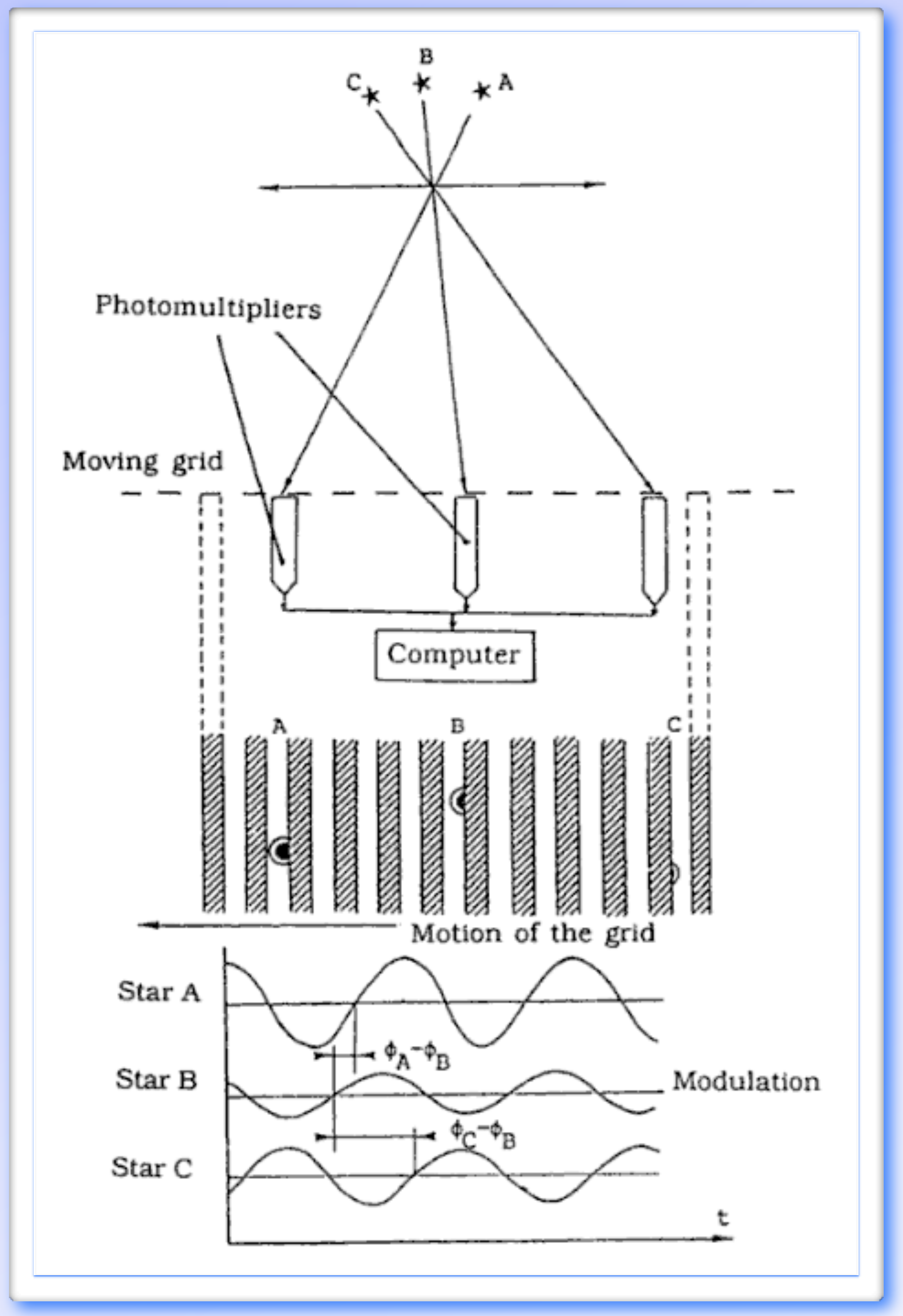
model based on relative proper motion and relative parallax

note also the additional constraints set by the timing of the measurements!

From Relative to Absolute Astrometry

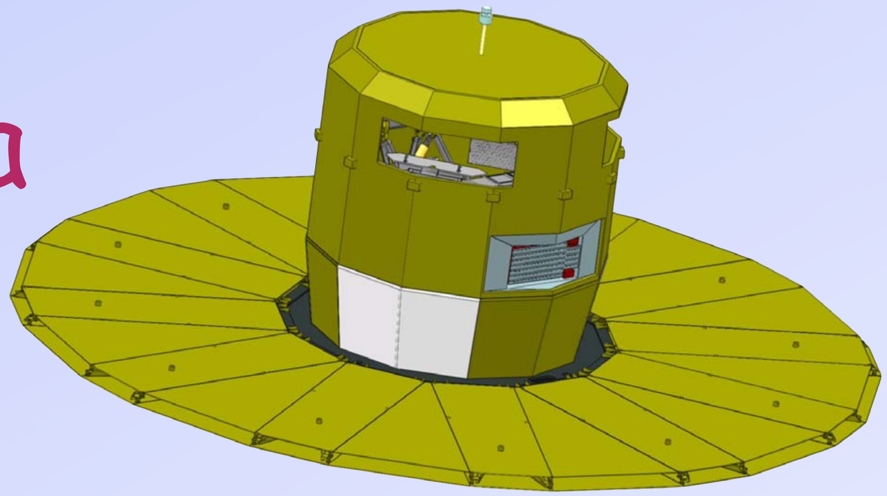
- if desired, one can finally try to convert relative astrometry into absolute astrometry
- this step is usually rather uncertain, and one might wish not to do it
- requires outside knowledge about reference frame objects, e.g. photometric or spectroscopic parallaxes, or statistical assumptions about the reference stars (e.g., random proper motions)

The Multichannel Astrometric Photometer



- completely different approach to relative astrometry, using a fine ruling and a mask with one photomultiplier per star
- Gatewood, Allegheny Observatory (Pittsburgh)
- grid is moved over the star field, gives 1-dim relative positions of stars in field
- accuracy achieved is around 1 mas

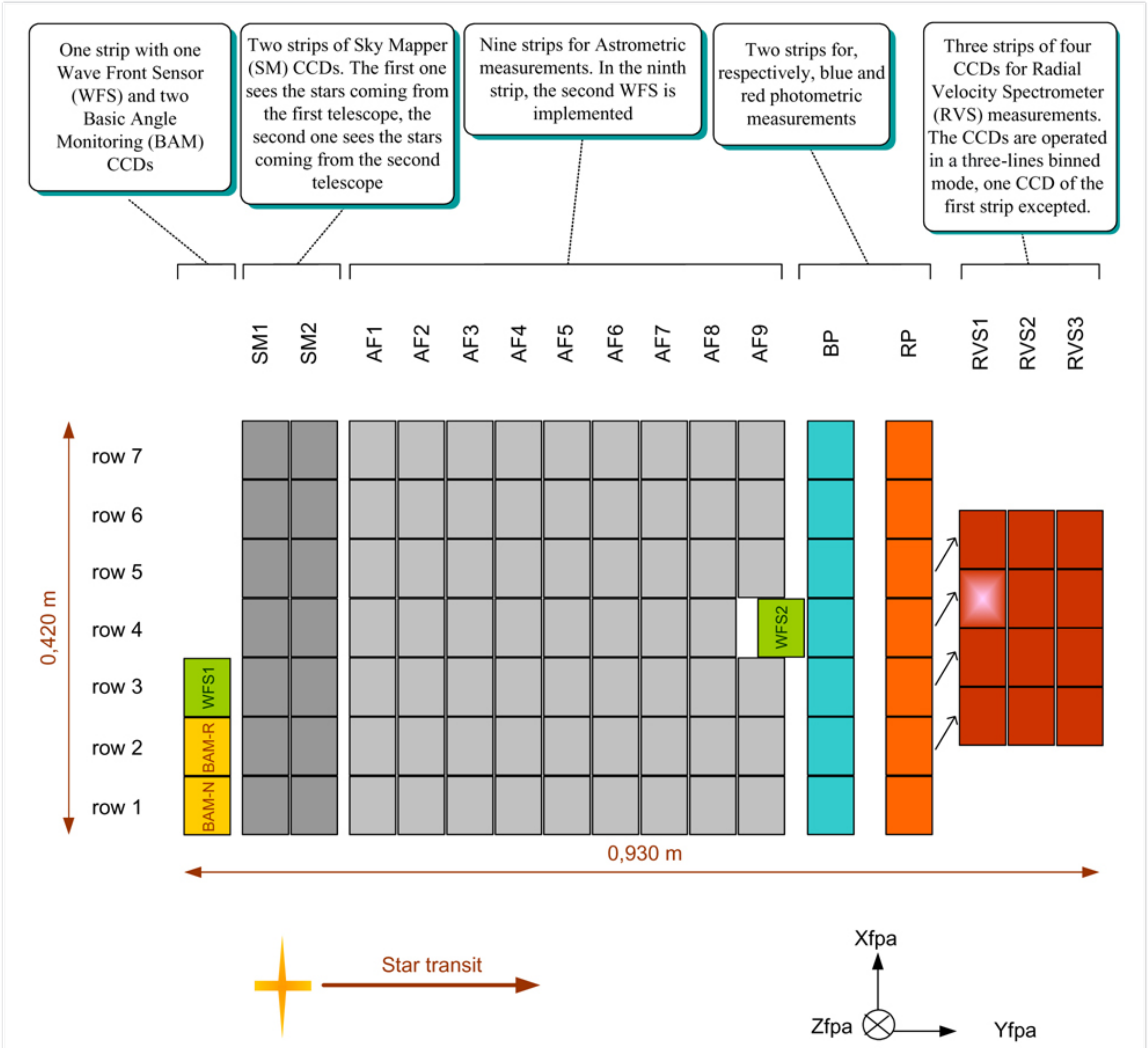
Principle of Hipparcos & Gaia



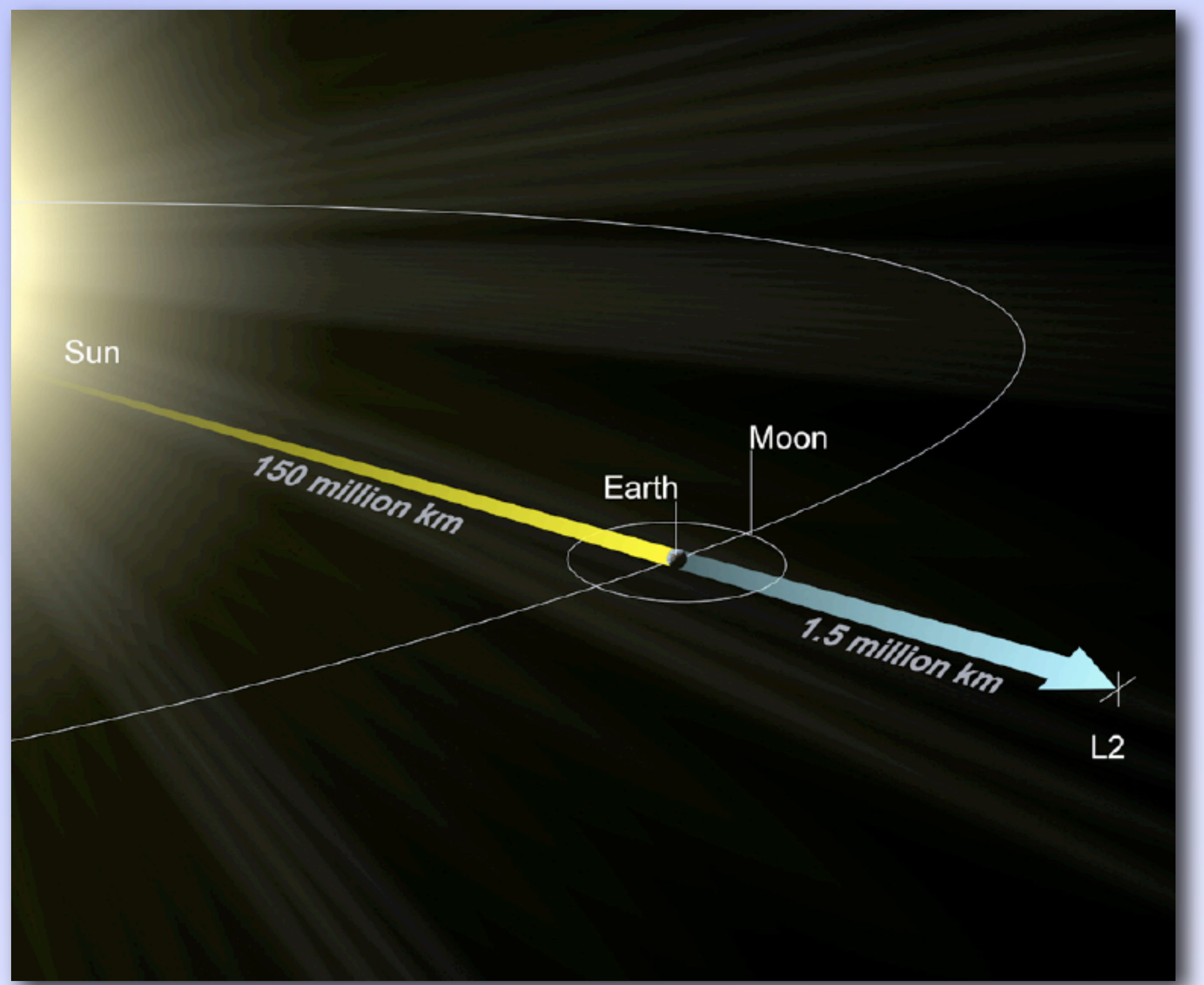
Copyright: EADS Astrium

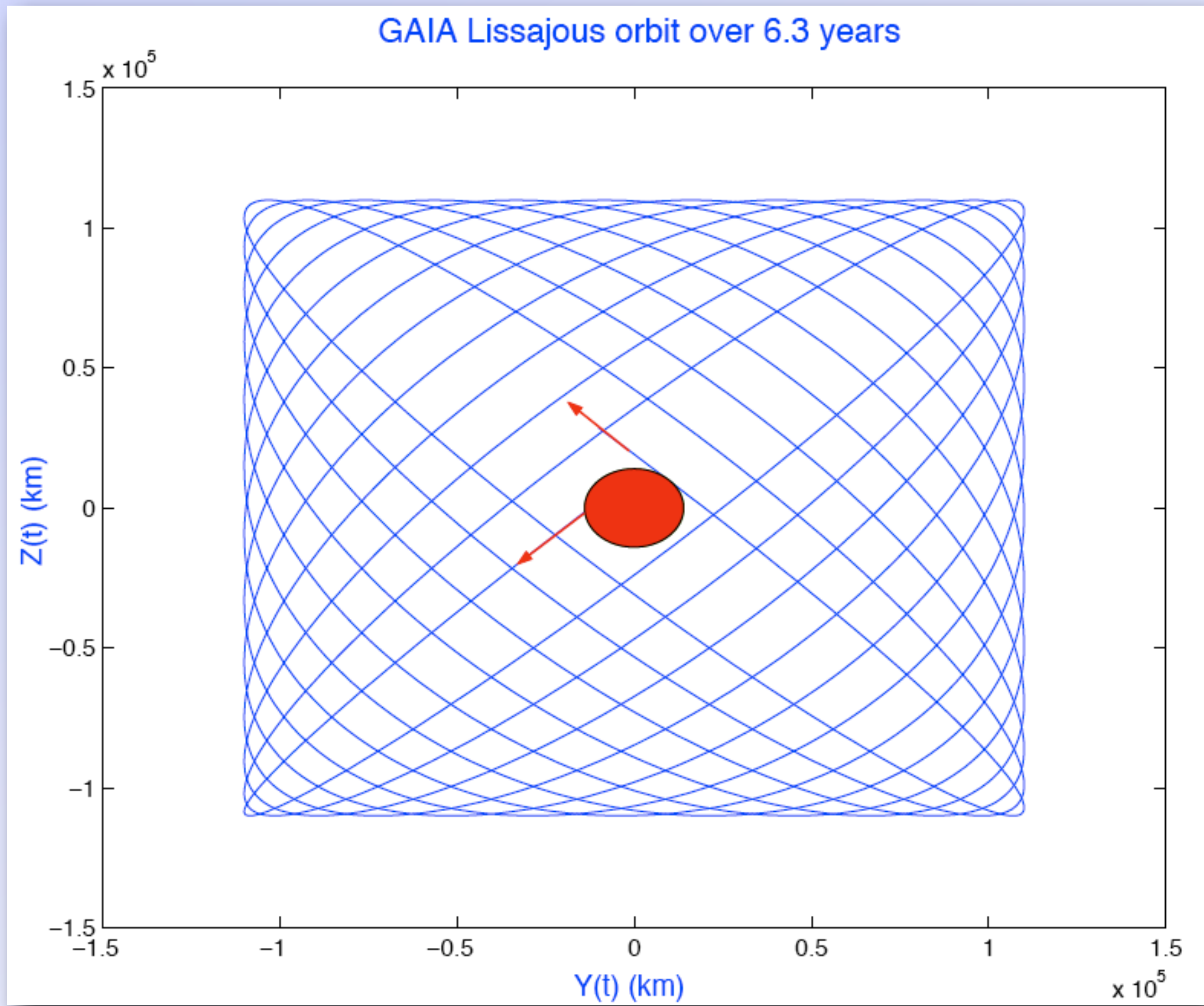
- this is absolute astrometry now!
- survey missions scanning the whole sky and tying all measurements together into a common absolute reference frame
- totally different concept from relative astrometry with single telescope
- TDI mode (time delayed integration):
  exposure time is synchronized with satellite spin rate
- measurement is essentially a timing measurement (compare to transit circles!)

Gaia Focal Plane



Gaia Scanning Law

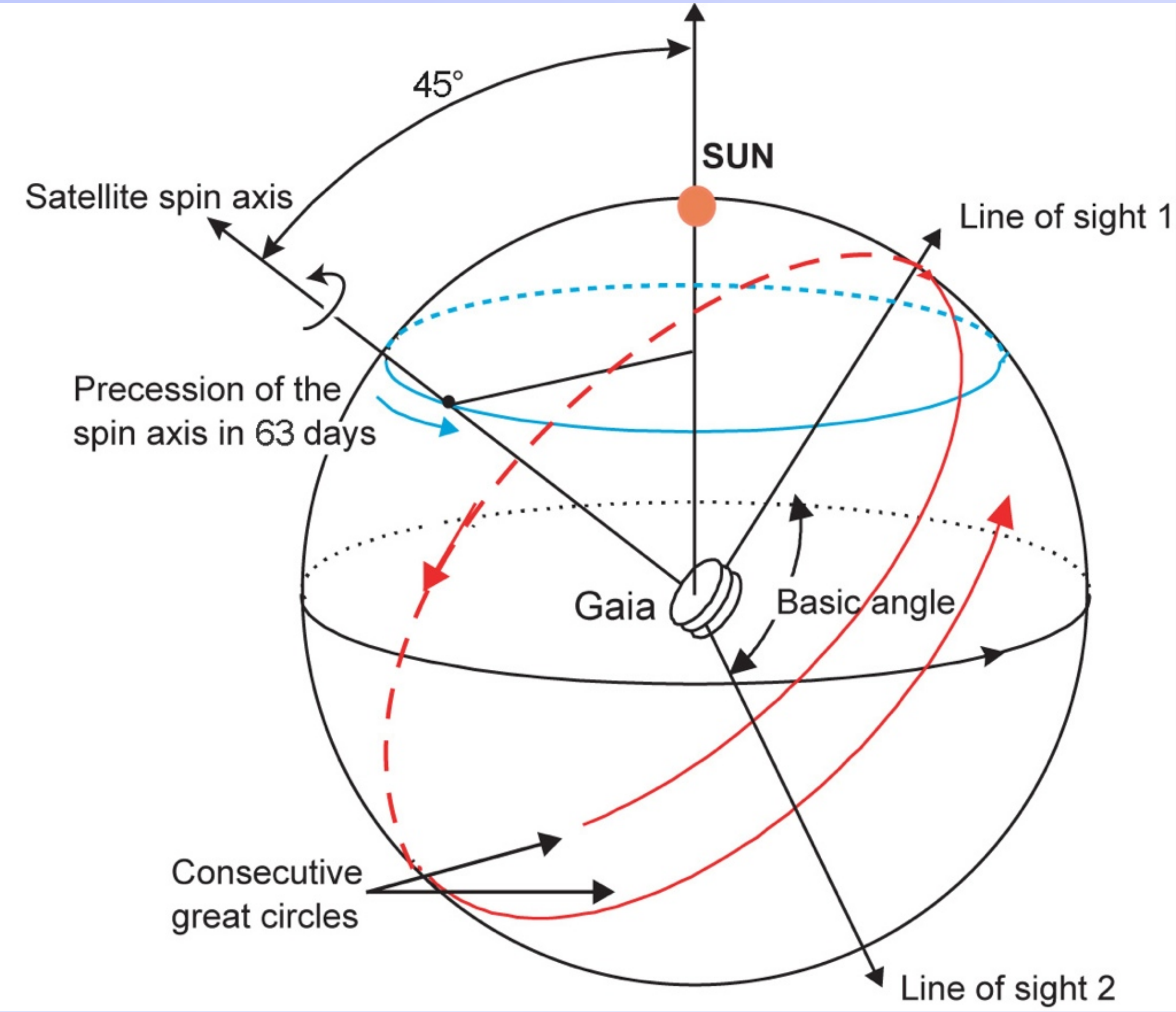




CREDIT: F. MIGNARD

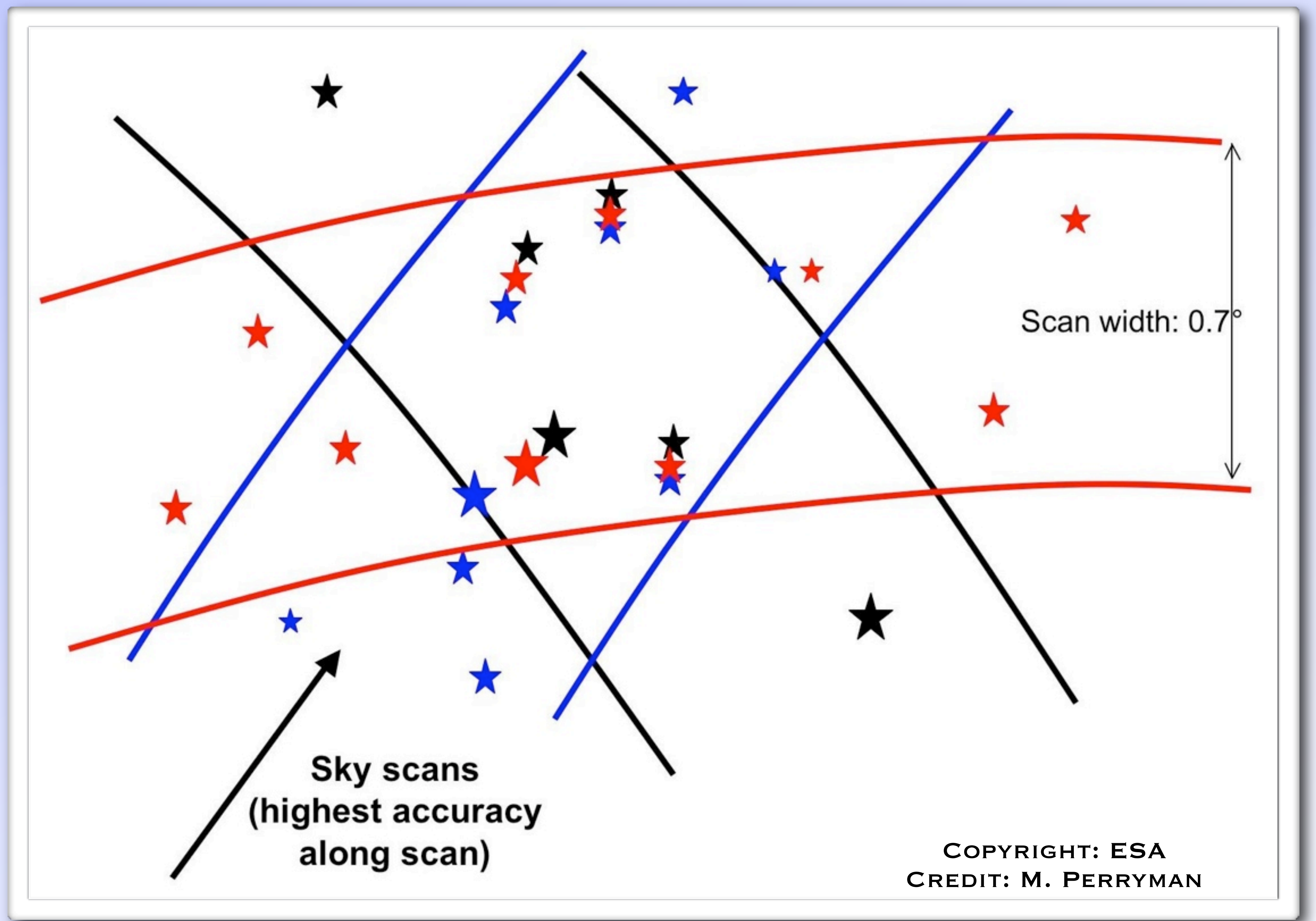
Gaia will be operated from around L2,
staying out of the Earth shadow for at least 6 years

Spinning Satellite



- scans the sky continuously
- constant angle to the Sun
- two fields of view! (106.5 deg)
- scannning speed is 60"/sec -> 6 hours/great circle
- great circles about 1 deg apart
- on average, each star is observed 70 times over 5 years

Measurement Principle



one region observed in several scans

Hipparcos Data Reduction Steps

(very similar for Gaia)

1. **Great Circle Reduction**

   estimating the relative positions of all stars in one great circle (typically 2000) using the primary observables (for Hipparcos intensities and phases of a signal modulated with a grid) as well as attitude information

   strong closure condition exists (circle must add up to 360 deg!)

2. **Sphere Solution**

   interconnection and locking

   combining data from a larger number of great circles (typically a few months), the zero points of the relative positions on each great circle can be estimated, turning them into absolute, but still one-dimensional positions

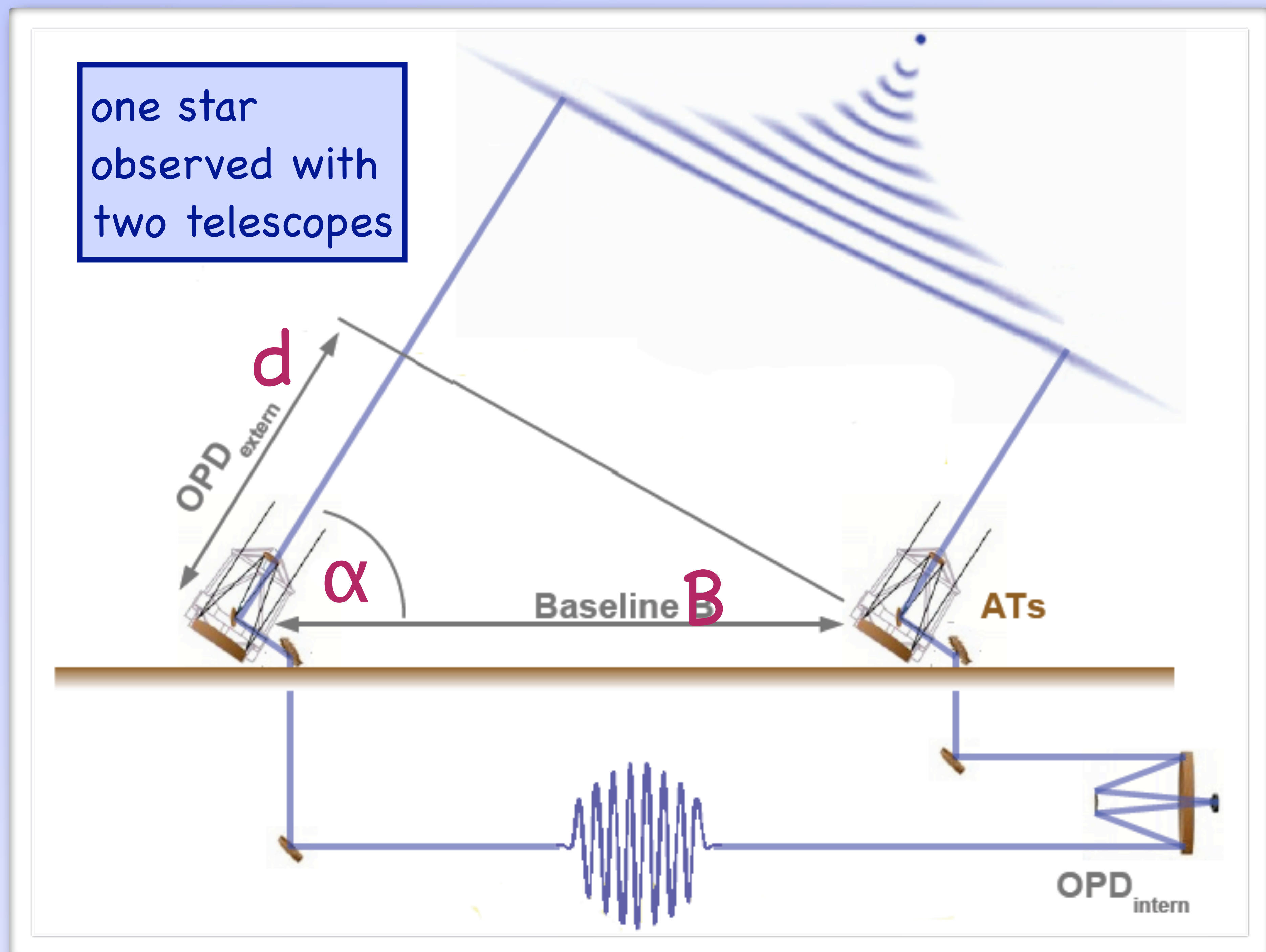
3. **Astrometric Catalog**

   modelling all absolute positions for one star (typically about 100 1-dim observations) in terms of all five astrometric parameters - iteration required!

Hipparcos Reference System

- this procedure allows Hipparcos to establish its own reference system
- the zero point for that reference system is arbitrary, but if you know the position of just one star you know all others
- Hipparcos system is linked to extragalactic reference frame (ICRF, defined by the positions of several hundred extragalactic radio sources) through auxiliary, ground-based observations of a handful of quasars
- the axes of the two systems are aligned to about 0.6 mas, and relative rotation of the two systems is less than 0.25 mas/year

Single Star Interferometry



Credit: R. Geisler

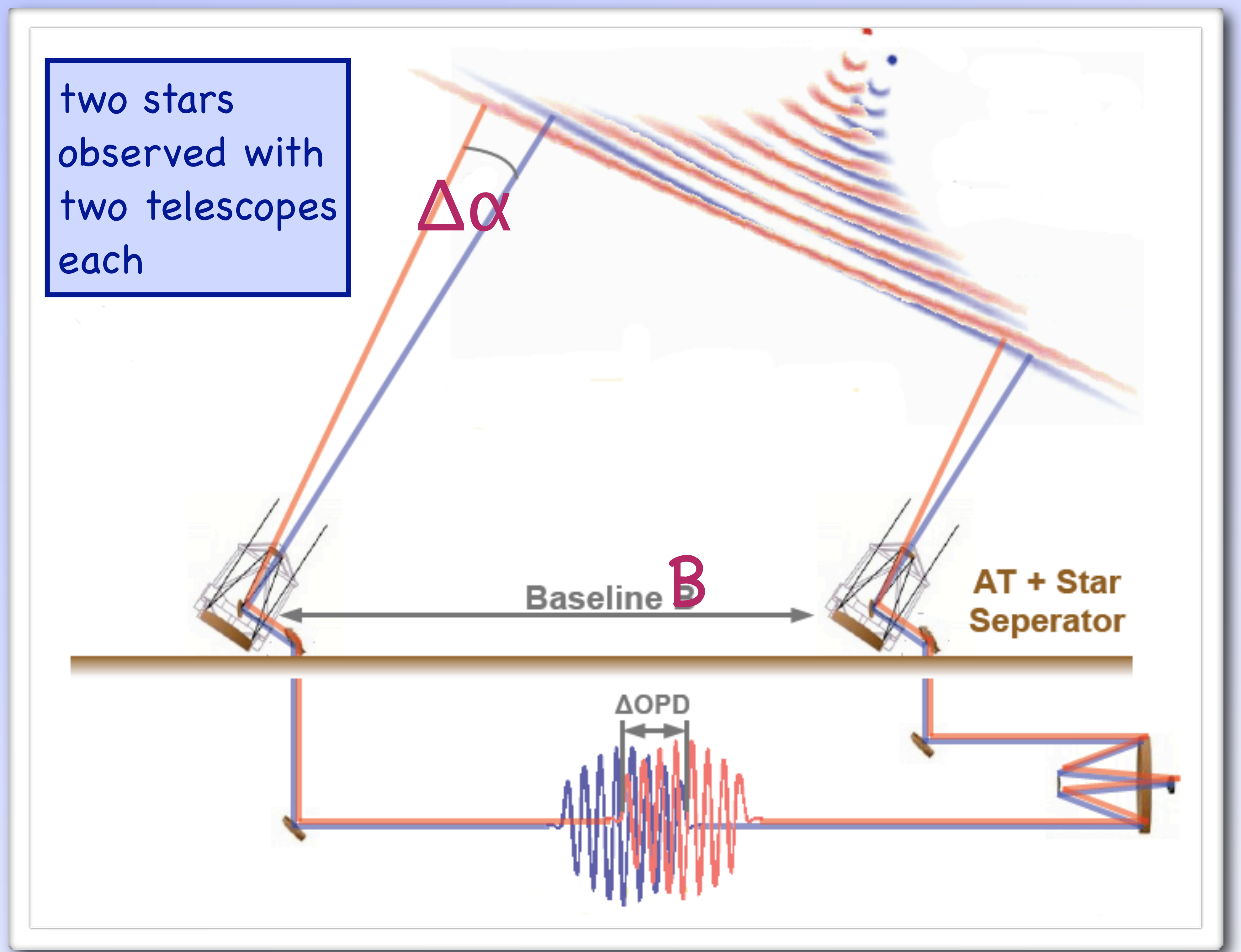
d: delay
measured

B: baseline
measured using known star positions

α: position
to be determined

$$d = B \sin(\alpha)$$

Dual Star Interferometry: PRIMA



Credit: R. Geisler

Δd: differential delay measured

B: baseline measured using known star positions

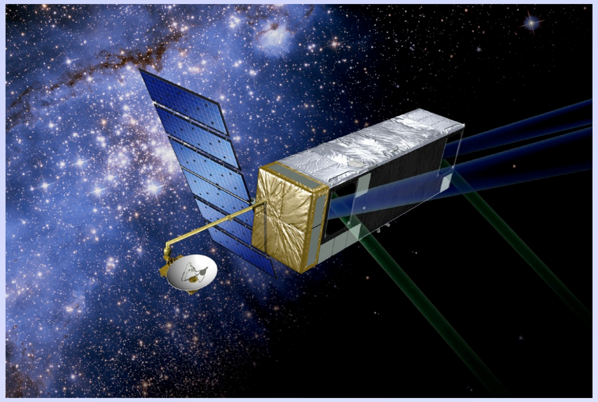
$\Delta\alpha$: relative position to be determined

$$d = B \sin(\alpha) \longrightarrow \Delta d = B \sin(\Delta\alpha)$$

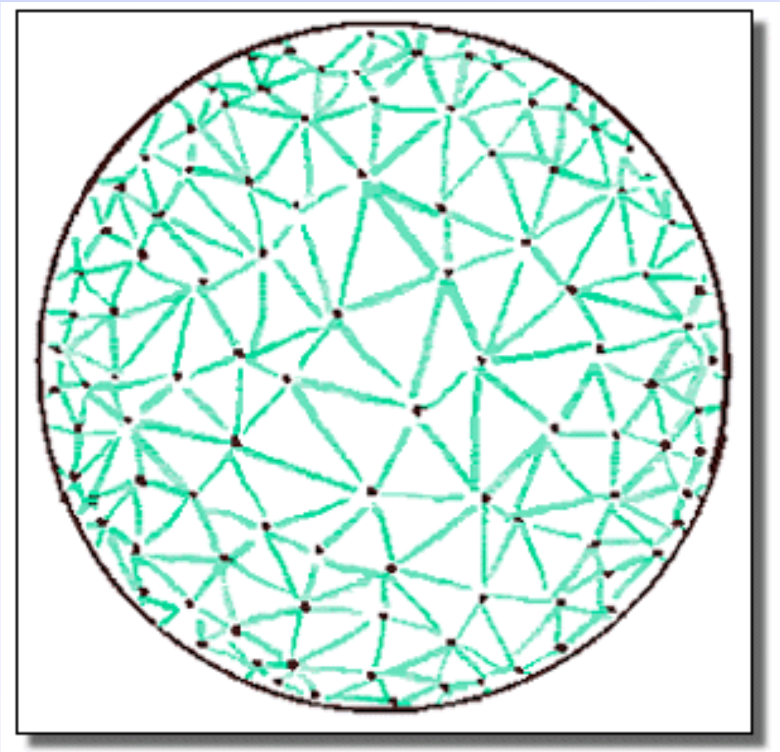
PRIMA

- with PRIMA, one can measure 1-dim relative star positions with an accuracy of about 10 microarcseconds
- large baselines help, have to be calibrated with baseline calibration stars
- stars have to be located within the isoplanatic patch (10-20" in K-band)
- large effort to select suitable target and reference stars (magnitude limitations)
- many measurements over a long time baseline needed to be scientifically useful

Space Interferometry Misson: SIM Planetquest

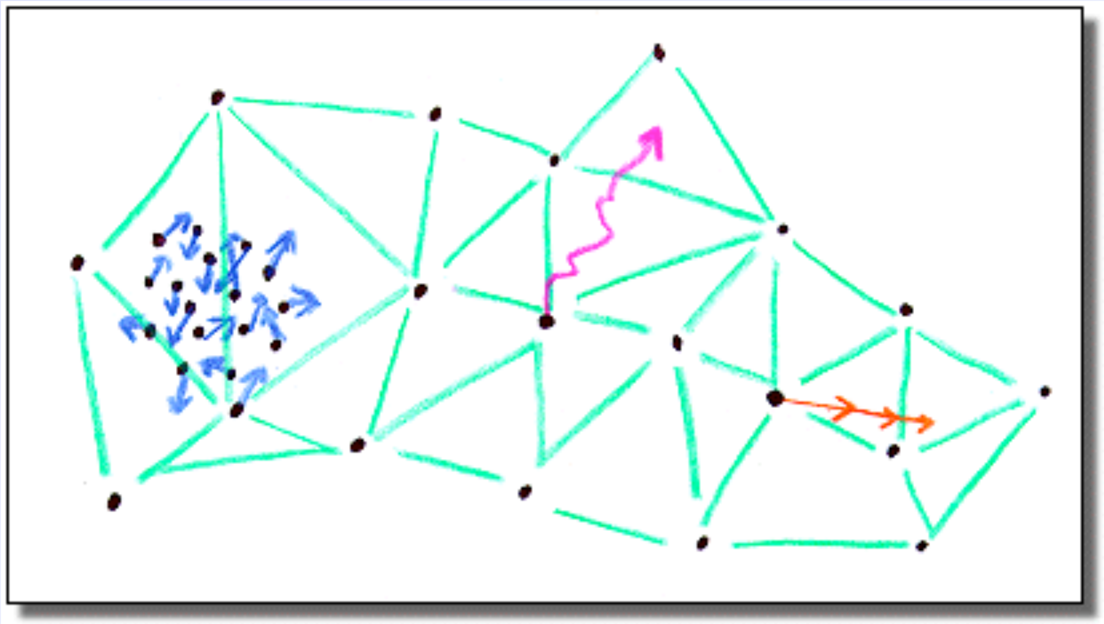


- phase-referenced interferometry, just like PRIMA
- however, SIM does absolute astrometry by establishing its own reference frame
- it will repeatedly observe a grid of several thousand stars, into which all other measurements are tied



global grid

Credit: NASA



local grid

Astrometric Methods Overview

	Ground	Space
Single Telescope	transit circles imaging interferometry (e.g. NPOI) MAP	Hipparcos Gaia
Dual Telescopes	PRIMA PHASES	SIM/SIM

1 mas

accuracy

1 μas

relative astrometry

absolute astrometry

Collection of Astrometric Measurement Principles

- Timing Methods
  - Transit Circles
  - Scanning Satellites (Hipparcos, Gaia)
- Imaging
  - measuring image centroids (many ground-based projects)
- Grid/Ruling
  - using a fine ruling or a grid (Hipparcos, MAP)
- Interferometry
  - using the resolution of a wavelength to make exact position measurements (PRIMA, SIM)

Summary

This talk addressed the questions ...

- ... why astrometry is complicated
  (remember the movies from the beginning!)

- ... how positions can be measured precisely
  (relative vs. absolute astrometry, ground vs. space, single telescope vs. dual telescopes)

PRIMA Astrometric Data Reduction

Sabine Reffert
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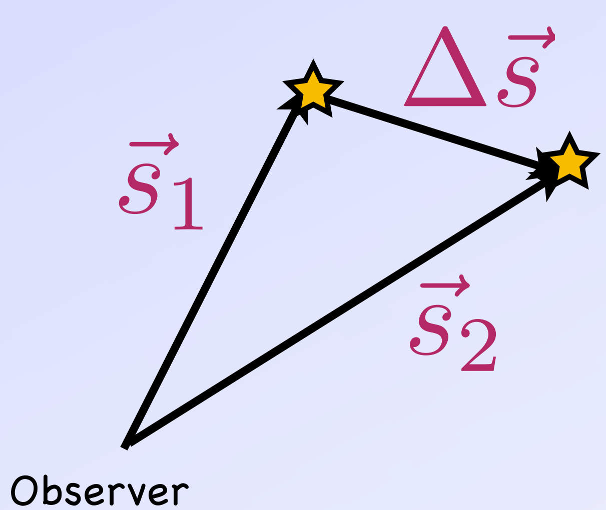
This talk explains:

- the various astrometric effects which can be present in relative astrometric data, at mirco-arcsecond accuracy
- how to take those unwanted things out of the data!

Differential Delays

- for the following, it is enough to think of the differential delays Δd which we measure as equivalent to a projected angular separation between two stars on the sky
- more precisely, we do not determine the angular separation, but the separation vector $\Delta\vec{s}$:



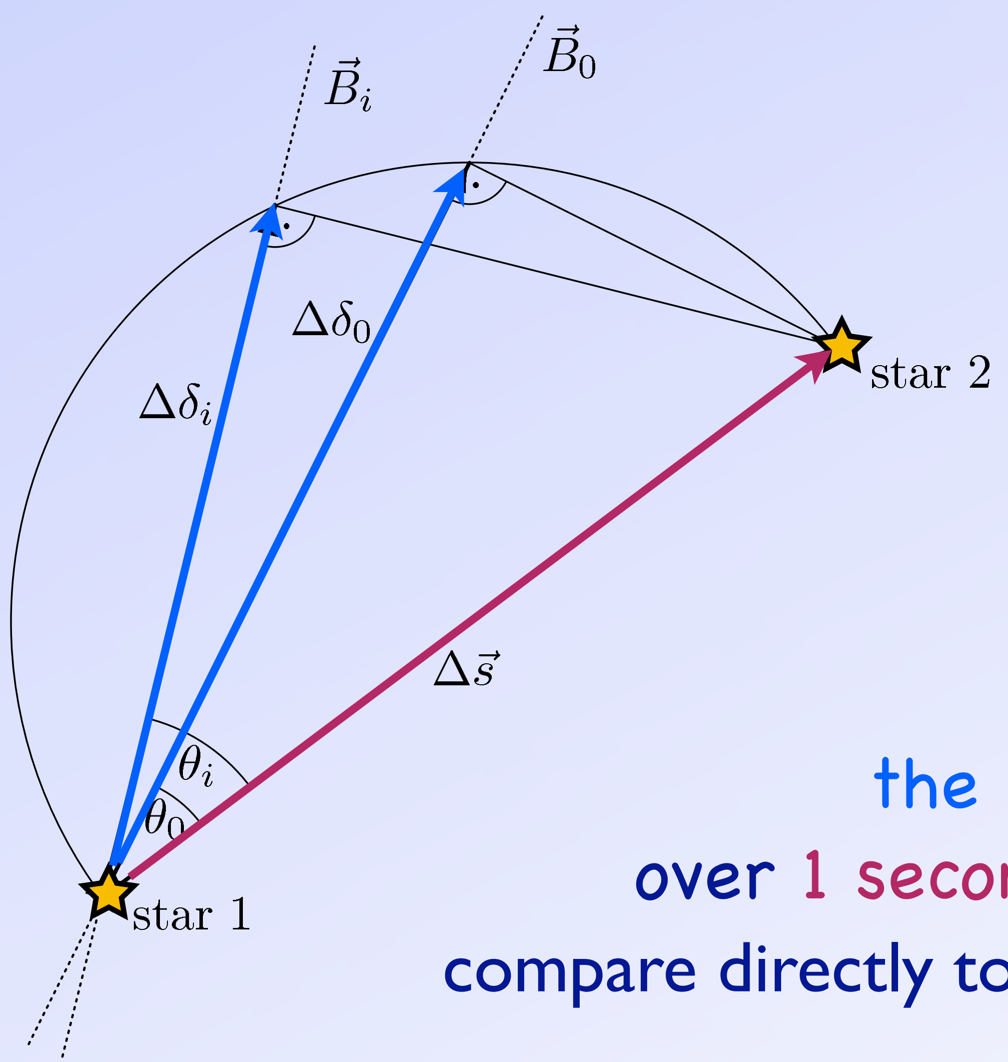
$$\Delta\vec{s} = \vec{s}_2 - \vec{s}_1$$

$$\boxed{\Delta d = \Delta\vec{s} \cdot \vec{B}}$$

$\vec{B}$: baseline vector

Rotating Baseline

- the differential delay is recorded during up to one hour
- of course, the baseline, projected on the sky, will rotate during that time
- when the differential delays are averaged, this rotation has to be taken into account:



$$\Delta d(t_0) = \Delta\vec{s} \cdot \vec{B}(t_0)$$

$$\Delta d(t_i) = \Delta\vec{s} \cdot \vec{B}(t_i)$$

the effect can become quite large:
over 1 second, baseline rotates by 15 arcseconds
compare directly to measurement accuracy of 10 micro-arcsecond!

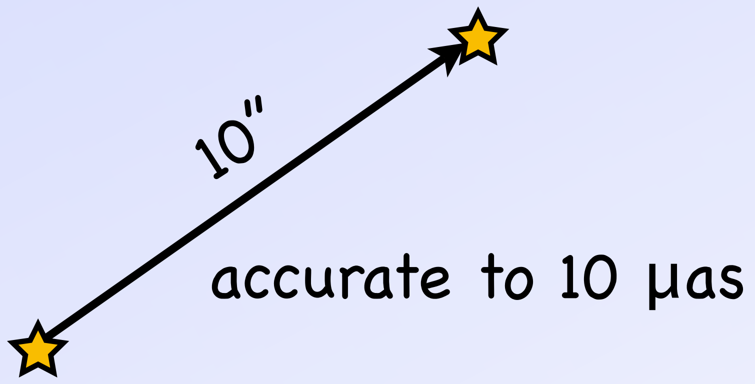
Measurement Accuracy

we want to measure angular separation with an accuracy of 10 micro-arcseconds

$\Delta\vec{s}$ is of order 10″ ≈ 5 · 10^{-5} radians

$\vec{B}$ is of order 100-200m

Δd is thus of order 1cm



if we measure the differential delay with 5nm precision, i.e. with 5nm/1cm = 5 · 10^{-7} relative precision,

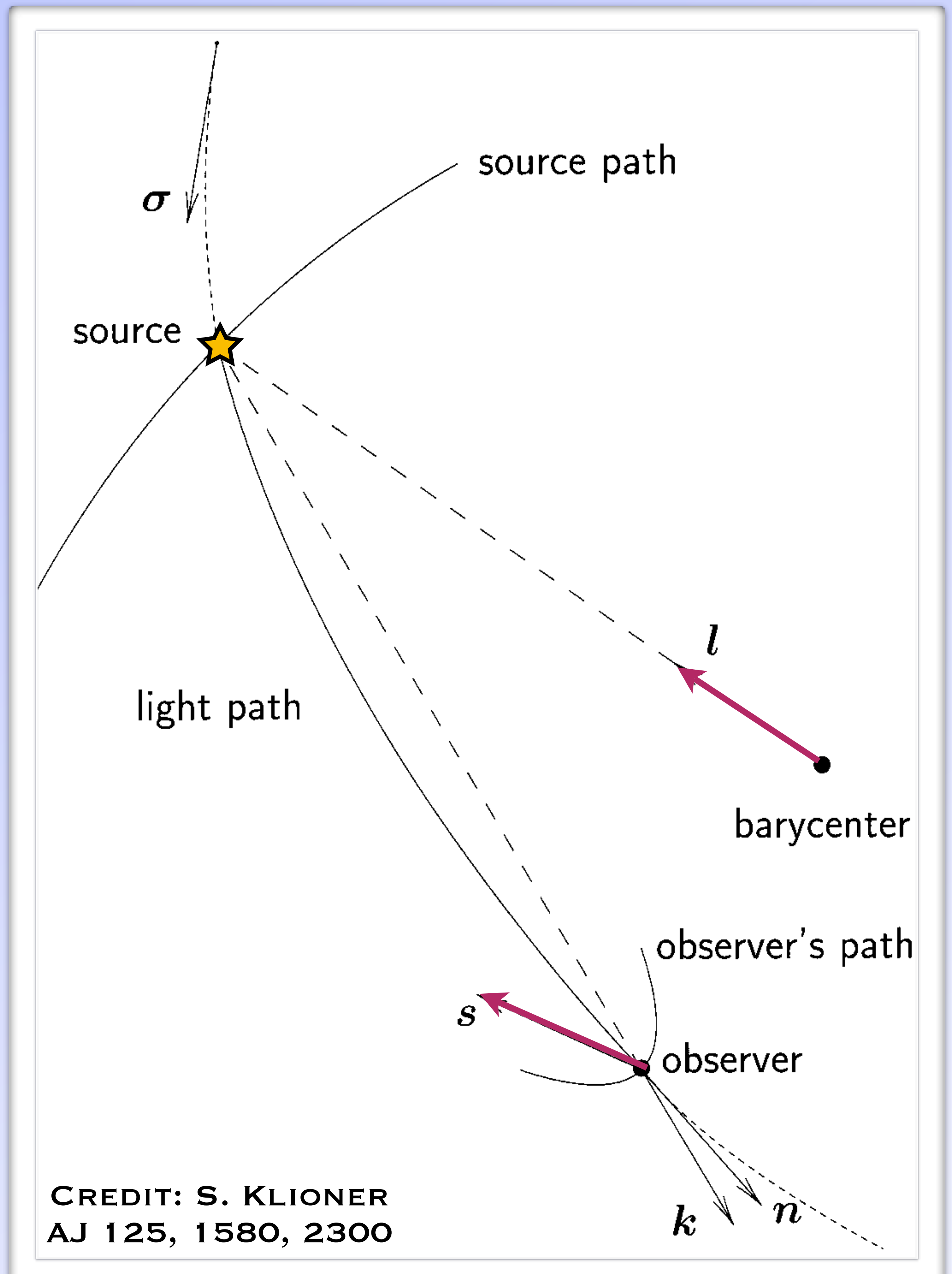
the angular separation will be accurate to
10 arcseconds · 5 · 10^{-7} = 5 micro-arcseconds!

$$\Delta d = \Delta\vec{s} \cdot \vec{B}$$

Astrometric Effects

- the effects which we have to remove in the course of the data reduction are the following:
  - aberration
  - light time delay
  - relativistic light deflection
  - annual and diurnal parallax
  - epoch transformations (proper motion etc.)
- as we have just seen, those corrections to the differential delays should be accurate to better than 1 μas

Observed Directions



s: observed unit direction vector

n: unit tangent vector to the light ray at $t = t_{obs}$

σ: unit direction of propagation at $t = -\infty$

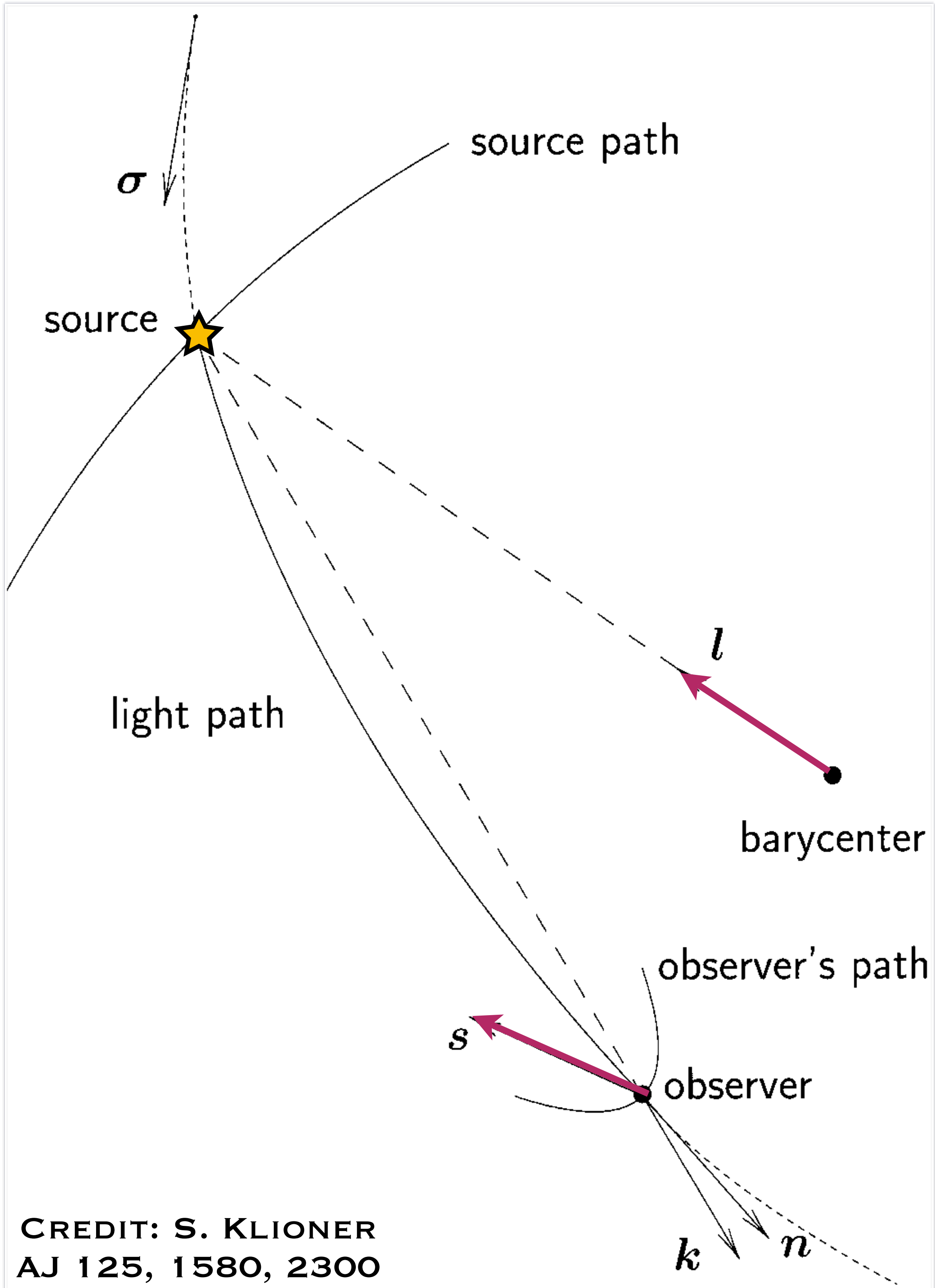
k: unit coordinate vector from source to observer

l: unit vector from barycenter to observer

observed: s

want: l

Observed Directions



CREDIT: S. KLIONER
AJ 125, 1580, 2300

s

↓ aberration

n

↓ gravitational light deflection

σ

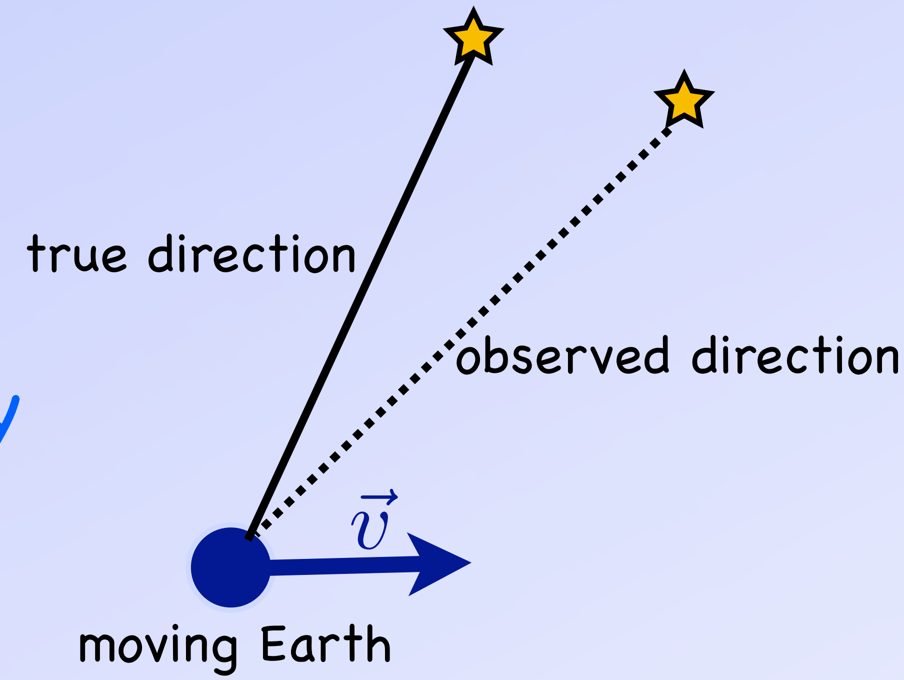
↓ finite source distance

k

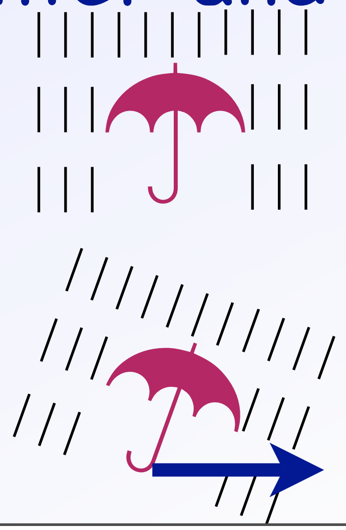
↓ parallax, proper motion

l

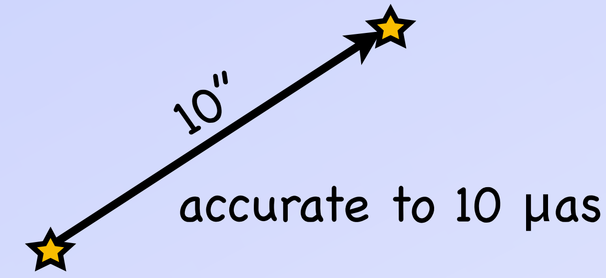
Aberration



- aberration is caused by relative velocity of source and observer and the finite speed of light
- during the light travel time, the relative positions between observer and source change, and the light seems to be coming from a different direction than when it was emitted
- Newtonian theory: annual and diurnal aberration, due to the orbital motion of the Earth around the barycenter and Earth rotation, respectively
- relativistic theory: distinction no longer possible



Yearly Aberration



The size of the absolute effect is 20.5″ maximum:

$$a_{\text{year}} = k_{\text{year}} \cdot \sin\gamma$$

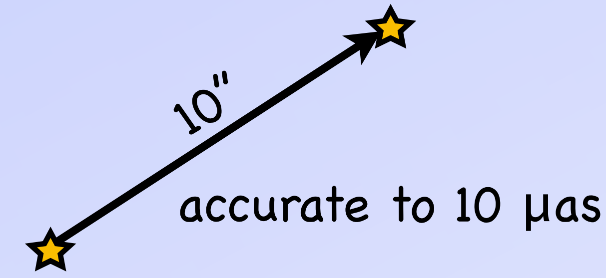
where k_{year} = v/c is the yearly aberration constant and γ is the angle between the velocity of the Earth and the direction of the light coming from the star

How big is the relative effect?

$$\Delta a_{\text{year}} = k_{\text{year}} \cdot (\sin\gamma_1 - \sin\gamma_2)$$

for a maximum assumed separation of target and reference star of 30″, relative aberration can reach the oder of 3 mas and needs to be corrected for!
(6 mas over half a year, and 1.1 µas over half an hour)

Daily Aberration



The size of the daily effect is:

$$a_{\mathrm{day}} = k_{\mathrm{day}} \cdot \sin\gamma$$

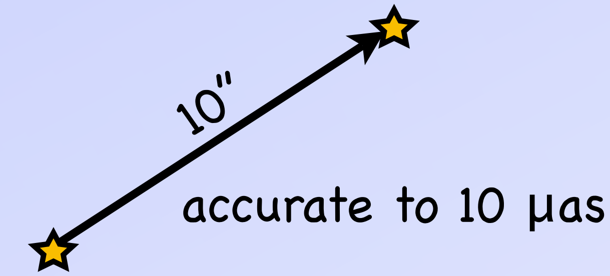
where $k_{\mathrm{day}} = 0.32'' \cos\phi$ is the daily aberration constant (ϕ is the latitude of the position of Earth) and γ is the angle between the velocity of the Earth and the direction of the light coming from the star

How big is the relative effect?

$$\Delta a_{\mathrm{day}} = k_{\mathrm{day}} \cdot (\sin\gamma_1 - \sin\gamma_2)$$

for a maximum assumed separation of target and reference star of 30″, relative daily aberration can reach 42 μas over one day, and 5.5 μas over half an hour!

Relativistic Aberration



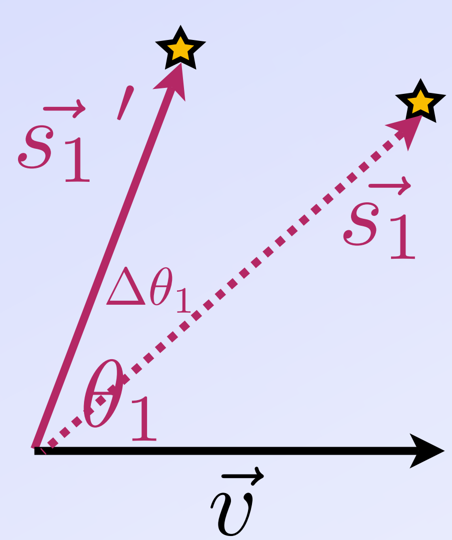
Precise derivation needs to take into account relativistic addition of velocities:

$\gamma = \sqrt{1 - v^2/c^2}$

c : speed of light

$$
\begin{aligned}
\sin \Delta\theta_1 &= \frac{(v/c)\sin\theta_1 + 1/2(v/c)^2 \sin 2\theta_1/(1+\gamma^{-1})}{1+(v/c)\cos\theta_1} \\
&= \frac{v}{c}\sin\theta_1 - \frac{1}{4}\left(\frac{v}{c}\right)^2 \sin 2\theta_1 + \frac{1}{4}\left(\frac{v}{c}\right)^3 \sin 2\theta_1 \cos\theta_1 + \ldots
\end{aligned}
$$

This is the correction of the direction to one of the stars due to aberration. The third order time is of the order micro-arcseconds.



$\vec{s_1}\,'$ true direction

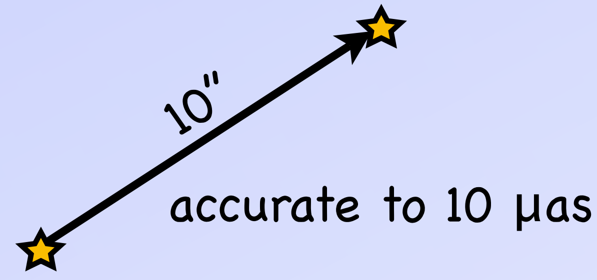
$\vec{s_1}$ apparent (observed) direction

$\vec{v}$ velocity of Earth

θ_1 angle between $\vec{s_1}\,'$ and $\vec{s_1}$

$\Delta\theta_1$ angle between $\vec{s_1}\,'$ and $\vec{v}$

Light Time Delay



Because the distance of the two stars to us is changing by a different amount (difference in radial velocities), the light from the two stars was emitted at different times, and this time difference will change over time.
This, together with proper motion, results in a steady change of the observed angular separation.

Time delay is typically of the order of hours to years.
The resulting change in angular separation, depending on the proper motion, would typically be about 10-100 μas, and of the order of 100 mas for the most extreme assumptions.

Size of Light Time Delay: Example

$\Delta V_{rad} = 50$ km/s, $\Delta T = 5$ years

➡ After 5 years, the two stars are $8 \cdot 10^9$ km further apart than before. This corresponds to a light travel time of about 7 hours.

$\Delta\mu = 500$ mas/yr

➡ In 7 hours, one star travels 400 µas with respect to the other. This is the change of their angular separation over 5 years due to light time delay.

Light travel time can also be thought of as an aberration component caused by the proper motion of the star.

Light Time Delay: Precise Formula

correction for one star:

$$\Delta\vec{s}_1 = (f_1 - 1)\vec{s_1} - f_1\zeta_1(t)\Delta T_1\vec{s}_1 - f_1\vec{\mu}_1(t)\Delta T_1$$

with:

- $\Delta\vec{s}_1$: change in position due to light time delay
- $\vec{s}_1$: apparent (observed) direction of star 1
- f_1 : f-factor, close to 1, explained later
- ζ_1 : radial velocity of star 1 in proper motion units
- ΔT_1 : light travel time difference for star 1
- μ_1 : proper motion of star 1

Light Time Delay: Precise Formula

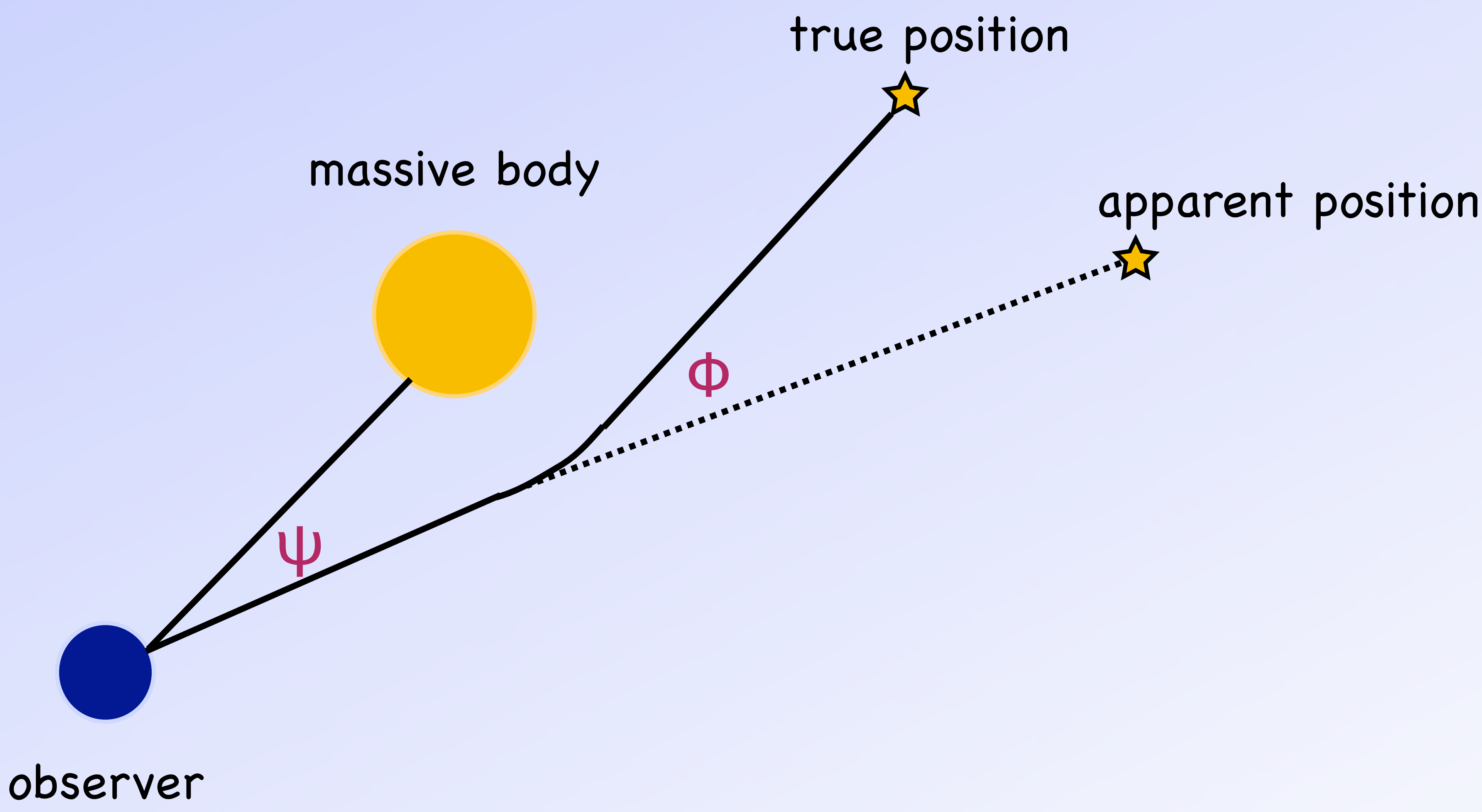
correction in angular separation:

$$
\begin{aligned}
\Delta\vec{s}\,' &= \Delta\vec{s} + (f_2 - 1)\vec{s_2} - f_2\zeta_2(t)\Delta T_2\vec{s}_2 - f_2\vec{\mu}_2(t)\Delta T_2 \\
&\quad -(f_1 - 1)\vec{s_1} + f_1\zeta_1(t)\Delta T_1\vec{s}_1 + f_1\vec{\mu}_1(t)\Delta T_1 \\
&\approx \Delta\vec{s} + f_1\vec{\mu}_1(t)\Delta T_1 - f_2\vec{\mu}_2(t)\Delta T_2
\end{aligned}
$$

the approximation reflects our earlier simple size considerations for light time delay to be dependent on proper motion and and light travel time difference!

Gravitational Light Deflection

Light is deflected in gravitational fields, and thus we have to correct observed positions if the observation is taking place close to a massive object.



The light is always deflected towards the massive body.

Gravitational Light Deflection

The general formula for relativistic light deflection is:

$$\phi = \frac{2GM_\circ}{c^2 R_\circ} \cdot \cot \psi/2$$

4 mas for the Sun

Φ: angle by which light is deflected

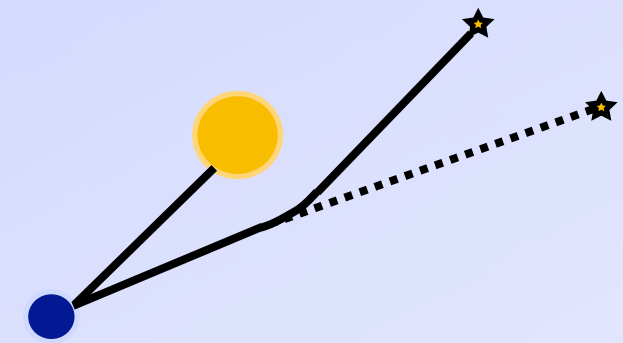
M_0, R_0: mass and distance of massive object

G, c: gravitational constant, speed of light

ψ: angular separation between star and massive object as seen from observatory

Gravitational Light Deflection

Examples for the absolute effect in the grazing case:



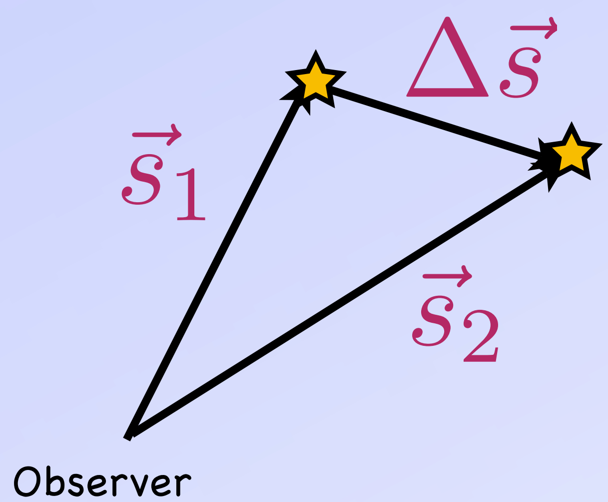
	symmetric mass distribution	quadrupole moment
Sun	1.75″	1 μas
Jupiter	16 mas	240 μas
Mars	0.1 mas	0.2 μas

The differential effect is of course always much smaller!

Gravitational Light Deflection

For completeness, here is the formula for the differential effect:

$$\Delta\vec{s}\,' = \Delta\vec{s} + \phi_2 \cdot \frac{\vec{s}_2 - \vec{s}_\odot}{|\vec{s}_2 - \vec{s}_\odot|} - \phi_1 \cdot \frac{\vec{s}_1 - \vec{s}_\odot}{|\vec{s}_1 - \vec{s}_\odot|}$$



Φ_1, Φ_2: absolute light deflection for star 1/2

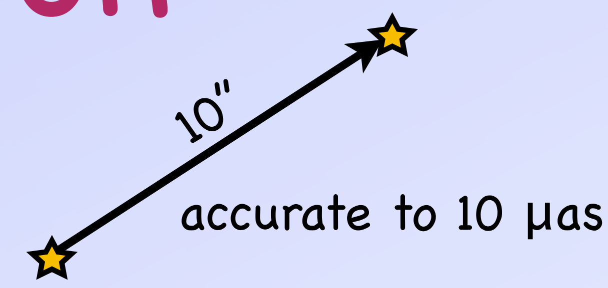
$\Delta\vec{s}$: apprent angular separation

$\Delta\vec{s}\,'$: corrected angular separation

$\Delta\vec{s}_0$: direction unit vector to massive body

For the Sun, the differential effect is always smaller than about 10 μas for typical PRIMA observations. For Jupiter, the differential effect reaches 10 μas for separations around 1 deg.

Gravitational Light Deflection



Note that in order to precisely correct for the (differential) effect of gravitational light deflection, one needs accurate solar system body ephemerides!

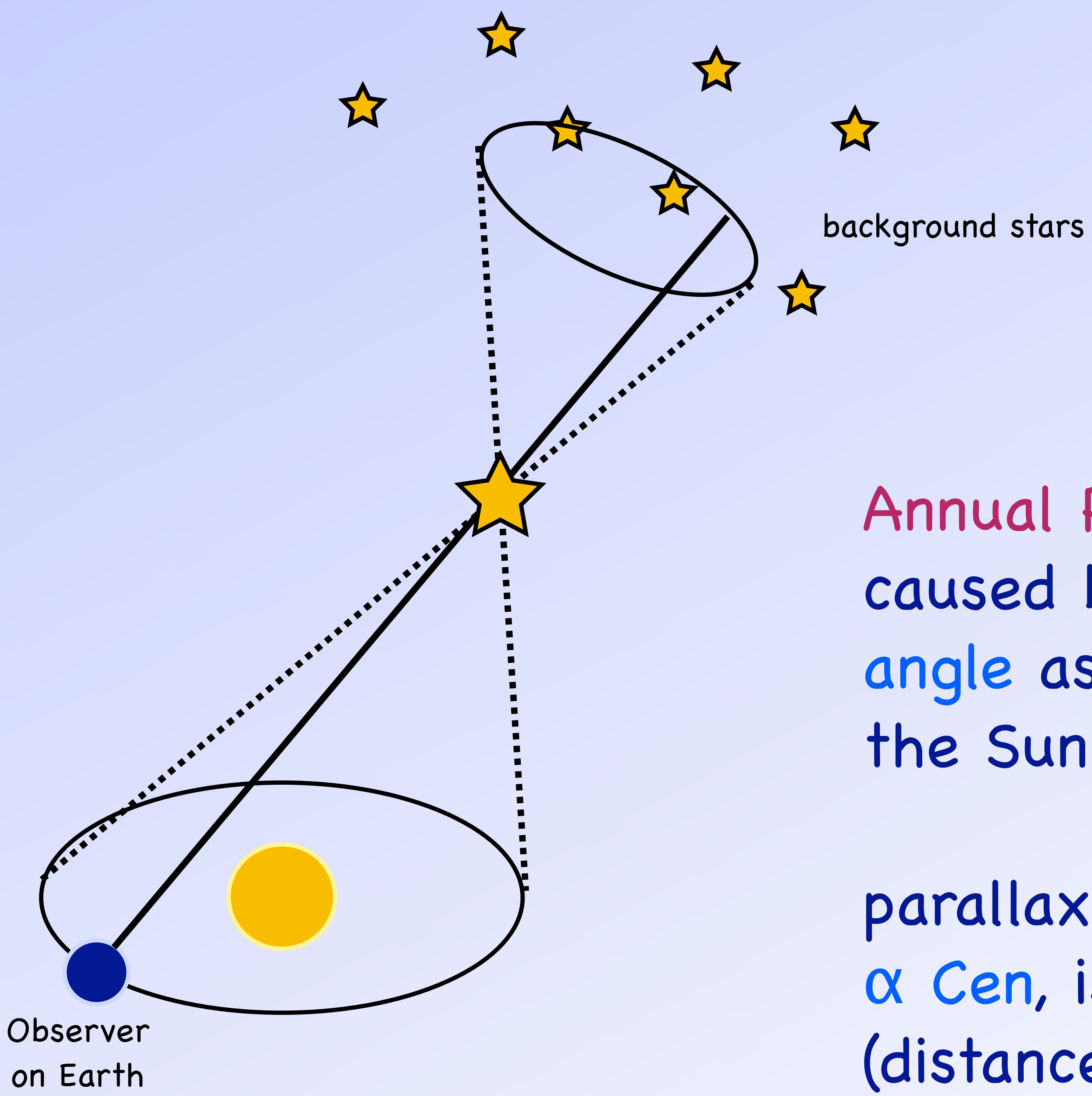
This is usually available for the larger solar system bodies (planets and biggest satellites), but for minor bodies this is not always the case.

In unlucky circumstances, one could observe very close to a very small solar system body and not notice! This would introduce astrometric noise at the level of typically 1 μas or less.

Parallax

distance [pc] = 1000 / parallax [mas]

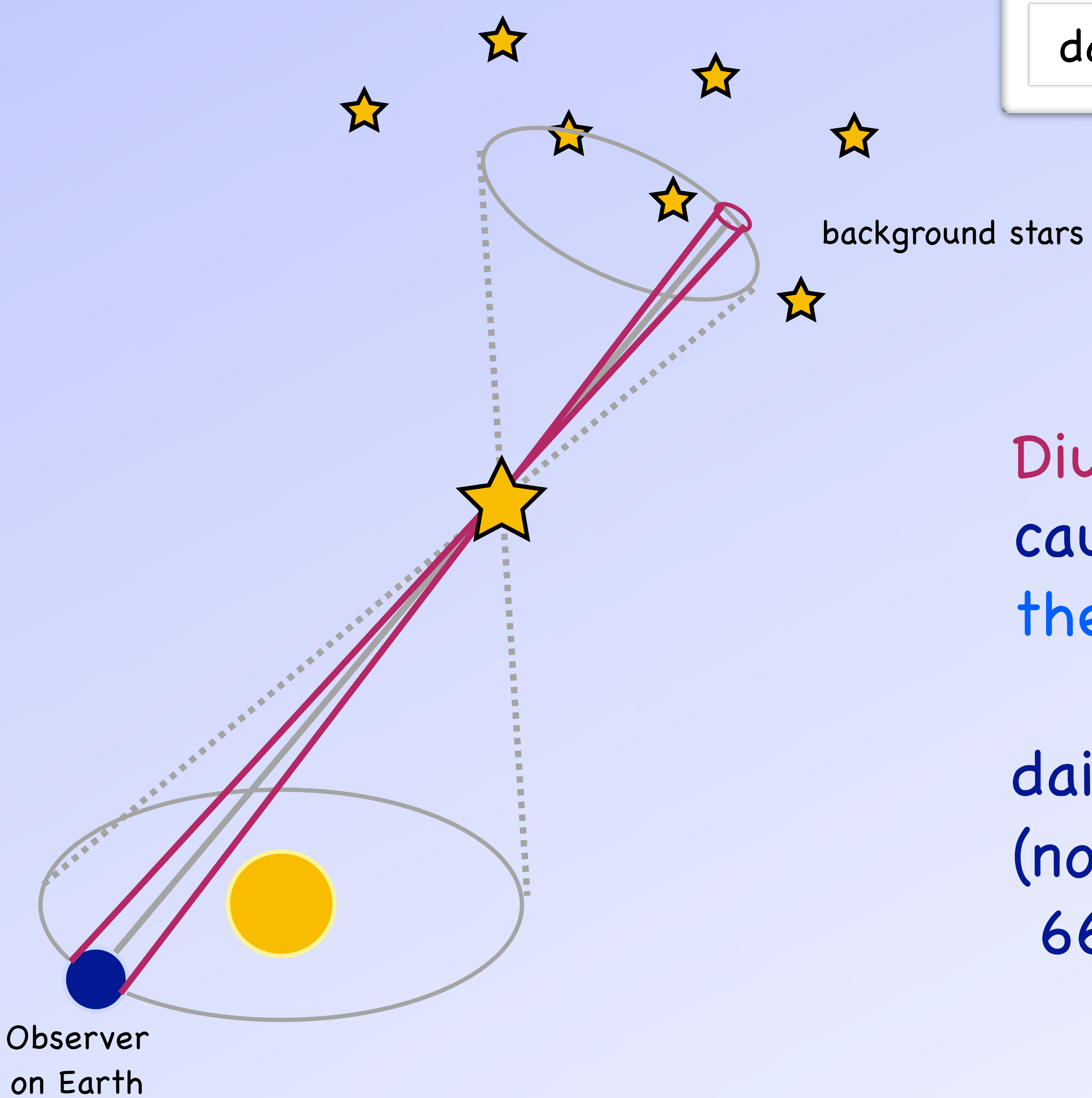
(due to definition of the parsec)



Annual Parallax:
caused by the change in viewing angle as the Earth goes around the Sun

parallax of most nearby star, α Cen, is about 772 mas (distance is 1.3 pc)

Diurnal Parallax



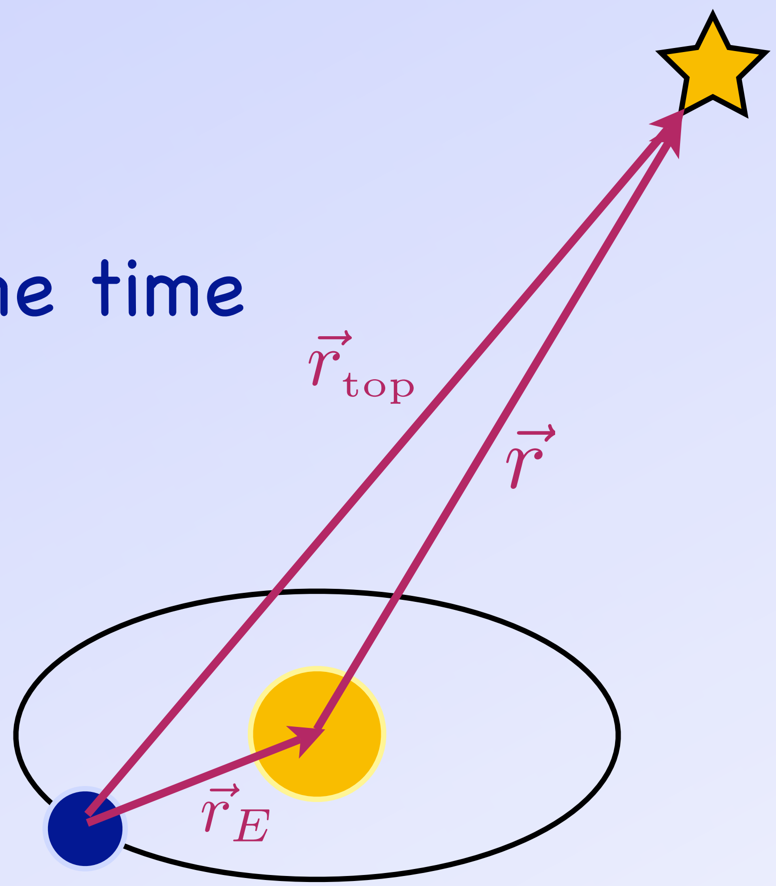
daily parallax = annual parallax * R_{Earth}/AU

Diurnal (daily) Parallax:
caused by the daily rotation of the Earth

daily parallax of α Cen is 33 μas
(note that it's changing fast,
66 μas over half a day!)

there even is a montly parallax due to the Moon!

Correcting for Parallax



Easiest to get rid of all parallaxes at the same time by using precise Earth ephemerides:

$$\vec{r} = \vec{r}_{\text{top}} + \vec{r}_E$$

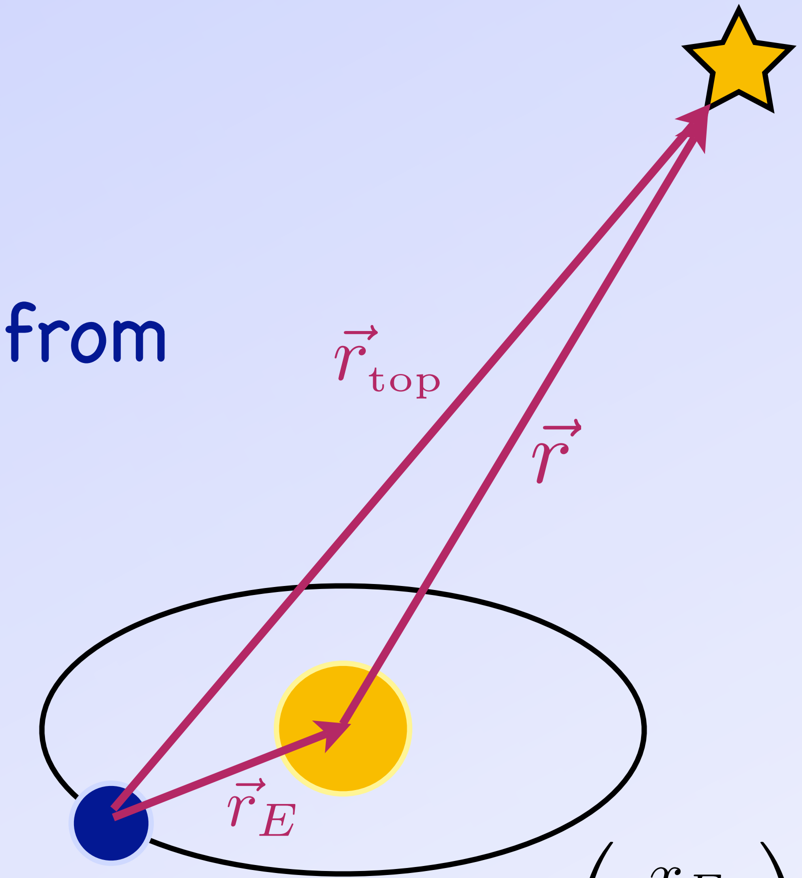
$\vec{r}_{\text{top}}$ is the observed, topocentric position,
$\vec{r}_E$ is the precise position of the Earth with respect to the solar system barycenter,
and $\vec{r}$ is the barycentric position which we want to calculate (as if we had observed from the solar system barycenter)

$$\vec{r} = \rho \begin{pmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{pmatrix}$$

α: right ascenscion
δ: declination
ρ: distance

Parallax Formula

Inserting everything into the simple equation from before, one gets for the corrections to be applied to right ascension, declination and distance:



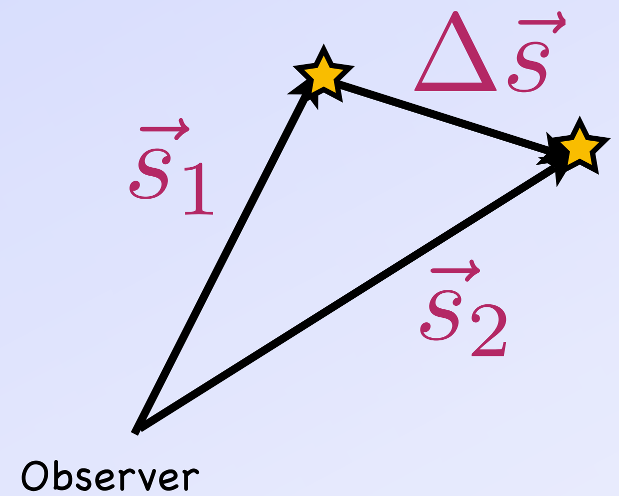
$$\vec{r}_E = \begin{pmatrix} x_E \\ y_E \\ z_E \end{pmatrix}$$

$$\begin{aligned}
\Delta\alpha \cos\delta &= \frac{x_E}{\rho}\sin\alpha - \frac{y_E}{\rho}\cos\alpha \\
\Delta\delta &= \left(\frac{x_E}{\rho}\cos\alpha + \frac{y_E}{\rho}\sin\alpha\right)\sin\delta - \frac{z_E}{\rho}\cos\delta \\
\Delta\rho &= -x_E\cos\alpha\cos\delta - y_E\sin\alpha\cos\delta + z_E\sin\delta
\end{aligned}$$

Parallax Correction in Angular Separation

Inserting the corrections for both stars, one gets:

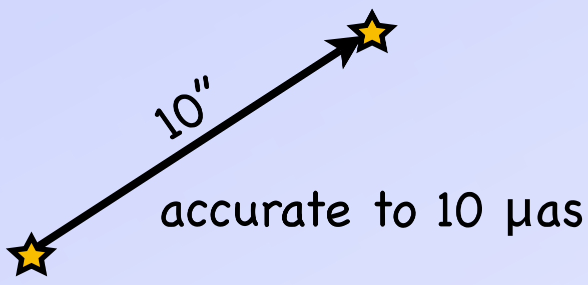
$$\Delta\vec{s}\,' = \Delta\vec{s} + \begin{pmatrix} -\Delta\alpha_2 \sin\alpha_2 \cos\delta_2 - \Delta\delta_2 \cos\alpha_2 \sin\delta_2 + \Delta\alpha_1 \sin\alpha_1 \cos\delta_1 + \Delta\delta_1 \cos\alpha_1 \sin\delta_1 \\ \Delta\alpha_2 \cos\alpha_2 \cos\delta_2 - \Delta\delta_2 \sin\alpha_2 \sin\delta_2 - \Delta\alpha_1 \cos\alpha_1 \cos\delta_1 + \Delta\delta_1 \sin\alpha_1 \sin\delta_1 \\ \Delta\delta_2 \cos\delta_2 - \Delta\delta_1 \cos\delta_1 \end{pmatrix}$$



$\Delta\vec{s}$ observed angular separation

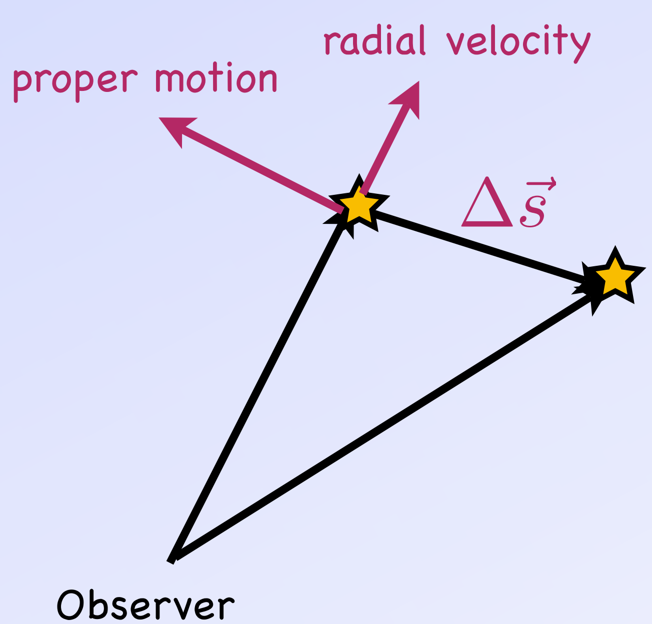
$\Delta\vec{s}\,'$ angular separation corrected for parallax

Epoch Transformations



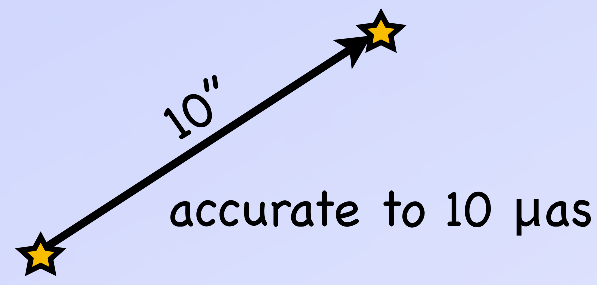
most obvious influence is the proper motion:
due to relative proper motion of the two stars,
angular separation will change

proper motion is a huge effect:
for nearby stars, it can reach 5000 mas/year, corresponding
to about 10 µas/minute!



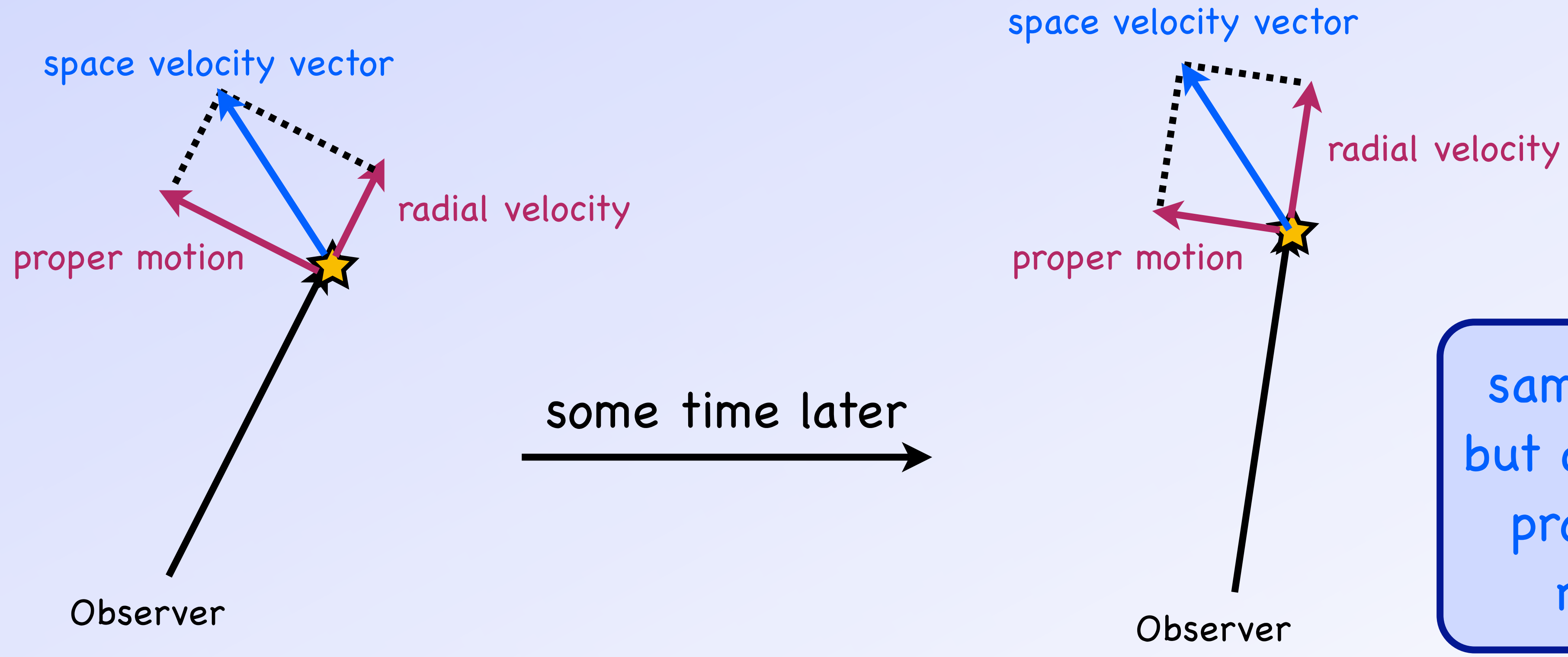
for PRIMA, proper motion
is thus easily noticeable over
the course of a half hour
observation!

Epoch Transformations



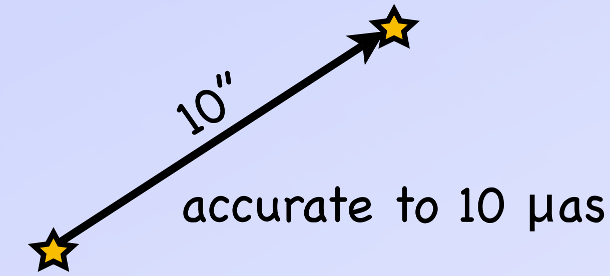
but the angular separation will also change due to radial velocity:

think of it as projection effect:
because the star at a later time is observed at a diferenct position and different distance, its total space motion vector is projected differently on radial and tangential directions



same space velocity, but different distance, proper motion and radial velocity!

Epoch Transformations



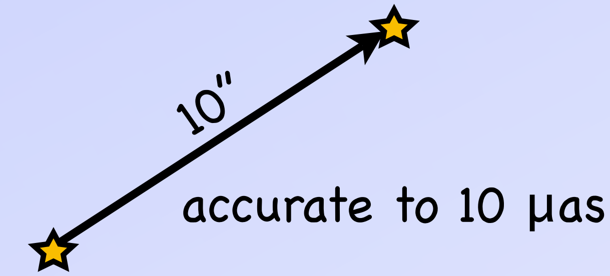
if the barycentric coordinate vector to a star at time t_0
is $\Delta\vec{s}_0$, at some other time t it will be

$$\vec{s}(t) = [\vec{s}_0(1 + \zeta_0(t - t_0)) + \vec{\mu}_0(t - t_0)]f$$

where ζ_0 is the radial velocity (in proper motion units)
and $\vec{\mu}_0$ is the proper motion vector, both at time t_0

f is the so-called f-factor from before;
it's a normalization factor, so that the result is a unit vector;
its value is usually close to 1

Epoch Transformations



for the change in angular separation, we obtain by just plugging in the formula from before:

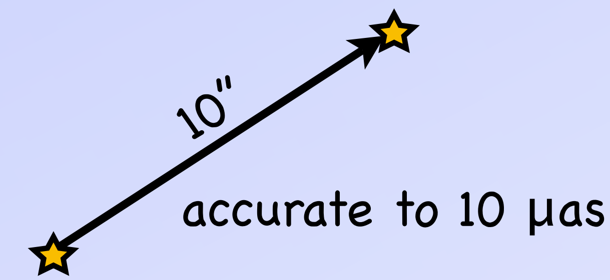
$$
\begin{aligned}
\Delta\vec{s}\,' &= \vec{s}_2' - \vec{s}_1' \\
&= \Delta\vec{s} + \vec{s}_2(t_0)(f_2 - 1) + \vec{s}_2(t_0) f_2 \zeta_2(t_0)(t - t_0) + \vec{\mu}_2(t_0) f_2 (t - t_0) \\
&\qquad - \vec{s}_1(t_0)(f_1 - 1) - \vec{s}_1(t_0) f_1 \zeta_1(t_0)(t - t_0) - \vec{\mu}_1(t_0) f_1 (t - t_0)
\end{aligned}
$$

with: $\Delta\vec{s}$ observed angular separation

$\Delta\vec{s}\,'$ angular separation transformed to another epoch

includes corrections for perspective acceleration!

Epoch Transformations

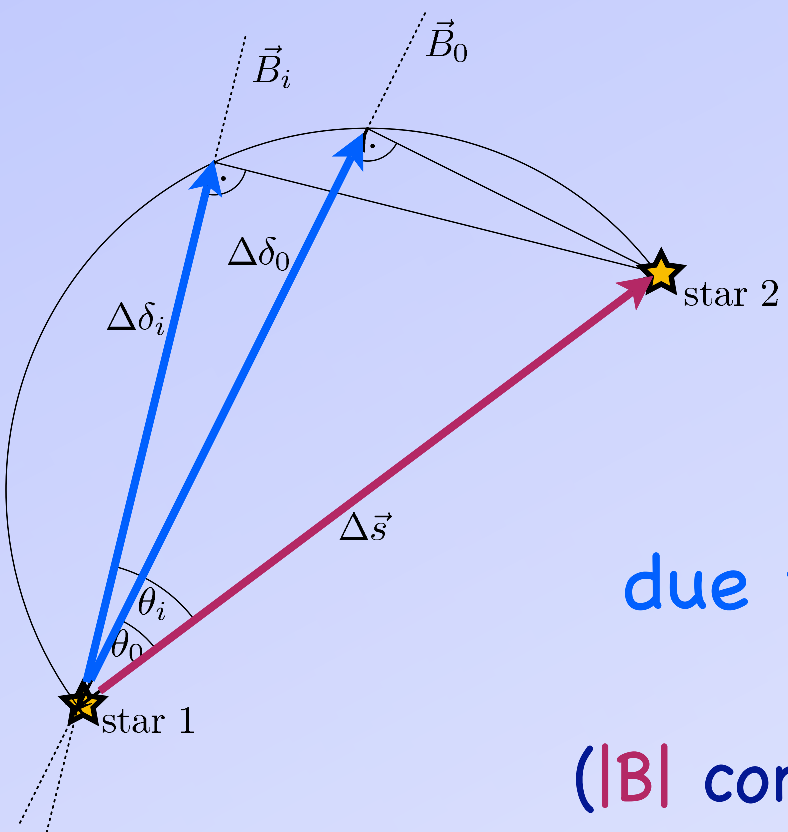


In order to apply the epoch transformations correctly, we need very accurate parallaxes, proper motions and radial velocities (micro-arcsecond accuracy).

Fortunately, we need only the relative quantities with high accuracy, so that we can (and have to) determine those quantities directly from our PRIMA observations.

This means that we will need a lot of observations to determine all these parameters, and our data reduction will improve over time.

Correction for Rotating Baseline



$$\Delta d = \Delta\vec{s} \cdot \vec{B} = |\Delta s| \cdot |B| \cdot \cos\theta$$

highly time-dependent: angle θ

due to Earth rotation, orbital motion, precession, nutation, polar motion

(|B| constant, |Δs| constant after astrometric corrections)

in order to correct for the rotating baseline, we need very precise models for the Earth rotation, as provided by the IERS (International Earth Rotation Service) based on VLBI measurements of polar motion etc.

Calculating the Rotated Baseline Vector

baseline vector $\vec{B}$ needs to be converted from Earth-fixed system (ITRS) to space-fixed system (ICRS):

$$[ICRS] = \mathcal{Q}(t)\mathcal{R}(t)\mathcal{W}(t)[ITRS]$$

Q(t), R(t) and W(t) are the following rotation matrices:

$$\begin{aligned}
\mathcal{Q}(t) &= \mathcal{R}_3(-E)\,\mathcal{R}_2(-d)\,\mathcal{R}_3(E)\,\mathcal{R}_3(s) \\
\mathcal{R}(t) &= \mathcal{R}_3(-\theta) \\
\mathcal{W}(t) &= \mathcal{R}_3(-s')\,\mathcal{R}_2(x_p)\,\mathcal{R}_1(y_p)
\end{aligned}$$

E, d: coordinates of CIP in ICRS
s: position of CEO on equator of CIP
θ: Earth rotation angle
s′: position of TEO on equator of CIP
x_p, y_p: coordinate of CIP in ITRS (polar motion)

CIP: Celestial Intermediate Pole
R_1, R_2, R_3: rotation matrices around the 3 spatial directions

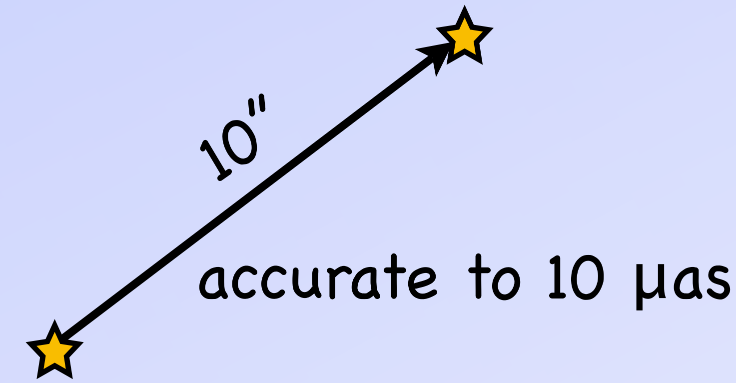
Take-Away Message

Reduction of Astrometric Data at the micro-arcsecond level tedious at times, but it can be done if you have all the required accurate auxiliary data:

- accurate Earth position and velocity
  aberration, parallax, baseline rotation
- micro-arcsecond relative parallax, proper motion
  epoch transformations
- absolute positions, proper motions, radial velocity, parallax
  parallax, epoch transformations, light time delay,
  all other corrections
- ephemerides of solar system bodies
  relativistic light deflection

can be obtained from fit to data

Literature:



good introductory book

Fundamentals of Astrometry

Kovalevsky & Seidelmann, Cambridge University Press, 2004

epoch transformations

Hipparcos Catalogue

Vol 1: Introduction and Guide to the Data, ESA, 1997

A Practical Relativistic Model for Microarcsecond Astrometry in Space

application to GAIA

S. Klioner, AJ 125, 1580, 2003

Microarcsecond Astrometry and Beyond...

Sabine Reffert
Landessternwarte Heidelberg, Germany

If you your astrometric measurement accuracy was better than 1 micro-arcsecond:

- Which additional steps in the data reduction would you have to take?
- Which additional phenomena would be present in your data?

Recap: Astrometric Effects

	absolute	differential
aberration	20.5″	3 mas
light time delay	typically 10–100 μas, extreme 100 mas	same as absolute
gravitational light deflection	1.75″	typically 10 μas
parallax	772 mas	same as absolute
epoch transformations	up to 10″/year	same as absolute

Sub-Microarcsecond Data Reduction

For sub-mircoarcsecond accuracy, all needed input parameters would have to be of adequate precision:

- positions, parallaxes, proper motions
- Earth position and velocity
- ephemerides of solar system bodies

The reduction steps would be the same as before.
The formulae for light time delay, parallax and epoch transformations are geometrically exact.
The formulae for aberration and gravitational light deflection had to be used in their relativistic forms already for microarcsecond astrometry.

Additional Astrometric Effects?

YES!

But before we get to those effects, consider the following:

Data = Signal + Noise

Whether to call something a scientifically interesting astrometric signal or disturbing astrometric noise entirely depends on the measurement accuracy and the questions you ask. E.g., proper motion could either be an interesting quantity to measure in its own right, or disturbing when you look for the much smaller astrometric signature of a planet.

Additional Astrometric Effects

- galactic orbits -> secular aberration
- non-stationary gravitational potential
- lumpiness of gravitational potential
- stellar surface structure
- gravitational waves
- ...?

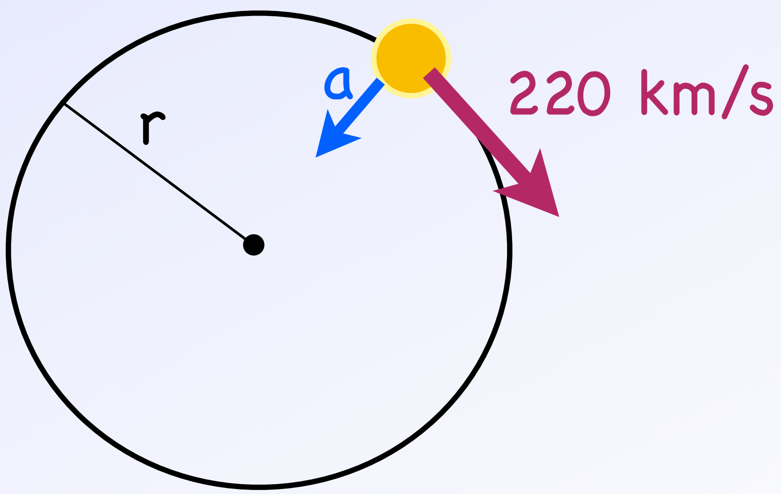
Galactocentric Acceleration

- stars in the Galaxy move on orbits around the galactic center

- circular velocity of Sun about v=200-220 km/s, distance r=8.5 kpc from center
  -> needs 250 Myears for one orbit, has made ≈20 orbits

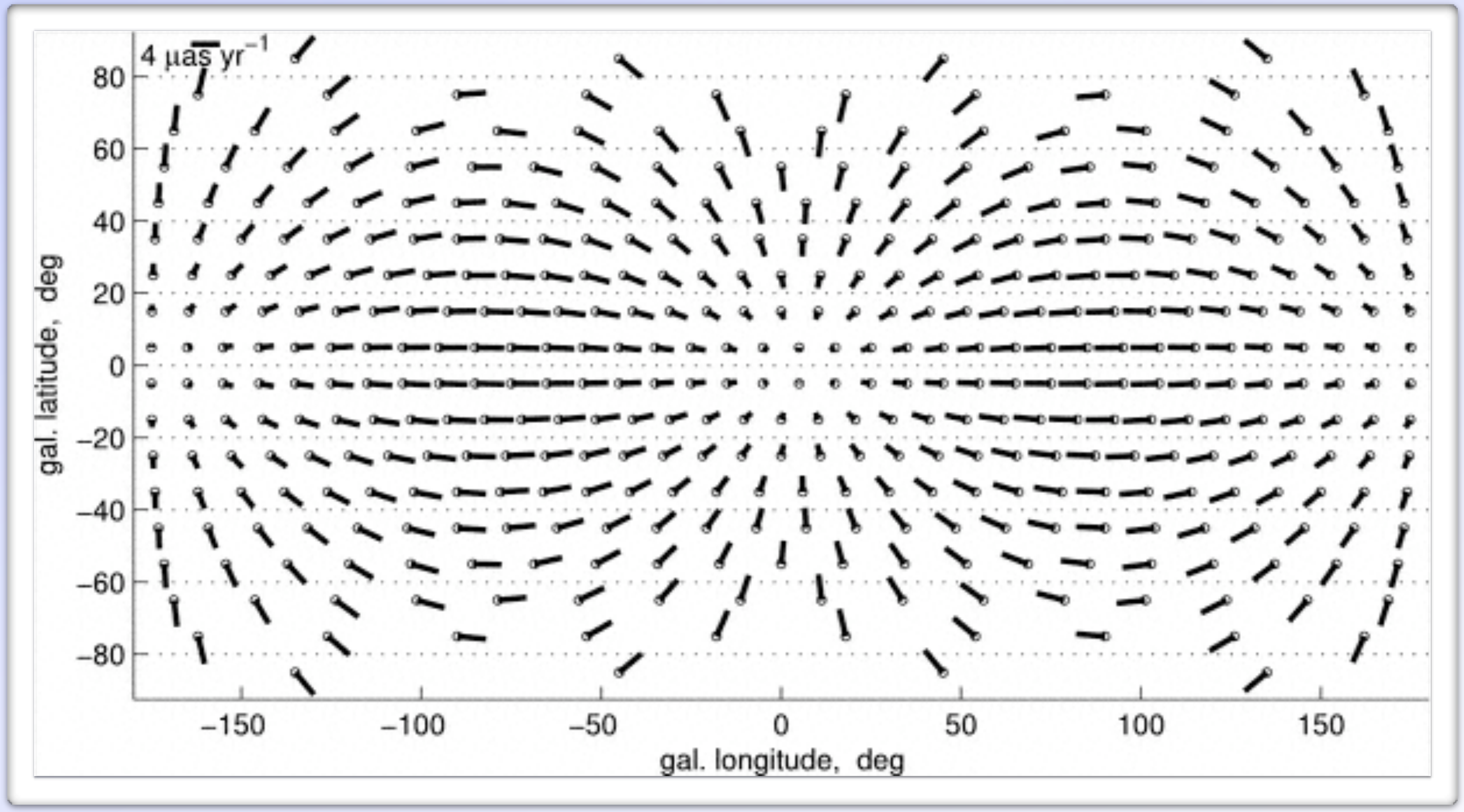
- galactocentric acceleration $a=v^2/r$ for Sun is about
  0.2 nm/s^2 or 6 mm/s/yr

- Sounds small, but which effects does that introduce?



Galactocentric Acceleration

- barycenter of solar system is zero-point of our coordinate system
- if there is acceleration, it will lead to an apparent proper motion of quasars and other distant objects of the order of 4 µas/yr, independent of distance:



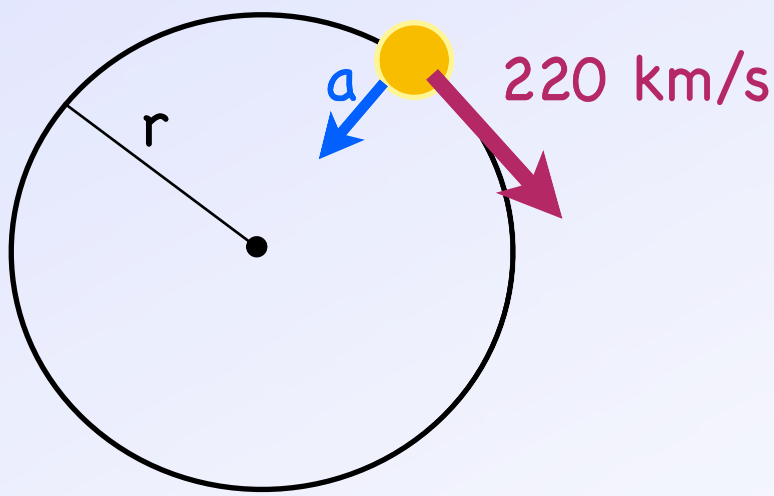
Kopeikin & Makarov, 2006, AJ 131, 1471

Galactocentric Acceleration

- it could also be viewed as secular aberration, because it is caused by an (unmodeled) motion of the observer

- alternatively, one could think about using a different coordinate system (with the center of the Galaxy as origin instead of the Sun)

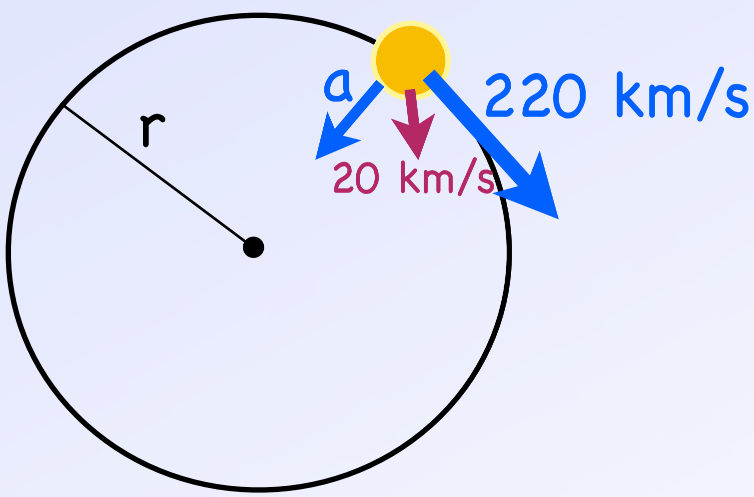
- would required precise knowledge of velocity of Sun and its galactocentric distance



Signal or Noise?

Galactocentric Acceleration

- there is also a component due to peculiar motion of the Sun with respect to the local standard of rest (LSR)
- however, this motion is about 20 km/s, a factor 10 smaller than the circular velocity
- becomes relevant at sub-microarcsecond astrometric accuracy



Non-stationary Gravitational Potential

- on average the gravitational field of the Galaxy is constant, but on smaller scales, it is variable because objects move
- because light is bent in gravitational fields, this gives rise to a small random gravitational light deflection effect
- introduces astrometric jitter, similar to atmospheric seeing
- Sazhin (1996) estimates the astrometric jitter to have a characteristic amplitude of 1-2 μas and a characteristic time scale of tens of years, but there are outliers with much larger astrometric jitter

→ **astrometric catalogs at microarcsecond level have to be revised every 10-30 years!!!**

Sazhin 1996, Astronomy Letters 22, 573

Lumpiness of Gravitational Potential

- the gravitational potential is not smooth; there is structure stemming from spiral arms, molecular clouds, clusters, and possibly black holes

- they will all slightly change the apparent direction to a star through light bending

- example1: Taurus cloud at 140 pc, assume mass of $2 \cdot 10^5$ $M_\odot$

  -> bends light by an additional 0.1 µas maximum

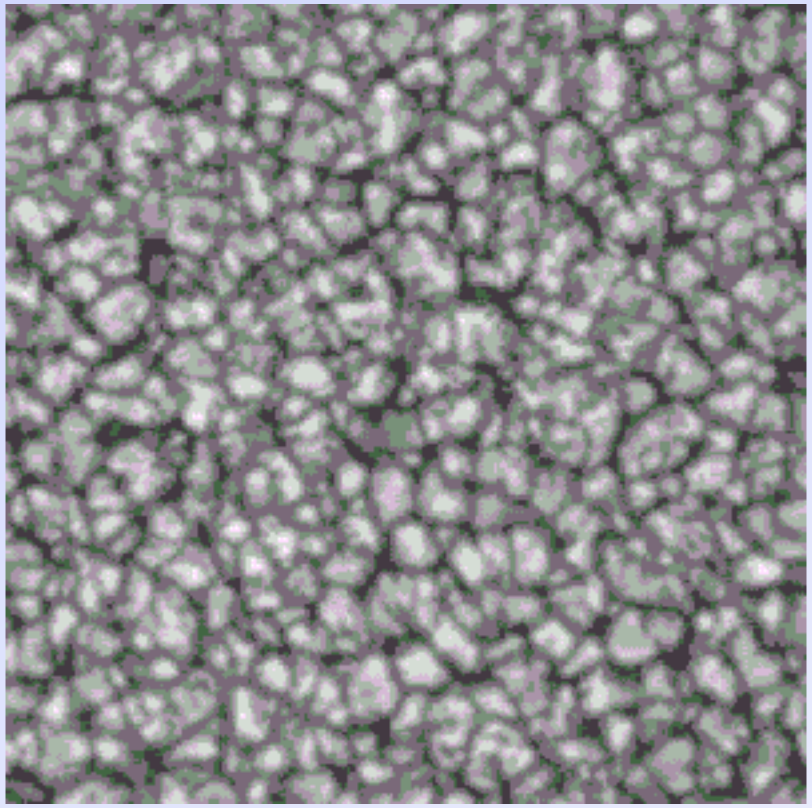
- example2: assume black hole at 10-100 pc, mass of 100 $M_\odot$
  -> bends light by microarcseconds

→ difficult to correct for,
introduces sub-microarcsecond astrometric jitter

Stellar Surface Structure

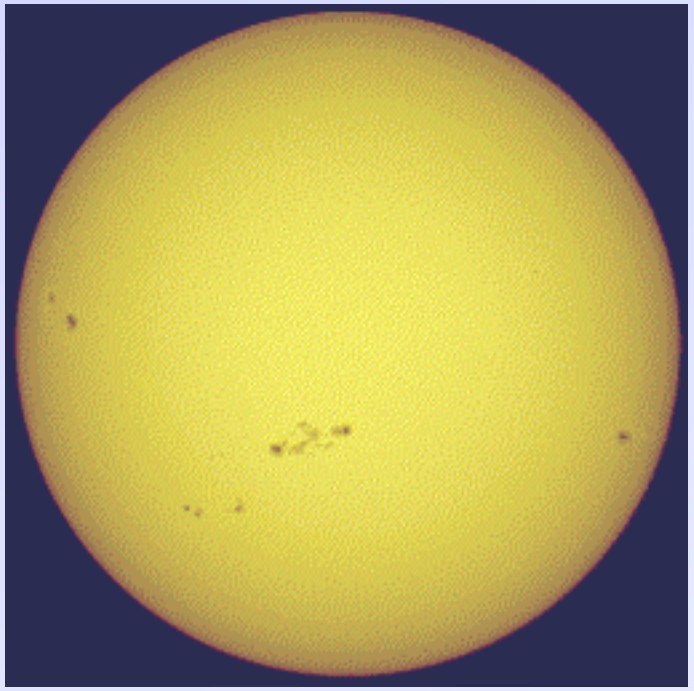
Stars have spots, plages, and granulation. This all introduces shifts in their photocenters.

solar granulation



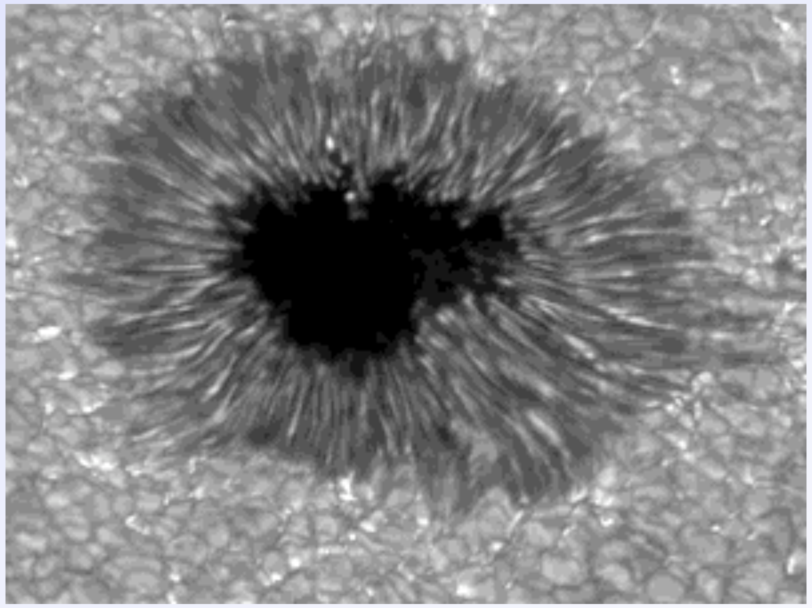
Credit: MSFC/NASA

rotating Sun



Credit: NASA

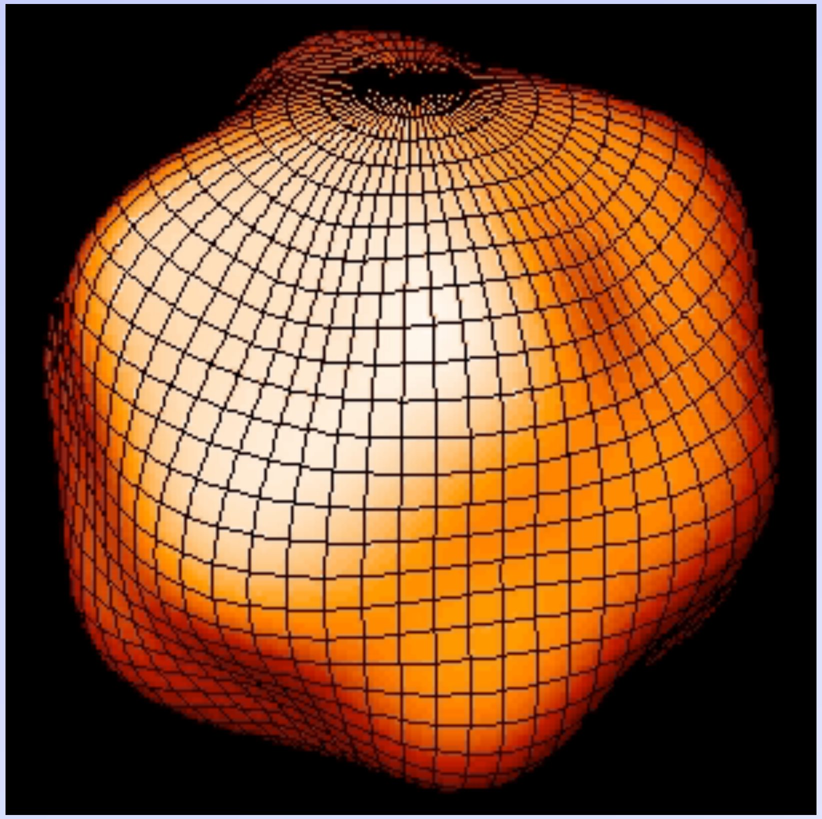
sunspot with boiling granulation



Credit: Peter Sütterlin, DOT Team, SIU

Pulsation Patterns

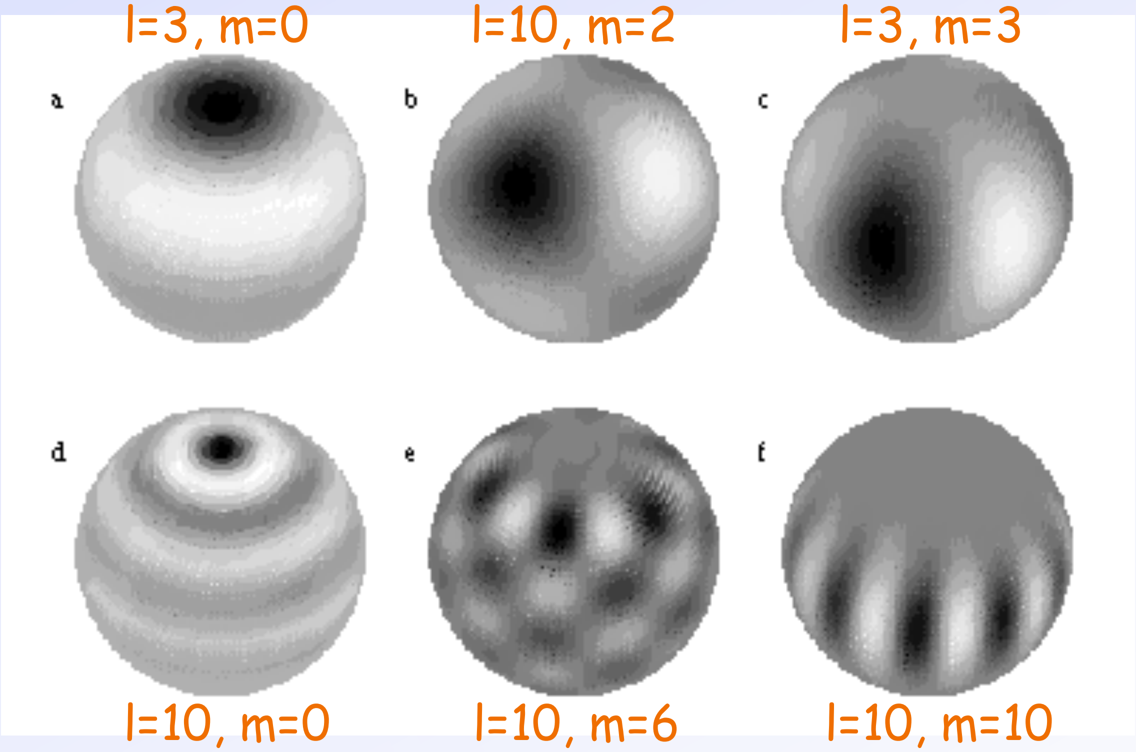
m=-4 p-mode oscillation



Credit: R.H.D. Townsend

Pulsations can also induce shifts in the photocenter of a star!

various non-radial pulation patterns



Credit: E.J. Kennelly

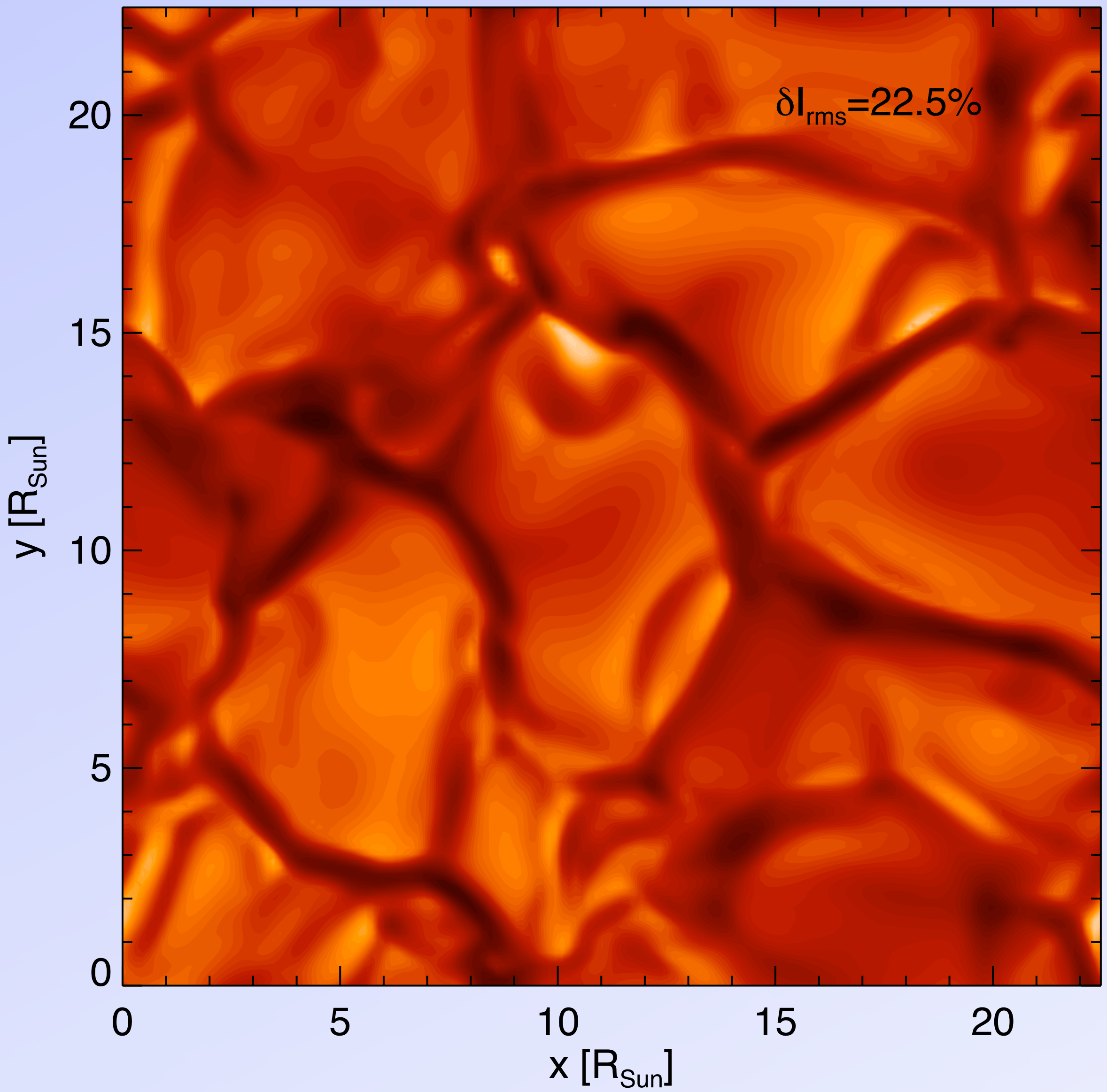
Typical Sizes of Astrometric Jitter

for a distance of 10 pc	lower limit (only from granulation)	upper limit (photometric variability)
early main-sequence	0.03 µas	12 µas
mid-late main-sequence	0.01 µas	1-3 µas
K giants	5 µas	20-50 µas
F supergiants	10 µas	0.4-2 mas
M supergiants	30-300 µas	10 mas

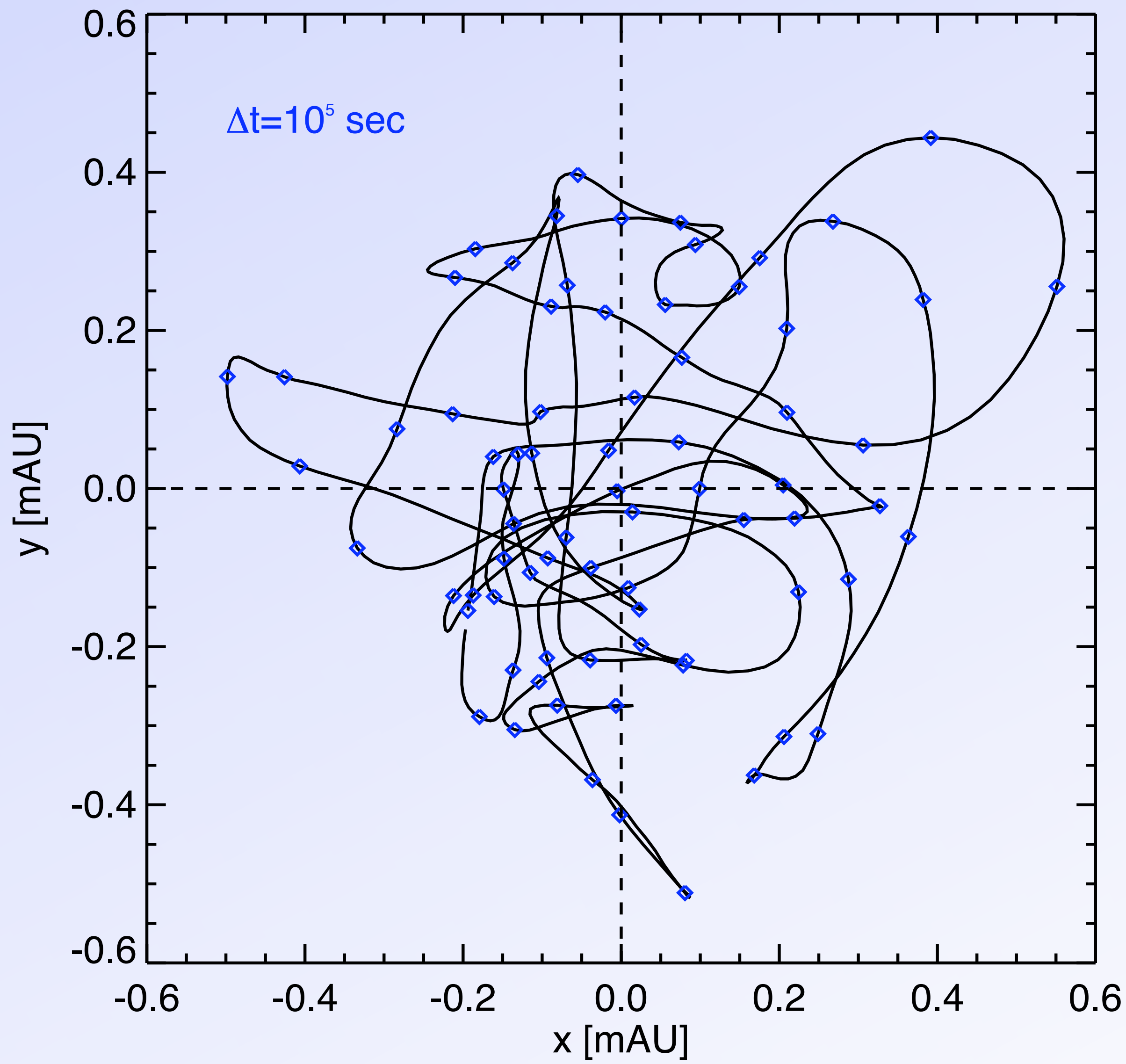
Eriksson & Lindegren, A&A 476, 1389, 2007

Simulations of Granulation

simulated red giant granulation



simulated astrometric jitter

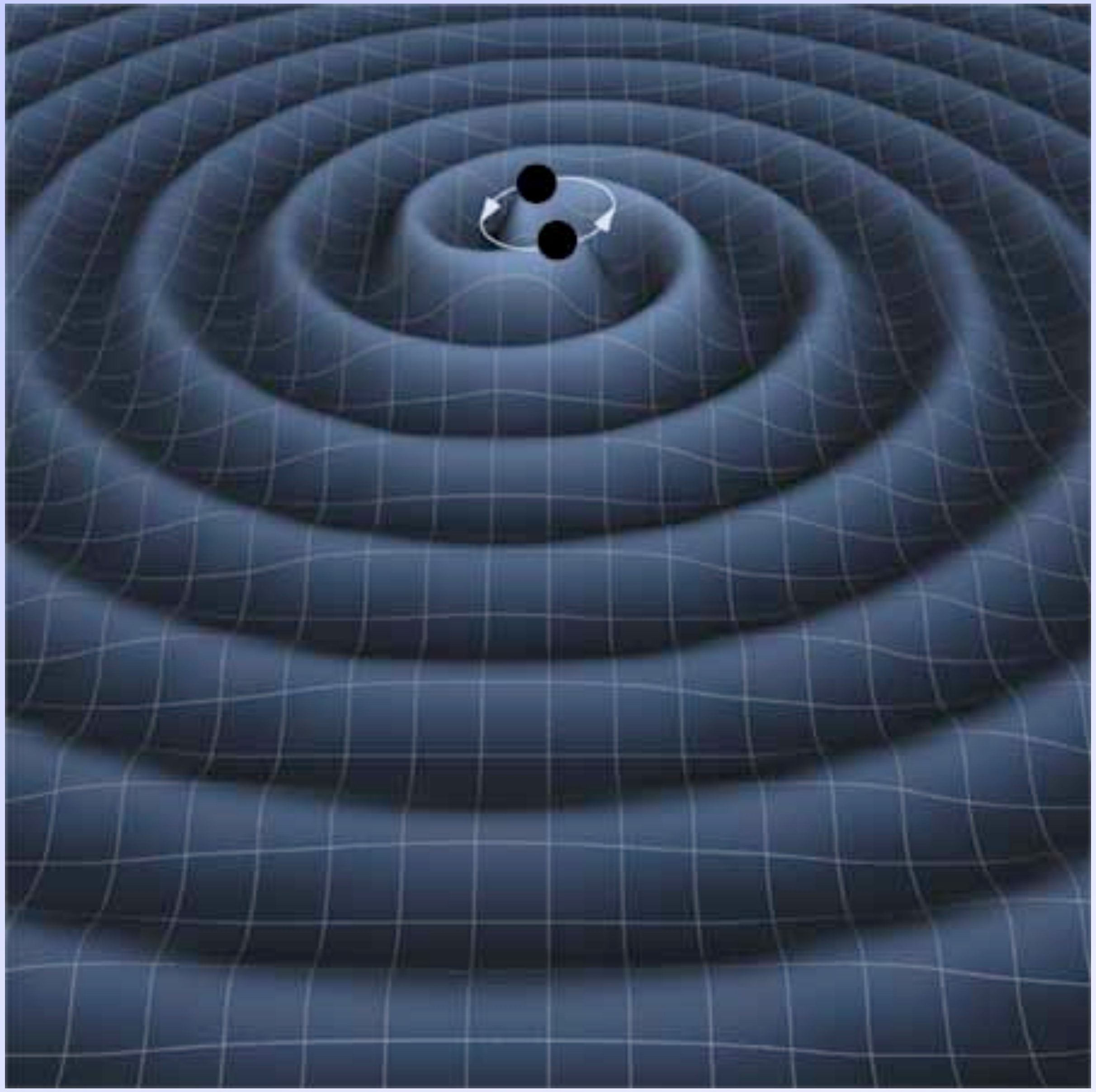


at 10 pc, the amplitude is about 50 μas

Svensson & Ludwig, ESA SP-560, 979, 2005

Gravitational Waves

gravitational wave emitted
by a rotating binary



Credit: K. Thorne (Caltech), T. Carnahan (NASA GSFC)

- gravitational waves are periodic fluctuations in the curvature of spacetime
- so far only indirect confirmation of the existence of gravitational waves
- gravitational waves are emitted if a not spherically symmetric mass distribution is accelerated

Gravitational Waves

- gravitational waves passing over a telescope will cause a time-variable shift in the position of an observed object, i.e. an apparent proper motion
- angular displacement is of order h, the magnitude of the gravitational wave at the telescope
- observed coherently, for all sources on the sky
- current constraints on gravitational wave energy come from Big Bang nucleosynthesis and millisecond pulsar timing
- based on those, expected proper motions are smaller than 0.1 μas/yr and 0.002 μas/yr, respectively

Gravitational Waves

- however, let's assume GAIA will measure 1 billion objects with an accuracy of 100 µas/yr, i.e. it would be sensitive to systematic motions of the order of 3 nano-arcsec
- this would place the tightest limit on gravitational wave energy ever obtained, at least in the follwing frequency band:
- if astrometric measurements are taken over a number of years T, one is sensitive to gravitational wave frequencies of 10^{-8} - 10^{-11} Hz (1/T)
- so it's totally possible (although not likely)that GAIA will not only set the tightest limit, but for the first time detect gravitational waves directly

Summary: Size of Additional Effects

	astrometric jitter
galactic orbits	4 μas/yr
non-stationary gravitational potential	≈ 1-2 μas
lumpiness of gravitational potential	< 1 μas
stellar surface structure	0.01 μas – 10 mas
gravitational waves	< 0.1 μas/yr
.....???	???

Astrometry: Practical Session

Sabine Reffert
Landessternwarte Heidelberg, Germany

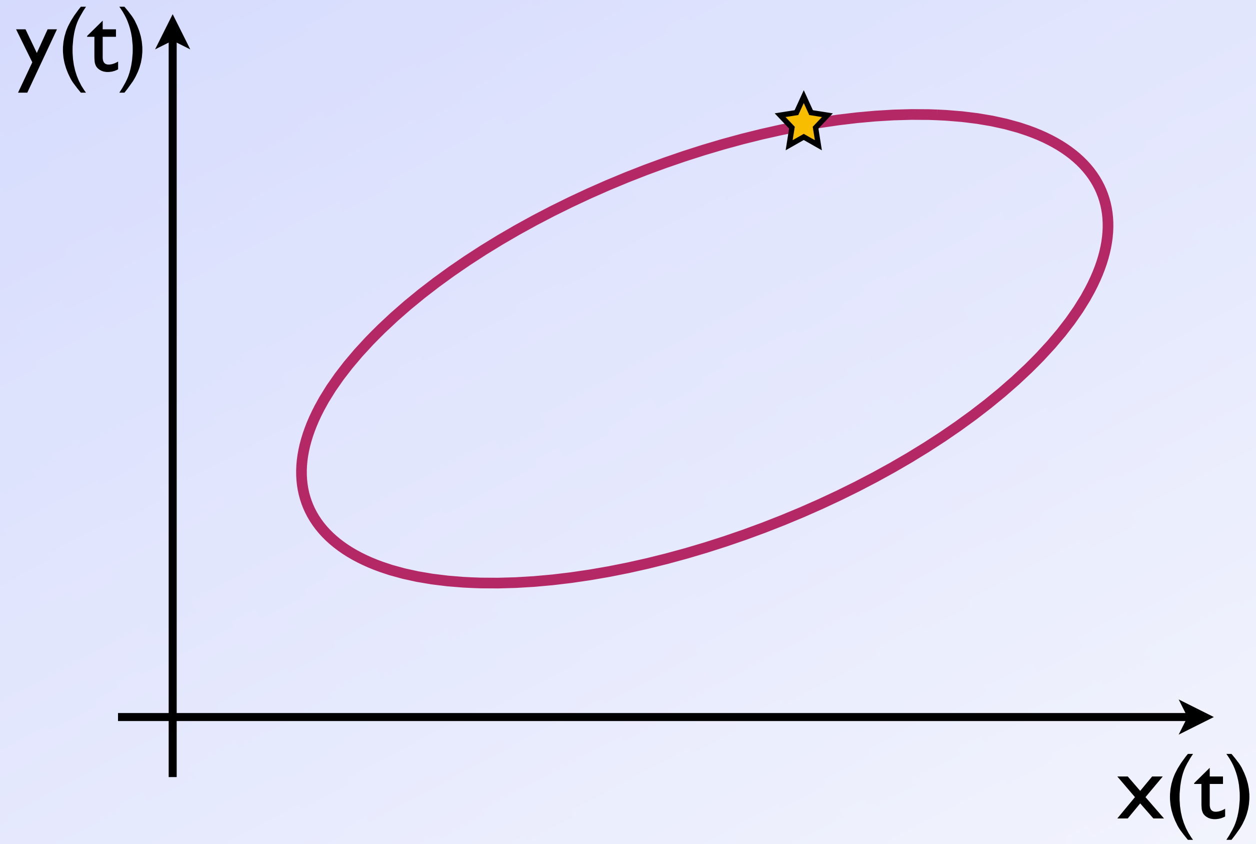
In this session, you will:

- learn how to describe the photocenter motion of a star with an unseen companion
- fit an astrometric model to simulated, 2-dim astrometric measurements
- play a little bit with the program to get a feeling for the various effects and relevant numbers

Modeling Photocenter Motion

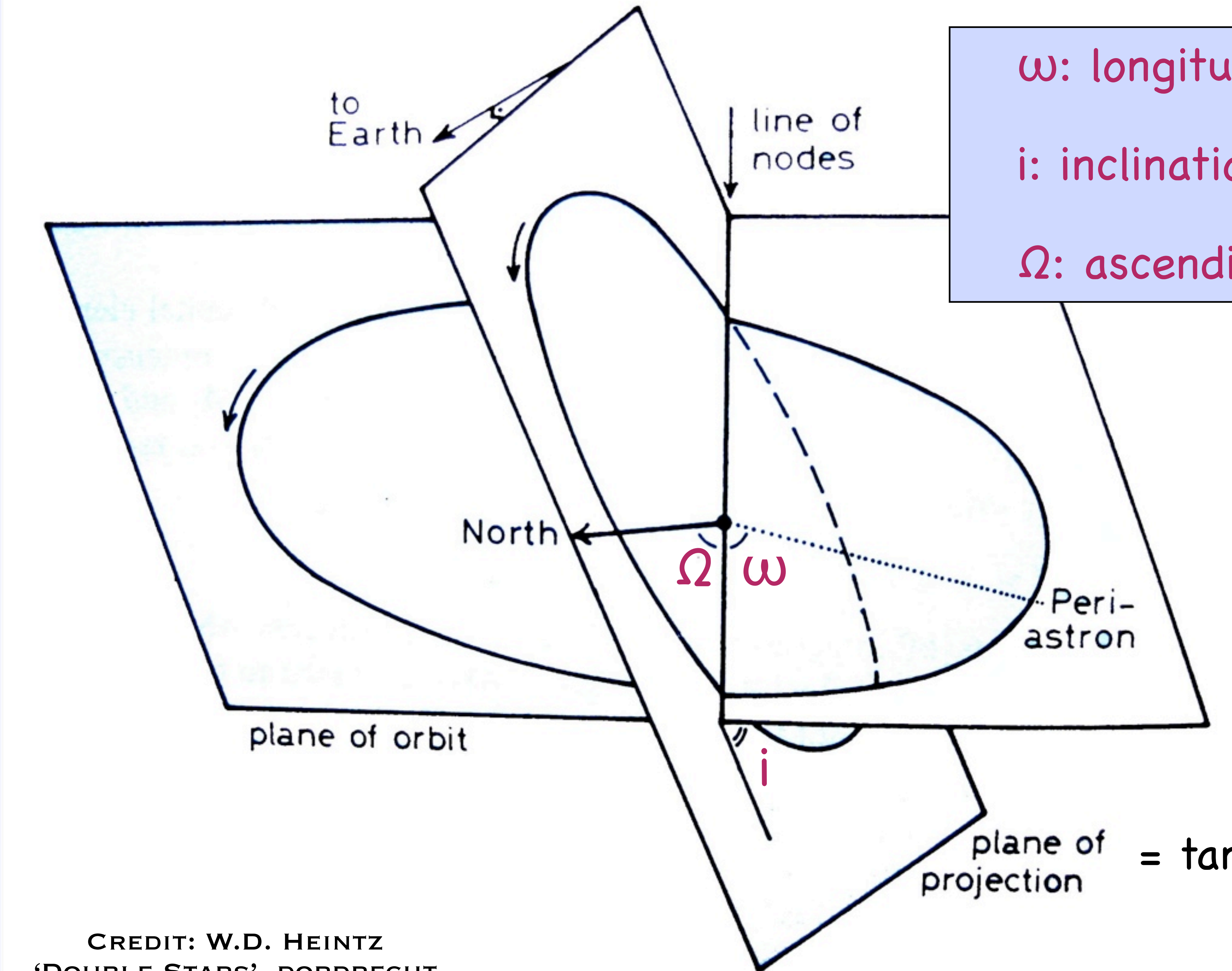
Assume that you have all 7 orbital parameters describing binary orbital motion:

- period P
- periastron time T_0
- eccentricity e
- longitude of periastron ω
- semi-major axis $a1$
- inclination i
- ascending node Ω



How can you calculate $x(t)$ and $y(t)$ for all times, describing the photocenter motion?

Plane of Orbit and Tangential Plane



Credit: W.D. Heintz
'Double Stars', Dordrecht

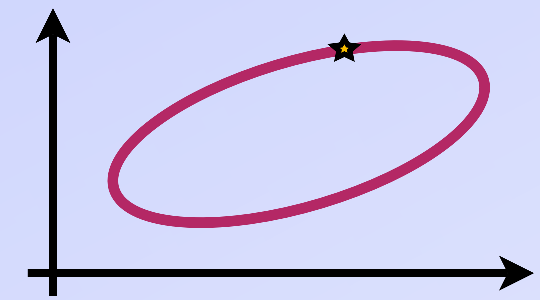
ω: longitude of periastron

i: inclination i

Ω: ascending node

= tangential plane

Kepler Equation



the first step is to solve the Kepler equation to determine the true anomaly:

$$M = \frac{2\pi t}{P}$$

M: mean anomaly

transcendental Kepler Equation

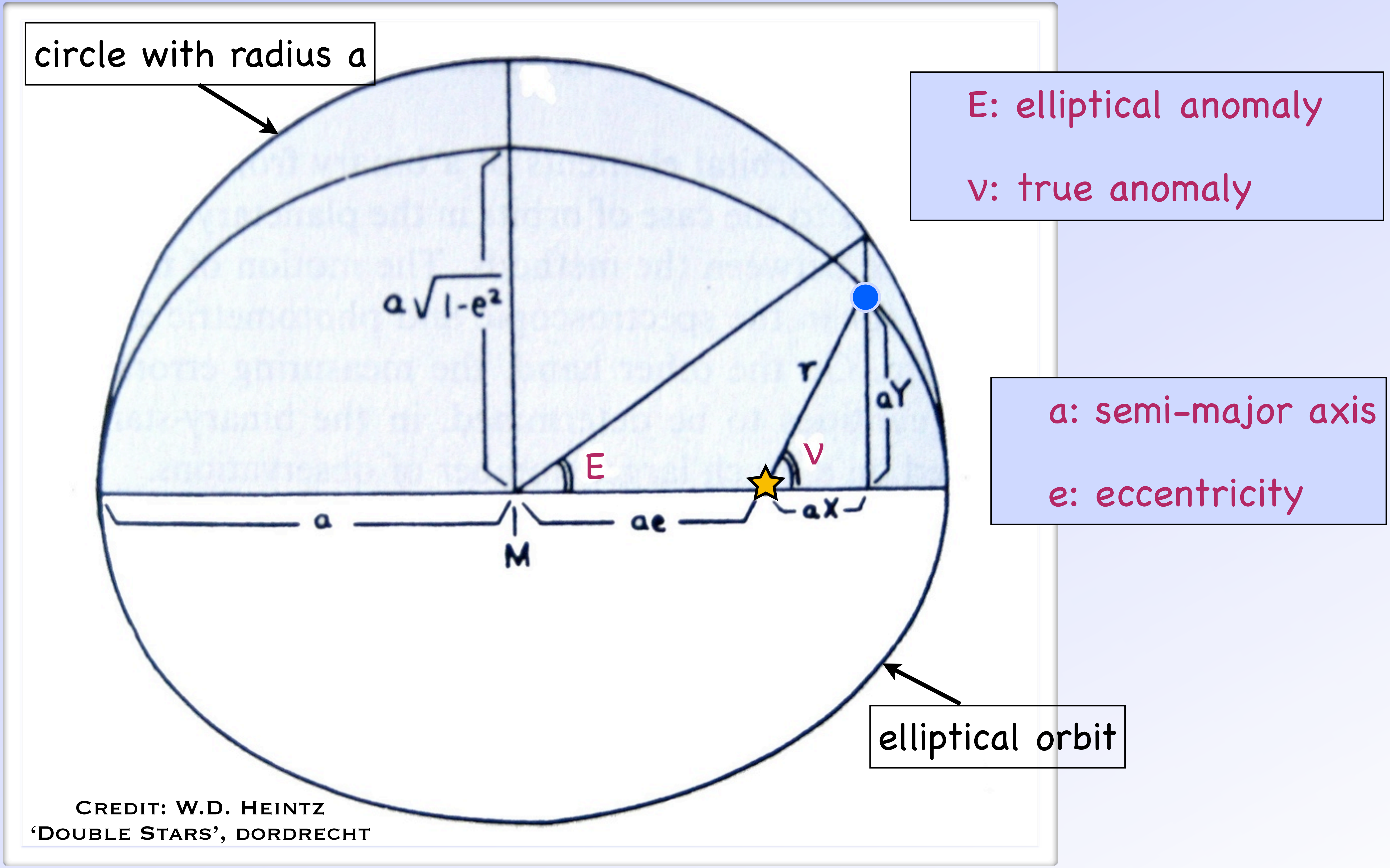
$$M = E - e \cdot \sin E$$

E: eccentric anomaly

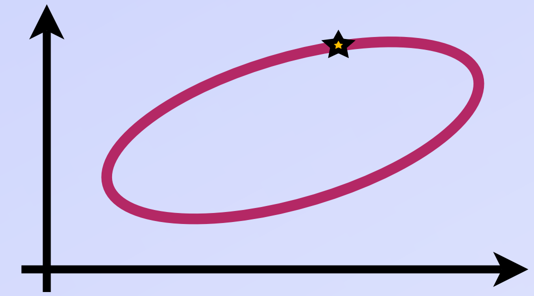
$$\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \cdot \tan\frac{E}{2}$$

ν: true anomaly

Elliptical Orbit



Kepler Equation



from the eccentric anomaly, one can calculate the time-dependent factors $f_x(t)$ and $f_y(t)$:

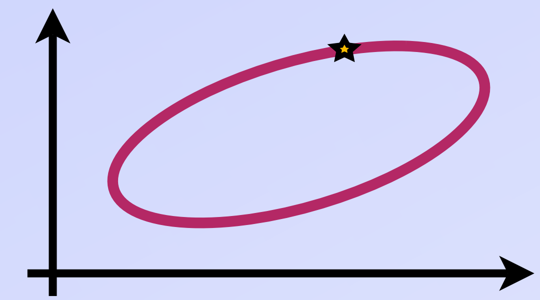
$$\begin{aligned} f_x(t) &= cos E - e \\ f_y(t) &= \sqrt{1-e^2} \cdot \sin E \end{aligned}$$

now, the position x(t) and y(t) can be derived from:

$$\begin{aligned} x(t) &= B \cdot f_x(t) + G \cdot f_y(t) \\ y(t) &= A \cdot f_x(t) + F \cdot f_y(t) \end{aligned}$$

A, B, F and G are the Thiele-Innes constants

Thiele-Innes Constants



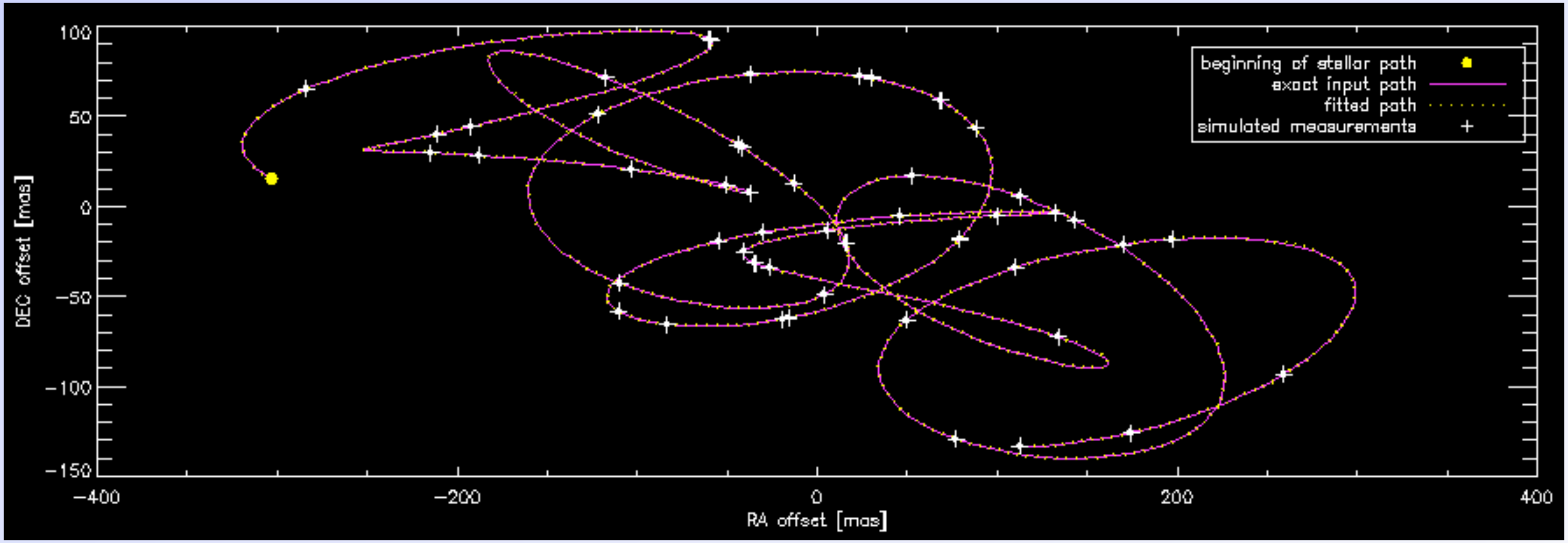
the Thiele-Innes constants are just another parameterization of the orbital parameters a, ω, i, and Ω:

$$
\begin{aligned}
A &= a(\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i) \\
B &= a(\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i) \\
F &= a(-\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i) \\
G &= a(-\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i)
\end{aligned}
$$

the unit of A, B, F and G is the same as those of a, which could be either [AU] or [mas]

Simulation & Fitting Program

program simulates 2-d astrometric measurements including proper motion, parallax and orbital motion and errors, then tries to fit for the relevant 12 parameters



Simulation & Fitting Program

start program with

idl> go

you can modify parameters in the program 'go.pro' in the section 'set input parameters'

alternatively, you can run the program with an input file:

idl> go,'epseri'

which will read input data from 'epseri.input'.
You can add additional input files as you wish.

But before it will work...

... you have to finish writing 'get_thiele_innes.pro'!

$$
\begin{aligned}
A &= a(\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i) \\
B &= a(\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i) \\
F &= a(-\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i) \\
G &= a(-\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i)
\end{aligned}
$$

Summary of Exercises

1. Finish writing thiele_innes.pro!
2. How many measurements do you need to get masses to a few percent, with reasonable accuracy (default)?
3. How accurate do 50 measurements of ϵ Eri have to be to get masses to a few percent?
4. How accurate can you get the mass when you observe only half an orbit? Does it help to make those measurements more precise?
5. How good do the starting values have to be? I.e., are there many local minima?
6. How big would the astrometric signature of Jupiter around the Sun be when viewed from a distance of 10pc?