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Adverse effects in dual-feed interferometry

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Outline

- Introduction: astrometry with interferometers
- Narrow angle astrometry and the atmospheric limit
- Error allocation
- Systematic terms
  - Baseline errors
  - OPD errors
  - Fringe measurement errors
- Random terms
  - SNR
- Review of PTI experiment
- Final discussion

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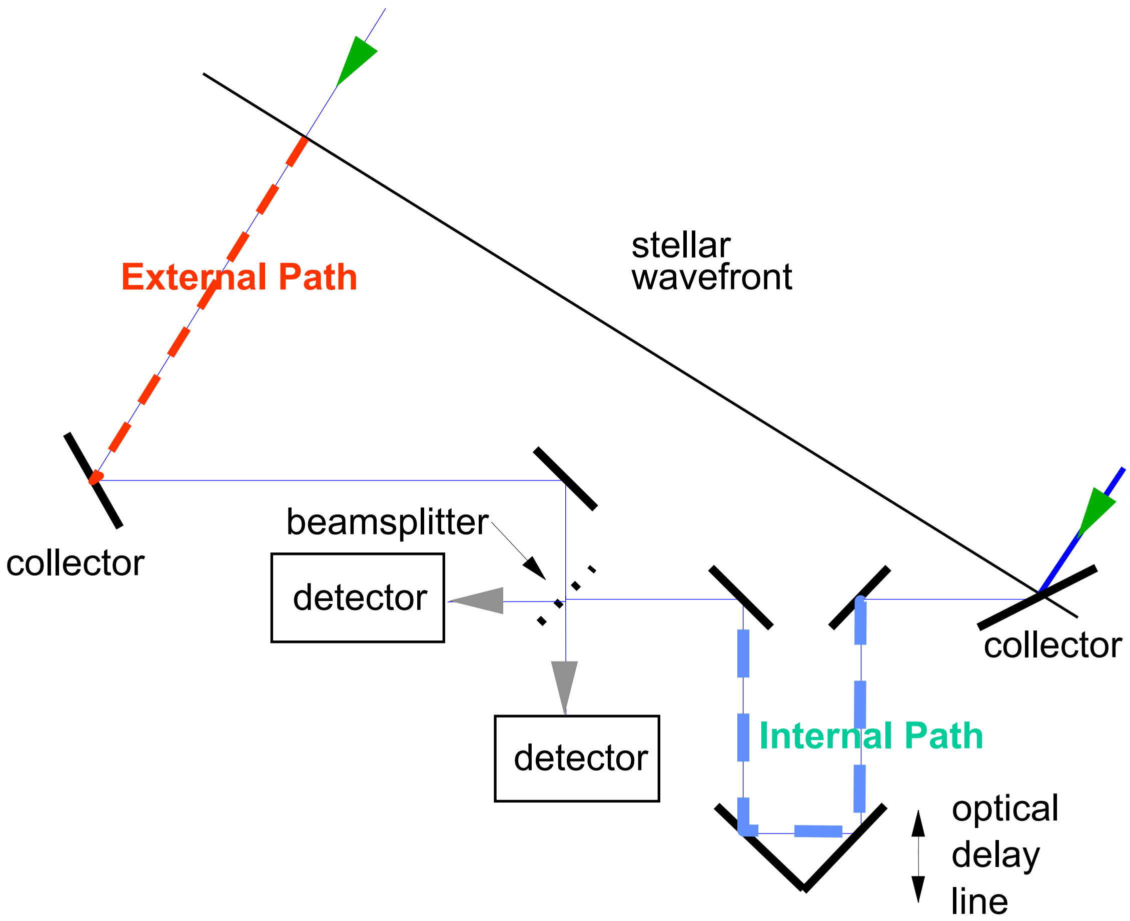
Detecting fringes with an interferometer, 1

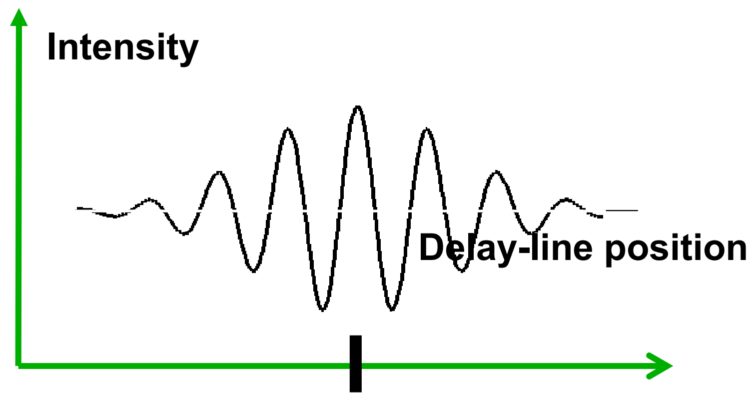


- We detect fringes when the external path delay — which we can't measure directly

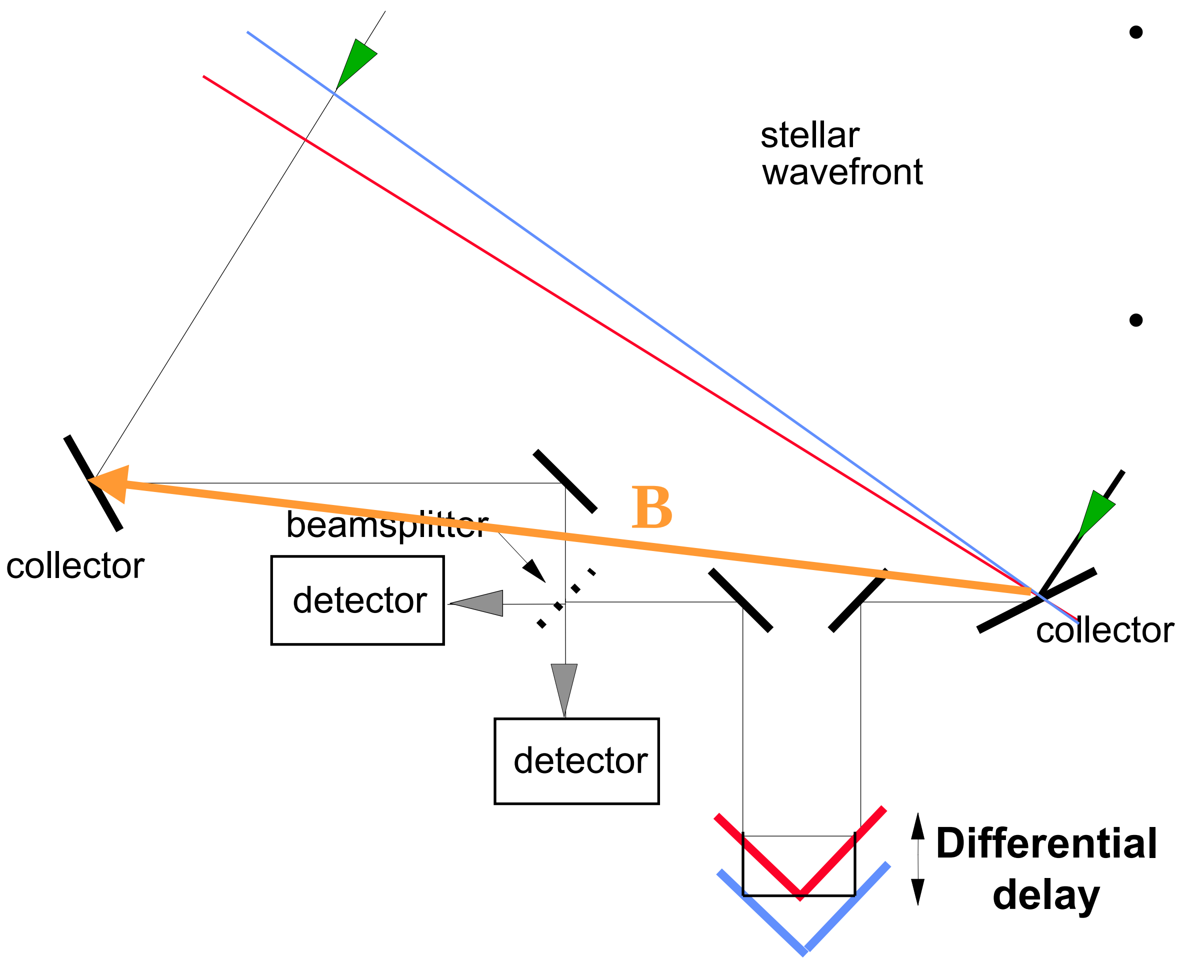
  equals

  the internal path delay — which we can measure

  How accurately we can do so affects our astrometric accuracy



Detecting fringes with an interferometer, 2



- Interferometric astrometry is usually differential: our accuracy requirements are over a switching cycle
- Measurements are with respect to the interferometer baseline: how is this defined?

Astrometric equation

- Astrometry with interferometers is based on just one equation
  - **$\mathbf{x = B.s + C}$**

    x = delay (m)

    B = baseline 3 vector

    s = star unit vector

    c = “constant” term (can also incorporate into delay)
- In terms of understanding where the errors come in, it’s perhaps more clear to say x is the (unmeasurable) external delay determined by the star-baseline geometry

- Errors come in when we try to measure x, typically as
  - $x = \ell + (\lambda/2\pi)\phi$

    ℓ = laser monitored path

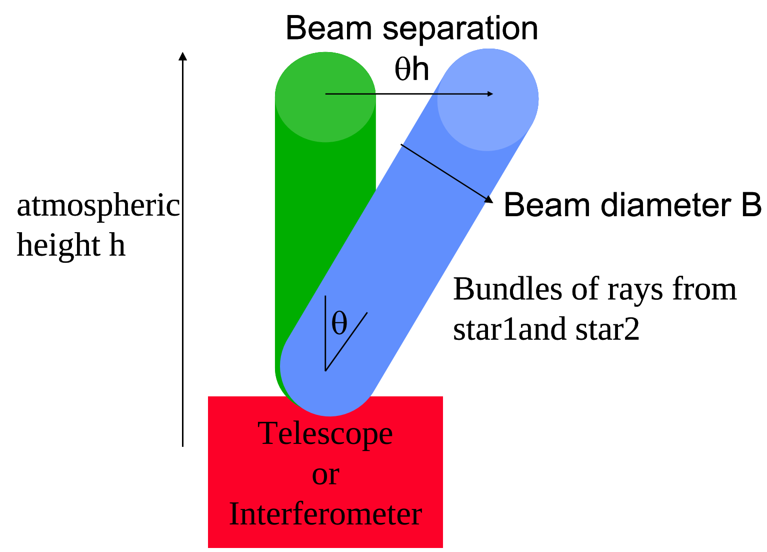
    ϕ = residual phase

Sources of error

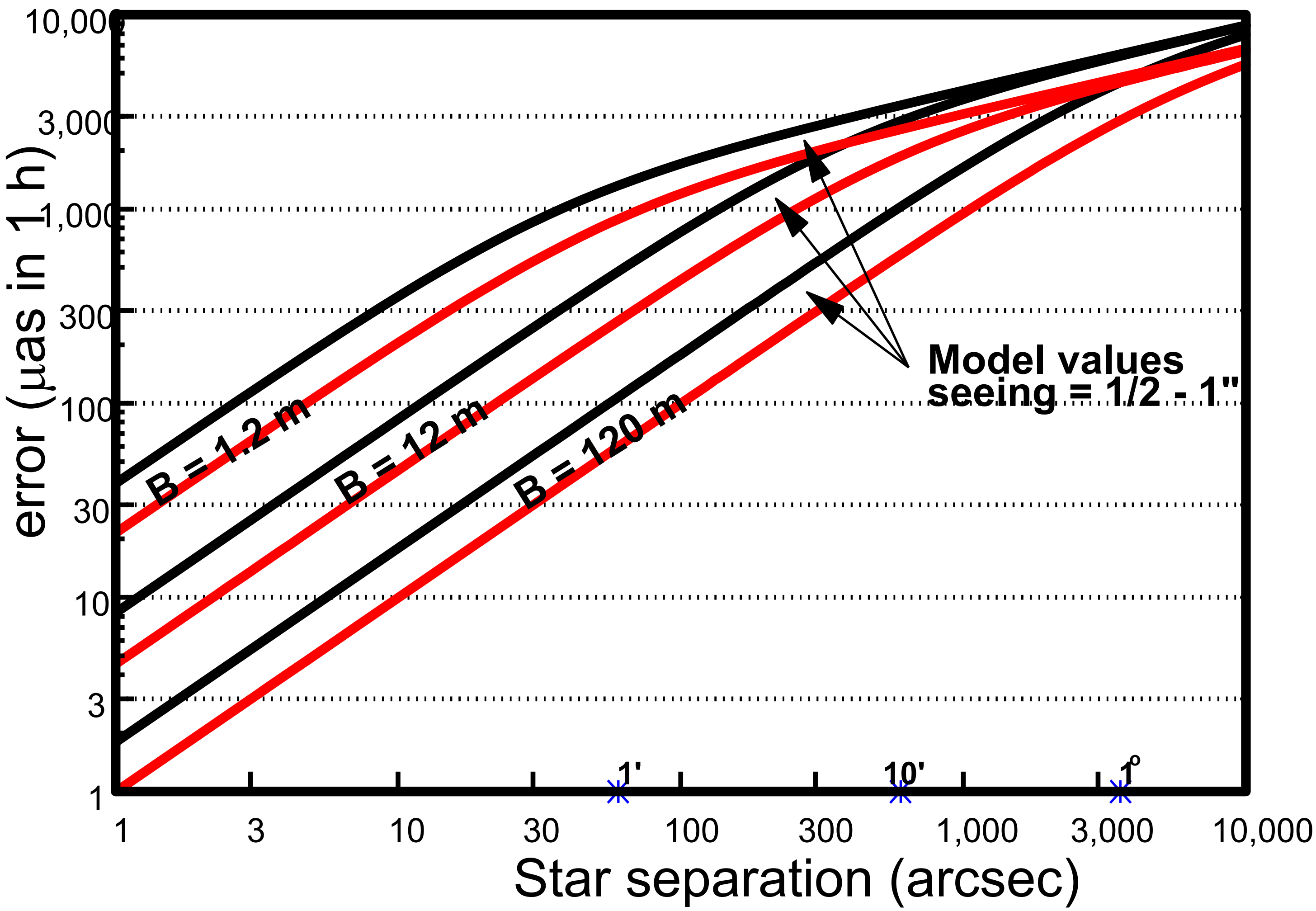
- Anything the affects s, B, or our estimate of x
- Star unit vector s
  - Atmospheric noise
  - Classical refraction
- Baseline vector B
  - Knowledge/stability
- Delay measurement x
  - Metrology stability and correlation with starlight
  - Fringe measurement accuracy, including SNR
  - Internal atmospheric and dispersive effects

Atmospheric noise: narrow angle astrometry

- Rays from different stars traverse different paths through the atmosphere
- Intuitively, expect error (difference in measured angle between two stars) to depend on separation in atmosphere, as well as on amount of overlap of the beams with respect to
  - Single-telescope: diameter
  - Interferometer: baseline length B
- This is in fact the case:
  - The error behavior becomes very favorable when θh < B, especially when B is large

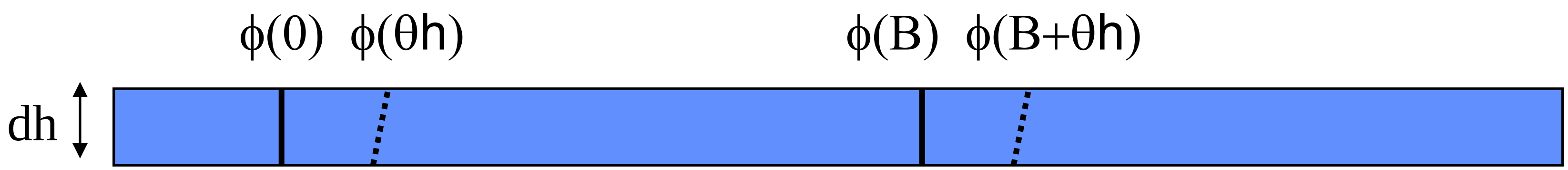


Atmospheric limits to a narrow-angle measurement



Deriving the error variance

- Compute error by layer, $\delta = (\phi(0) - \phi(B)) - (\phi(\theta h) - \phi(B+\theta h))$



- Can derive the error variance as

$$\varepsilon^2(T) \propto \frac{1}{B^2}\int \mathrm{d}h \int \mathrm{d}\kappa \Phi(\kappa, h)\big(1-\cos(B\kappa)\big)\big(1-\cos(\theta h\kappa)\big)$$

$$\propto \frac{1}{B^2}\int \mathrm{d}h C_n^2(h) \int \mathrm{d}\kappa \kappa^{-11/3}\big(1-\cos(B\kappa)\big)\big(1-\cos(\theta h\kappa)\big)$$

- The last two terms are filter functions, proportional to κ^2 near origin
- Astrometric error behavior depends on relative sizes of B and θh

Error variance for very narrow-angle case
$\theta h < B$

$$\varepsilon^2(T) \propto B^{-4/3}\theta^2\left(\int \mathrm{d}h C_n^2(h)h^2\right)T^{-1}$$

- Notes
  - Error is white
  - High altitude turbulence weighted as h^2
  - Error standard deviation linear with star separation
  - Nearly linear baseline dependence
- For a Mauna Kea turbulence profile

$$\varepsilon(T) \approx 300B^{-2/3}\theta T^{-1/2}\text{arcsec}$$

Example performance

- For θ = 15” separation
  - 100 m baseline: ~32 uas in 1000 sec
  - 200 m baseline: ~20 uas in 1000 sec

- Results are better with a finite outer scale
  - Dependence changes from $\Theta B^{-2/3}$ to $\Theta L_0^{1/3} B^{-1}$

- For θ = 15” separation, 40 m outer scale [probably right order]
  - 100 m baseline: ~24 uas in 1000 sec
  - 200 m baseline: ~12 uas in 1000 sec
- These are very interesting performance levels!

Fundamental aspects driving the design

- With the long baselines of an interferometer, 10’s of uas accuracy possible over small fields
- With the small fields of a narrow-angle measurement, the requirements on baseline knowledge are greatly decreased
- The measurement of the two stars must be essentially simultaneous to exploit the common-mode nature of the atmosphere over small fields
- While one of the stars will typically be bright, small fields mean that the second star will be faint

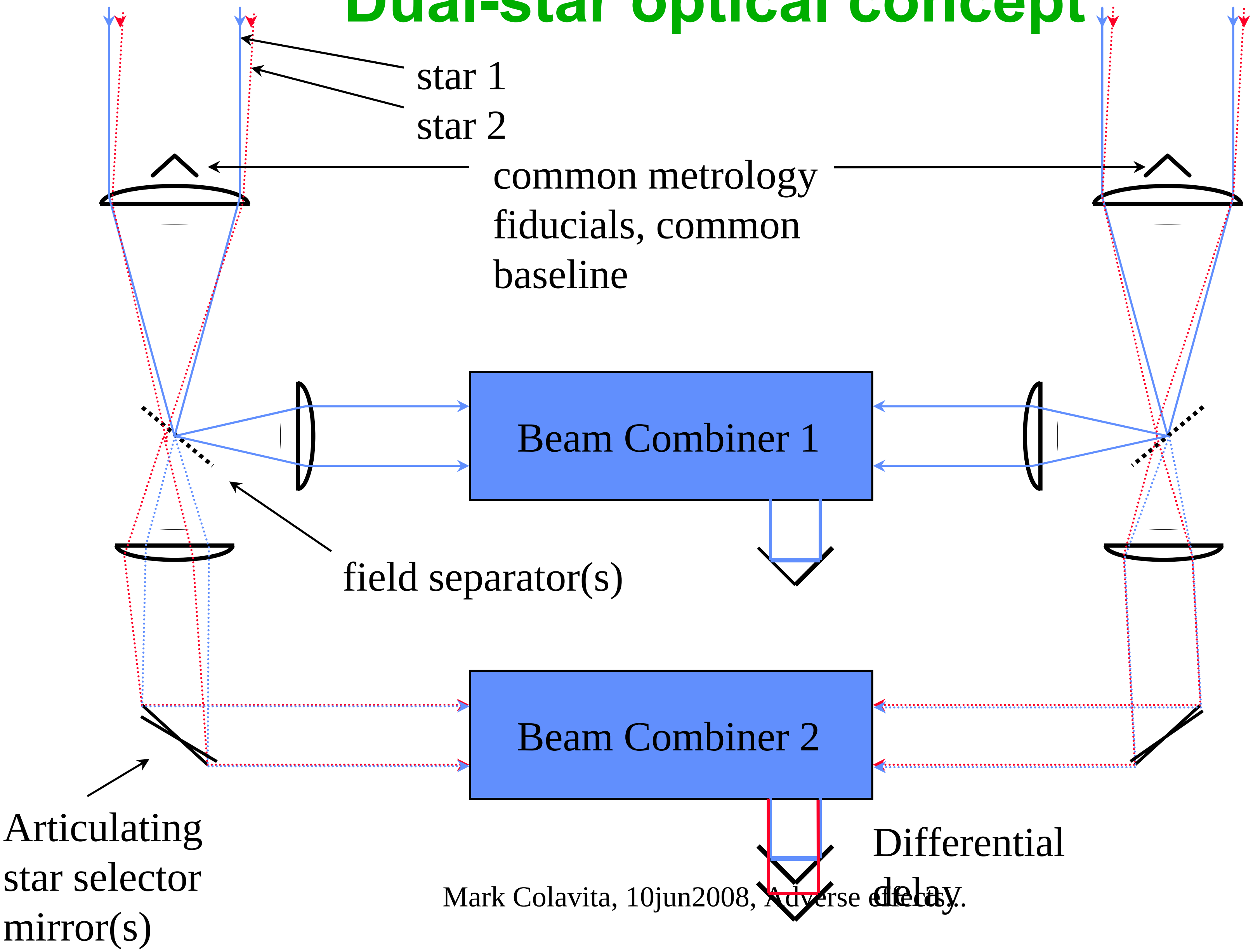
Practical aspects driving the design

- Interferometers usually pass only small fields of view
- Thus a simultaneous differential measurement will require the ability to observe two separate fields of view
  - Will require 2 separate beam trains
  - Will require laser metrology to “tie” together

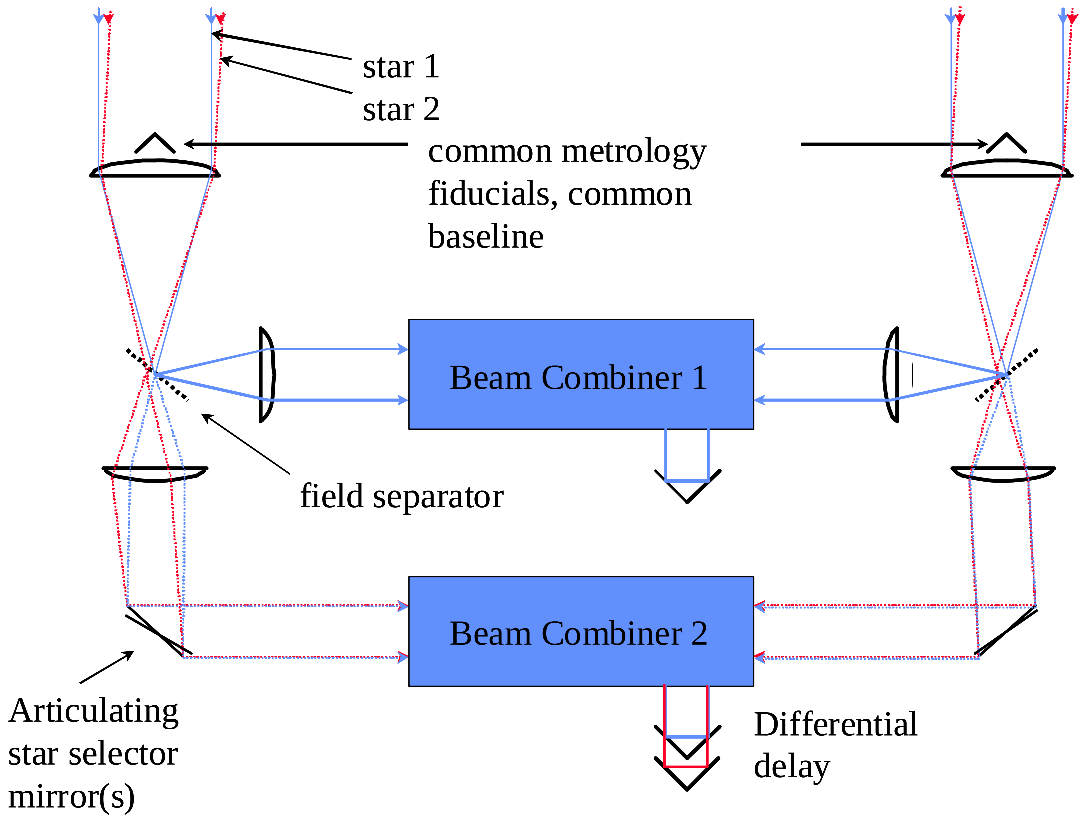
Dual-star approach

- Two interferometers, sharing common baseline and apertures
- Two stars: one bright (target, nearby); one faint (astrometric reference, far away)
- Observe target star on 1st interferometer
  - Use as phase reference for stars within its isoplanatic patch; feed forward to second interferometer
- Observe astrometric reference star on 2nd interferometer
- Work in the infrared (2.2 um) for its larger isoplanatic angle
  - Increases solid angle over which to find astrometric reference stars (~20 arcsec radius)
- Use 2-m class, or larger, apertures to provide sensitivity for adequate sky coverage
  - AO (D > 2m) or fast tip tilt (D < 2m) needed to correct aperture

Dual-star optical concept



A dual-star observational approach



- Beam combiner 1 continually tracks the bright star – provides the phase referencing
- Beam combiner 2 makes the differential measurement, switching between the bright and faint stars
- Implications
  - Beam combiners 1 and 2 can be different
  - Metrology continuity (or absolute metrology) required
  - Star separator has to pass both stars
- Other approaches possible

Limitations to performance

- Perhaps surprisingly, the random atmospheric term will be least bothersome aspect of a dual-star system
- The finite SNR of the measurement will be quite important, as the small fields allowed by phase referencing require faint reference stars – we'll get to that at the end
- However, astrometry is very much about the control of systematic errors, and we'll talk about that, at length, below

- For simplicity, I'm usually going to be assuming λ = 2.2 um & 15" or 20" separation between stars, and B=100 m
  – B = 200 m will be better, as well see, as long as we don't resolve the bright star

A 2-d sensitivity analysis

-Write the astrometric equation for a differential measurement:

$$\Delta x = \vec{B} \cdot \Delta \hat{s}$$

-Rewrite in 2-d for estimating the differential angle Θ in terms of length l, phase ϕ, baseline B

$$l + k^{-1}\phi = B\Theta$$

-A simple sensitivity analysis illustrates the required accuracies:

$$\delta\Theta = \frac{\delta l}{B} + \frac{k^{-1}\delta\phi}{B} - \frac{\delta B}{B}\Theta$$

- All three terms inversely dependent on baseline
- Baseline term proportional to FOV

Systematic errors

- For now, let's do a simple error budget for $\delta\Theta$ = 20 uas with equal $\delta\Theta$=10 uas allocations for
  - 1) Baseline noise δB
  - 2) Measurement noise $\delta\ell$
  - 3) Fringe measurement noise δf
- Assume
  - B = 100 m and Θ = 20 as
- Then
  - 1) $\delta B = B\,(\delta\Theta\,/\,\Theta)$ = 50 um
  - 2) $\delta\ell = B\,\delta\Theta$ = 5 nm
  - 3) $\delta\phi = B\,\delta\Theta$ = 5 nm
- These are small, and yet to be suballocated!

Objectives of following error discussion

- I'm not going to do a formal error budget
- The objectives are to
  - do some suballocation to give a feel for what terms are important
  - to indentify some of the mechanisms by which systematic errors get introduced or can be controlled
  - to give some examples of the magnitudes of the underlying effects

Baseline Noise: 1) wide-angle baseline solution

- **Baseline knowledge required to $\delta B = B\,(\delta\Theta\,/\,\Theta) = 50$ um**
- Suballocate equally among

1. **Wide angle baseline solution – 25 um**
2. Unmodeled baseline noise – 25 um
3. Wide-angle baseline identification – 25 um
4. Narrow-angle to wide-angle baseline transfer – 25 um

Wide-angle baseline solution

- Everyone who uses an interferometer has had to deal with solving for the baseline in order to find fringes. We have to do it a bit more accurately here.
- But, tolerances are reasonable:
  - 25 um requirement / 100 m baseline = 50 mas
- Contributors
  - a) Input star position accuracy: ~20 mas
  - b) Wide angle atmospheric accuracy: ~20 mas
  - c) DCR: < 1 um
  - d) External H20: << wide-angle error
  - e) Internal H20: < 1 um

a) Hipparcos catalog accuracy

- Quoting from: http://ad.usno.navy.mil/star/star_cats_rec.shtml#hip
- “This catalog contains 118,218 stars that were observed by the European Space Agency's Hipparcos Satellite, operational from late 1989 to 1993.
- It is complete to V=7.3
- The positional accuracies of 1 to 3 mas at epoch 1991.25 are unsurpassed in the optical.
- Proper motion accuracies, of around 1 to 2 mas/yr, remain state of the art.
- Thus typical positional errors at a 2005 epoch are around 15 mas. **[assume 20 mas for 2010]**
- By international agreement, the Hipparcos catalog is the standard reference catalog for optical astrometry,…”

b) Wide Angle Accuracy

-With an infinite outer scale, Komogorov turbulence (p=5/3), the rms fringe fluctuations are:

$$\sigma = 0.42\lambda\left(\frac{B}{r_0}\right)^{p/2}$$

-But, p is usually slightly sub-Kolmogorov (p = 1.5 or less) and the outer scale L_0 is usually < B, in which case

$$\sigma \approx 0.42\lambda\left(\frac{L_0}{r_0}\right)^{p/2},$$

-Ex: L_0 = 50 m, r_0 = 0.2 m @ 0.55 um, σ = 15 um rms = 30 mas.

-So, in the right ballpark.

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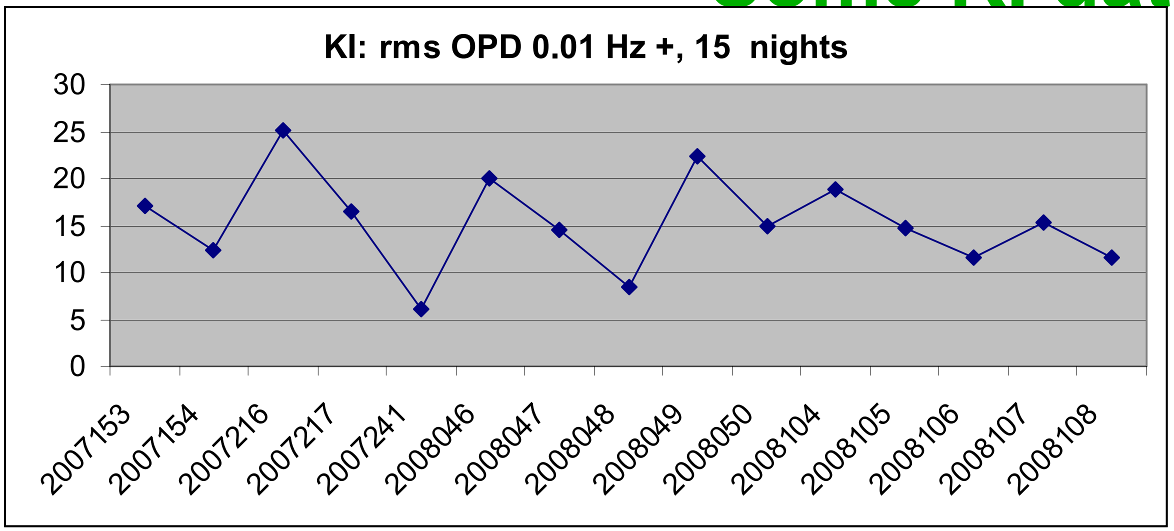
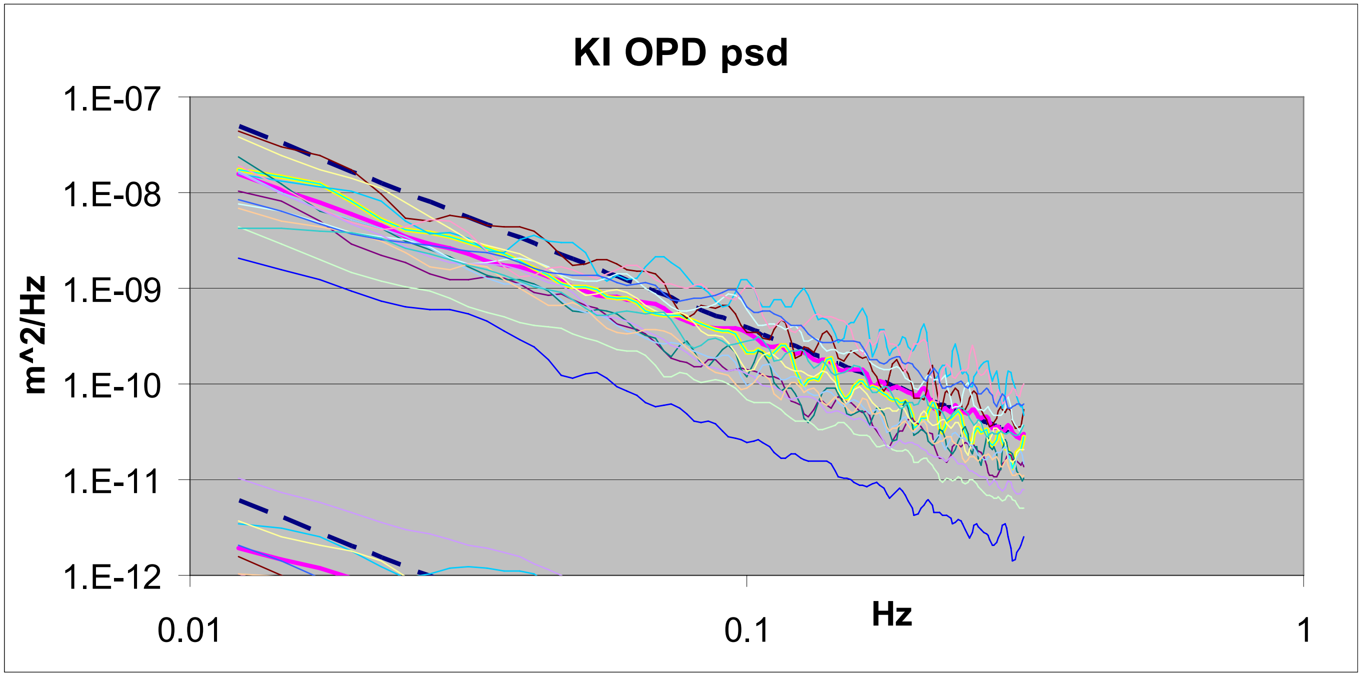
Some KI data

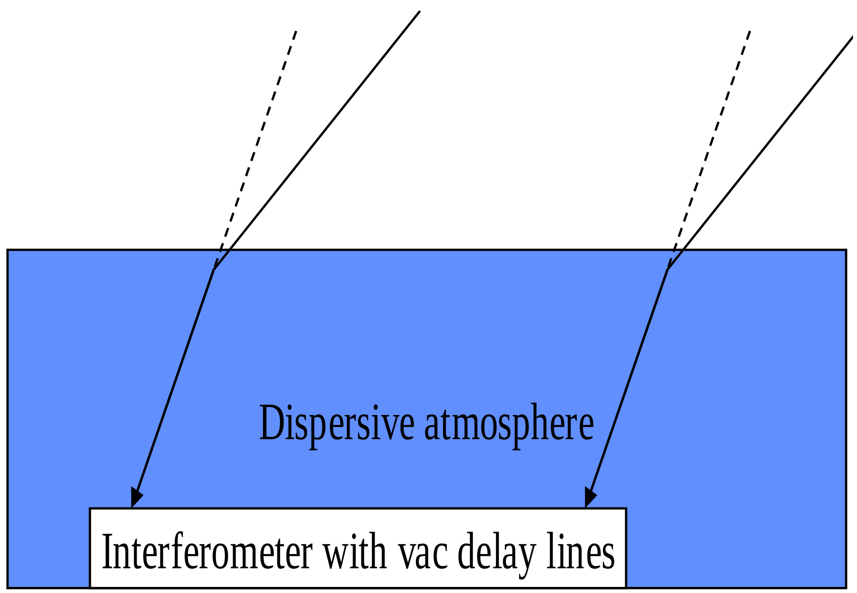


Discussion
-Baseline rolloff
-Averaging
-Focus offloads



c) Differential Chromatic Refraction (DCR)

- Not an issue for an interferometer with vacuum delay lines (in limit of plane parallel atmosphere)
  - Extra OPD from being off zenith introduced (and compensated) in vacuum
  - Pathlengths of the two arms of the interferometer through the atmosphere remain matched



DCR, continued

- Over 100 m baseline, there is a small curvature (3 as) [=100 m / 1852 m/nmi * 60]
  - One atmospheric path is ~0.25 m longer
  - Small effect for wide-angle astrometry
- RE: “delay lines in vacuum”
  - Strictly, the statement applies if you measure the delay position with ruler (or a linear encoder)
  - However, it’s equivalent to measure the path with a laser at the same wavelength as starlight

DCR on the measurement of the internal path

- If $\lambda_{metrology} \neq \lambda_{science}$
  - 1st order: a scale factor error
    - » $N \approx 270 \times 10^{-6}$
      - For a pathlength x = 100 m, dispersive component = 27 mm
    - » $\Delta N \approx 3.5 \times 10^{-6}$ between HeNe and K
      - ΔN is approximate scale factor error if don't correct
  - What about temperature changes?
    - » $\Delta\Delta N \approx 12 \times 10^{-9}$ for a 1K temperature change
      - $\delta x \approx 1.2$ um for a 100 m path
      - Small effect for wide-angle astrometry

DCR, continued

- If $\lambda_{metrology} \neq \lambda_{science}$
  - 2nd order: DCR
    - » $\Delta N \approx 3.0\times10^{-9}$ between 2.20 and 2.21 um, i.e., a 10 nm uncertainty
      - Δx = 300 nm for a 100 m path
      - Small effect for wide-angle astrometry
    - » Note that the dispersion over that small 0.25 m external path is negligible for both wide and narrow-angle

d) e): Water vapor dispersion

- The wavelength dependence of the refractivity of dry and water vapor are different
- d) External effects
  - – Water vapor contributes to turbulent fluctuations, even in the visible, at 1/10 to 1/20 of the level of dry air
    - » External H20 small effect compared with dry-air seeing
- e) Internal (quasistatic) effects: metrology errors if metrology and science wavelengths different
  - – $\Delta N \approx 4\text{x}10^{-6}$ between HeNe and K
  - – $\Delta\Delta N \approx 1.5\text{x}10^{-9}$ for a 1% change in relative humidity (RH)
    - » $\delta x \approx 150$ nm for a 100 m path
    - » Small effect for wide-angle astrometry

Backup spreadsheet

OPD:	100		
		ppm	um
	N(HeNe)	271.8	27180
	N(K)	268.3	26827
	dN	3.530	353
1	dN/dT	0.012	1.20
	N(2.1)	268.3	26830
	N(2.3)	268.2	26824
0.01	dN per 10nm	0.003	0.30
	NQ(He;17% RI	-0.146	-14.6
	NQ(K; 17% RH	-0.172	-17.2
	dN	0.026	2.6
1%	dN per 1%	-0.0015	-0.2
	NQ(2.1)	-0.169	-16.9
	NQ(2.3)	-0.176	-17.6
0.01	dN per 10nm	0.000	0.04
curvature	3.240	1.6E-05	
	10,000	2.2E-01	

Wide-angle baseline solution: summary

- Contributors
  - a) Input star position accuracy, epoch 2010: ~20 mas
  - b) Wide angle atmospheric accuracy: ~20 mas
  - c) DCR: < 1 um
  - d) External H20: << wide-angle error
  - e) Internal H20: < 1 um

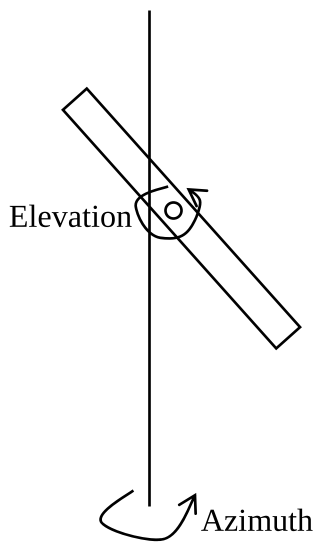
- No major issues

Baseline Noise: 2) Unmodeled baseline noise

- **Baseline knowledge required to $\delta B = B\ (\delta\Theta / \Theta) = 50$ um**
- Suballocate equally among

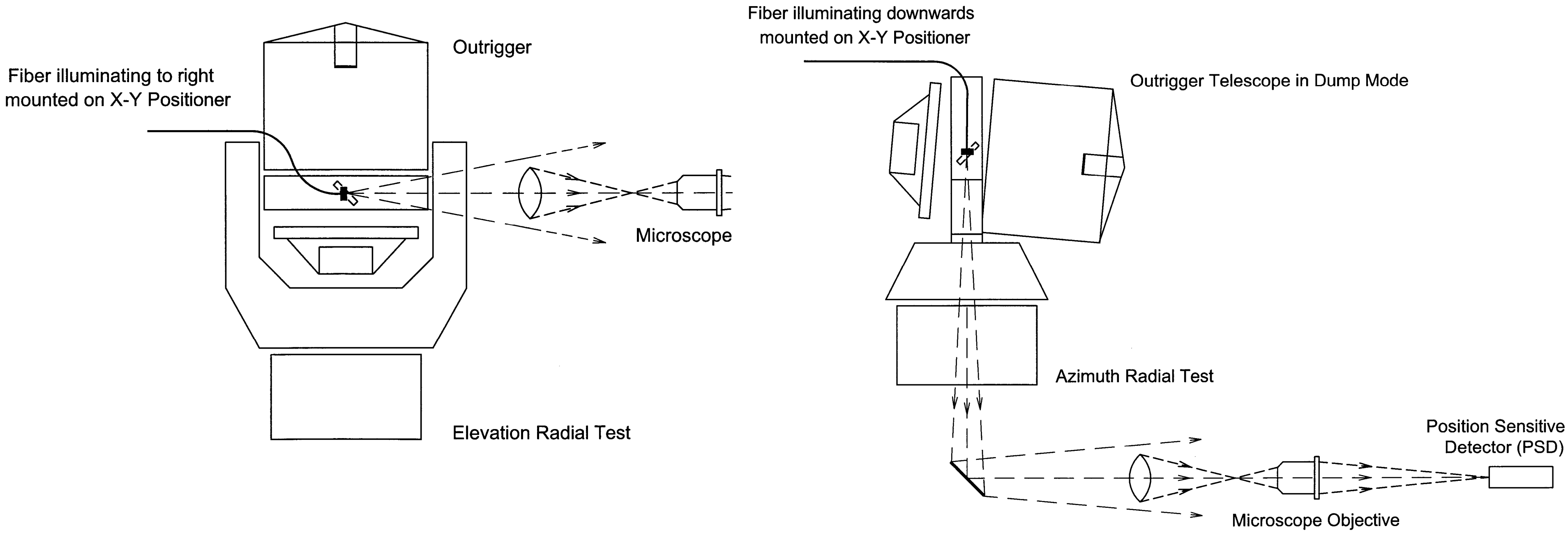
1. Wide angle baseline solution – 25 um
2. **Unmodeled baseline noise – 25 um**
    - **17 um for each telescope**
3. Wide-angle baseline identification – 25 um
4. Narrow-angle to wide-angle baseline transfer – 25 um

Unmodeled baseline noise



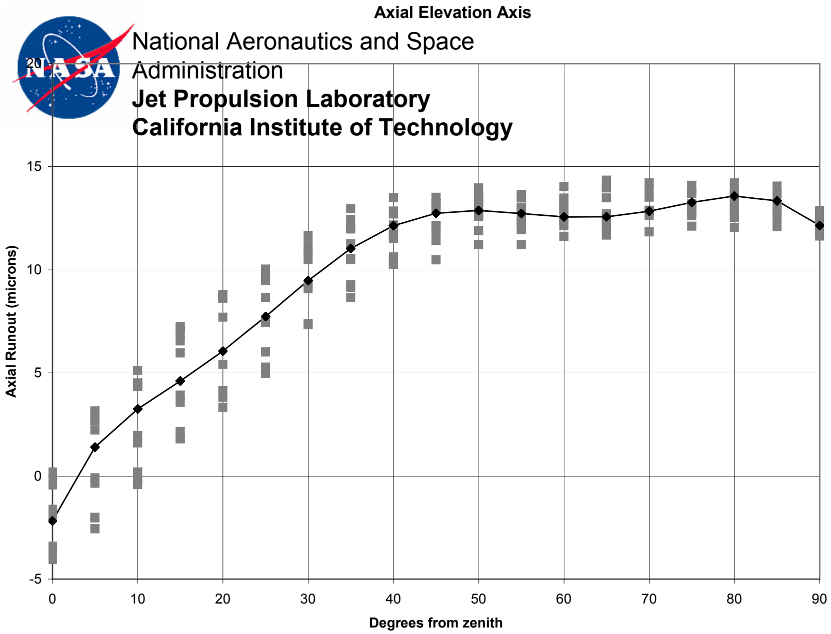
- Addresses “mechanical” quality of wide-angle baseline
  - I.e., the noise in the “pivots”, assumed perfect, above, when doing the baseline solution
- It’s a knowledge requirement:
  - The requirement is that the unmodelable component be <25 um total
- Siderostat example
  - There’s not a well-defined pivot if azimuth axis does not intersect elevation axis, and mirror surface does not intersect elevation axis
  - But, if you can measure the offsets, you can define a geometric model such that your uncertainty is just the bearing noise (runout)
  - How to measure?

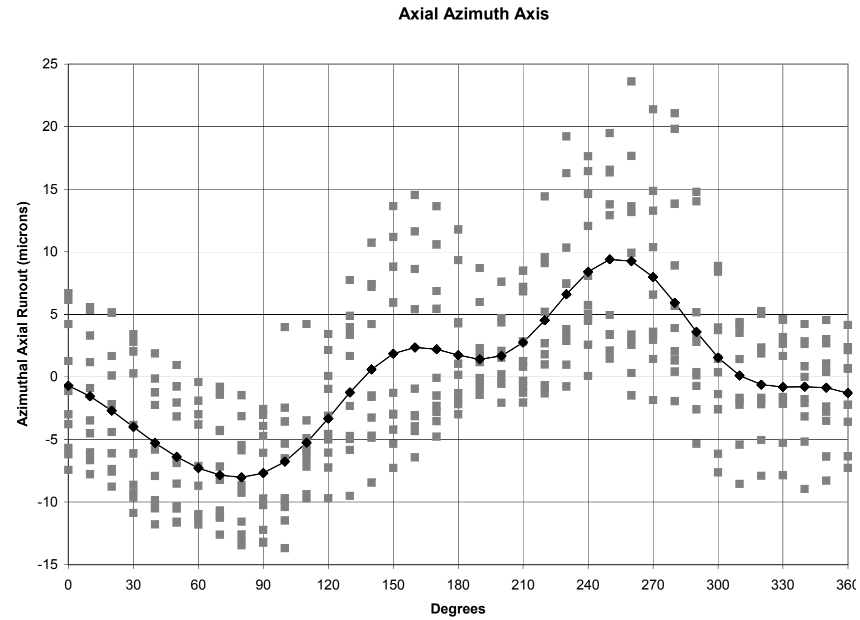
Measuring pivot noise on the OTs

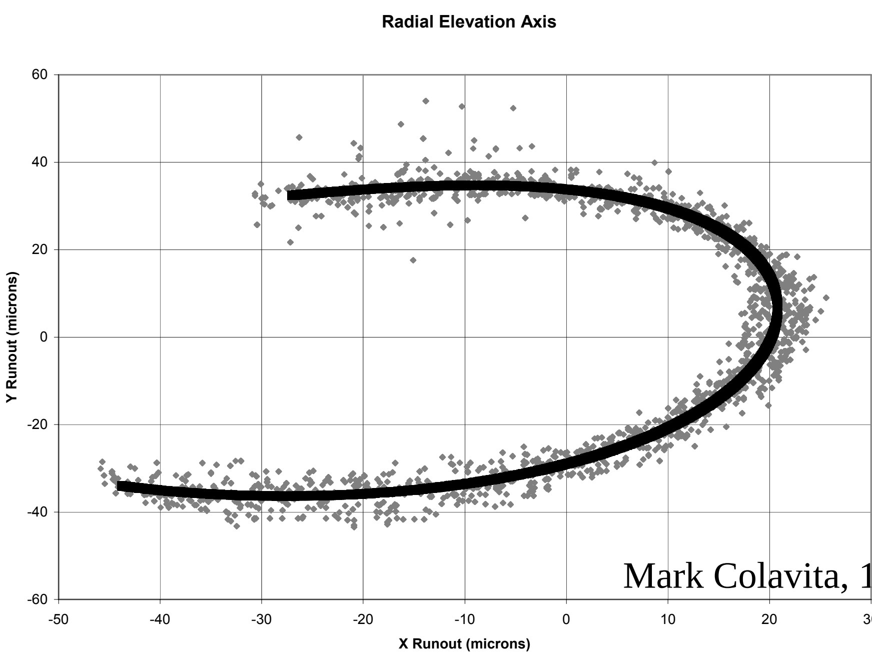


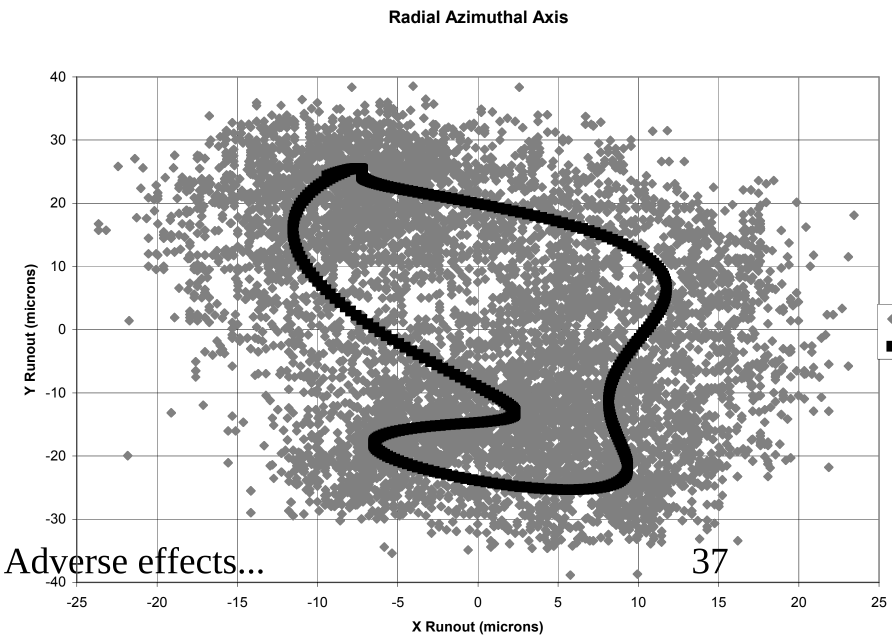
- The OTs included a pivot beacon on the tertiary mirror that could be monitored with metrology
- On next page are results of measurements and model residuals

Baseline monitoring for astrometric interferometry
M. A. Hrynevych, E. R. Ligon, M. M. Colavita,
2004, Proc. SPIE, 5491, 1649.


Axial Elevation Axis
Axial Runout (microns)
Degrees from zenith


Axial Azimuth Axis
Azimuthal Axial Runout (microns)
Degrees


Radial Elevation Axis
Y Runout (microns)
X Runout (microns)


Radial Azimuthal Axis
Y Runout (microns)
X Runout (microns)

Unmodeled pivot noise

$$f(\theta) = c + b_0\theta + \sum_{n=1}^{n=4} a_n \cos(n\theta) + b_n \sin(n\theta)$$

	Axial um rms	**Radial X um rms**	**Radial Y um rms**	**Total um rms**
Elevation	1.42	2.19	3.07	4.03
Azimuth	7.31	4.64	5.01	10.00
Combined				10.78

- For these telescopes, the residual runout after modeling was ~10 um, less than the 17 um tolerance
- With good design, and an appropriate modeling and observing strategy, pivot noise should be controllable

Unmodeled baseline noise: summary

- Addresses “mechanical” quality of wide-angle baseline
  - I.e., the noise in the “pivots”, assumed perfect, above, when we doing the baseline solution
- No major issues

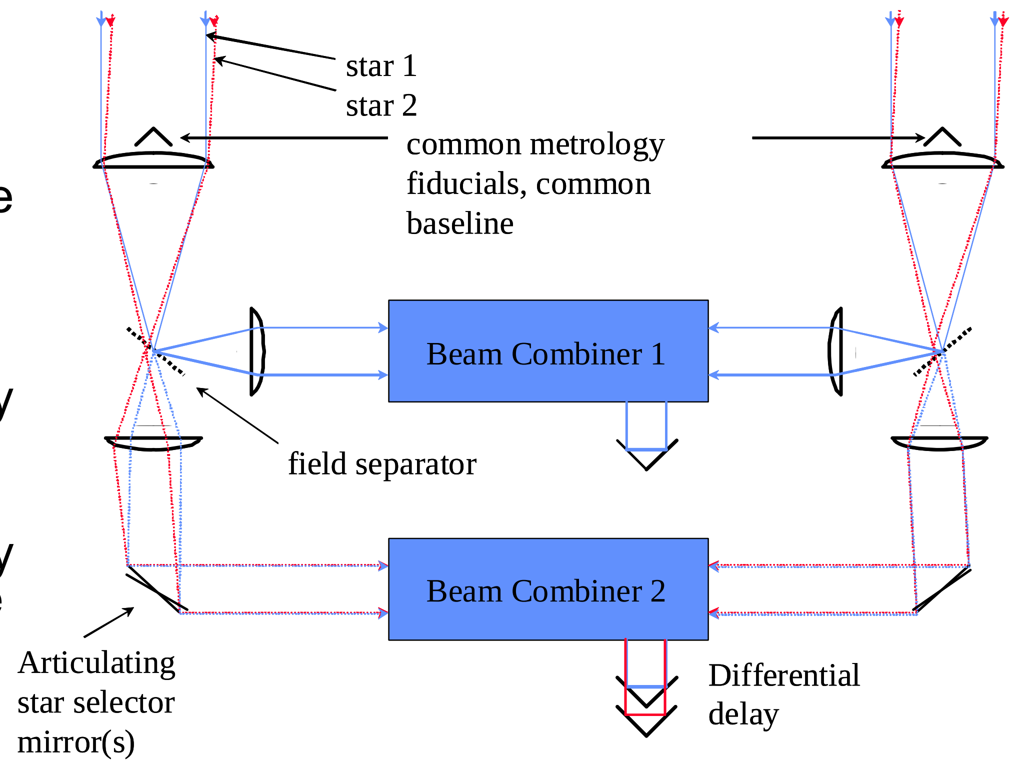
Baseline Noise: 3,4) Narrow-angle vs. wide-angle baseline

- **Baseline knowledge required to $\delta B = B\ (\delta\Theta\ /\ \Theta) = 50$ um**
- Suballocate equally among

1. Wide angle baseline solution – 25 um
2. Unmodeled baseline noise – 25 um
3. **Wide-angle baseline identification – 25 um**
4. **Narrow-angle to wide-angle baseline transfer – 25 um**

What is the narrow-angle baseline?

- We've been carefully skirting this issue: time to address it now
- Recall the dual-star schematic from before
  - For wide-angle astrometry, we articulate between stars by repointing the telescopes
  - For narrow-angle astrometry, we articulate between stars by tilting a mirror in the star separator
  - There's no a priori reason why these baselines should be the same, i.e., the wide angle baseline we carefully solved for earlier may be different than the one we really care about



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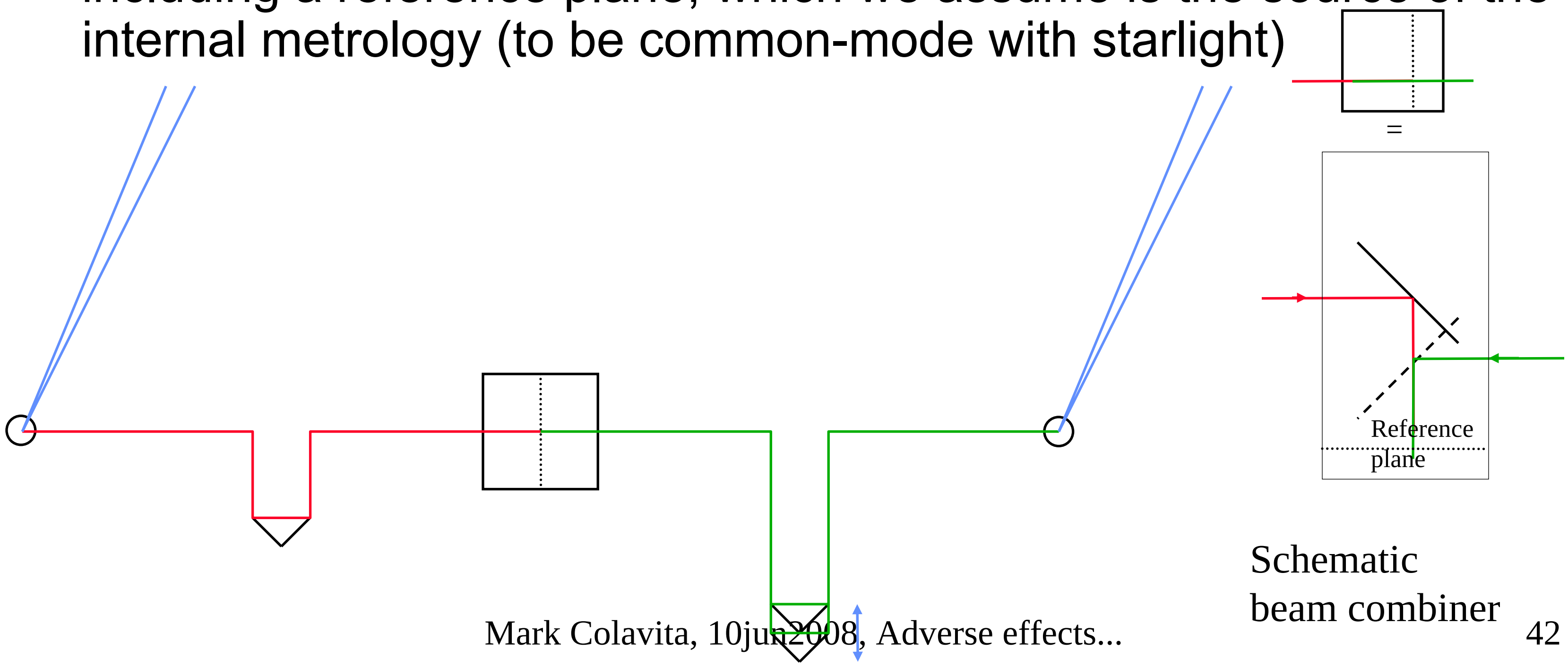
Wide angle baseline with metrology

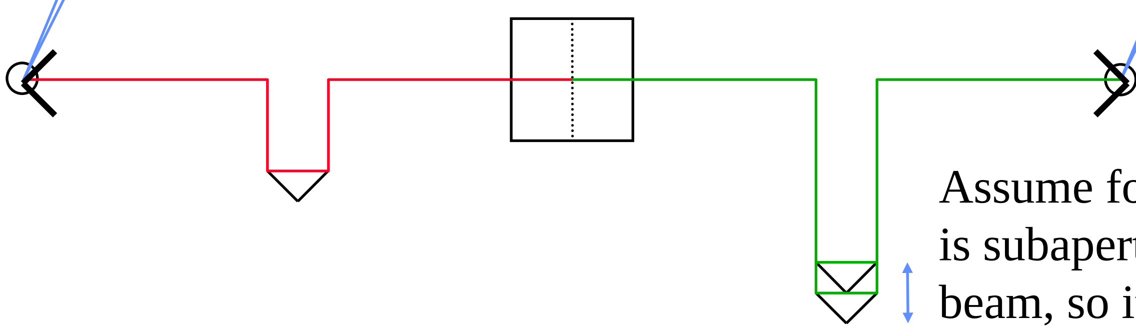
- Normally the wide angle baseline connects the telescope pivot points
- But we need to have end-to-end, or nearly end-to-end metrology: how does this impact the baseline definition?
- Consider the schematic interferometer, below, with the beam combiner including a reference plane, which we assume is the source of the internal metrology (to be common-mode with starlight)



Schematic
beam combiner

Wide angle baseline with metrology, 2

- Intuitively, you put the metrology end point at the pivots, about which the telescopes articulate, so you exactly measure the internal path
  - This is by far the best approach.
    - » You're left with one error term, the accuracy in locating the fiducial on the real pivot
    - » If the both dual-star interferometer see this, then the narrow-angle baseline and the wide-angle baseline are the same
    - » But, is the pivot accessible in this way?



Assume for now the metrology is subaperture, in the center of the beam, so it doesn't block starlight

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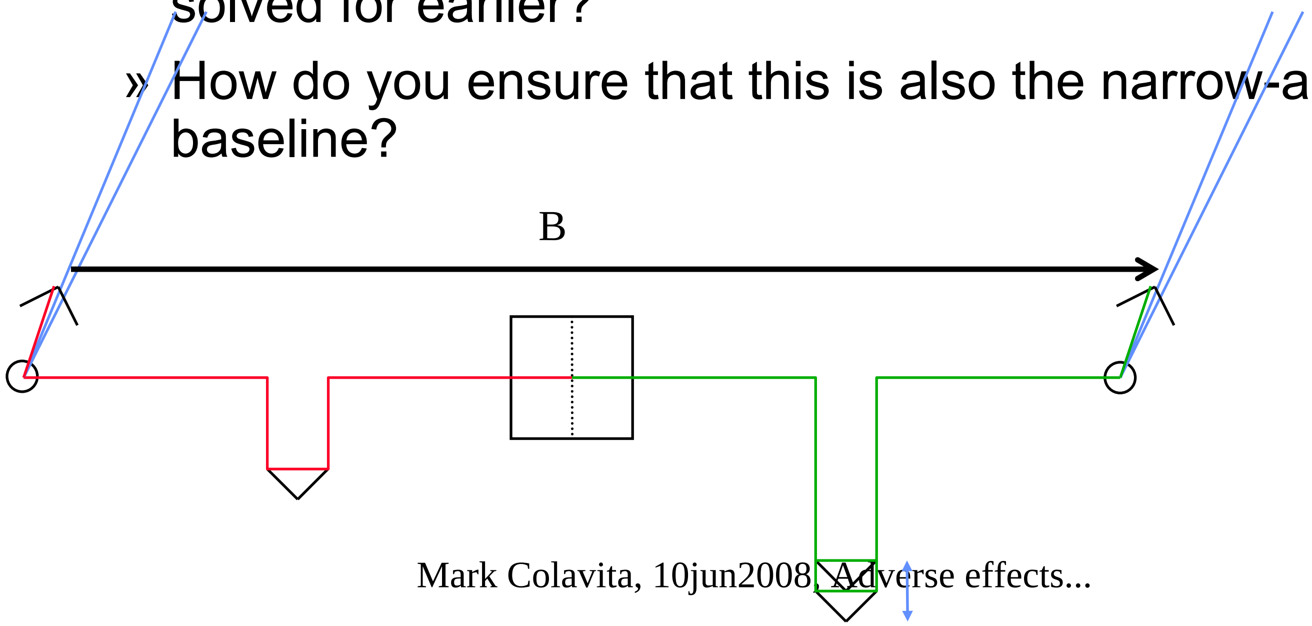
Wide angle baseline with metrology, 3

- However, we can put the corner cube anywhere in input space (i.e., prior to the first optic), if we define the baseline as the vector connecting the CC vertices
  - But two questions, and the two remaining baseline error budget terms
    - » How do you ensure the baseline shown below is the one solved for earlier?
    - » How do you ensure that this is also the narrow-angle baseline?



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Solving for the narrow-angle baseline directly

- Can you solve for it directly, like for wide angle?
  - Not in an absolute sense, unless you had a priori knowledge of star positions at the 10’s of uas accuracy
  - However, you could imagine an approach which uses an unknown, but stable reference pair(s), and a ratiometric approach
- For the next few slides, we discuss the approach we had intended for the Keck OTs
  - We used a beacon near the real pivot (conceptually, located at the tertiary surface)
    - » We would survey the beacon as we articulated the telescope to transfer the beacon to the wide angle baseline (3)
      - This would be done similarly to the data shown earlier
  - We used a CC in the star selector that we aligned to an image of the beacon
    - » This transferred the wide angle baseline to the narrow angle one (4)
      - Discussed on next few slides

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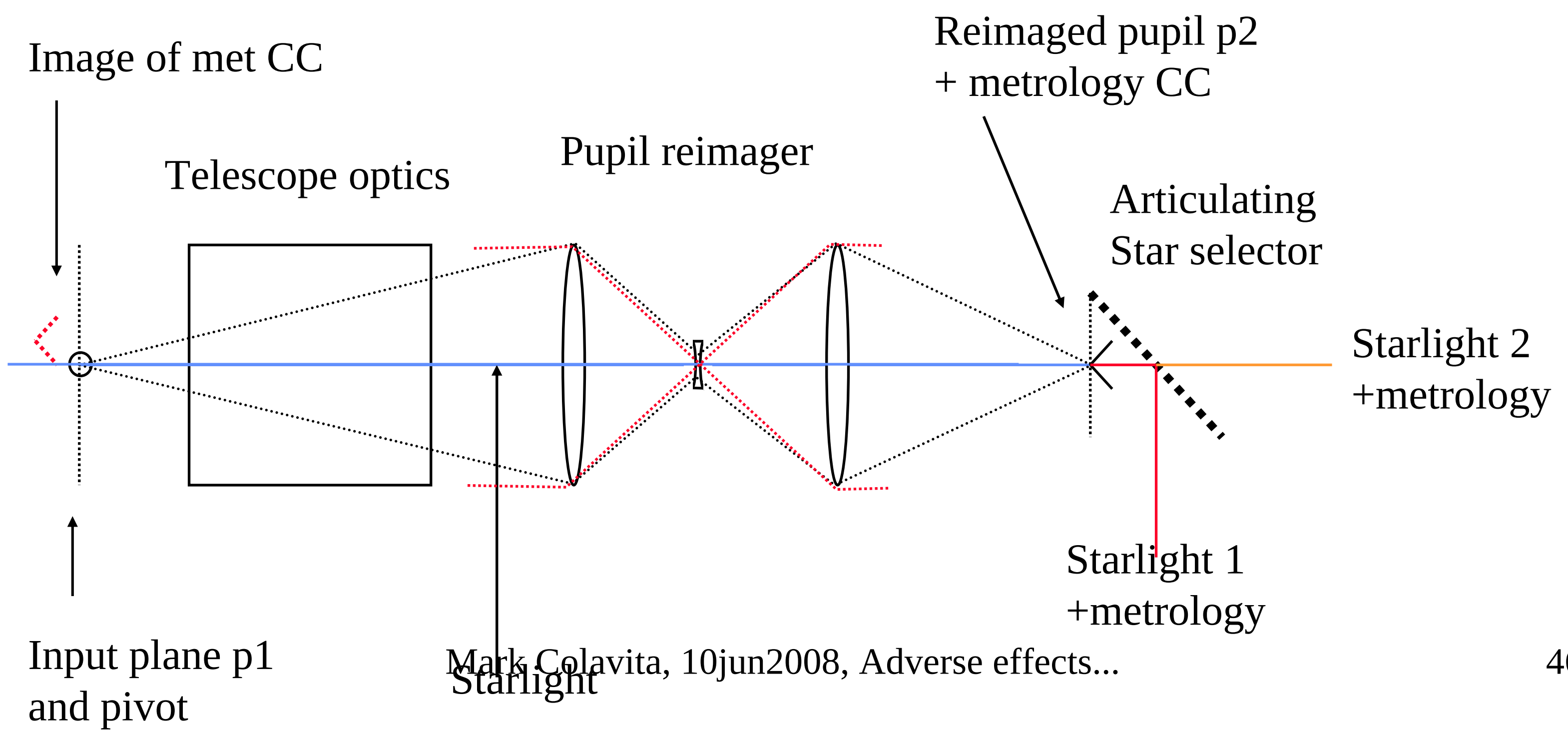
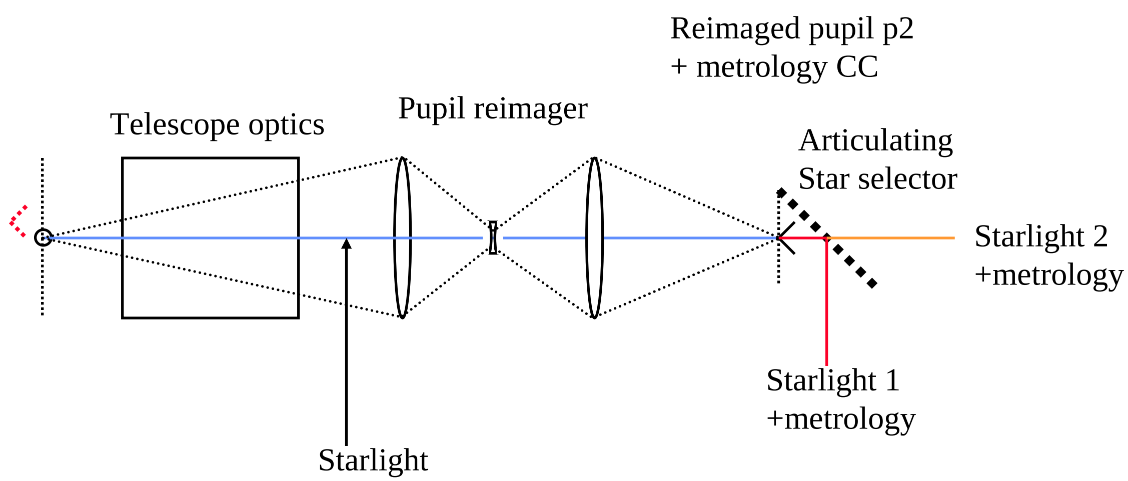
Reimaging the pupil, 1

- Shown is a schematic star selector for a large telescope
- Input plane p1 is, e.g., the plane including tertiary mirror & telescope pivot, seen from the starlight side
- “Telescope Optics” represents M1,M2…
- The pupil reimager is part of the star selector: the reimaged plane includes the star selector mirror and metrology CC



Reimaging the pupil, 2



- If the pupil mapping is exact, the metrology CC is projected onto the pivot
- If the pivot defines the wide angle baseline (i.e., it's observable)

and

if the aberration and beamwalk are acceptable in the reimager

then

the narrow angle baseline = the wide-angle baseline

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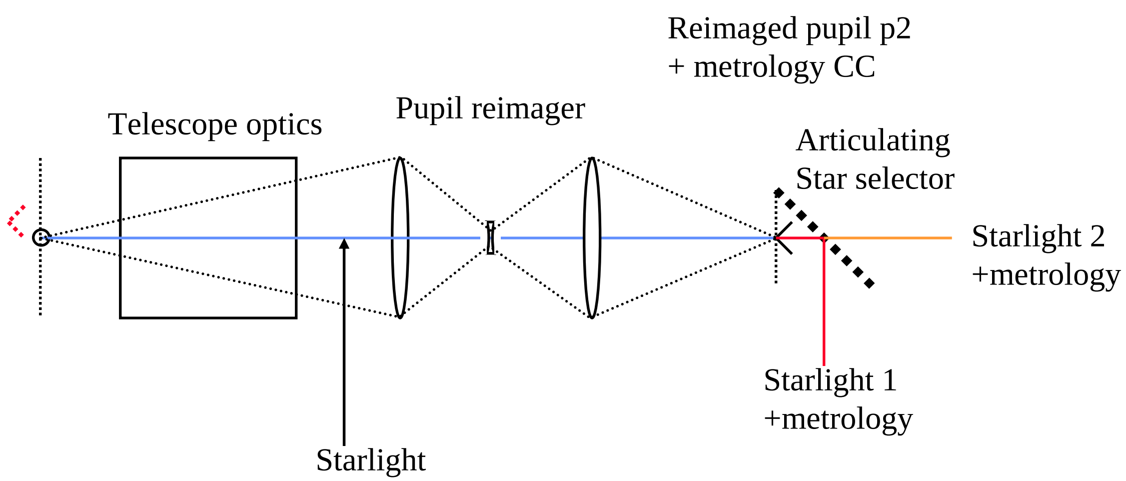
Reimaging the pupil, 3



- Tolerances
  - Lateral mapping x,y
    - » Wide-angle tolerances, e.g., 25 um
  - Longitudinal
    - » Loose: require
    - » $\frac{1}{2}\Theta^2 z \ll 5$ nm
      $z \ll 1$ m
- How?
  - Lateral: illuminate CC and pivot
  - View with camera
  - Adjust to tolerance

Baseline baseline baseline

- This all gets a bit confusing: let's recap
- It's straightforward to solve for a wide-angle baseline, which will predict where you'll find fringes when you articulate the telescope over large angles
- You can build telescopes that have an adequately stable pivot after some modeling
- With internal metrology, the baseline is where the CC is in input space
- The narrow angle and wide angle baselines are not guaranteed to be equal
- One approach was shown here to force this equality
  - Identify a fiducial which you can tie to the physical pivot through surveying or other measurements
  - Transfer the narrow-angle baseline physically or virtually to this fiducial
- Other approaches are certainly possible

Narrow-angle vs. wide-angle baseline: summary

- Baseline knowledge required to $\delta B = B\ (\delta\Theta\ /\ \Theta) = 50$ um
- Suballocate equally among

1. Wide angle baseline solution – 25 um
2. Unmodeled baseline noise – 25 um
3. **Wide-angle baseline identification – 25 um**

   Errors associated in tying something observable to the wide-angle baseline, i.e., the errors in the telescope survey
4. **Narrow-angle to wide-angle baseline transfer – 25 um**
   Errors in carrying out the transfer step, i.e., the lateral mapping tolerance on the previous slide

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Measurement noise

- For now, let's do a simple error budget for $\delta\Theta$ = 20 uas with equal $\delta\Theta$=10 uas allocations for
  - 1) Baseline noise δB
  - **2) Measurement noise $\delta\ell$**
  - 3) Fringe measurement noise δf
- Assume
  - B = 100 m and Θ = 20 as
- Then
  - 1) $\delta B = B\,(\delta\Theta / \Theta)$ = 50 um
  - **2) $\delta\ell = B\,\delta\Theta$ = 5 nm**
  - 3) $\delta\phi = B\,\delta\Theta$ = 5 nm

Measurement noise

- Terms here
  – Laser metrology accuracy
  – Beamwalk errors
  – Thermal stability
  – DCR
  – Material dispersion
  – Environmental stability

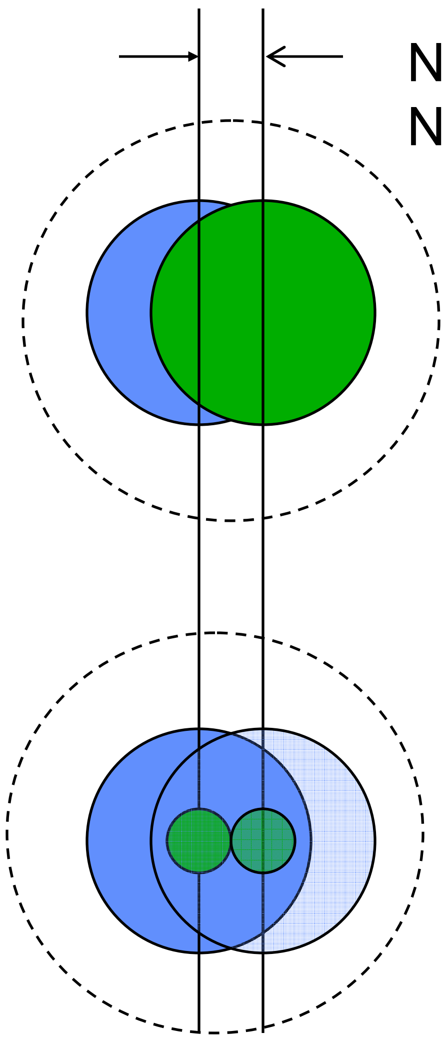
Laser metrology accuracy

- To first order
  - If want 1 nm accuracy over 100 m, require laser stability of 10^{-11}
    - » Not impossible, but hard
      - E.g. workhorse stabilized HeNe laser is 10^{-8}
- Is there a better approach? Yes: design the experiment so the metrology needs to be accurate only over the delay articulation range:
  - Θ = 20 as * 100 m = 10 mm
  - Now require only stability of 10^{-7} for narrow-angle astrometry
  - With a 10^{-8} laser, wide-angle accuracy is 1 um, plenty adequate
- Basis for most approaches is to use same laser for all of the beams
  - Errors in the large common-mode paths between primary and secondary then drop out

Beam walk errors

- Optical surfaces are not smooth at the nm level
  - λ/20 surface in reflection: ~13 nm rms surface
  - 16 surfaces: ~50 nm rms per arm if errors can be rss'd
- If these effects are static, they just impact Strehl
- For astrometry they cause errors when one beam translates with respect to another, caused by
  - Beamwalk in star-separator as switch from star to star
  - Drifts in beamtrain with time, temperature, and tilt compensation (drives switching architecture described earlier)
- A particular issue is if the metrology and starlight view the optics differently
  - Typically metrology beam diameter << starlight beam diameter

Beam-walk scenarios



Normalized translation Δ,
Normalized beam diameters D, d

Case 1: Change in OPD as a beam walks across an optic

Case 2: Change in OPD between starlight beam (larger diameter) and metrology beam (smaller diameter) as they both walk across an optic

Filter function analysis

Given phase maps, one can do numerical computations: the following gives an approximate analysis to give a feel for the error

Assuming a probabilistic description of the wavefront, you can write the variance of the OPD error as: $\sigma_e^2 = 2\pi \int f df W(f) H(f)$,

where W(f) is the power spectrum of the wavefront, and H(f) is a filter function specific to the problem.

Let the surface power spectrum be $W(\mathbf{f}) = \frac{w^2}{8\pi} f^{-2.5}$,

where w is the total wavefront variation, -2.5 is a typical slope, and the ad-hoc normalization used here puts half the variance at spatial frequencies < 1 cycle/optic

You can show that the filter functions for the two cases are

$$H(f) = 2(1 - J_0(2\pi f \Delta))A^2(\pi f D), \qquad A() = 2J_1()/()$$

$$H(f) = 2(1 - J_0(2\pi f \Delta))(A(\pi f D) - A(\pi f d))^2$$

which have plausible-looking forms.

Filter function analysis, 2

Results

Case 1:

$$\varepsilon^2 \approx 1.5 w^2 \Delta^2 D^{-1.5}, \quad \Delta << D$$

Case 2:

$$\varepsilon^2 \approx 1.5 w^2 \Delta^2 d^{-1.5}, \ 2\Delta << d << D$$

$$\varepsilon^2 \approx 2.2 w^2 \Delta^{0.5}, \ \mathrm{d} << 2\Delta << D$$

Example, caveats, implications

- Example:
  - 20 cm optic
  - 10 cm starlight beam (D=0.5)
  - 2 cm metrology beam (d=0.1)
  - 1 cm shear (Δ=0.05)
  - w = 50 nm rms (from previous example)
- Then
  - Change in starlight path: e_1 ~ 5 nm rms
  - Change in starlight vs. metrology: e_2 ~ 25 nm rms
- Caveats
  - Approximate analysis only (~2X), likely pessimistic. Also, may only care about beam shear over one or two surfaces. However
    - » Large aspheres can have significant zonal errors, which could be a larger effect than given above
    - » Superpolished (λ/100) optics available for critical locations
- Implications
  - You need to pay attention to this effect in the star separator design, where beams must walk, in how you implement the metering and what optics it includes, and how you specify the optics
    - » This was one of the drivers for KI OT design with a virtual CC

D	0.5
d	0.1
delta	0.05
w	50

5.1	case1
17.2	case2a
35.1	case2b
25	sqrt(2a x 2b)

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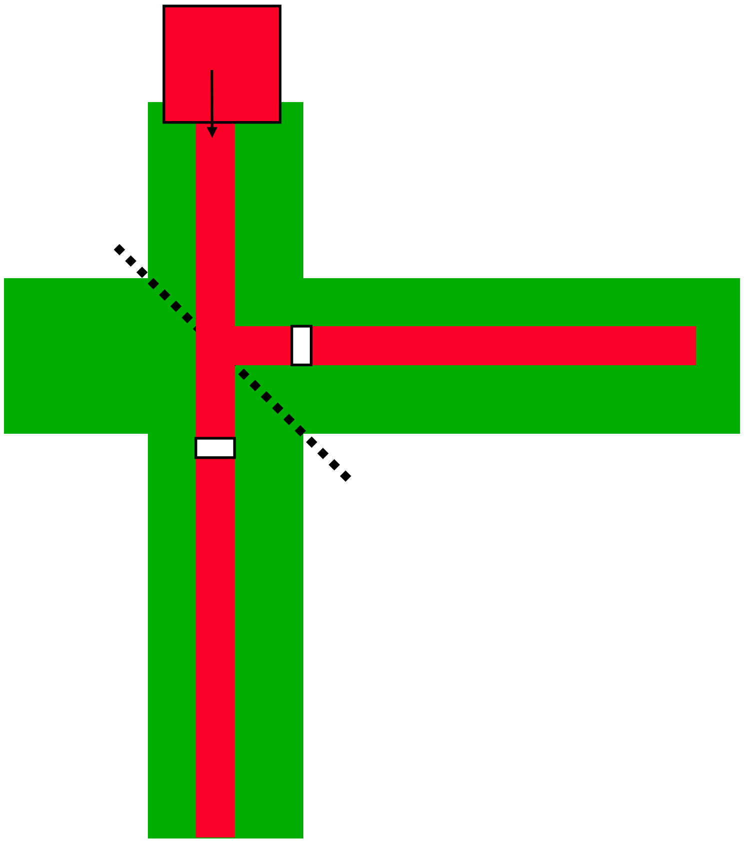
Thermal stability: non-common optics



- Even if you meter everything, you still need to introduce the metrology into the starlight path in a way that doesn't introduce its own complications
- Example, suppose you introduce the metrology from behind the starlight beamsplitter into the center of pupil, shown schematically on the left, and add two small polarizers in the beam
  - How stable do they need to be?

Non-common thermal effects

- Thermo-optical constant measures the change in OPD with temperature as an optic changes size (cte) and index, i.e., G ~ N cte + dN/dT
- BK7, a common optical glass, has G = 7×10^{-6}
  - » For 10 mm thickness, ΔOPD = 70 nm per K
  - » Note
    - Better glasses exist
    - Maybe can design out of system
    - Add local insulation to increase time constant
    - Fast switching keeps ΔT small and avoids problems
- In previous example, the paths were different. What, now, if they're the same but dispersive
  - – Offset doesn't matter, but now the chromaticity of the thermo-optical constant matters for different metrology and starlight wavelengths
    - » This is probably much smaller than above, and again avoided by fast switching

DCR

- While vacuum delay lines make most of this problem go away (leaving only the small sky pathlength difference due to curvature of the Earth – small effect), many interferometers are using air delay lines these days
- We noted earlier:
  - $\Delta N \approx 3.0 \times 10^{-9}$ between $\lambda = 2.20$ and $\lambda = 2.21$ um, i.e., $\delta\lambda = 10$ nm uncertainty [for dry air; water vapor term smaller]
    - » $\Delta x = 300$ nm for a 100 m path
    - » Implies we need 100X better wavelength accuracy, $\delta\lambda = 0.1$ nm ?
- Worth noting that DCR is a major problem for all ground-based astrometry
- What to do
  - 1st, path may be less than 100 m if work closer to zenith and account for reduced atmospheric pressure
  - 2nd, take advantage of the fact that we have a spectrometer

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DCR with a spectrometer

- In principle, with a 10 nm spectral channel, the change in effective wavelength for a change in stellar temperature from 5000 to 6000 K is ~0.001 nm, << less than requirement
- Absolute calibration also not required
- Thus, the primary requirement is spectrometer stability to ~0.1 nm, i.e., ~1% of channel width, over a switching cycle
- Approach
  – Stable camera design and environment
  – SM fiber to ensure stable MTF
  – Use a fixed weighting per spectral channel to compute the phase estimate
  – Account for spectral slopes to do a second order correction
  – Fast switching essential: to reduce stability time scale and allow averaging of errors over multiple cycles

Material dispersion

- There's likely, accounting for windows, etc., similar amounts of glass dispersion as air dispersion
- See previous discussion on air dispersion

Environmental stability

- We noted earlier:
  - $\Delta\Delta N \approx 12\text{x}10^{-9}$ for a 1K temperature change [HeNe metrology, K band]
    - » $\delta x \approx 1.2$ um for a 100 m path
  - $\Delta\Delta N \approx 1.5\text{x}10^{-9}$ for a 1% change in relative humidity [“”}
    - » $\delta x \approx 150$ nm for a 100 m path
- These seem tight; what to do?
  - 1st, path may be less than 100 m if work closer to zenith and account for reduced atmospheric pressure
  - 2nd, IR metrology: move metrology closer to science wavelength
  - 3rd , and a big one
    - » The light from the two stars is likely traversing the same air in the lab: the beams are likely side-by-side, 10’s cm apart
      - In this case, most of the error, except over a 10 mm OPD difference, drops out, leaving only variations over 10 cm scales
  - 4th, take advantage of the fact that we’re making a differential measurement

Environmental stability, cont

- Approach
  - This is probably not too bad, as the beams are side-by-side
  - IR metrology helps: as laser wavelength gets closer to science wavelength, smaller effects
    - » Recall, if wavelengths match, drops out
  - Add environment monitoring
  - Use fast switching
    - » Note in particular, fast switching with full-bracketed calibration eliminates linear trends

Measurement noise, summary

- Terms here
  - Laser metrology accuracy
    - » Use (effectively one laser): then need accuracy over only 10 mm
  - Beamwalk errors
    - » Pay attention if you have subaperture metrology and translating beams
  - Thermal stability
    - » Be careful with non-common optics
  - Air and material dispersion
    - » Needs good camera stability over a switching cycle
  - Environmental stability
    - » Much of this will end up being common mode
  - <u>Fast switching greatly reduces effect of last three terms</u>

Fringe measurement noise

- For now, let's do a simple error budget for $\delta\Theta$ = 20 uas with equal $\delta\Theta$=10 uas allocations for
  - 1) Baseline noise δB
  - 2) Measurement noise $\delta\ell$
  - **3) Fringe measurement noise $\delta\phi$**
- Assume
  - B = 100 m and Θ = 20 as
- Then
  - 1) $\delta B = B\,(\delta\Theta / \Theta)$ = 50 um
  - 2) $\delta\ell = B\,\delta\Theta$ = 5 nm
  - **3) $\delta\phi = B\,\delta\Theta$ = 5 nm**

Fringe measurement noise

- Absent camera stability, which we already discussed, if you work at null, there are no errors in this category
  - However, you won't be working exactly at null
- One effect is that even if your phase referencing is perfect, there would still be group delay fluctuations due to water vapor turbulence as well as to the increase in dry-air path with earth rotation
  - You might want to consider "group delay referencing", too

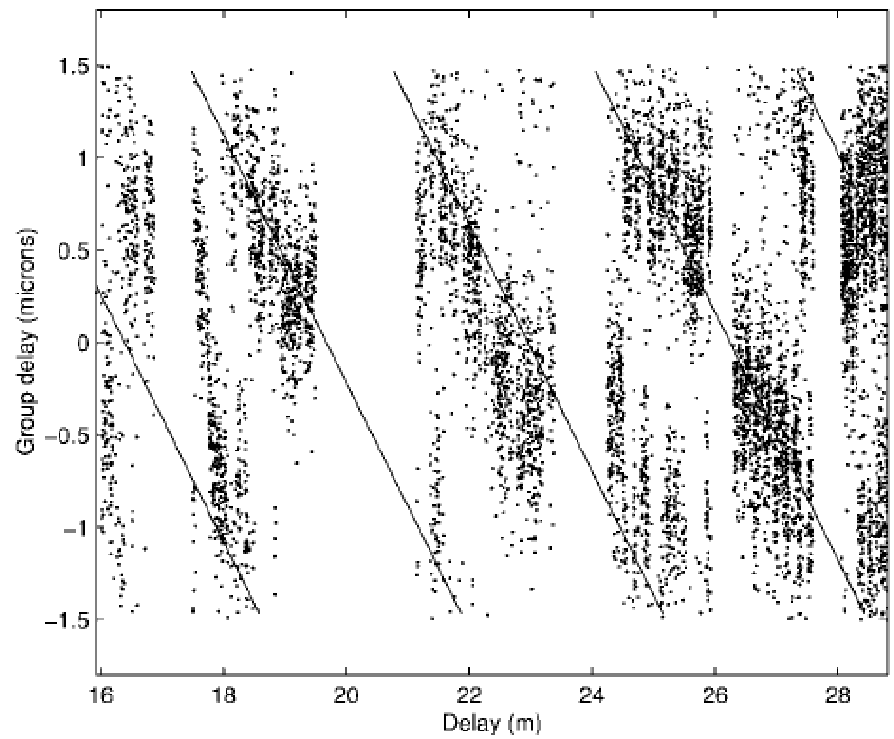

FIG. 5.—Group delay minus phase delay vs. total delay at PTI (adapted from Akeson et al. [2000]).

Required fringe-measurement accuracy

- To first order you need accuracy (including linearity) of 5 nm over a range of 2.2 um from all effects
  - Accuracy also needs to apply in the presence of small rates
- Sources of error
  - Wavelength calibration
  - Finite coherence
    - » Addressed partially through narrow-spectrometer channels
  - Phase measurement linearity
    - » Includes linearity of OPD stroke
- Approach
  - Accuracy needed to 0.2% / 5 nm: should be achievable with care
    - » Might be a place where a dither could be useful to provide some degree of cyclic averaging

Systematic errors: summary

- Main contributors
  - 1) Baseline noise δB
  - 2) Measurement noise $\delta \ell$
  - 3) Fringe measurement noise $\delta \phi$

- Main points
  - Challenging tolerances: but take advantage of the fact that we're making a differential measurement
  - You can get the wide angle baseline accurately enough: need to be sure the narrow-angle baseline also known accurately
  - Almost all of the very tight linear measurement problems can be accommodated by fast switching
    - » First, fast switching reduces the stability time scale to minutes, rather than hours
    - » And if you switch right, errors are further reduced by $\sqrt{N}$
    - » But don't forget about the $\sqrt{2}$
  - The remaining terms you must accommodate in the design
  - It's all doable

Sensitivity

- The squared phase SNR is given by the usual formula

$$S^2 = \frac{4}{\pi^2} \frac{N^2 V^2}{N + B + 4R}$$

which assumes a 4-bin algorithm
  - N is total photons, both apertures
  - V is fringe visibility
  - B is background photons
  - R is detector read noise variance
- The astrometric error e for SNR S is given by

$$e = \frac{\lambda}{2\pi S B}$$

- For λ = 2.2 um and B = 100, 20 uas requires SNR=36.
- This is for the faint star: the error on the bright star will be negligible.
- Let's go through the coherence terms. We assume a single-mode combiner Where applicable, will also assume a 15" off axis angle

Detected photons

- Flux N = αSF
  - F = flux at first mirror
  - α = effective instrument throughput
    - » Includes warm & cold loss, mode match, encircled energy, duty cycle, etc.
  - S = Strehl, product of three terms
    - » S_b = beam-train Strehl
      - For 100 nm rms per arm, S_b ~ 90%
    - » S_w = residual wavefront error after correction
      - For tip/tilt correction, r0=15 cm, est. S_w ~ 50% [larger if higher-order correction]
    - » S_a = anisoplanatism error from off-axis wavefront correction
      - For θ_0 ~2", est. S_a ~ 70%
    - » [Backup on following slides]

S_w: residual wavefront error after correction

- I'll do the tip/tilt only case as it's easy, and probably adequate for a D=1.8 m telescope
- For Kolmogorov turbulence, if you full correct tilt, the wavefront variance decreases from 1.03× to 0.134 × $(D/r_0)^{5/3}$ rad^2.
  - In reality, some tilt will remain due to
    - » coma anisoplanatism (i.e., tilt sensing errors from using a centroid rather than a wavefront sensor)
    - » finite bandwidth
    - » sensor noise.
  - Assume the residual is 5% of variance of the tilt component,
  - Then for D=1.8m, r_0=15cm, the rms residual is 300 nm, and Strehl S_w =50% at 2.2 um: not bad
  - However, does degrade quickly for poorer seeing / short wavelengths

D	1.8	
r0	0.15	
k	0.05	resid tilt va
lambda	2200	
h/o residua	254	nm
tilt residual	147	nm
total	294	nm
Strehl	50%	

S_a: anisoplanatism error from of-axis wavefront correction

- The faint star will be off-axis from the bright star used for tilt correction, and thus there will be a tilt isoplanatism error.
- We estimate the Strehl from tilt errors as [1]
  - $S=1/(1+\sigma^2_{TILT})$
    where
  - $\sigma^2_{TILT} = 2\,\sigma^2_1/(0.637\lambda/D)^2$
    and
  - σ^2_1 is the 1-axis tilt error

- [1] Alloin, D. M, & Mariotti, J.-M., 1994, Adaptive Optics for Astronomy, Springer, Berlin

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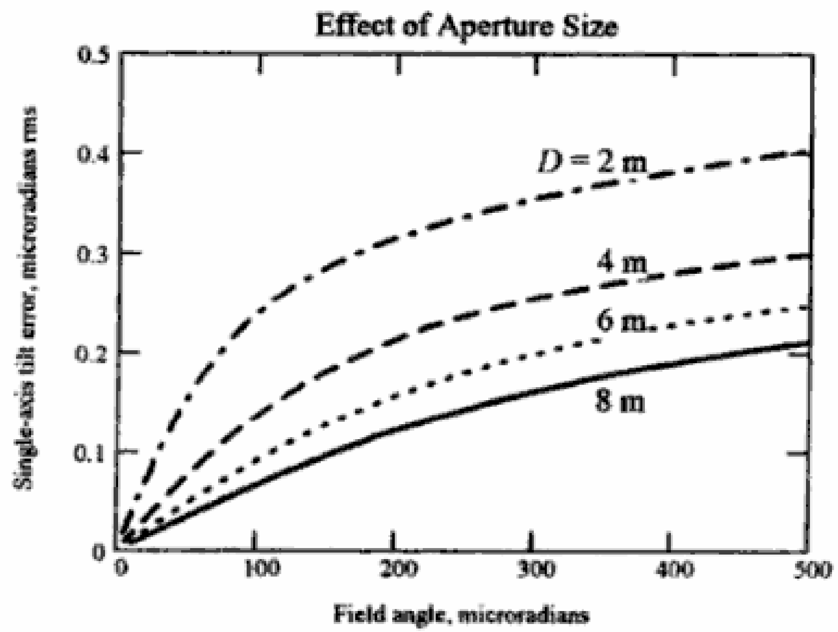
S_a = anisoplanatism, continued

- Scaling some data from a 4” isoplanatic angle model to a more typical 2”, estimate σ_1 = 0.35 urad at 15” (75 urad) off-axis
  - S_a ~ 70%



HV 20/20 model has r_0=20 cm & θ_0 = 20 urad [4”]

Figure 7.38 Anisoplanatic tilt angular error for apertures of 2, 4, 6, and 8 m. HV, Hufnagel Valley.

From: John W. Hardy, Adaptive optics for astronomical telescopes / Oxford U Press, 1998

1-axis err	0.35
r^2_tilt	0.404191213
Strehl	0.712153723

Visibility

- This term represents coherence loss: product of 3 terms
  - – Instrument coherence loss, i.e., V^2 you measure on an unresolved star, say $V^2_i = 80\%$
  - – Piston anisoplanatism
    - » Estimate isopistonic angle at Paranal ~10” at 2.2 um for $\lambda/10$ incoherence (for a 2m telescope; ~16” for an 8m telescope)
      - For 15” star separation, variance is -1.5 $\lambda/10$
      - $V^2_p = \exp(-\sigma^2_{rad}) = 40\%$ [computed like Strehl]
  - – Cophasing time delay
    - » Assume 200 nm rms
      - $V^2_d = 70\%$
  - – [Backup on following slides]

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Isoplanatic/Isopistonic angle

- Anisoplanatism between the phase reference star and the target star reduces fringe visibility, reducing sensitivity
  - Coherence $V = \exp(-0.5\ \sigma^2)$
- Formulas for differential piston
  - 1) Simplest formula: point aperture, infinite baseline & outer scale [cf. 1,2]
    - » $\sigma^2 = 2(\theta/\theta_0)^{5/3}$ rad^2
      - $\theta_0 = 0.31\ r_0/h^*$, where $h^* = [\int C_n^2\ h^{5/3}\ dh\ /\ \int C_n^2\ dh\ ]^{-5/3}$
  - 2) More sophisticated formula: finite aperture d , finite baseline B, finite outer scale L_0 [3, 4]
    - » NB: isopistonic angle in literature sometimes given for λ /10 rms, vs. 1 radian $(\lambda/\ 2\pi)$ which is more common
    - » $\sigma^2 = k\ (d/r_0)^{-1/3}\ (\theta/\theta_0)^2$ rad^2 [5,6]
      - Constant k depends on outer scale: for $L_0=\infty$, k=1.3
    - » Typical values assuming $\theta_0 = 2''$ in visible, L_0=50 m
      - ~10” for a 2 m telescope, ~15” for an 8 m telescope
      - All very site and seeing dependent!

Values of isopistonic angle

- Outer scale has larger effect for larger apertures
- For small telescope, reasonable L_0, can work out to close to isoplanatic angle with low coherence loss
  - If background-limited, can go 2.4X further out with 1 mag sens hit
- Usual atmospheric caveats
  - Structure constant isn't, along with all other atmospheric params
  - Ref [4] shows larger values for finite outer scale, big telescopes than Ref [3] – adopting Ref 3 here
  - There's limited data in exactly the right configuration to confirm [all] values

	K band	V band
r_0 (cm)	83	14.1 [3]
0_0 (")	11	1.9

Approx. K-band isopistonic angle in arcsec for 0□10 rms (V =0.82, V^2=0.67)

	d = 1.8 m	**d = 8.2 m**
L_0 = inf	7.0	9.1
L_0 = 1000 m	7.8	11.0
L_0 = 100 m	9.2	15.2
L_0 = 50 m	10.0	19.2

-These values derived using [3, eq. 5] and a single layer in order to get values for d=1.8m.

-For comparison, computed values for 8.2 m this way, too. The value of 19.2" for L_0=50 is close to the value of 16.1" given in [3, tab 2] for the full multilayer Integral.

Isopistonic references

[1] Fried, D. L., 1979, Opt. Acta, 26, 597

[2] Roddier, F, Gilli, J. M., Vernin, J., 1982, J. Optics (Paris), 13, 63

[3] Esposito, S., Riccardi, A. & Femenia, B., 2000, A&A, 353, L29

[4] Elhalkouj, T. et al., 2008, A&A, 477, 337

[5] Same as a computation of the asymptotic form of [3, eq 5] for $L_0=\infty$.

[6] Same as [4, eq 24] in different units

Time-delay error

- In principle, this term is given by
  - $\sigma^2 = (T_d/\tau_{02})^{5/3}$ rad^2
    - » T_d is the end-to-end cophasing time delay
      - Depends on integration time on bright star, and overall control architecture
    - » τ_{02} is the first-difference interferometer coherence time
      - τ_{02} =0.2r_0/W ~ 16 ms at 2.2 um r_0(V) = 15 cm, W = 10 m/s
  - However, instrument vibrations also contribute
  - Adopt 200 nm rms, equivalent to T_d = 8ms
    - » V^2 = 70%

Background and read noise

- For long integration times, and faint stars, thermal background matters at 2.2 um, as the effective system emissivity will generally be high
  - For H-band observations, need to account for airglow
- Read noise depends on the number of pixels, and the effective read rate
  - When computing it, however, remember that you need to read out the detector fast enough to at least do some low-bandwidth tracking

Sensitivity: summary, 1

- For λ = 2.2 um and B = 100, 20 uas requires SNR=36.
- We care about the SNR on the faint star, which will be off-axis from the phase-referencing and tilt-correction star
- Flux N: need to account for Strehl terms, in addition to instrument throughput, which affect the flux N into the fiber
  - Corrected wavefront error
  - Tilt-correction isoplanatism
- Coherence V^2: need to account for phase-reference limitations
  - Phase-correction isoplanatism (isopistonic angle)
  - Phase-referencing time-delay
- Background and read noise need to be considered

Sensitivity: summary, 2

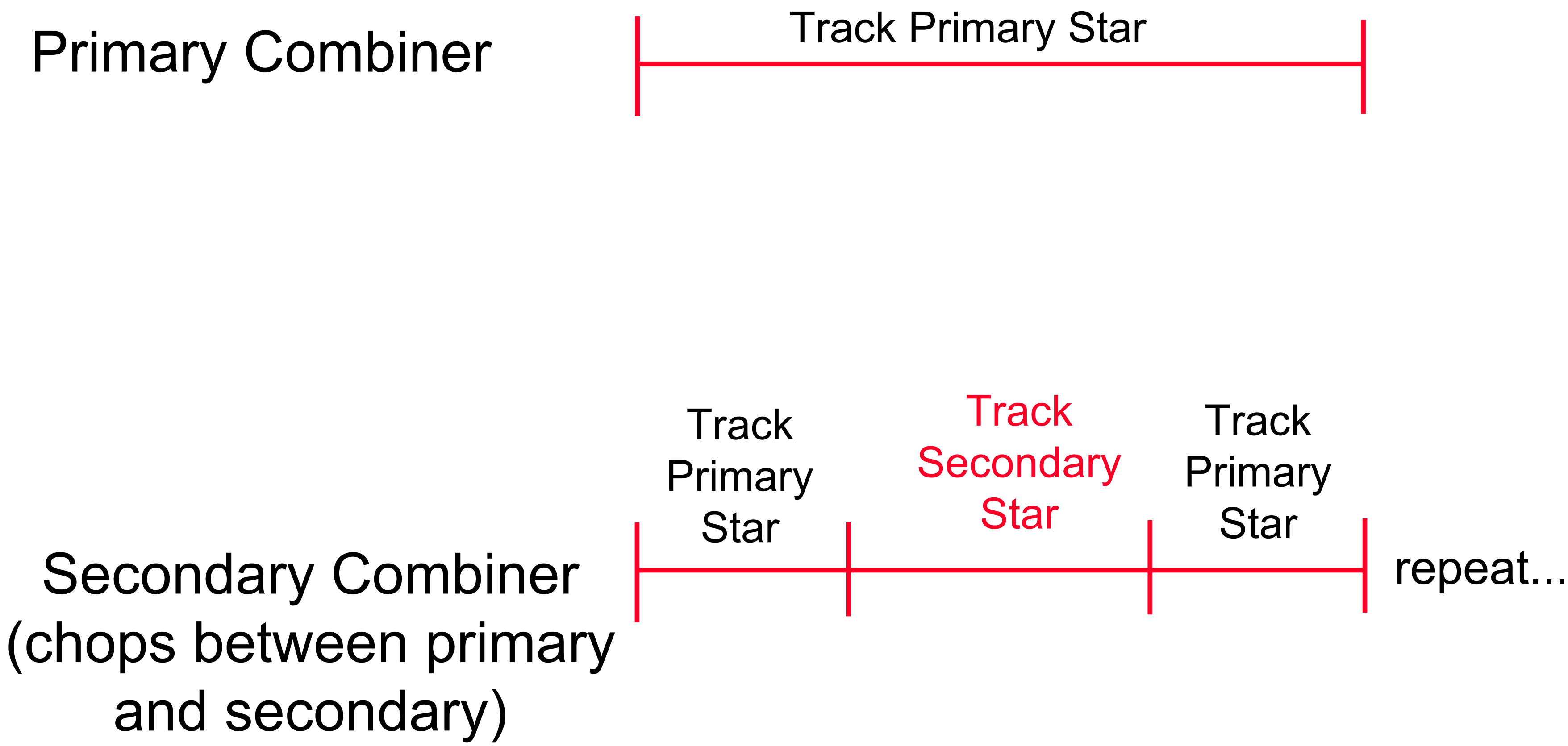
- Some of the fundamental isoplanatism terms you can't do much about; others you may be able to optimize

- Two main points
  - 1) SNR can dominate over the other terms: must optimize everything affecting throughput, Strehl, and coherence
    - » Consider simultaneous K+H observations by incorporating an ADC to get more light
    - » Consider higher order correction to improve Strehl, getting more light into the fiber
    - » Optimize your phase referencing system
    - » Optimize your system throughput
  - 2) Note, the big improvement with baseline: if you go to 200 m, need ½ the SNR, or ¼ the integration time

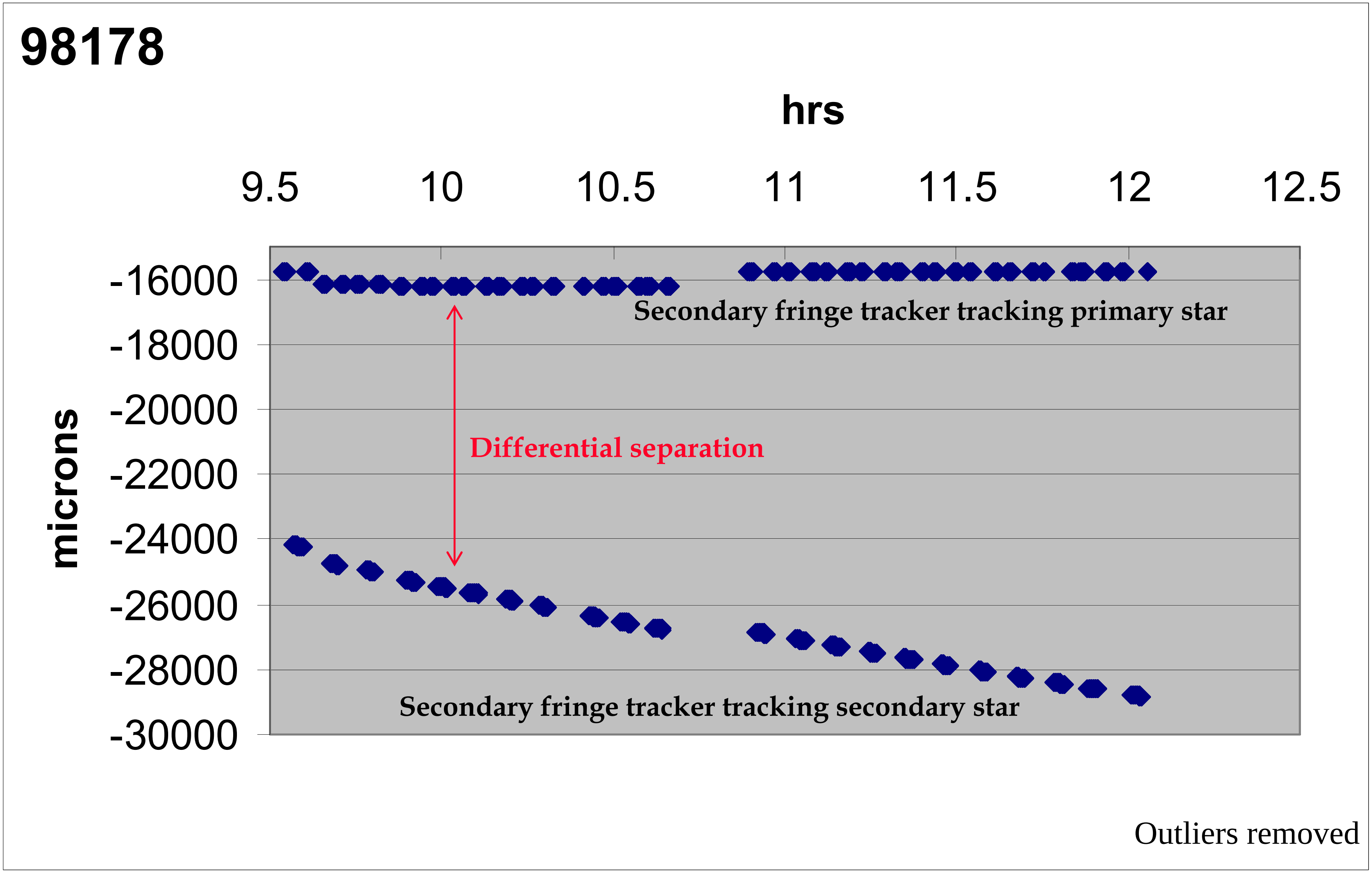
PTI (Palomar Testbed Interferometer) design issues

- On PTI, we moved away from an image-plane splitter (conceptually a pinhole at focus) to a quasi-pupil-plane splitter (KI OTs were to use pure pupil)
  - Avoided focused image on optic where dirt could cause metrology drop
  - Leaked some of the bright star so could do switching measurement
  - Maintained metrology continuity when switching between bright and faint star
- Metrology went end-to-end, stopping at CC's located at a precise offset from siderostat pivot
  - Probably not extensible to a telescope approach

PTI observing scenario for the dual-star tests



Raw data


98178
hrs
9.5
10
10.5
11
11.5
12
12.5
-16000
-18000
-20000
-22000
-24000
-26000
-28000
-30000
microns
Secondary fringe tracker tracking primary star
Differential separation
Secondary fringe tracker tracking secondary star
Outliers removed

Data processing

- Data selection step, to account for bad scans due to metrology drops
  - These occurred mostly due to some slew rate limits we had on the metrology in order to maintain SNR
- Correct laser-measured delays for fringe-tracking errors
  - We used group delay; but for faint stars, must use phase because of its better SNR
- Bracketed (vs. first difference) calibration, i.e., calibrate faint star against interpolated position of before and after measurements of bright star
  - Rejects linear trends

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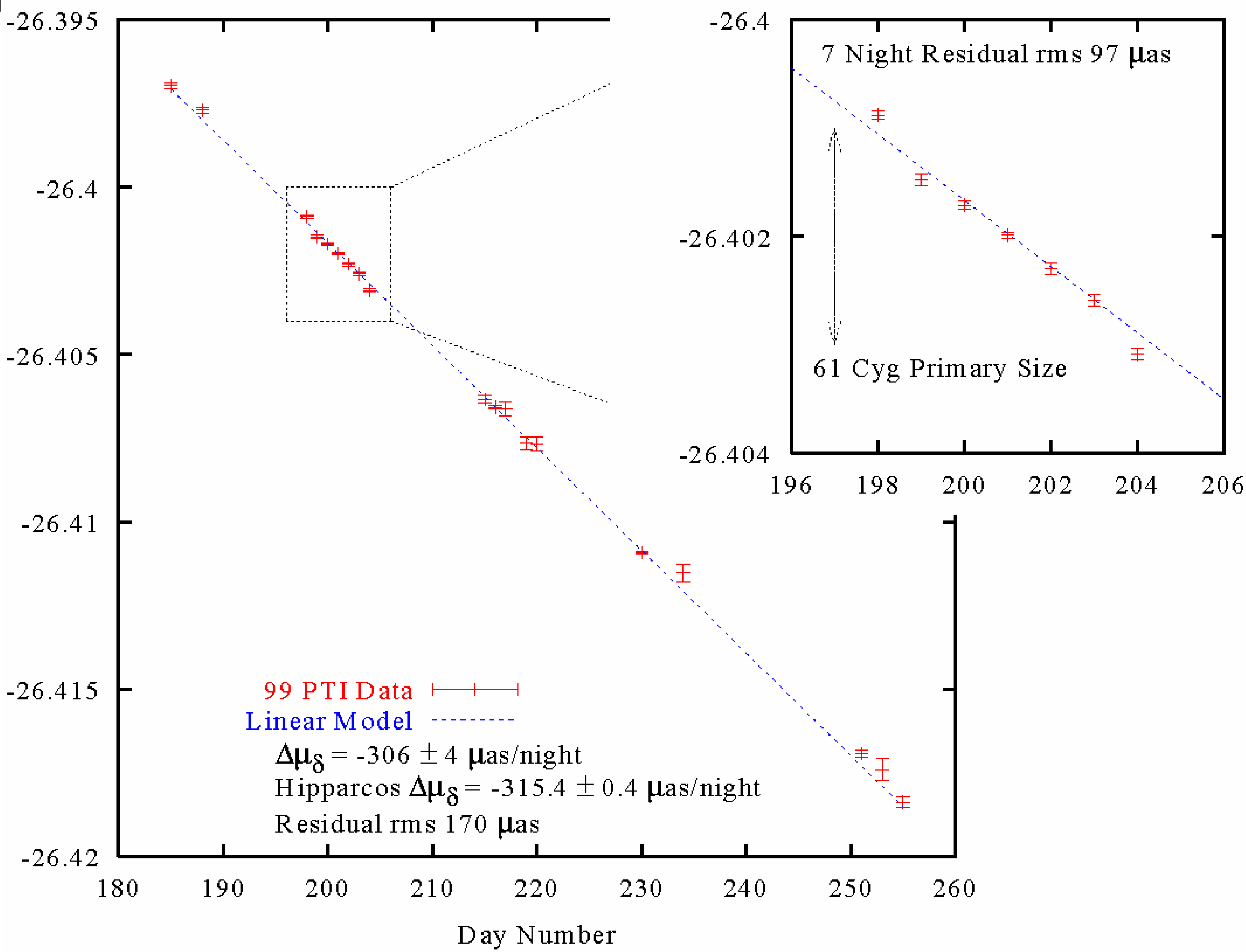

What have I left out ?

- Enough already!!!
- Binarity of reference stars?
- Availability of reference stars?
  - Note, depending on the anisoplanatism terms, you can consider slightly wider separations, as the atmospheric term is small
  - Availability strong function of galactic latitude
- Practical issues
  - Acquiring faint stars near bright ones
  - May need reasonable a priori positions on faint stars, as probably not enough SNR to search
- ?

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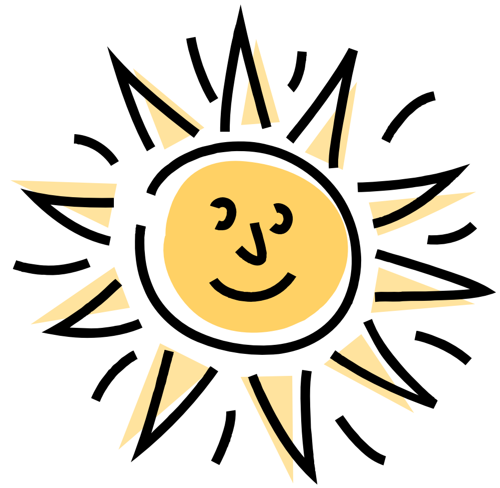
Summary



- Astrometry at 10's of uas is scientifically exciting
- The fundamental atmospheric limit is the easy part
- Careful control of systematics is required, for the baseline as well as for the delay measurement
- SNR is important to optimize as the stars will be faint
- Design recommendations
  - 1) Pay attention to the systematics. Be clever: there are likely different/better approaches to address them.
  - 2) Switching architecture is essential: first combiner tracks targets, does phase referencing; second combiner chops from target to reference
  - 3) Most errors are inverse with baseline. Most calculations shown were for B=100 m, but go as long as you can
    - » practical limit is where you resolve the bright stars
  - 4) Pay attention to effective throughput. Interferometers are notorious for low throughput. High SNR makes things easier and faster.