Analysing and interpreting interferometric *visibilities* by model fitting

Workshop: ***Astrometry and Imaging with the Very Large Telescope Interferometer***

June 2 - June 13

Jörg-Uwe Pott
(W. M. Keck Observatory ;-)

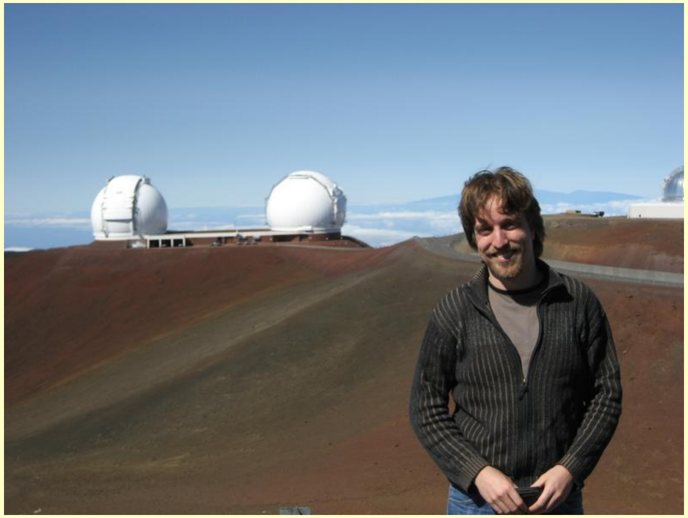
inspired by presentations of
F. Millour, J.P. Berger & D. Segransan

- Outline of this presentation

  - What is the visibility

  - Why and how do I model visibilities
    - What is your science goal
    - No images
    - Thinking in Fourier space, as easy as spectroscopy!

  - Repeat ideas, and other talks > intellectual branding



Interferometry in a nutshell

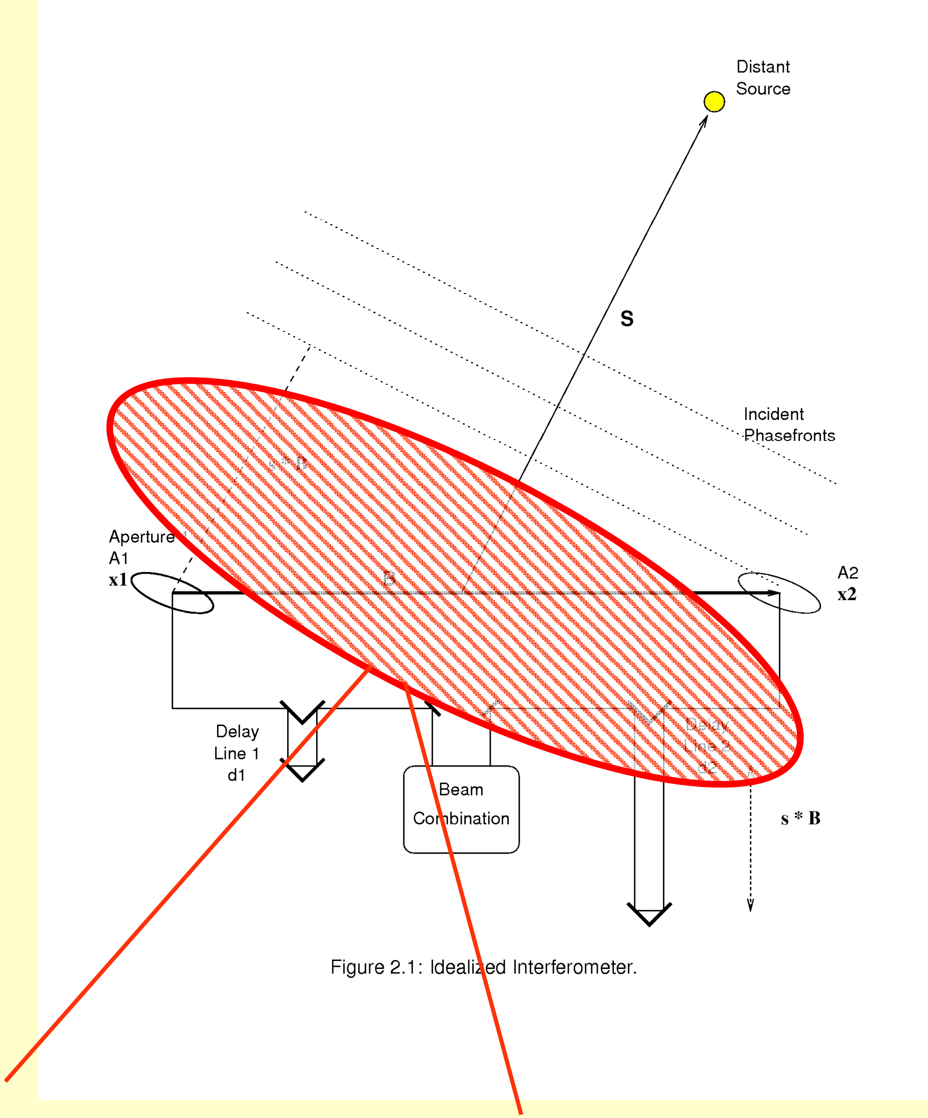


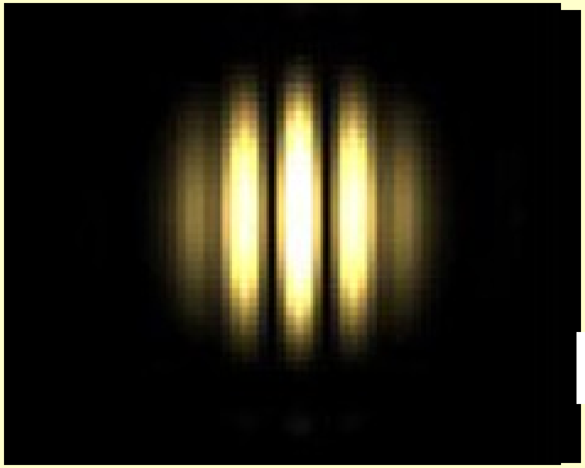
Figure 2.1: Idealized Interferometer.

IF simulates a large aperture telescope in terms of resolution

Interferometry in a nutshell

Measurand:

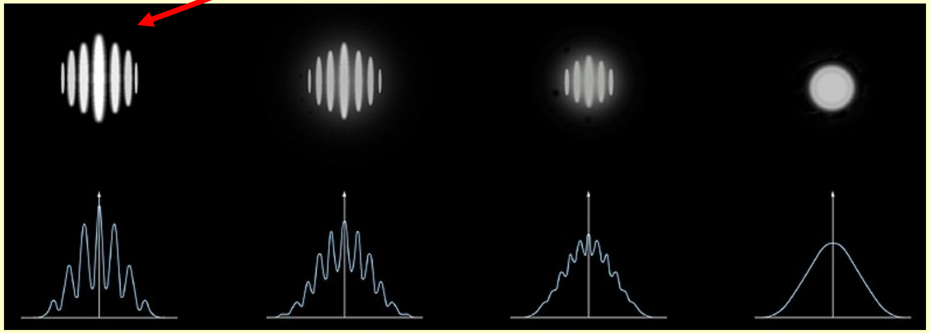
Fringe contrast
or
Visibililty,
(and phase)



$$\simeq 2I_{\rm tel}(\theta)\,[1 + V_{\rm UD}\cos(2\pi\theta B/\lambda)]$$

$$V_{\rm UD} = \frac{2J_1(\pi B\theta_{\rm UD}/\lambda)}{\pi B\theta_{\rm UD}/\lambda}.$$

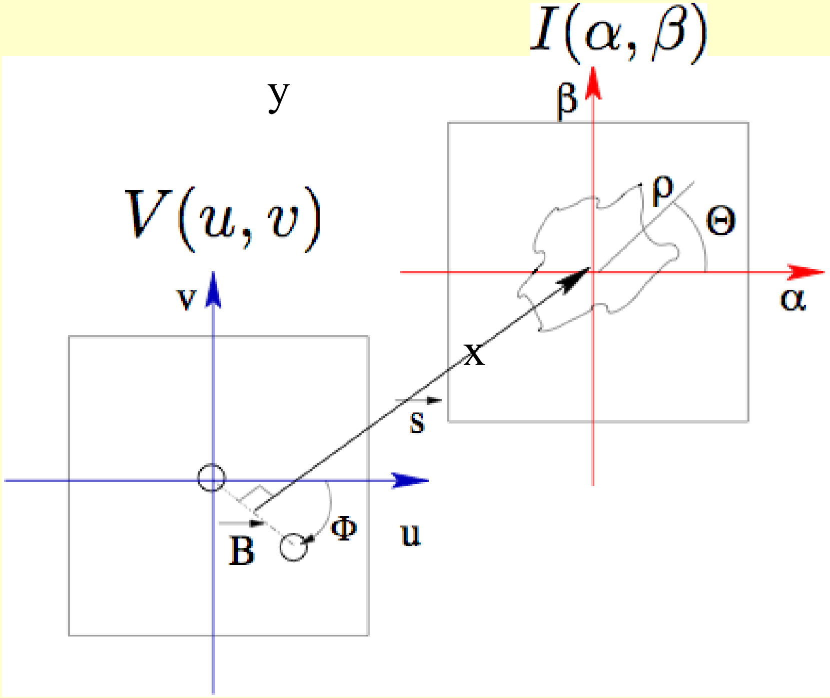
Larger size
=
Smaller visibility



IF simulates a large aperture telescope in terms of resolution

What is "visibility" ?

The Van-Cittert / Zernike theorem



The VCZ theorem links the intensity distribution (often: “brightness distribution”) of an object in the plane of the sky (in the far field) to the complex *visibility* measured by the interferometer.

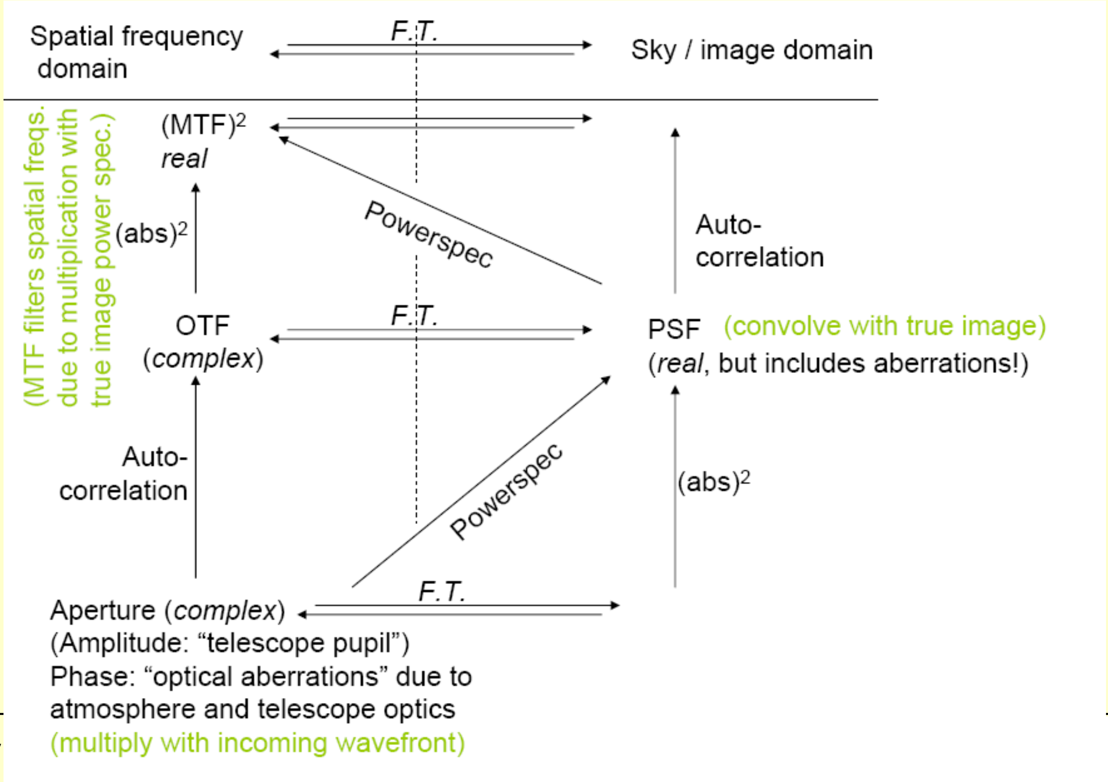
$$V(u,v) = \frac{\int\int I(\alpha,\beta)\exp^{-2i\pi(\alpha u+\beta v)}\, d\alpha d\beta}{\int\int I(\alpha,\beta) d\alpha d\beta}$$

This relation is a normalized **Fourier transform**
(i.e. total flux does not matter, only the *spatial concentration* of the flux).

Spatial frequency coordinates $u=B_x / \lambda$, $v=B_y / \lambda$
where B_x and B_y stand for *projected* baselines coordinates, projected onto the sky along the line-of-sight (pointing axis), i.e. the baseline as seen from the star.

- VanCittert-Zernike: Fourier transform?
  - Sounds like spectroscopy
  - The (complex) visibility of a source is the amplitude (phase) of a particular cos-wave
  - Since we analyse brightness distributions: the cos-wave is on the sky!
  - Coordinates in Fourier-space are *frequencies: '*Spatial frequencies', or u,v-coordinates, u = B/lam

- Concepts for interferometers
  - PSF is grill on the sky
  - The *actual projected* baseline defines the spatial frequency probed



Spatial frequency domain

F.T.

Sky / image domain

(MTF filters spatial freqs. due to multiplication with true image power spec.)

$(MTF)^2$
real

$(abs)^2$

Powerspec

Auto-correlation

OTF
(*complex*)

F.T.

PSF (convolve with true image)
(*real*, but includes aberrations!)

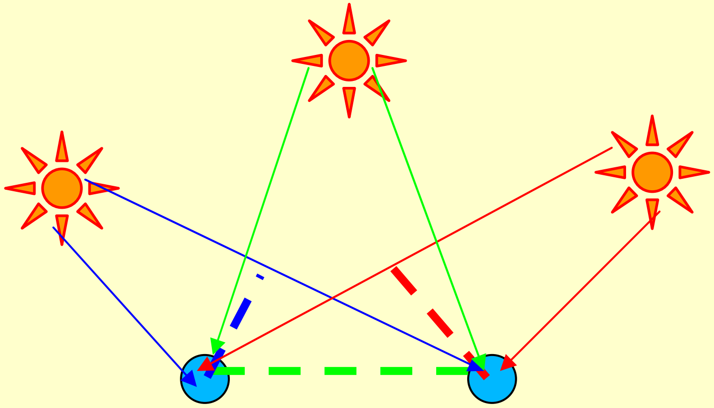
Auto-correlation

Powerspec

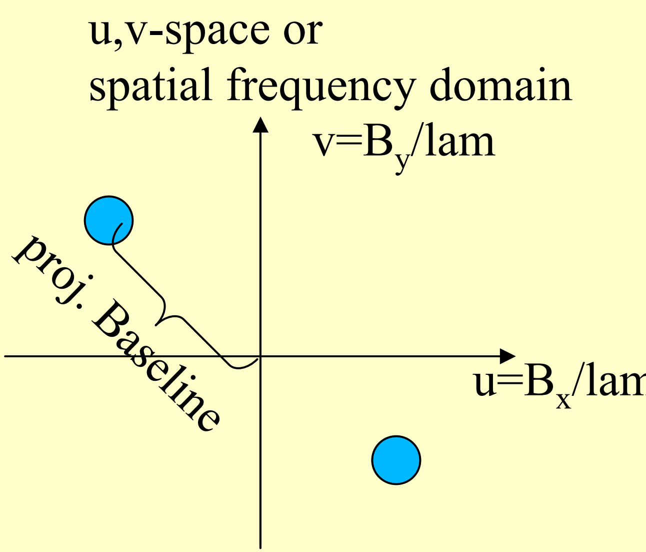
$(abs)^2$

F.T.

Aperture (*complex*)
(Amplitude: "telescope pupil")
Phase: "optical aberrations" due to atmosphere and telescope optics
(multiply with incoming wavefront)



- Projected baseline is what matters
  - Sidereal motion changes this projection
  - Changing the wavelength also changes the spatial frequency, Remember: u,v = B_p/lam
- Why does this matter:
  - Fourier transform of a delta-peak in Fourier space has no meaning
  - Actually we measure two points, so one interferometric measurement transforms into a cos-wave on the sky
  - We need several measurements (cosines) to locate and desribe the source



- Use Fourier transform properties
- Use basic intensity distribution functions

Important first step
towards modelling with
real physical models

Fourier transform properties:

- **Addition** $\mathrm{FT}\{f(x,y)+g(x,y)\}=F(u,v)+G(u,v)$
- **Convolution** $\mathrm{FT}\{f(x,y)\times g(x,y)\}=F(u,v).G(u,v)$
- **Shift** $\mathrm{FT}\{f(x-x_0,y-y_0)=F(u,v)\exp[2\pi i(ux_0+vy_0)]$
- **Similarity** $\mathrm{FT}\{f(ax,by)\}=\frac{1}{|ab|}F(u/a,v/a)$

Imaging and visibility

Example : **resolved binary star** (HIP 4849) observed with Speckle-interferometry (at the Special Astronomical Observatory, Zelentchouk):

Pair of Speckles interfere in the image plane, and resemble an interferometric measurement -> visibility measurements at a continuum of baselines between zero, and aperture diameter

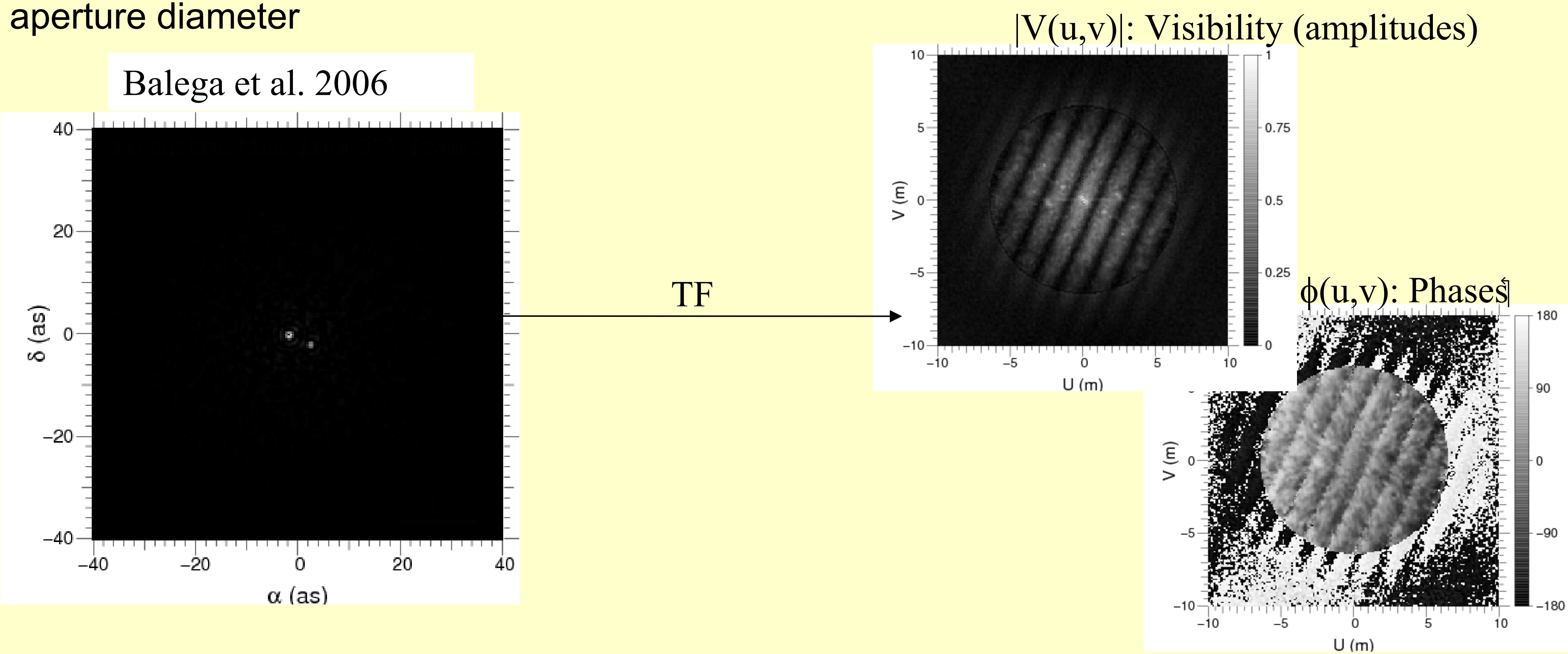
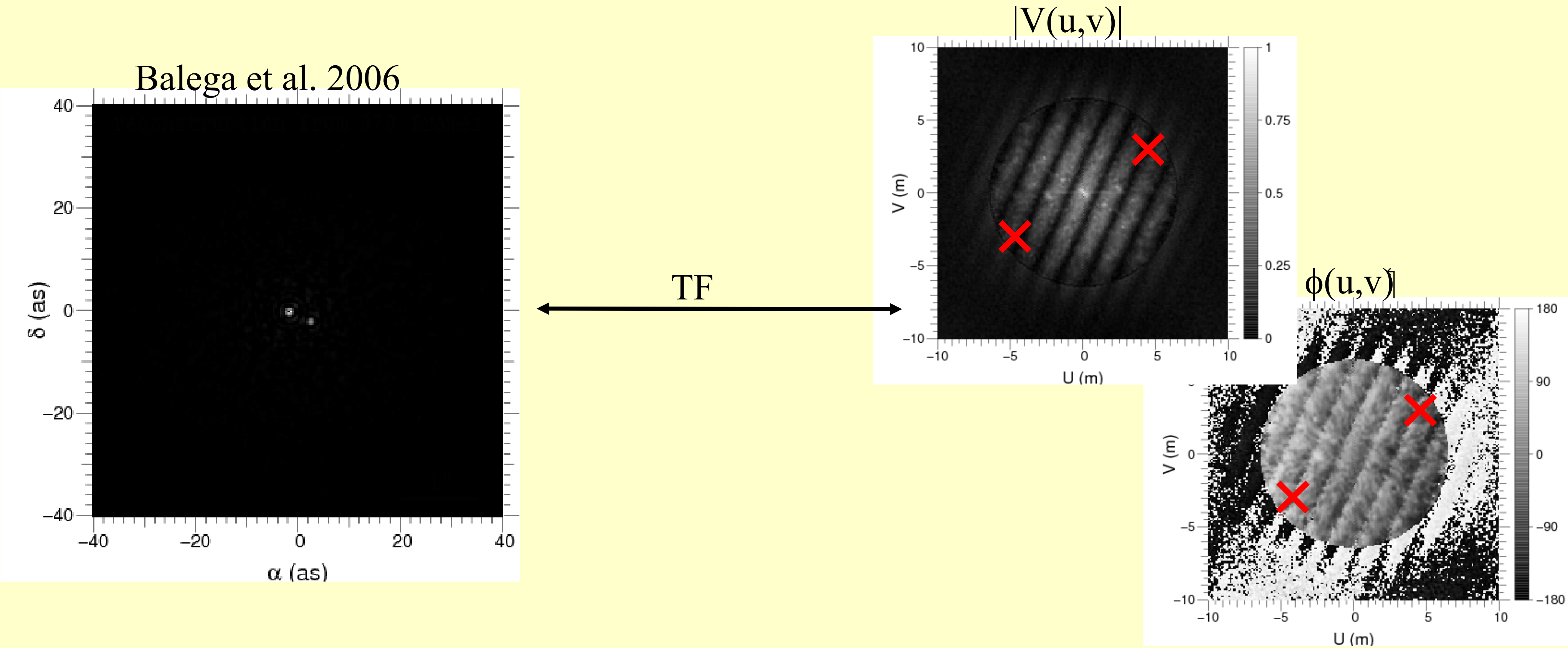


Image : $I(x,y)=O*PSF$ $|V(u,v)|$, $\varphi(u,v)$ & cut-off frequency at D/λ

What visibility does the VLTI produce ?

Only one (pair) per baseline



Balega et al. 2006

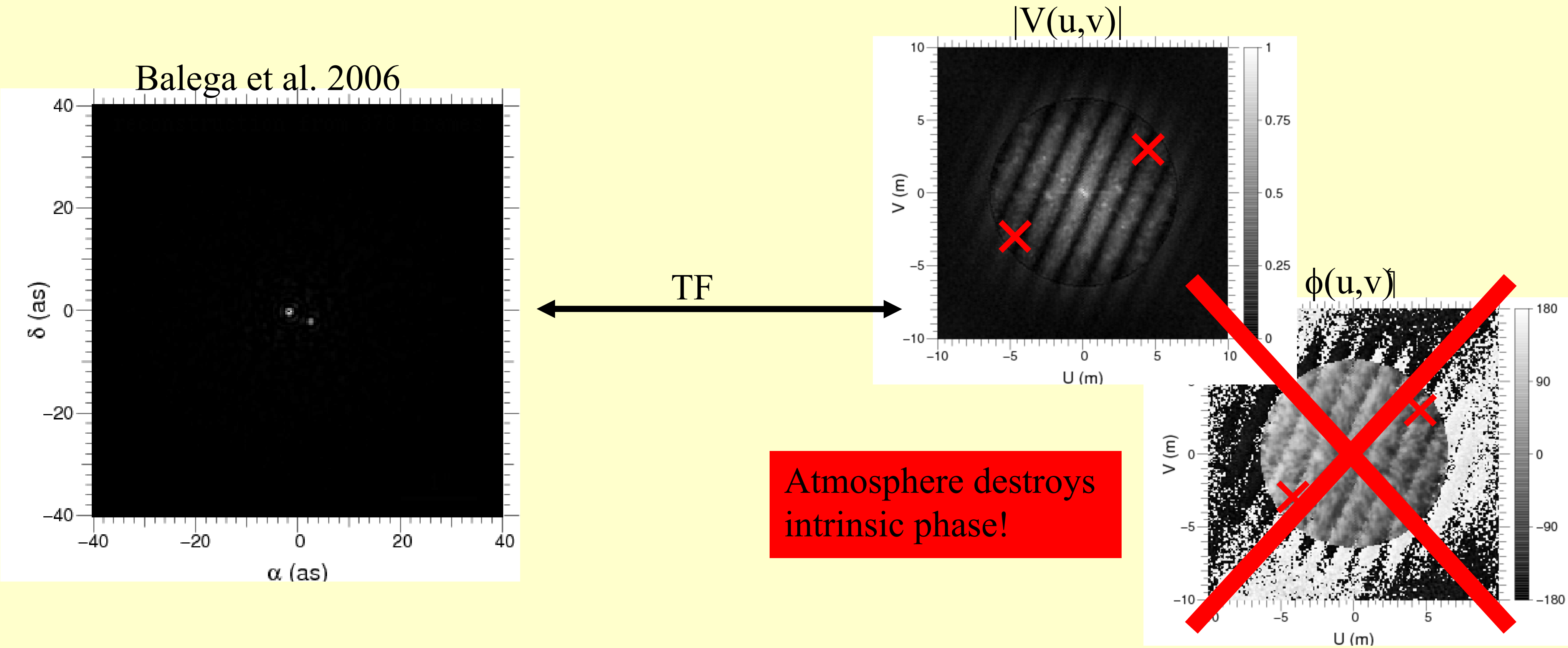
|V(u,v)|

ϕ(u,v)

Image : $I(x,y)=O*PSF$

$|V(u,v)|$, $\varphi(u,v)$ & cut-off frequency at D/λ

What visibility does the VLTI produce?



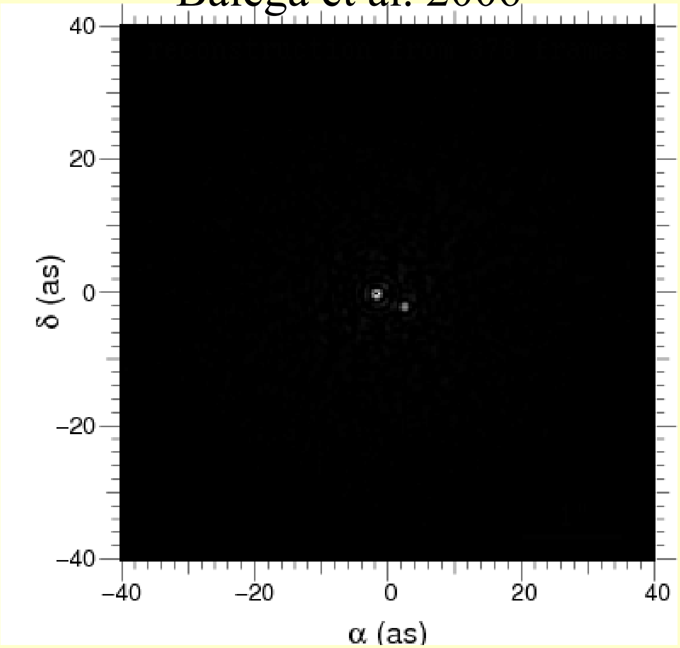
Atmosphere destroys intrinsic phase!

Image : I(x,y)=O*PSF

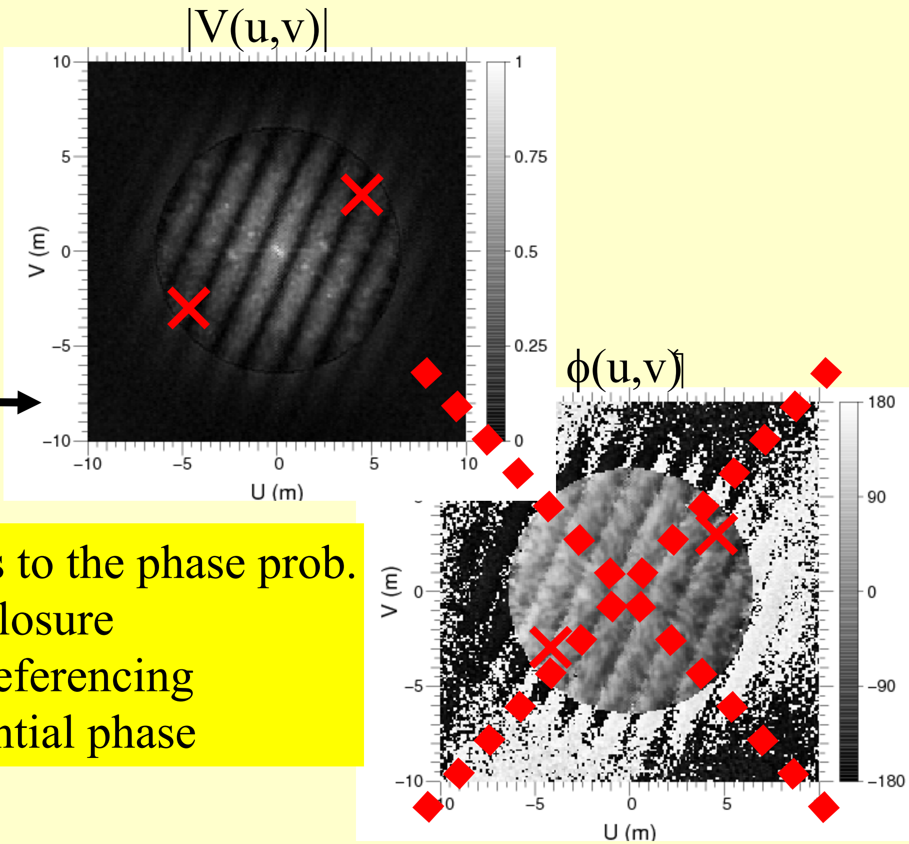
|V(u,v)|, φ(u,v) & cut-off frequency at D/λ

What visibility does the VLTI produce?

Balega et al. 2006



TF



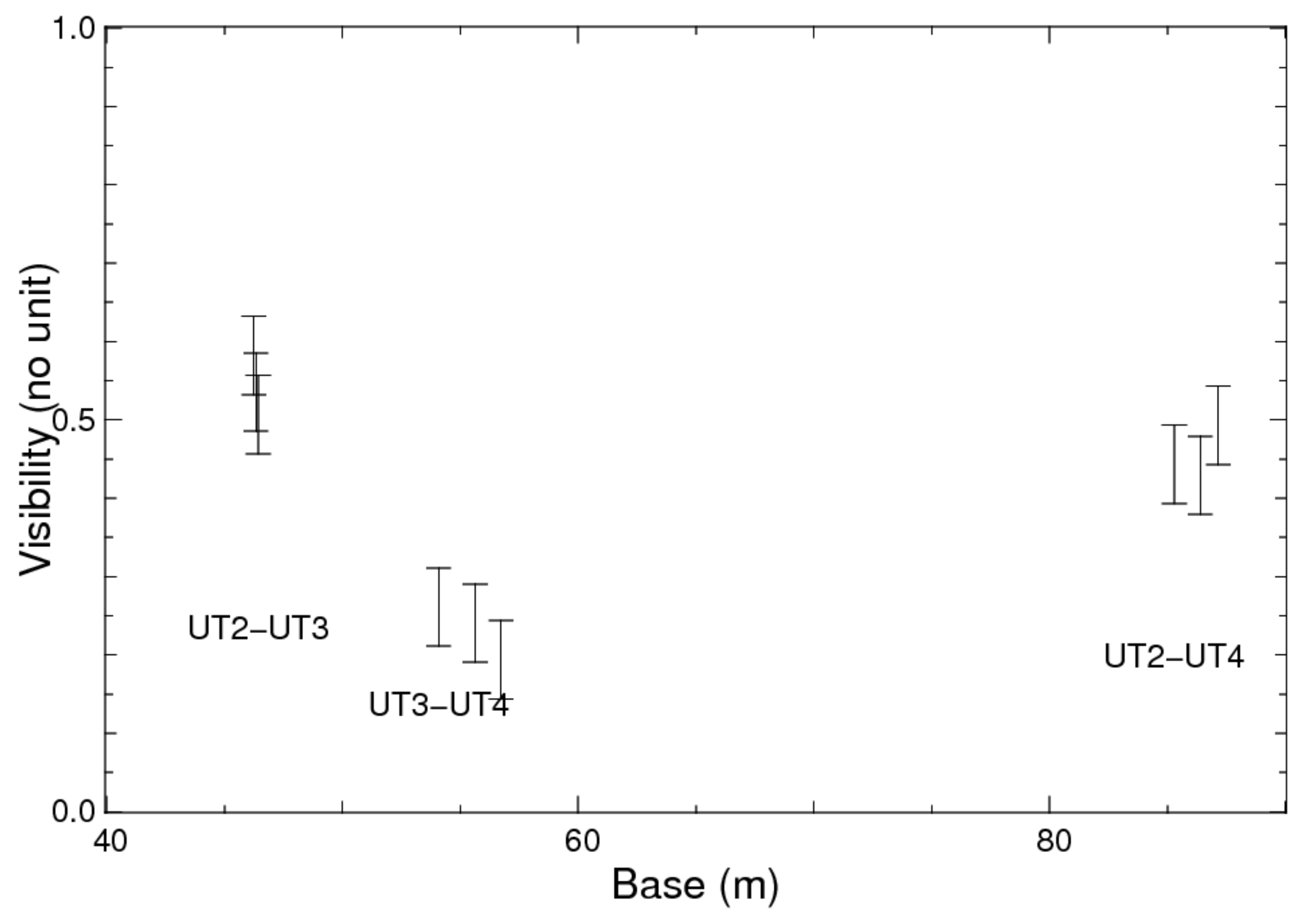
Solutions to the phase prob.

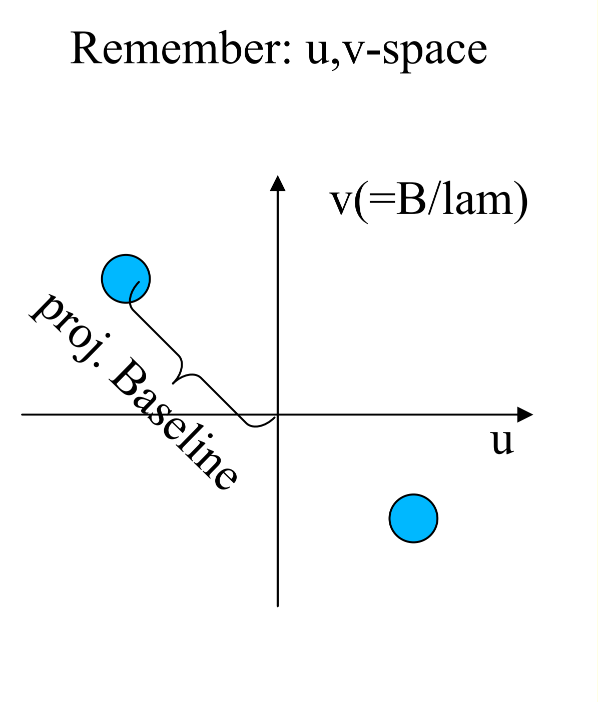
- Phase closure
- Phase referencing
- Differential phase

Image : I(x,y)=O*PSF

|V(u,v)|, φ(u,v) & cut-off frequency at D/λ

This session is about what you can do with that ...





Remember: u,v-space

Simple first step : parametric analysis using basic visibility functions.

What brightness distribution could (!) fit the data?

Model fitting in the Fourier / visibility domain:

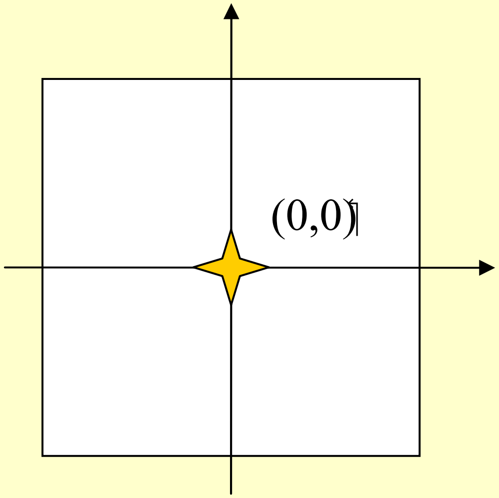
- Transfer your model from imaging in Fourier space, and not the visibilities from Fourier space back

- Domain where interferometric measurements are made
=> errors easier to take into account (ex: Gaussian noise)

- Is better when no easy imaging is possible (When (u,v) plane sampling is poor (almost always the case, in particular for variable source)

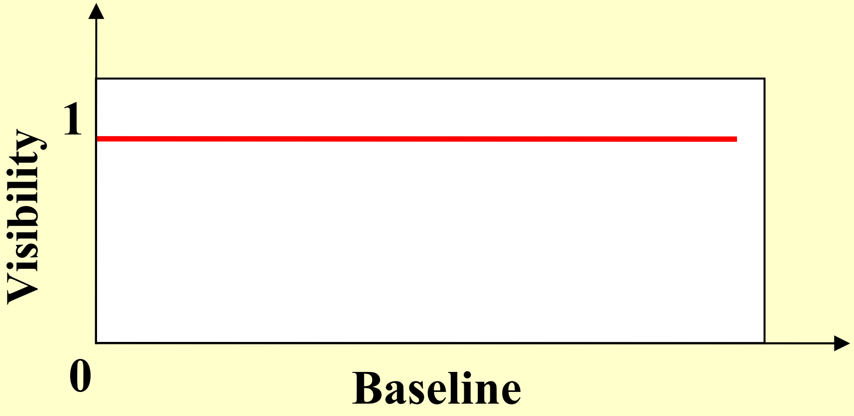
- > the VLTI AMBER and MIDI contexts

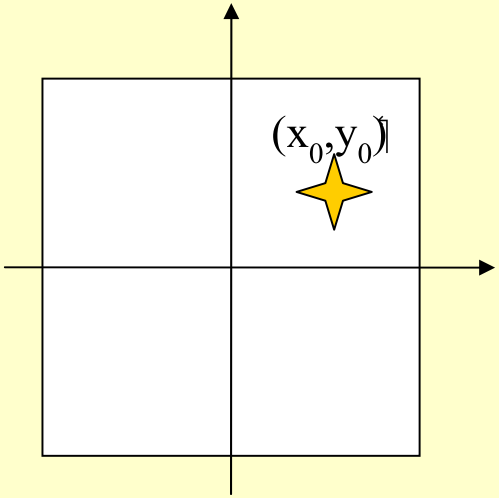
Example #1: Point source function



Centered source

$$I(x,y)=\delta(x,y) \Rightarrow V(u,v)=1$$





Off-axis source

$$I(x,y)=\delta(x-x_0)\delta(y-y_0) \Rightarrow V(u,v)=\exp[-2i\pi(x_0u+y_0v)]$$

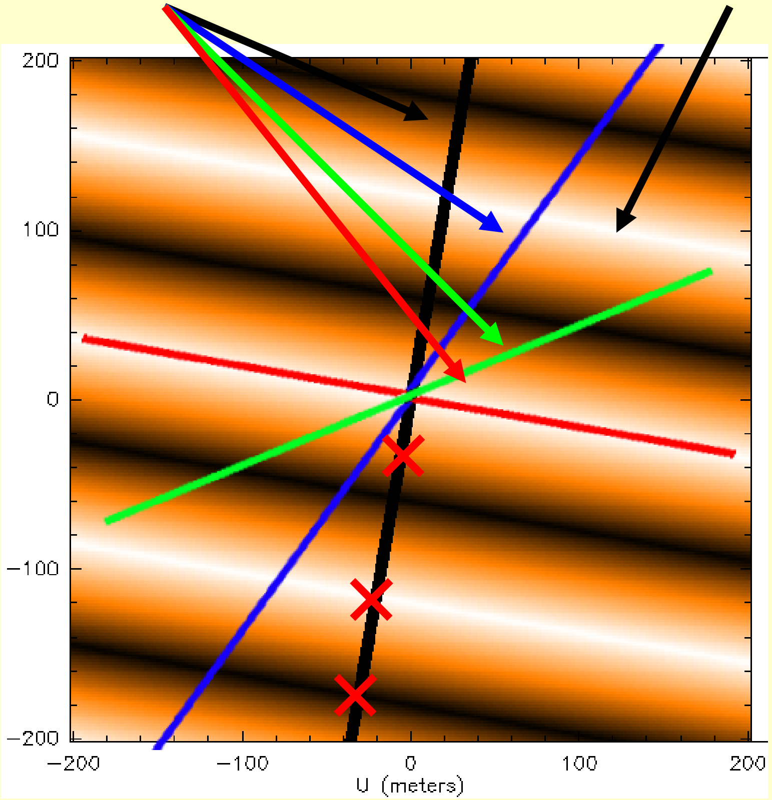
Amplitude =1 , linear dependence for the phase

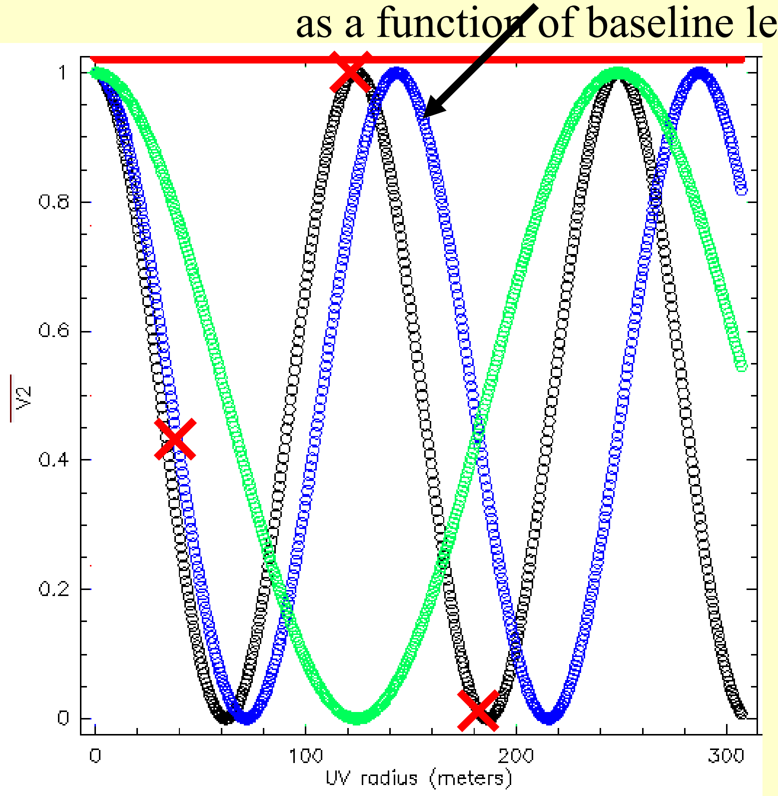
Example #2: Binary star

Projection of baseline in the plane of sky

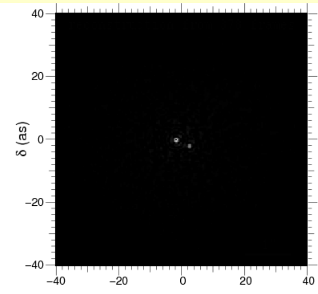
The visibility amplitude squared in (uv) plane

Squared visibility curves for three baselines
as a function of baseline length





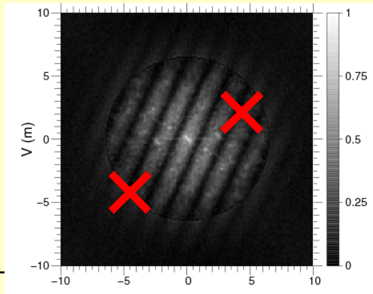
image



Remember:

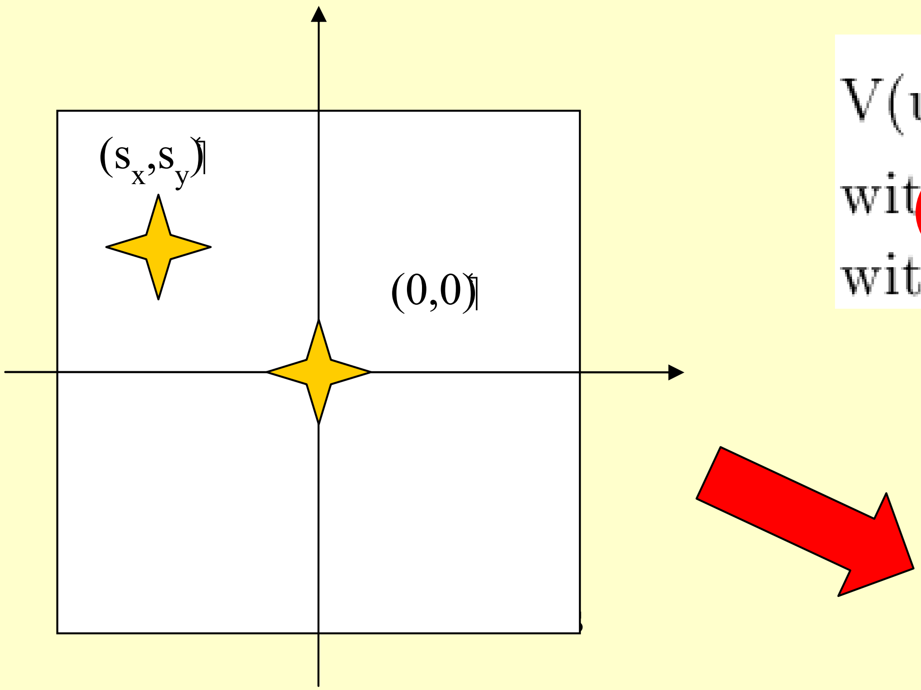
TF →

Visibility



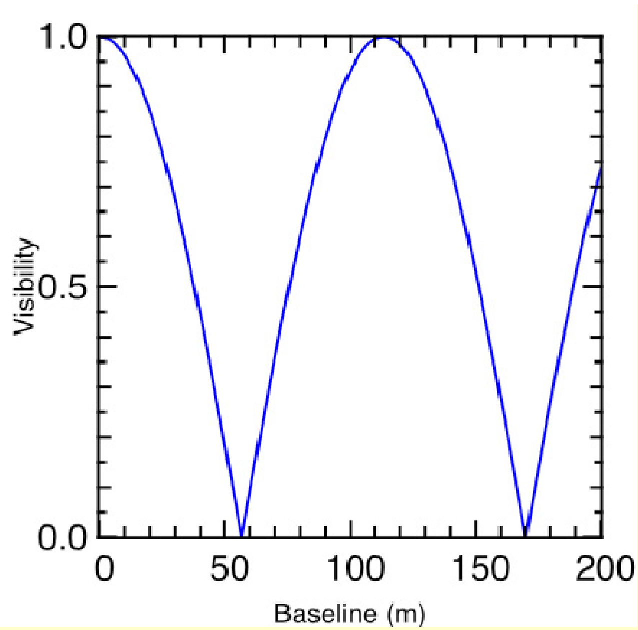
Example #2: Binary star (= two point sources)

$$A\delta(x,y) + B\delta(x - sx, y - sy) \text{ with } s = \sqrt{sx^2 + sy^2}$$



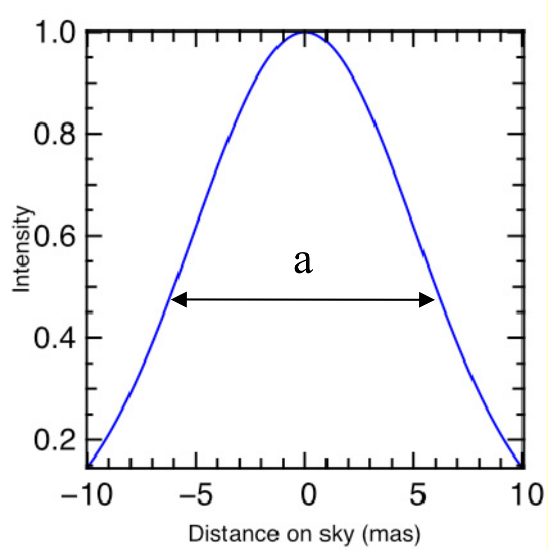
$$\begin{aligned} \text{V(u,v)} &= \sqrt{\frac{1+r_{ab}^2+2r_{ab}\cos 2\pi \vec{L}_b\vec{s}/\lambda}{1+r_{ab}^2}} \\ \text{with } \mathrm{r}_{ab} &= \mathrm{A/B} \\ \text{with } \vec{L}_b &= \text{Baseline} \end{aligned}$$

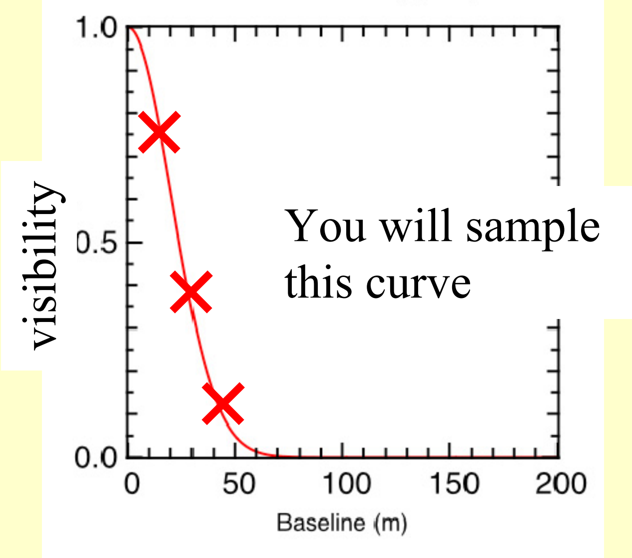
Brightness-ratio defines the strength of the cos-pattern



Example #3: Gaussian brightness distribution.

Use: Estimate for angular sizes of envelopes, disks, etc.





You will sample this curve

$$I(r) = \frac{I_0}{\sqrt{\pi/4\ln 2a}}\exp(-4\ln 2r^2/a^2)$$

Where a = FWHM intensity, I_0 = Peak intensity and

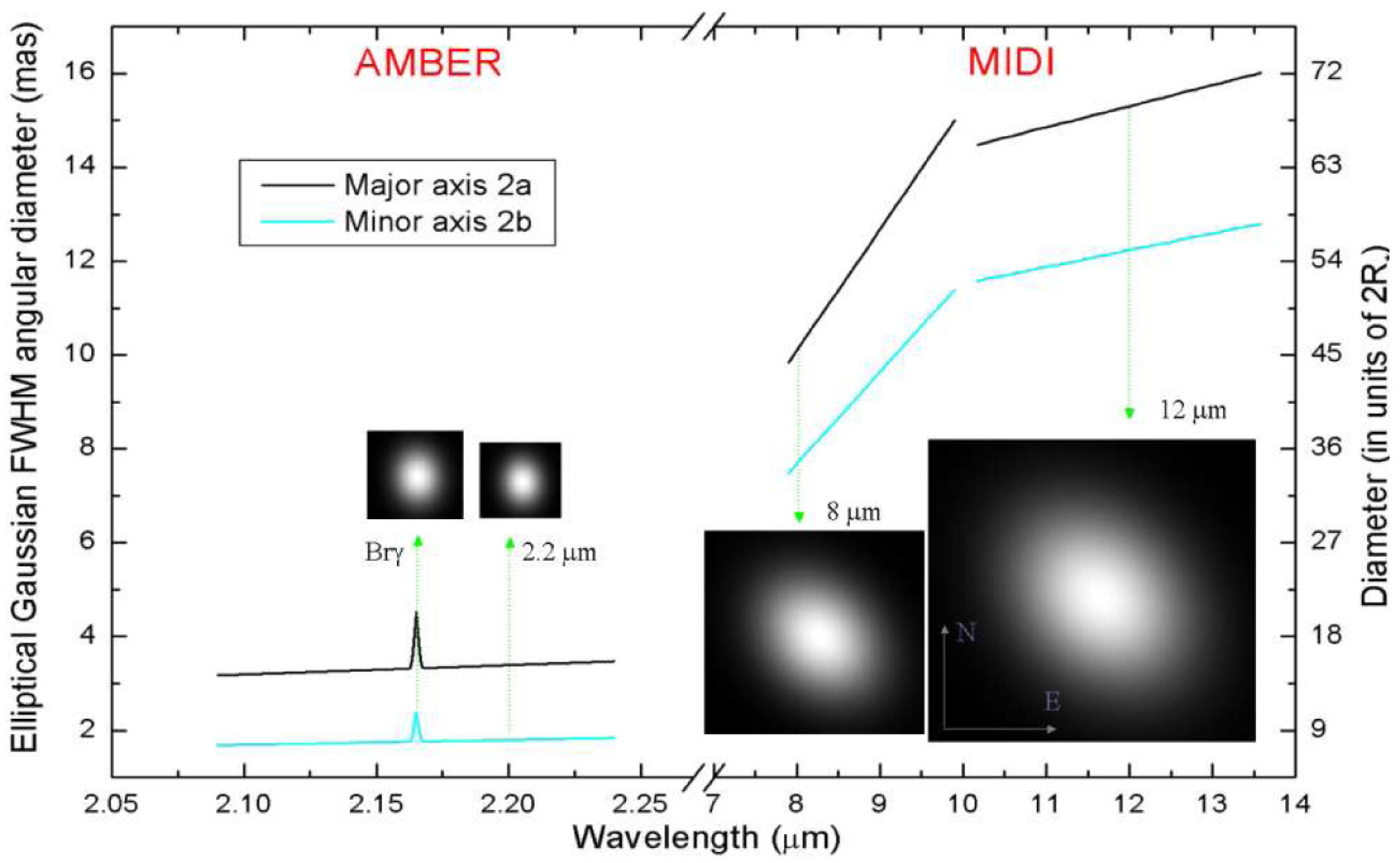
$$r = \sqrt{x^2 + y^2}$$

$$V(\rho) = \exp[-(\pi a\rho)^2/(4\ln 2)]$$

Where

$$\rho = \sqrt{u^2 + v^2}$$

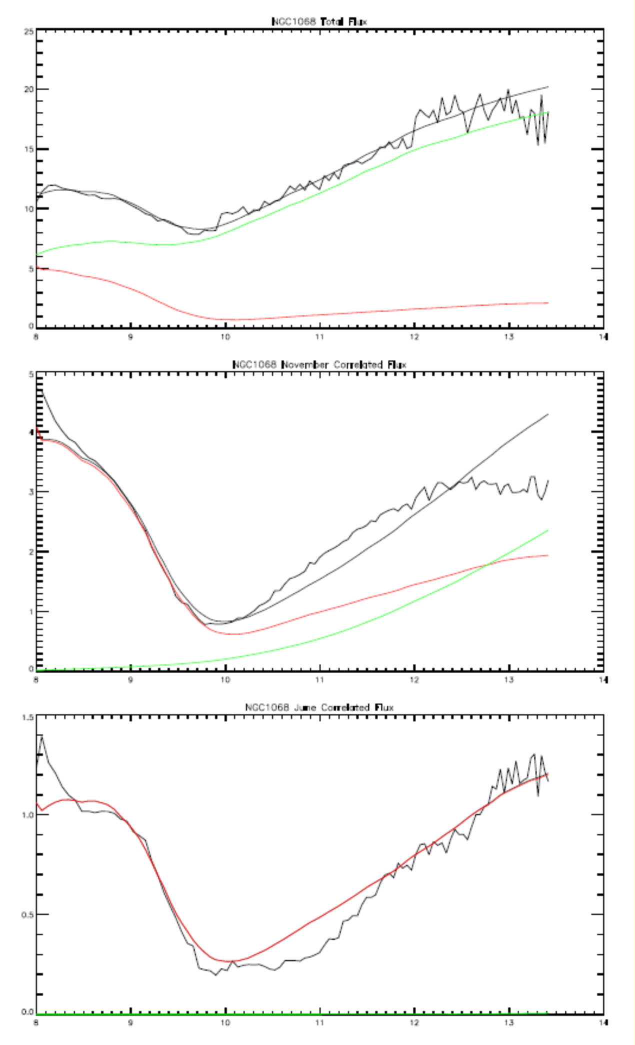
Example #3: Gaussian brightness distribution.



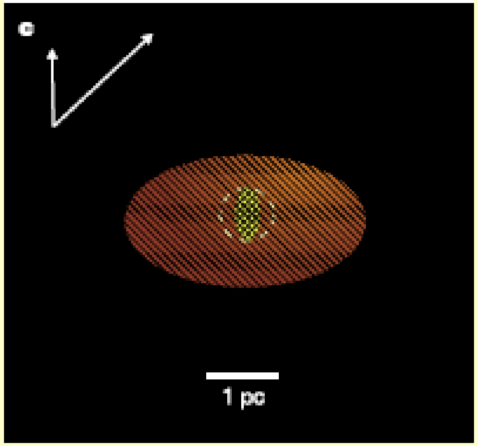
Dominiciano da Souza et al A&A 2007

Example #3:
Gaussian brightness distribution.

- First extragalaxtic optical interferometric observations:
- MIDI observations of NGC 1068
- 1st-order interpretation with a series of Gaussian disks



Jaffe et al 2004



- Note: spectroscopy is one of the keys

Example #4: Uniform disk

Use: aproximation for brightness distribution of photospheric disk.

$$\begin{aligned} \mathrm{I(r)} &= 4/(\pi a^2), \mathrm{if r} = \sqrt{\mathrm{x}^2+\mathrm{y}^2} \leq \mathrm{a}/2 \\ \mathrm{I(r)} &= 0 \ \ \mathrm{otherwise} \end{aligned}$$

$$F(\rho) = \frac{J_1(\pi a \rho)}{\pi a \rho} \mathrm{with} \rho = \sqrt{\mathrm{u}^2+\mathrm{v}^2}$$

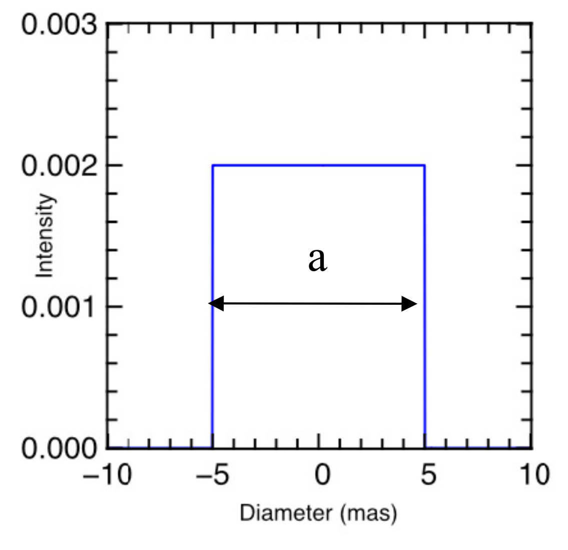
a = diameter

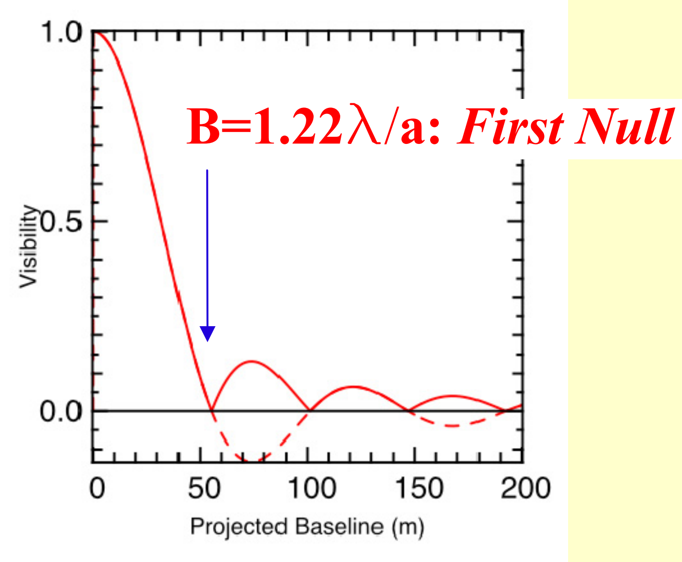
Sophistication of the model

I= f(r), limb darkening

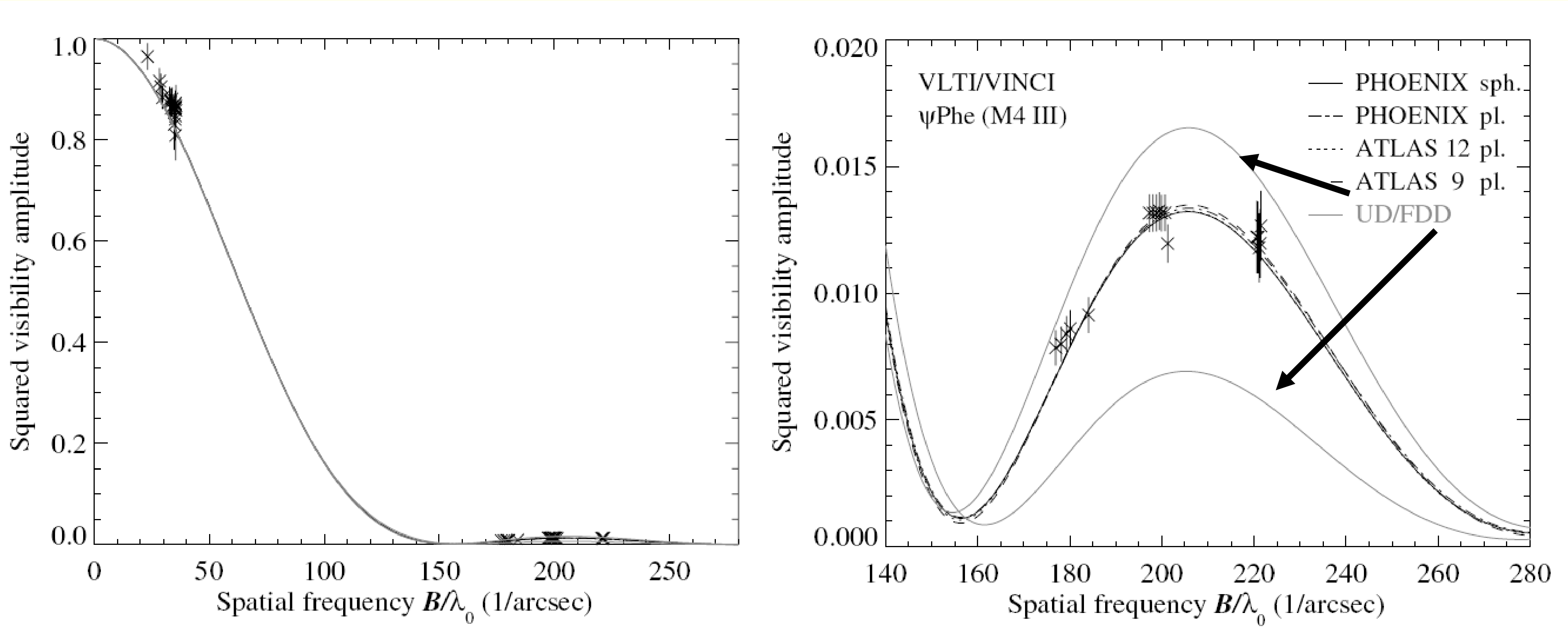
Cf Hankel transformation

(afterwards)




B=1.22λ/a: *First Null*

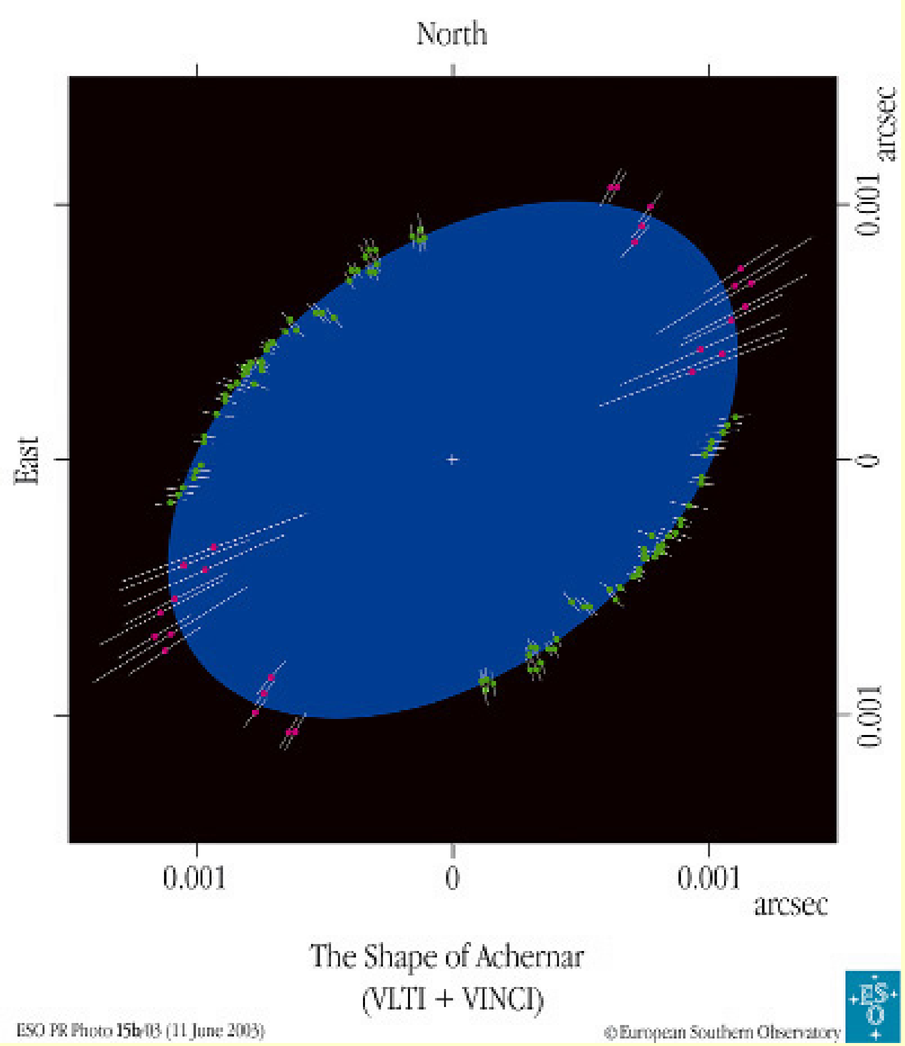
Example #5a: Deviations from uniform disks
Detailed modeling of the stellar photosphere



Wittkowski et al. 2003

- Comparison of ⍓ Phe VLTI/VINCI observations with uniform disk model (gray line)
- Second lobe points are the most constraining

Example #5b: Deviations from uniform disks



The Shape of Achernar
(VLTI + VINCI)

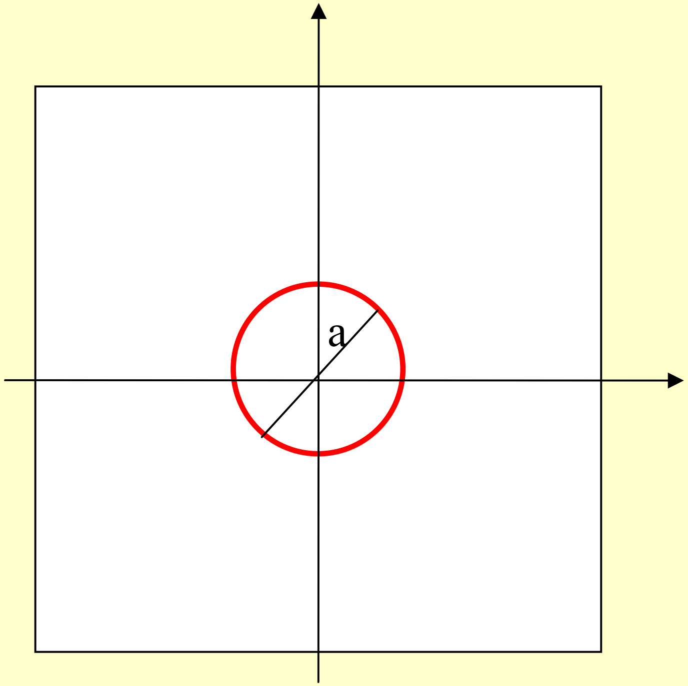
ESO PR Photo 15b/03 (11 June 2003)

© European Southern Observatory

- Determination of uniform diameter of Archenar (VLTI/VINCI)
- Different positions angles shows evidence for flattening due to fast rotation (can be interesting for chemistry and kinematical past)

Dominiciano da Souza et al A&A 2003

Example #6a: Ring

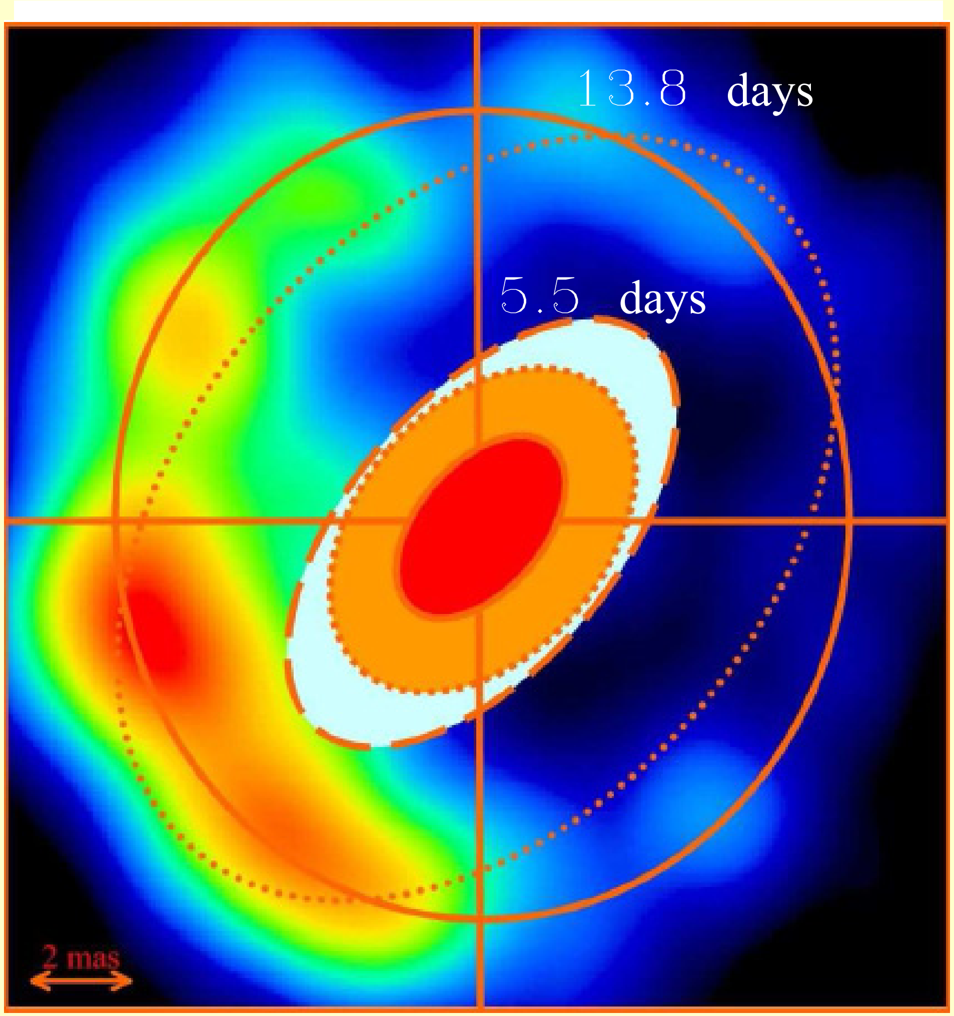


$$I(r) = 1/(\pi a)\delta(r - a/2)$$

$$V(\rho) = J_0(\pi a \rho)$$

Example #6a: Rings

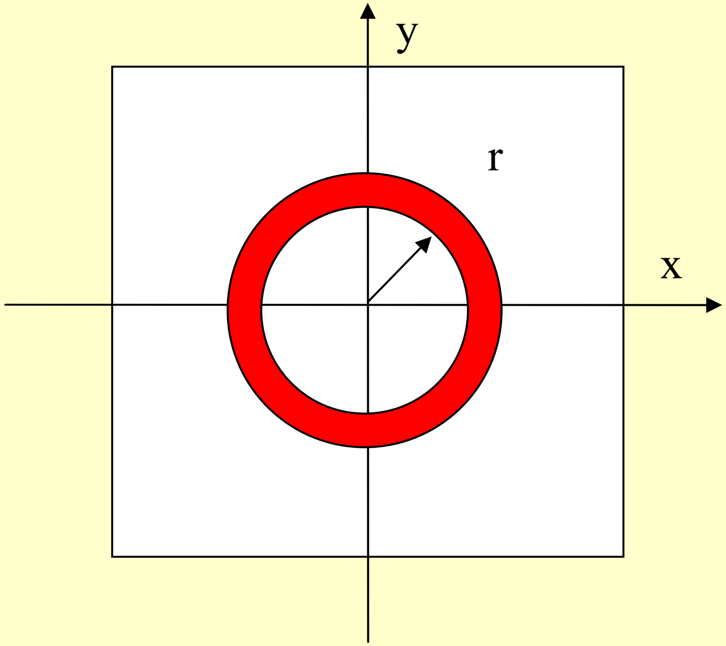
- RS Oph aspherical Nova explosion show rings of line emission (Brγ, HeI)
  Chesneau et al., A&A 2007
- hot inner edge of accretion disks
- proto-planetary disks



Example #6b: Circularly symmetric object

e.g: an accretion disk made of a finite sum of annulii with different effective temperatures

Circularly symmetric component I (r)
centered at the origin of the (x,y) coordinate system.



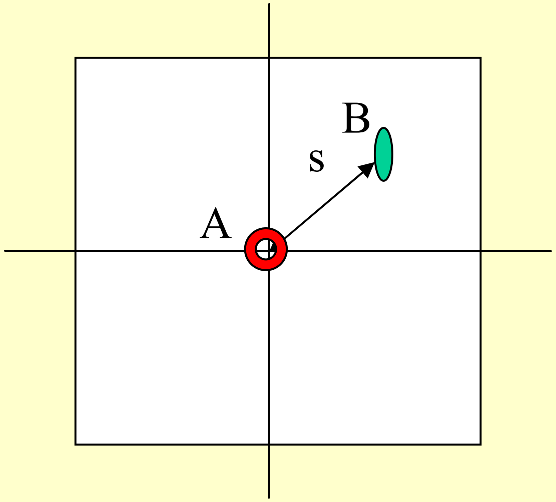
The relationship between brightness distribution and visibility is a ***Hankel function (=1d FT)***

$$V(\rho) = 2\pi \int_0^\infty I(r) J_0(2\pi r \rho) r dr$$

with $r = \sqrt{x^2 + y^2}$ and $\rho = \sqrt{u^2 + v^2}$

Example #7a: Resolved multi-structure

Describing any multicomponent structure.



$$V^2(u,v) = \frac{r_{ab}^2 * V_a^2 + V_b^2 + 2r_{ab}|V_a||V_b|\cos(2\pi \vec{L}_b \vec{s}/\lambda)}{(1+r_{ab}^2)}$$

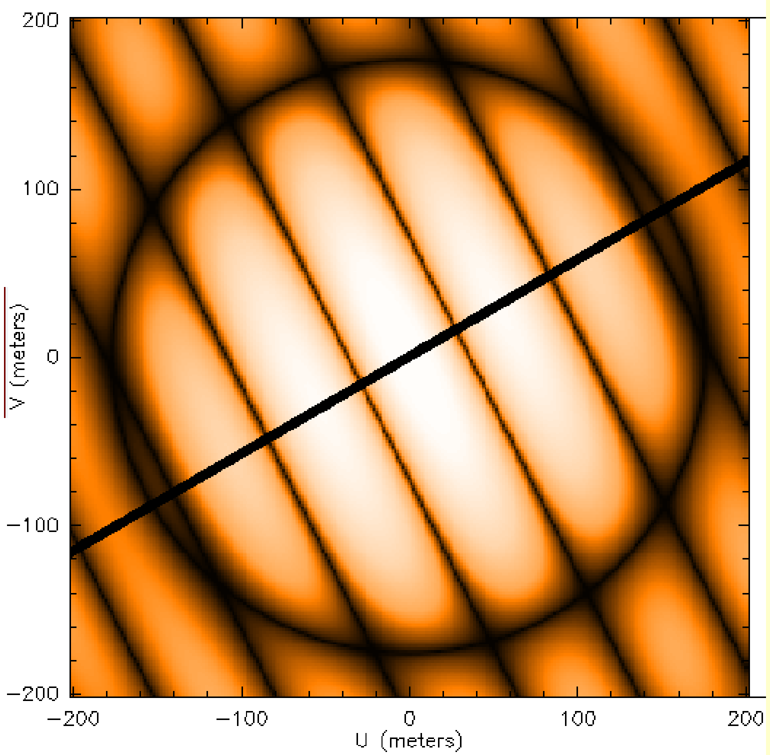
Where V_a and V_b are respectively the visibility of object A and B at baseline (u,v)

Generalization: $$V(u,v) = \frac{\sum_{i=1}^{k} F_i V(u_i,v_i)}{\sum_{i=1}^{k} F_i}$$

Example #7b: Resolved bi-structure

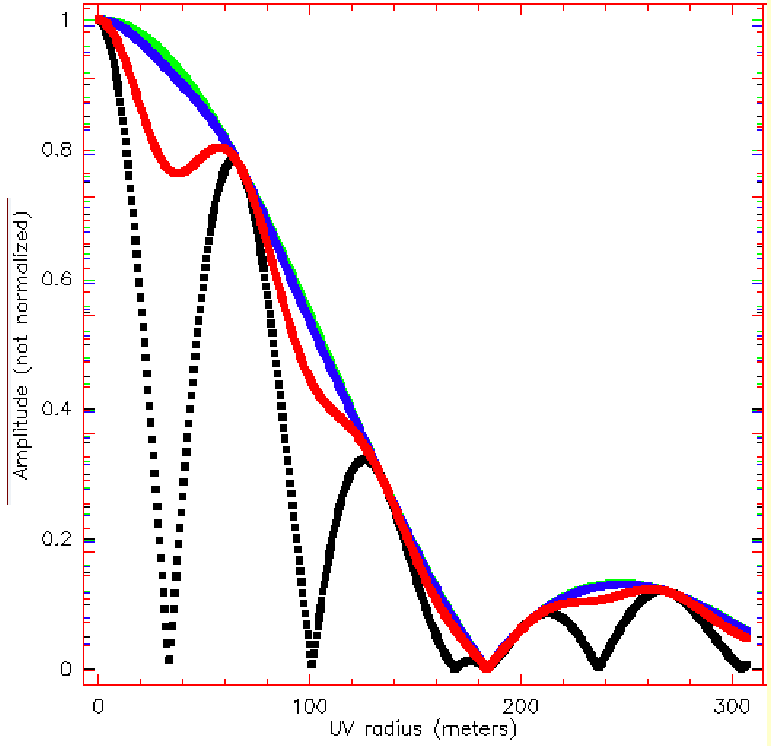
Binary made of two resolved photometric disks: d=3mas, PA: 35deg

Visibility in (u,v) plane



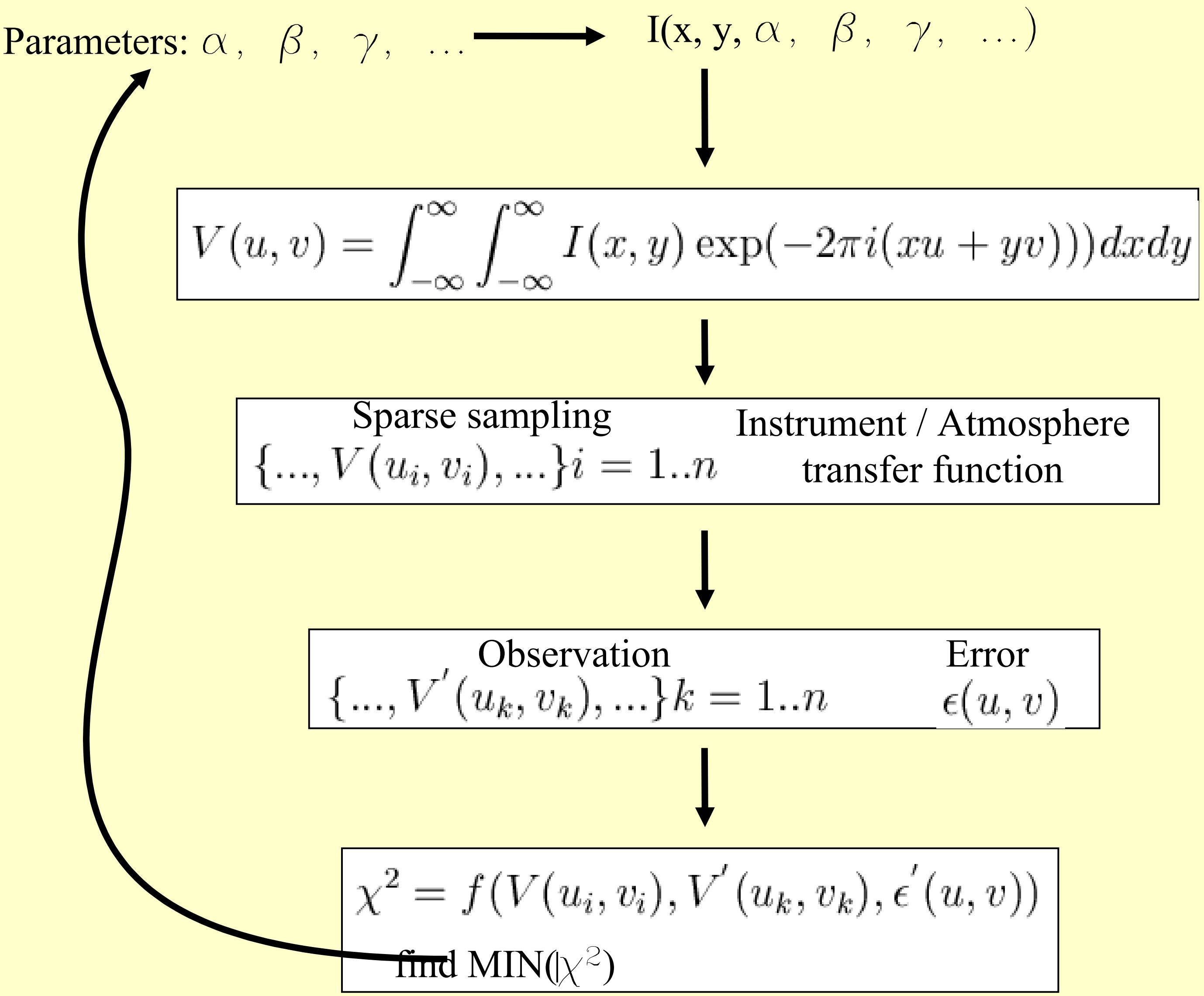
Visibility as a function of baseline for different flux ratios between *binary* and *disc*

NOTE: the inverse size-scale in Fourier-space



Pushing the limits by more sophisticated modelling

- Model/DOF
- Instrument / atmosphere
- Data
- Minimization



Parameters: $\alpha, \beta, \gamma, \ldots$ → I(x, y, $\alpha, \beta, \gamma, \ldots$)

$$V(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x,y)\exp(-2\pi i(xu+yv))dxdy$$

Sparse sampling $\{\ldots, V(u_i,v_i), \ldots\} i = 1..n$

Instrument / Atmosphere transfer function

Observation $\{\ldots, V'(u_k,v_k), \ldots\} k = 1..n$

Error $\epsilon(u,v)$

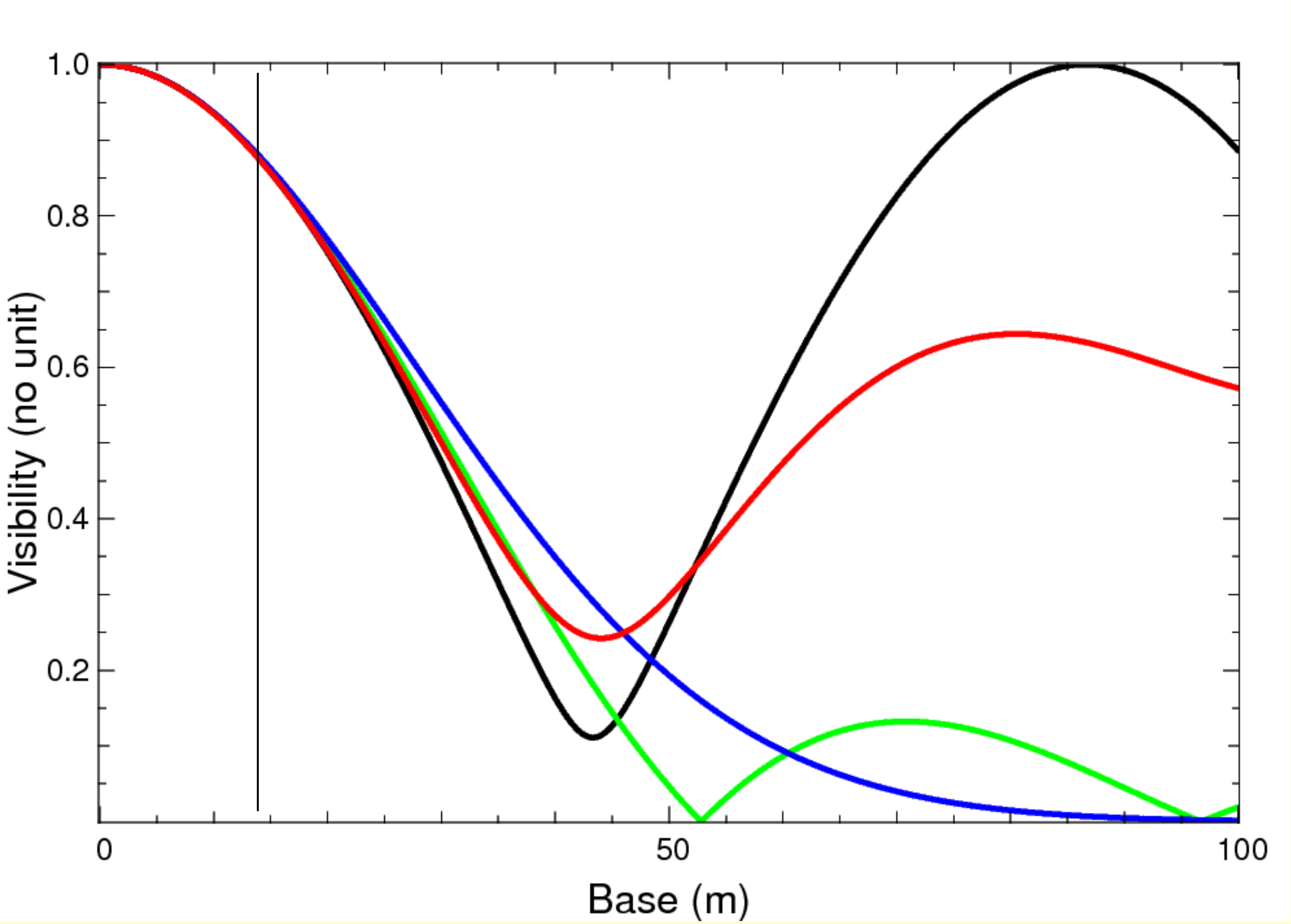
$$\chi^2 = f(V(u_i,v_i), V'(u_k,v_k), \epsilon'(u,v))$$

find MIN(χ^2)

Degeneracy at small baselines

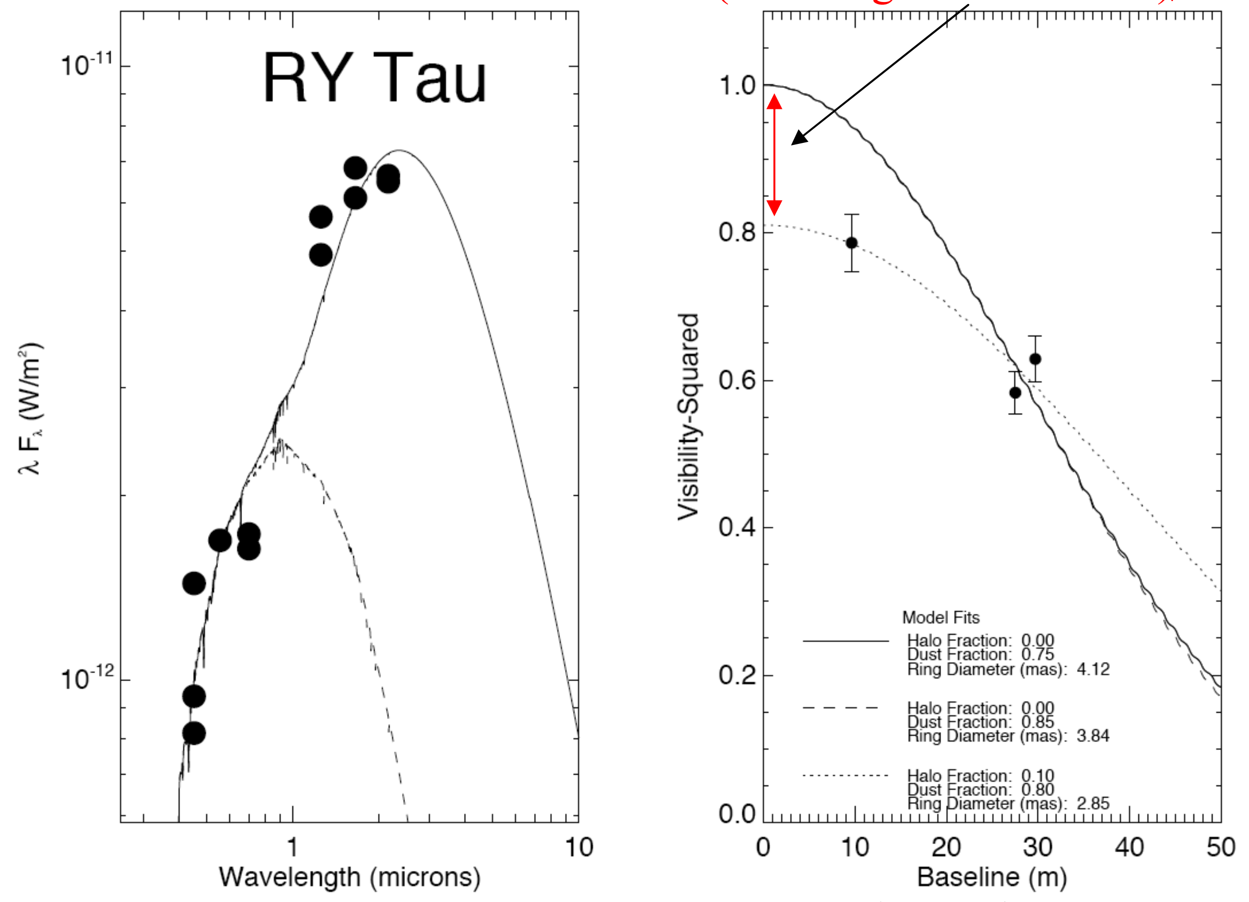
If the object is barely resolved the exact brightness distribution is not crucial
the dependance is quadratic for all the basic functions: visibility accuracy is mandatory

- Uniform disk (green)
- Binary (black)
- Gaussian disk (blue)
- Multiple object (red)



Detecting extended emission

Visibility drops rapidly: attributed to extended flux (10% of global emission)



Monnier et al, ApJ 2006

- Here a simple model of extended (totally resolved) dust emission + Gaussian brings the best fit

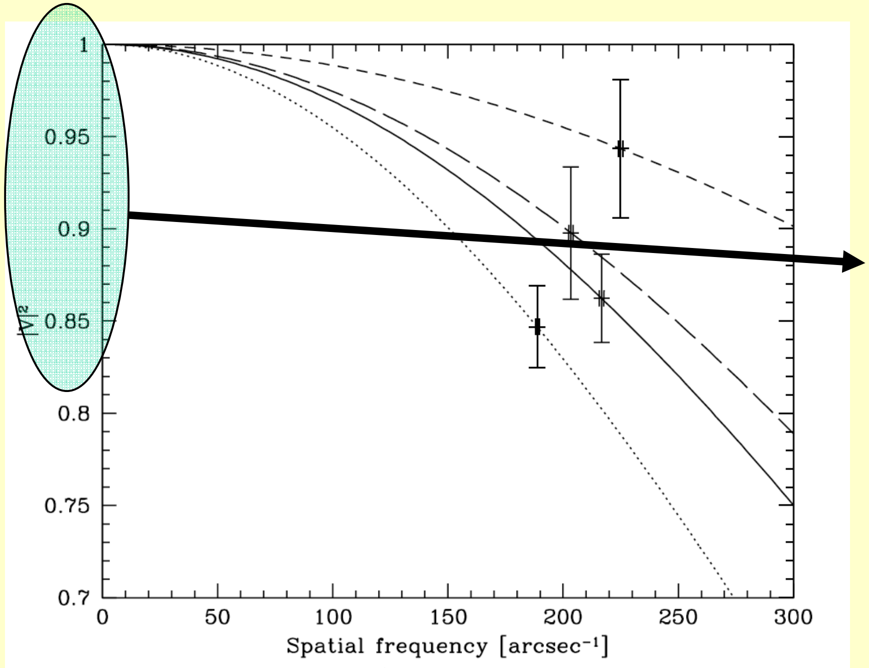
- Additional photometry data at other wavelengths is important

Small diameter estimation

Model fitting can also help to get results beyond the canonical resolution (the "beam" size") :

sizes estimates or positional uncertainties can be smaller than

=> **super resolution** (similar to standard imaging analysis)



Segransan et al, 2003

- First measurements of M dwarf star diameters
- Look how large visibilities are (i.e. how small the source is).
- No need for zero visibility measurements to retrieve diameters

Concluding remarks

✓Visibility study without imaging can be sufficient, depending on the (first-order) complexity of the observed brightness distribution

✓Limited allocated time means (very) limited (u,v) points, and strategic selection of baselines to be used.

✓Use basic models already to prepare your observation and determine what is the more constraining configuration.

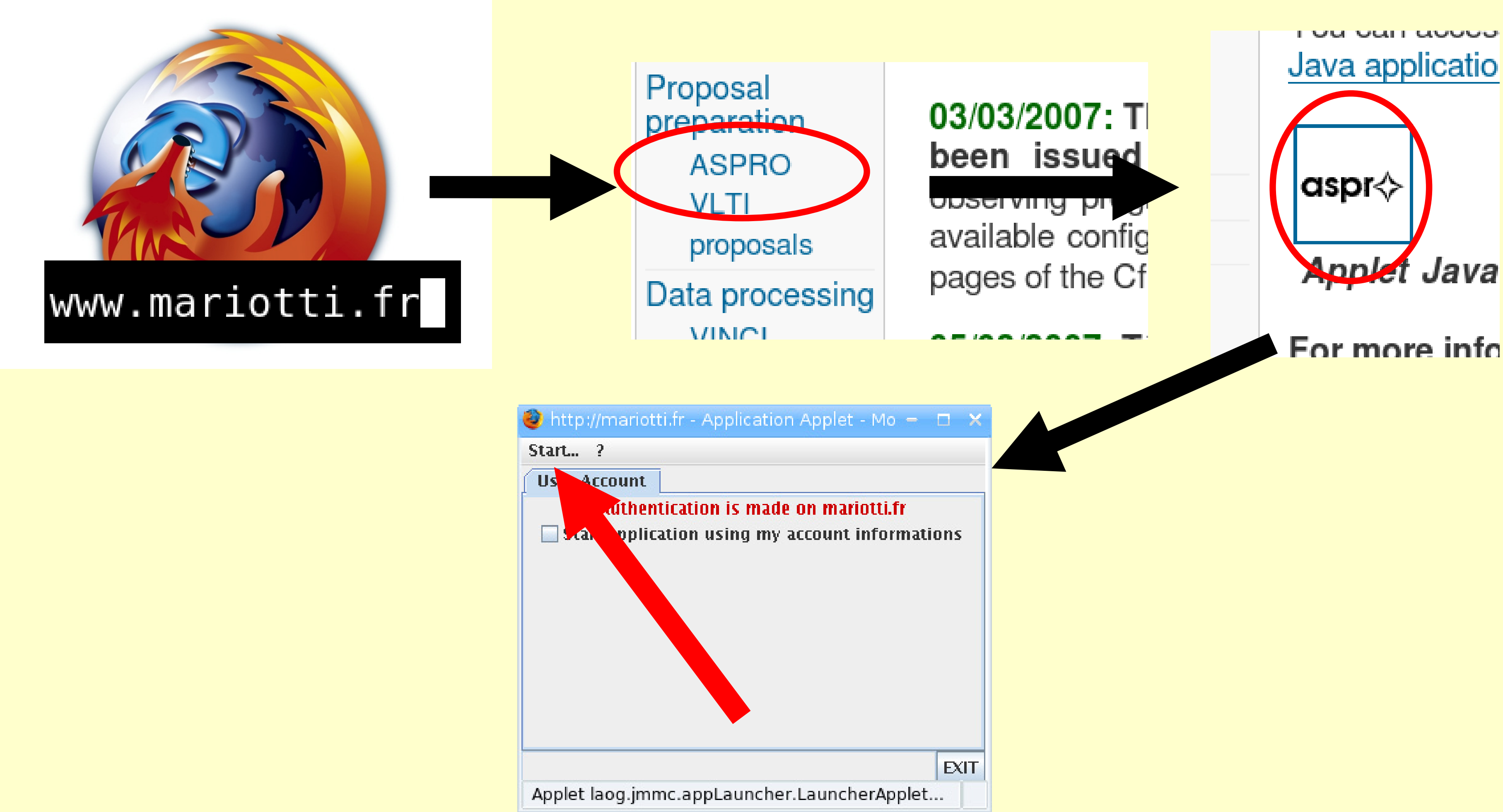
✓Visibility space is the natural place to understand the errors of the final result.

✓Always start by describing your observations in terms of basic functions. It brings quantitative information useful for further more detailed computations.

Do it yourself:

How to do simple modelling: launch ASPRO (on the web)

- Start your favourite browser
- Go to http://www.



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For more info

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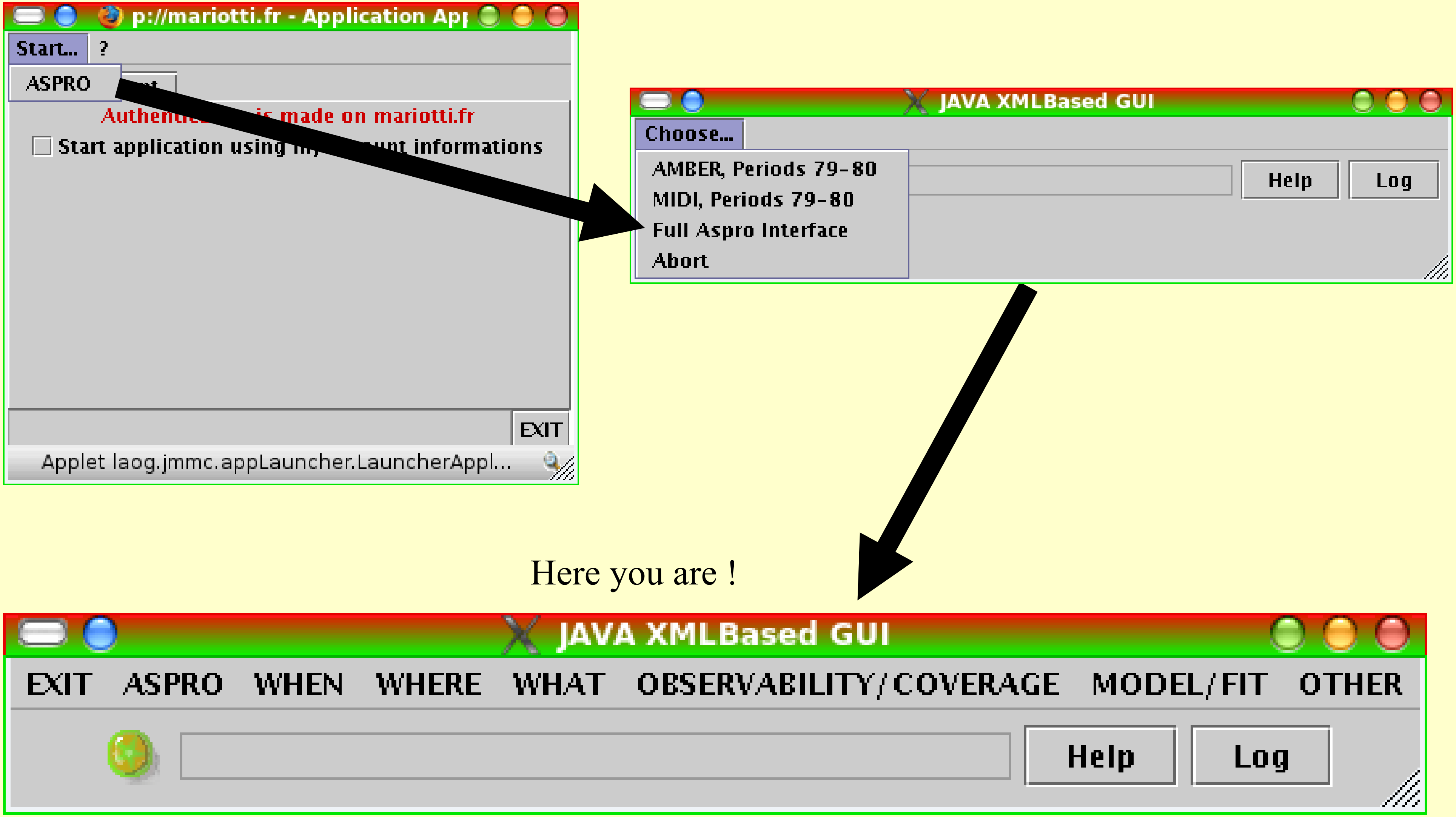
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JAVA XMLBased GUI
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Help
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Here you are !

Go for it!

Köszönöm!