Principle of Image Synthesis in Optical/IR Interferometry

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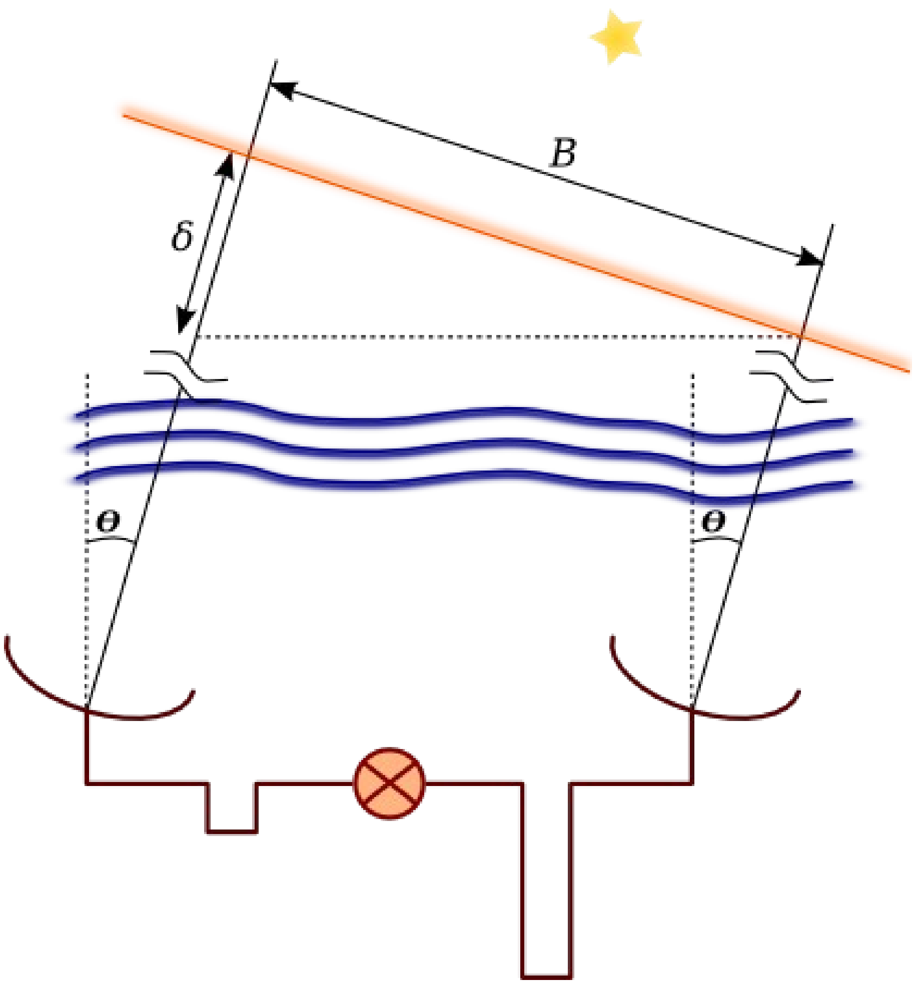
Interferometric Data

Principle of Optical Interferometry



output = instantaneous
complex visibility:

$$V_{j,k}(t) = \hat{I}(\boldsymbol{\nu}_{j,k}(t))\, g_j^*(t)\, g_k(t)$$

Fourier transform of object brightness distribution

“gains” = transfer functions of *j*-th and *k*-th telescopes

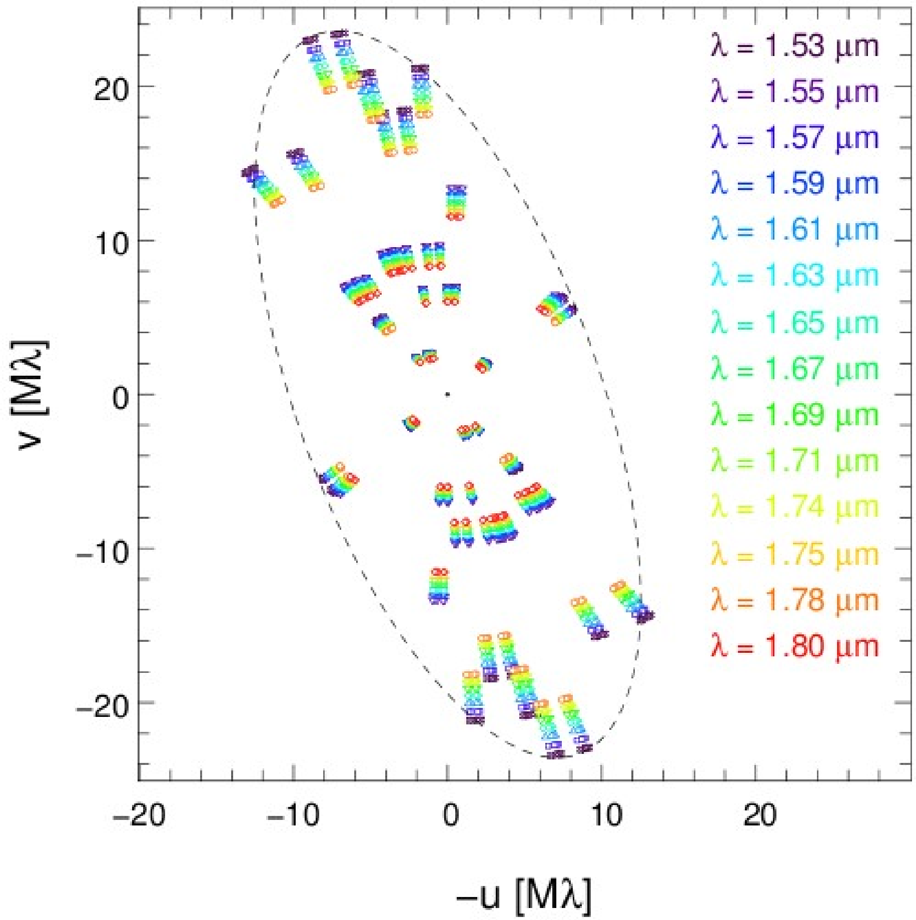
spatial frequency:

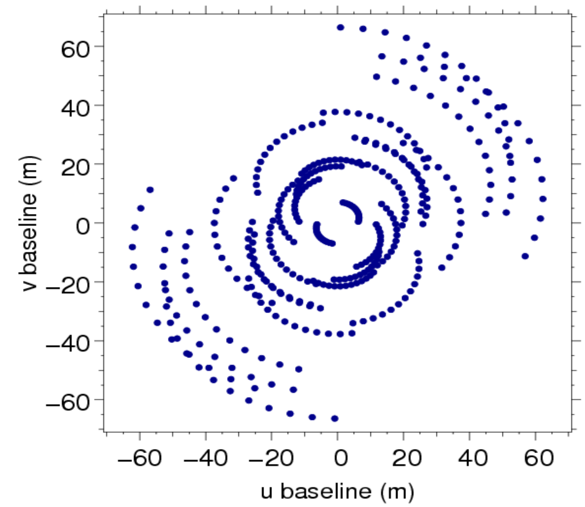
projected baseline ($\boldsymbol{B}$)

$$\boldsymbol{\nu}_{j,k}(t) = \frac{\boldsymbol{r}_k(t) - \boldsymbol{r}_j(t)}{\lambda}$$

wavelength

Sparse u-v coverage





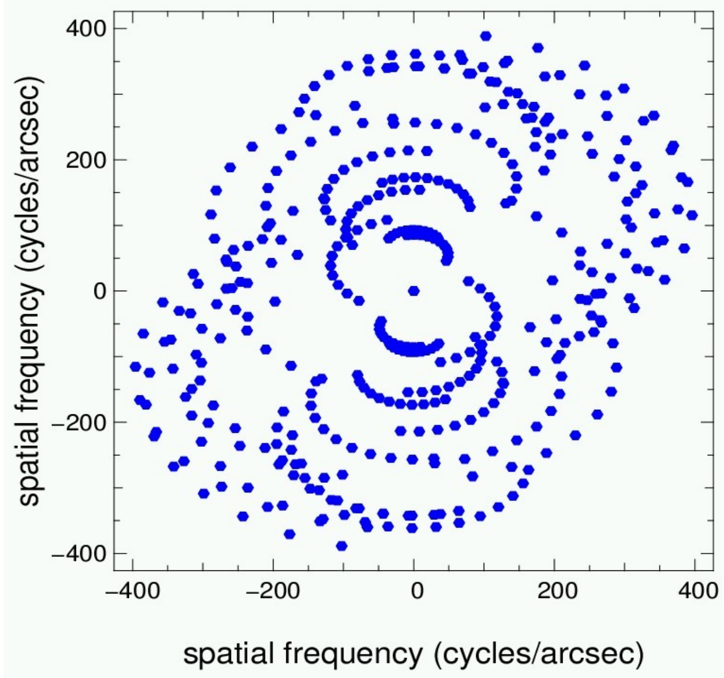


Image model and its Fourier transform

general linear expansion:

$$I(\boldsymbol{\xi}) = \sum_{k=1}^{k=N} x_k \, b_k(\boldsymbol{\xi}) \xrightarrow{\text{F.T.}} \hat{I}(\boldsymbol{\nu}) = \sum_{k=1}^{k=N} x_k \, \hat{b}_k(\boldsymbol{\nu})$$

parameters

functions basis

grid model:

$$I(\boldsymbol{\xi}) = \sum_{k=1}^{k=N} x_k \, b(\boldsymbol{\xi} - \boldsymbol{\xi}_k) \xrightarrow{\text{F.T.}} \hat{I}(\boldsymbol{\nu}) = \hat{b}(\boldsymbol{\nu}) \sum_{k=1}^{k=N} x_k \, \mathrm{e}^{-2\,\mathrm{i}\,\pi\,\boldsymbol{\theta}_k \boldsymbol{\nu}}$$

parameters
(N = number of «*pixels*»)

«*pixel*» function

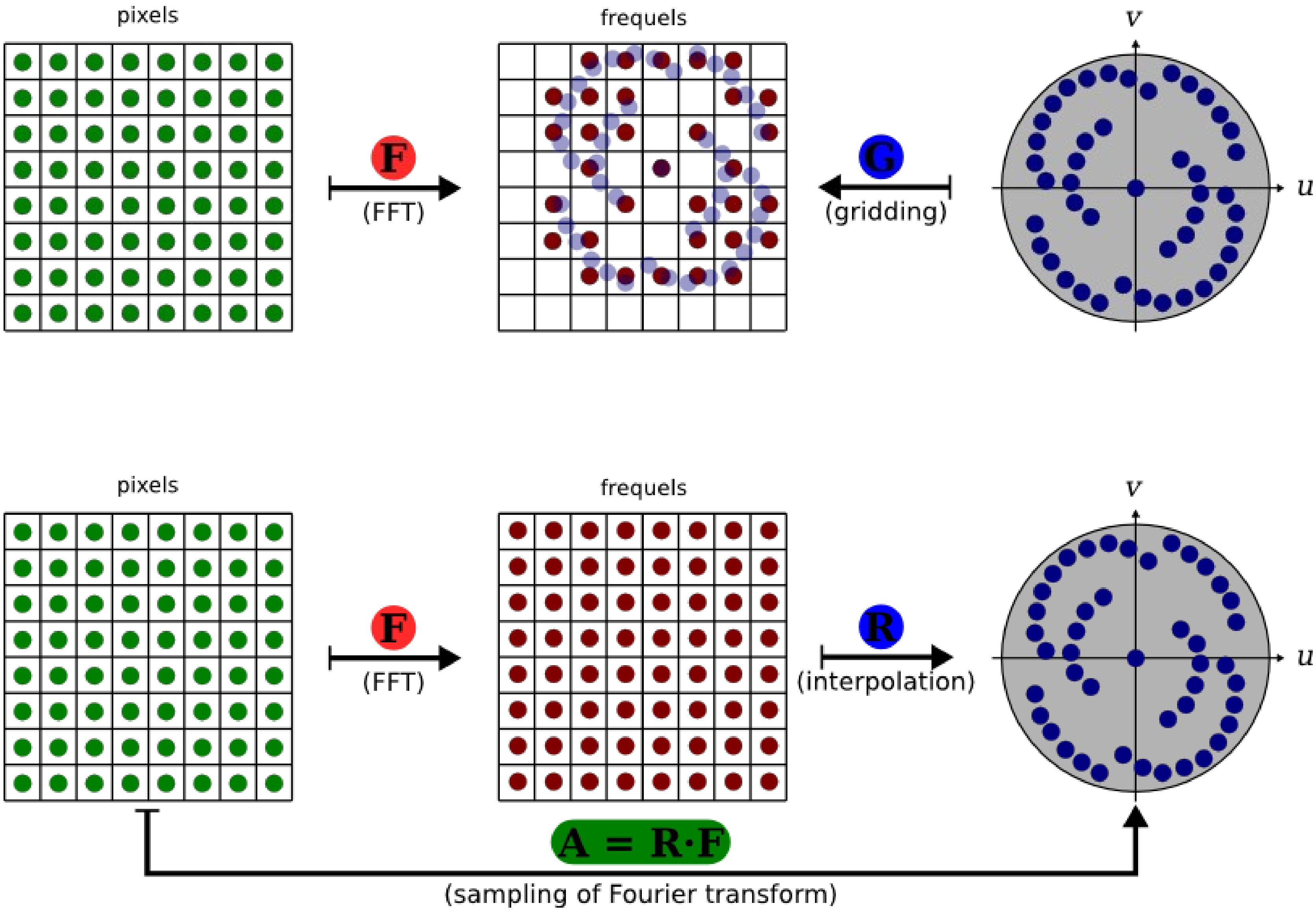
Complex visibility model

irregular spectral sampling: $$y_j \equiv \hat{I}(\boldsymbol{\nu}_j) = \sum_{k=1}^{k=N} A_{j,k}\, x_k$$

$$
\begin{aligned}
A_{j,k} &= \hat{b}_k(\boldsymbol{\nu}_j) && \text{general linear model} \\
&= \hat{b}(\boldsymbol{\nu}_j)\, \mathrm{e}^{-2\,\mathrm{i}\,\pi\,\boldsymbol{\theta}_k \cdot \boldsymbol{\nu}_j} && \text{grid linear model} \\
&= \mathrm{e}^{-2\,\mathrm{i}\,\pi\,\boldsymbol{\theta}_k \cdot \boldsymbol{\nu}_j} && \text{idem + sinc basis function}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{y} &= \mathbf{A}\cdot\boldsymbol{x} \\
&= \underbrace{\mathbf{R}}_{\text{spectral interpolation}}\cdot\underbrace{\mathbf{F}}_{\text{FFT}}\cdot\boldsymbol{x}
\end{aligned}
$$

Gridding *vs.* spectral interpolation



Inverse Problem Approach

The *direct* model

Model of data:

$$\boldsymbol{y} = \boldsymbol{m}(\boldsymbol{x}) + \boldsymbol{e}$$

$\boldsymbol{y} \in \mathbb{R}^m$ data

$\boldsymbol{x} \in \mathbb{R}^n$ parameters

$\boldsymbol{m}: \mathbb{R}^n \rightarrow \mathbb{R}^m$ model

$\boldsymbol{e} \in \mathbb{R}^m$ errors (noise + model approximations)

What are the best parameters? Maximum likelihood approach

in the ***maximum likelihood*** sense, the ***best*** parameters are the ones which maximize the probability to have obtained the data:

$$\boldsymbol{x}_{\mathrm{ML}} = \arg\max_{\boldsymbol{x}} \Pr\left(\boldsymbol{y} \middle| \boldsymbol{m}(\boldsymbol{x})\right)$$

with: ***x*** = parameters

m(***x***) = model

y = data

$$\boldsymbol{x}_{\mathrm{ML}} = \arg\min_{\boldsymbol{x}} f_{\mathrm{data}}(\boldsymbol{x})$$

$$f_{\mathrm{data}}(\boldsymbol{x}) = -c_1 \log \Pr\left(\boldsymbol{y} \middle| \boldsymbol{m}(\boldsymbol{x})\right) + c_0 \qquad c_1 > 0$$

ML = *Maximum Likelihood*

Likelihood penalty for complex visibility data

$$\boldsymbol{x}_{\mathrm{ML}} = \arg\min_{\boldsymbol{x}} f_{\mathrm{data}}(\boldsymbol{x})$$

$$\begin{aligned}
f_{\mathrm{data}}(\boldsymbol{x}) &= -2 \log \Pr(\boldsymbol{y}|\boldsymbol{m}(\boldsymbol{x})) + c_0 \\
&= (\boldsymbol{y}-\boldsymbol{m}(\boldsymbol{x}))^{\mathrm{T}}.\mathbf{W}.(\boldsymbol{y}-\boldsymbol{m}(\boldsymbol{x})) && \text{Gaussian statistics} \\
&= \|\boldsymbol{y}-\boldsymbol{m}(\boldsymbol{x})\|_{\mathbf{W}}^2 \\
&= \sum_j w_j (y_j - m_j(\boldsymbol{x}))^2 && \text{independent data} \\
&= \boxed{\sum_t \sum_{j,k} w_{j,k}(t) \left| V_{j,k}^{\mathrm{data}}(t) - V_{j,k}^{\mathrm{model}}(t) \right|^2} && \text{Goodman model}
\end{aligned}$$

with $\boldsymbol{V}_{\mathrm{model}} = \mathbf{A}.\boldsymbol{x}$

$$\mathbf{W} = \mathrm{Cov}^{-1}(\boldsymbol{e})$$

$$\boldsymbol{y} = \boldsymbol{V}_{\mathrm{data}}$$

$$\boxed{f_{\mathrm{data}}(\boldsymbol{x}) = \|\boldsymbol{y}-\mathbf{A}.\boldsymbol{x}\|_{\mathbf{W}}^2}$$

Maximum likelihood solution for complex visibility data

$$
\begin{aligned}
\boldsymbol{x}_{\mathrm{ML}} &= \arg\min_{\boldsymbol{x}} f_{\mathrm{data}}(\boldsymbol{x}) \\
&= \|\boldsymbol{y} - \mathbf{A}\,.\,\boldsymbol{x}\|^2_{\mathbf{W}}
\end{aligned}
$$

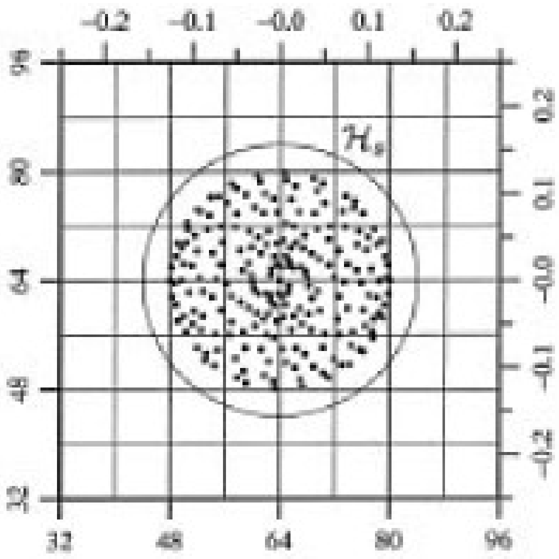
no unique solution (due to voids in *u-v* coverage)

using the generalized inverse:

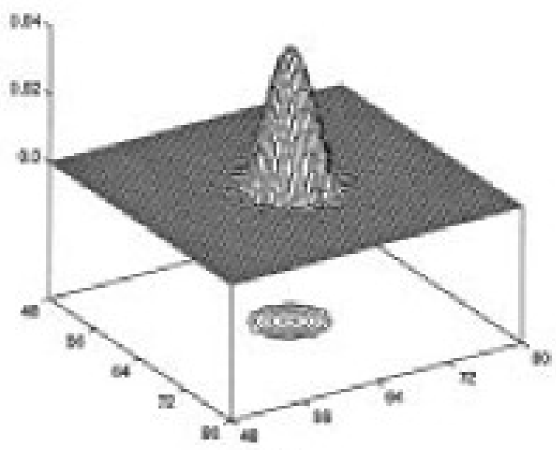
$$
\boldsymbol{x}_{\mathrm{ML}} = \left(\mathbf{A}^{\mathrm{T}}.\mathbf{W}.\mathbf{A}\right)^{+}.\mathbf{A}^{\mathrm{T}}.\mathbf{W}.\boldsymbol{y}
$$

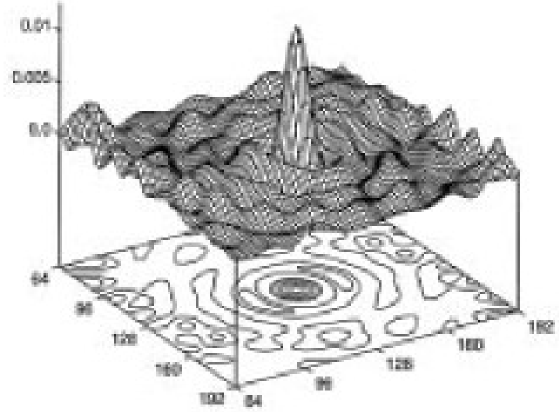
which is the ***dirty map***

Dirty beam and dirty map



(a)


(b)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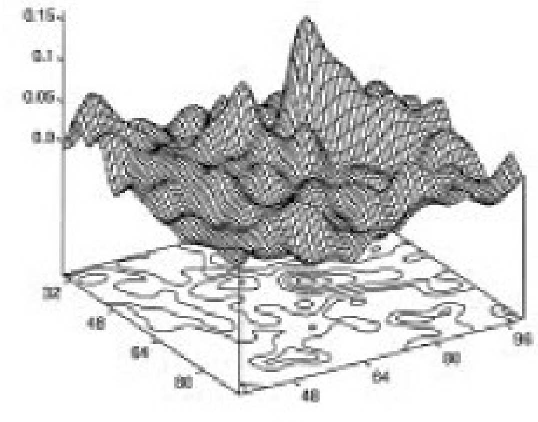
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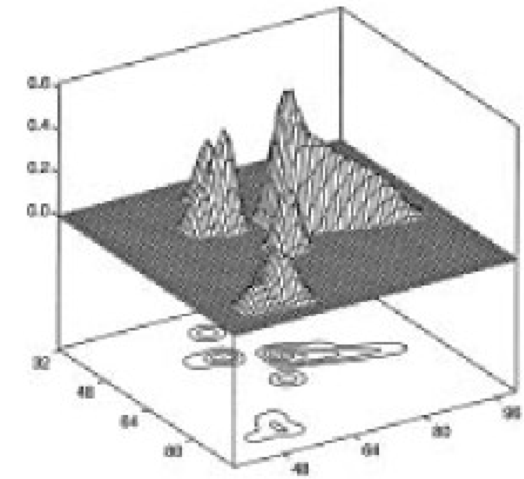
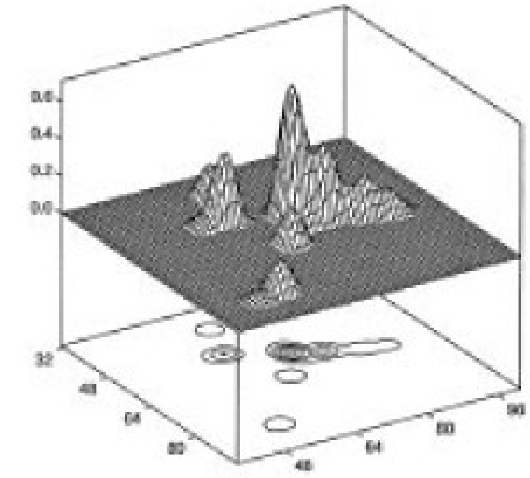
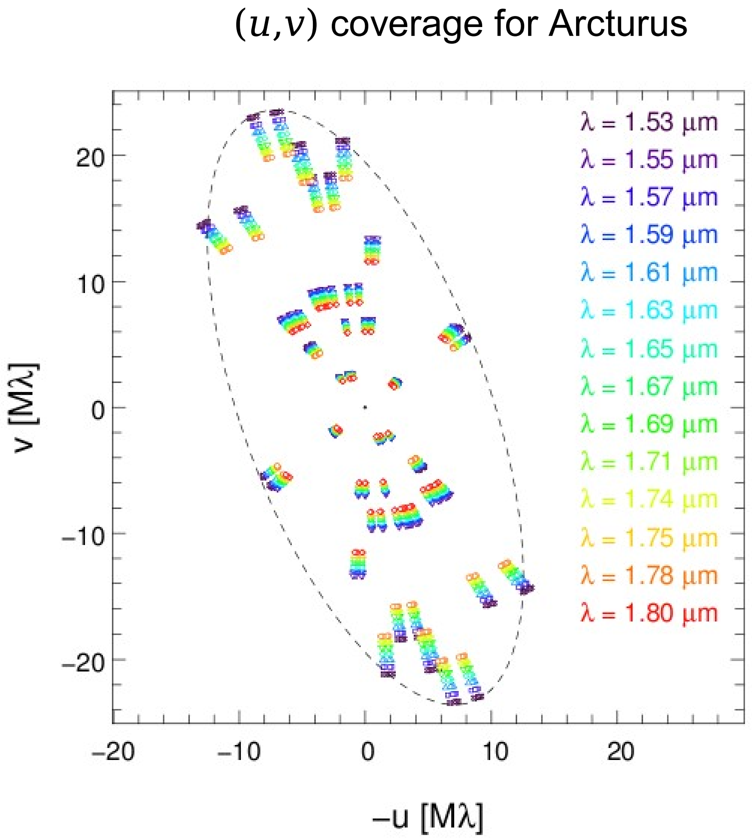
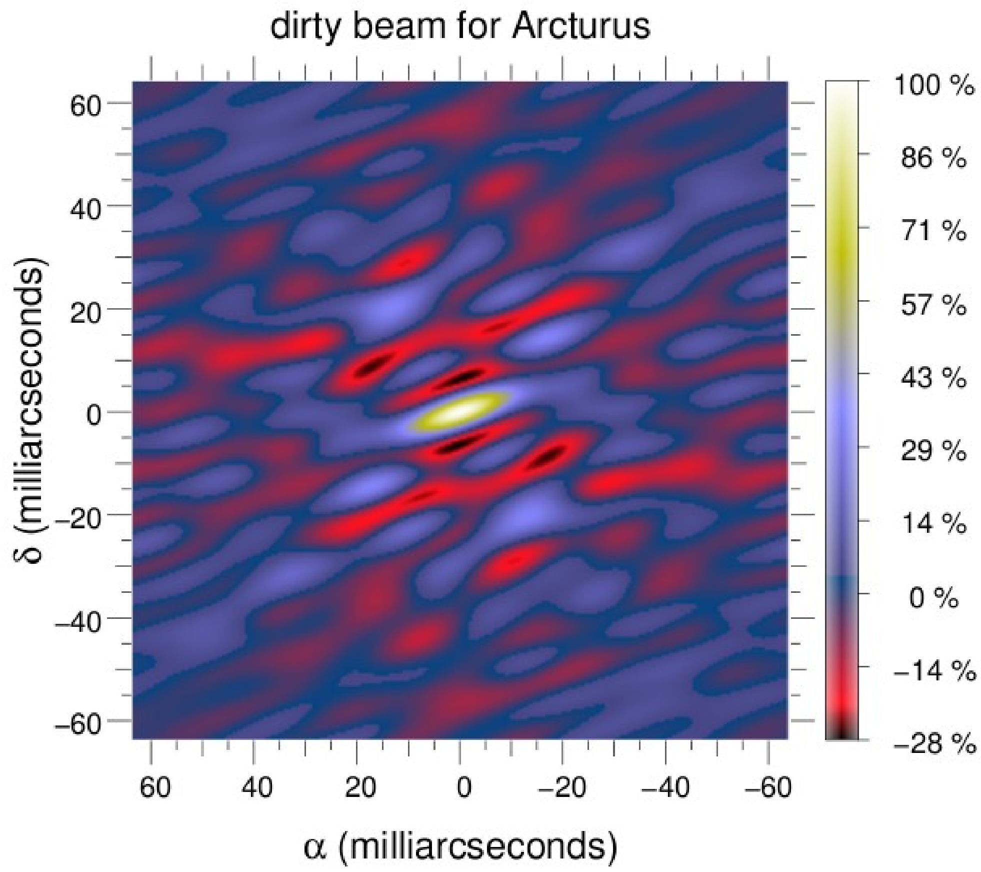

(c)


(d)

Lannes et al. (1997)

Dirty Beam = Point Spread Function





objet : α Boo (Arcturus)
IOTA/IONIC interferometer
source : S. Lacour *et al.* (2007)

Heuristics for Image Reconstruction

solving

$$\hat{I}(\boldsymbol{\nu}_{j,k}(t)) = V^{\text{data}}_{j,k}(t) \quad \forall j,k,t$$

for the image have ***no sense***

- maybe no solution
- maybe an infinite number of solutions (voids in u-v coverage can be assigned any value)
- no constraints such as positivity, support, flux
- fit the noise (amplification of noise when model is ill-conditioned)

intuitively

- model (F.T. of image) must be as close to data as allowed by noise level but no more
- image must be positive, normalized, *etc.*
- image must be as ***simple*** as possible

Maximum a posteriori: Bayesian Approach

the idea is to ***maximize the probability of the model given the data*** (MAP = *maximum a posteriori*):

$$
\begin{aligned}
\boldsymbol{x}_{\mathrm{MAP}} &= \arg\max_{\boldsymbol{x}} \Pr\left(\boldsymbol{m}(\boldsymbol{x}) | \boldsymbol{y}\right) \\
&= \arg\max_{\boldsymbol{x}} \frac{\Pr(\boldsymbol{y}|\boldsymbol{x}) \ \Pr(\boldsymbol{x})}{\cancel{\Pr(\boldsymbol{y})}} \qquad \text{(Bayes theorem)} \\
&= \arg\min_{\boldsymbol{x}} \{ \underbrace{-\log \Pr(\boldsymbol{y}|\boldsymbol{x})}_{f_{\mathrm{data}}(\boldsymbol{x})} \underbrace{- \log \Pr(\boldsymbol{x})}_{f_{\mathrm{prior}}(\boldsymbol{x})} \} \\
&= \arg\min_{\boldsymbol{x}} f_{\mathrm{MAP}}(\boldsymbol{x})
\end{aligned}
$$

$$
f_{\mathrm{MAP}}(\boldsymbol{x}) = f_{\mathrm{data}}(\boldsymbol{x}) + f_{\mathrm{prior}}(\boldsymbol{x}) \qquad \text{(penalty function)}
$$

Pragmatic Bayesian Approach (1)

- in practice:
  - the statistics of the data is ~ known
  - the a priori statistics is ***unknown***
- the priors must have some (qualitative) properties
  - solve degeneracies (ill-posedness)
  - avoid noise amplification (ill-conditioning)
  - supplement missing information (incomplete data)
- for instance:
  - imposing sufficient smoothness avoids noise amplification
  - imposing compactness helps filling voids in u-v coverage
- hence we may know the ***kind*** of required priors but do not know to what ***level*** they must be imposed

Pragmatic Bayesian Approach (2)

- we want to match the priors (*e.g.* the restored image must be *compact* or *smooth*) as much as possible

$$\min_{\boldsymbol{x}} \; f_{\text{prior}}(\boldsymbol{x})$$

- we want to be compatible with the data

$$f_{\text{data}}(\boldsymbol{x}) \leqslant \eta$$

where η is set according to the noise level

and accounting for constraints: $\boldsymbol{x} \geqslant 0$ $\sum_j x_j = 1$

Pragmatic Bayesian Approach (3) Penalized Likelihood

constrained optimization problem:

$$\boldsymbol{x}^{+} = \arg\min_{\boldsymbol{x}} f_{\text{prior}}(\boldsymbol{x}) \quad \text{s.t.} \quad f_{\text{data}}(\boldsymbol{x}) \leqslant \eta$$

Lagrangian: $L(\boldsymbol{x};\alpha) = f_{\text{prior}}(\boldsymbol{x}) + \alpha\,(f_{\text{data}}(\boldsymbol{x}) - \eta)$

if the constraint is *active*, $\alpha > 0$ and $f_{\text{data}}(\boldsymbol{x}^{+}) = \eta$;
otherwise, the data are unused!

$$\begin{aligned}
\boldsymbol{x}^{+} &= \arg\min_{\boldsymbol{x}} f_{\text{prior}}(\boldsymbol{x}) + \alpha\, f_{\text{data}}(\boldsymbol{x}) \\
&= \arg\min_{\boldsymbol{x}} f_{\text{data}}(\boldsymbol{x}) + \mu\, f_{\text{prior}}(\boldsymbol{x}) \\
&= \arg\min_{\boldsymbol{x}} f(\boldsymbol{x};\mu)
\end{aligned}$$

penalty function: $f(\boldsymbol{x};\mu) = f_{\text{data}}(\boldsymbol{x}) + \mu\, f_{\text{prior}}(\boldsymbol{x})$

with $\mu = 1/\alpha > 0$

Inverse Problem Approach

> **direct model:**

$$\boldsymbol{y} = \boldsymbol{m}(\boldsymbol{x}) + \boldsymbol{e}$$ general (non-linear) model

$$\boldsymbol{y} = \mathbf{M} \cdot \boldsymbol{x} + \boldsymbol{e}$$ linear model

- $\boldsymbol{y}$ = data
- $\boldsymbol{x}$ = parameters (*e.g.* restored image)
- $\boldsymbol{m}$, $\mathbf{M}$ = instrument response
- $\boldsymbol{e}$ = errors (noise + approximations)

> **direct inversion forbidden when**

- $\boldsymbol{m}^{-1}$ (or $\mathbf{M}^{-1}$) does not exist (even approximately)
- noise amplification: $\boldsymbol{x}^{+} = \mathbf{M}^{-1} \cdot \boldsymbol{y} = \boldsymbol{x} + \mathbf{M}^{-1} \cdot \boldsymbol{e}$

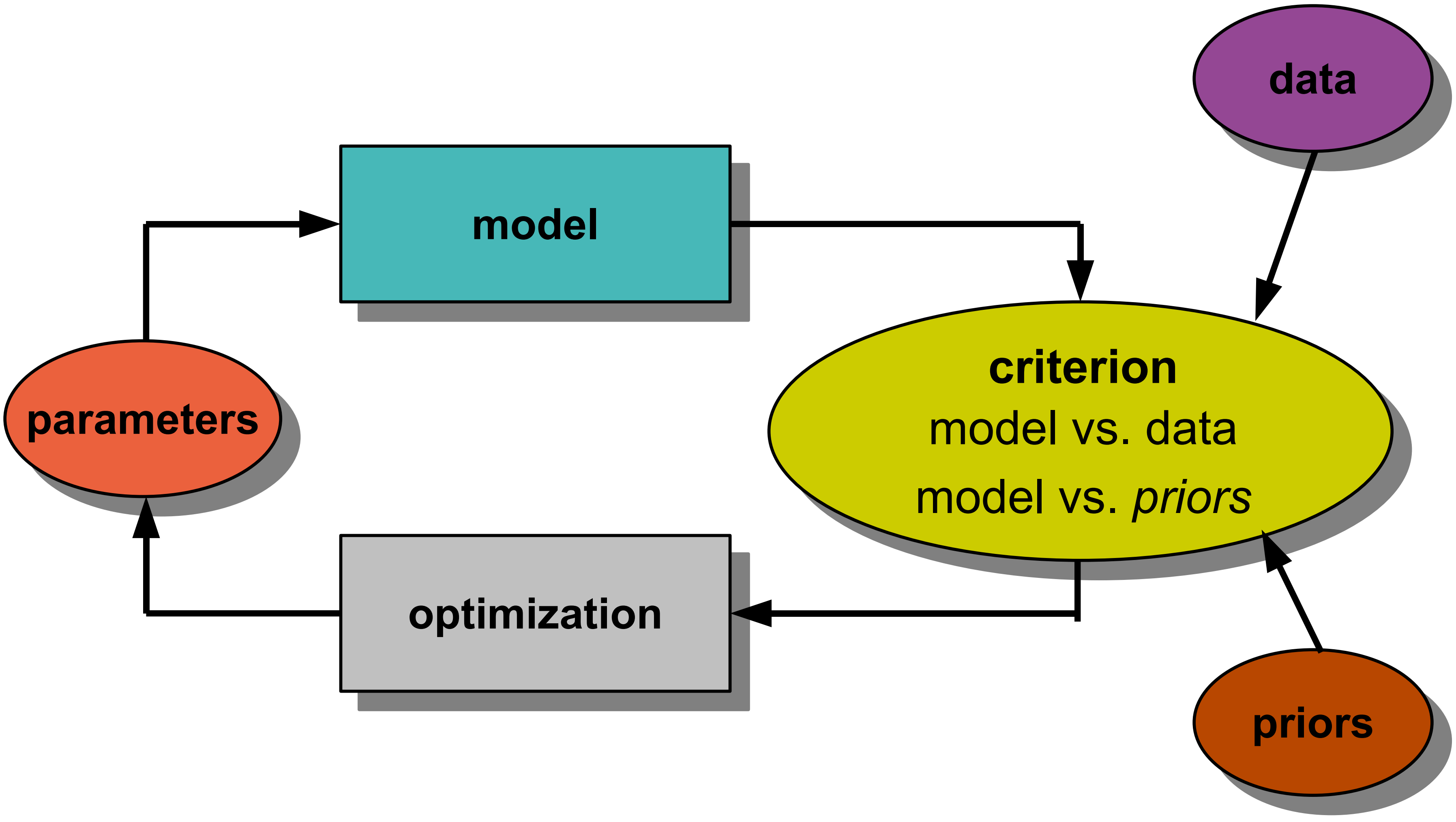
> **inverse problem approach required**

- account for *a priori* constraints (regularization)

$$\boldsymbol{x}^{+} = \arg\min_{\boldsymbol{x}} f_{\text{data}}(\boldsymbol{x}) + \mu\, f_{\text{prior}}(\boldsymbol{x})$$

Inverse Problem Approach

objective: find the ***best*** parameters given the data



Calibration and problems due to turbulence

Measurements in radio-astronomy

instantaneous complex visibility: $V_{j,k}(t) = \hat{I}(\boldsymbol{\nu}_{j,k})\, g_j^*(t)\, g_k(t)$

gain: $g_k(t) = \tau_k(t)\, \mathrm{e}^{\mathrm{i}\,\phi_k(t)}$

random phase shift: $\phi_k(t) = \dfrac{2\,\pi\,\delta_k(t)}{\lambda}$

$$f_{\text{data}}(\boldsymbol{x}, \boldsymbol{g}) = \sum_t \sum_{j,k} w_{j,k}(t) \left| V_{j,k}^{\text{data}}(t) - g_j(t)\, g_k^*(t)\, V_{j,k}^{\text{model}}(t) \right|^2$$

Self-calibration Algorithm

$$f_{\text{data}}(\boldsymbol{x},\boldsymbol{g}) = \sum_t \sum_{j,k} w_{j,k}(t) \left| V_{j,k}^{\text{data}}(t) - g_j(t)\, g_k^*(t)\, V_{j,k}^{\text{model}}(t) \right|^2$$

0. **initialization:** choose μ, and $\boldsymbol{x}^{(0)}$, set $n = 1$
1. **self-calibration:** $\boldsymbol{g}^{(n)} = \arg\min_{\boldsymbol{g}} f_{\text{data}}(\boldsymbol{x}^{(n\text{-}1)}, \boldsymbol{g})$
2. **image reconstruction:** $\boldsymbol{x}^{(n)} = \arg\min_{\boldsymbol{x}} f_{\text{data}}(\boldsymbol{x}, \boldsymbol{g}^{(n)}) + \mu\, f_{\text{prior}}(\boldsymbol{x})$
3. **convergence?** **yes:** terminate with solution $\boldsymbol{x}^{(n)}$;
   no: let $n = n + 1$ and loop to step 1.

Notes:

1. algorithm can be started with initial gains $\boldsymbol{g}^{(0)}$
2. image reconstruction can be any method (CLEAN, MEM, *etc.*)
3. self-calibration is generally non-convex
4. photometric calibration yields $|\boldsymbol{g}| = 1$, only the gain phases are to be found

Problem caused by *fast* turbulence

instantaneous ***complex visibility***: $V_{j,k}(t) = \hat{I}(\boldsymbol{\nu}_{j,k}(t))\, g_j^*(t)\, g_k(t)$

gain: $g_k(t) = \mathrm{e}^{\mathrm{i}\,\phi_k(t)}$ (after photometric calibration)

random phase shift: $\phi_k(t) = \dfrac{2\,\pi\,\delta_k(t)}{\lambda}$

coherence time of turbulence ~ 1 ms (in optical/IR)

simple ***integration***: $\langle V_{i,j}(t)\rangle_k = \hat{I}(\boldsymbol{\nu}_{i,j}(\bar{t}_k))\,\underbrace{\langle g_i^*(t)\;g_j(t)\rangle}_{0} = 0$

non-linear estimators insensitive to this defect must be used

Measurements in Optical/IR Interferometry

instantaneous complex visibility:

$$V_{j,k}(t) = \hat{I}(\boldsymbol{\nu}_{j,k})\, g_j^*(t)\, g_k(t)$$

$$g_k(t) = \mathrm{e}^{\mathrm{i}\,\phi_k(t)}$$

$$\langle V_{j,k}(t)\rangle = \hat{I}(\boldsymbol{\nu}_{j,k})\,\langle g_j^*(t)\, g_k(t)\rangle = 0$$

powerspectrum:

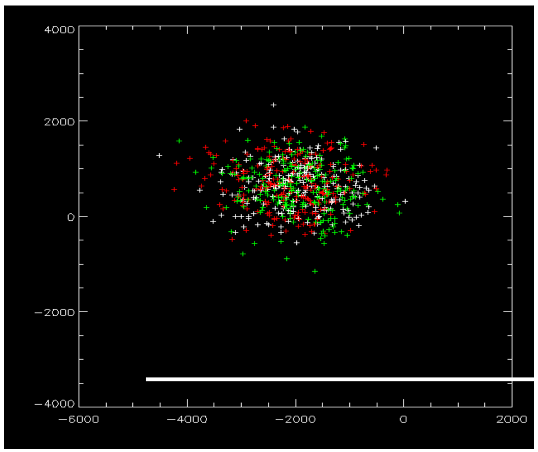
$$\langle |V_{j,k}(t)|^2\rangle = |\hat{I}(\boldsymbol{\nu}_{j,k})|^2$$

bispectrum (a.k.a. triple product):

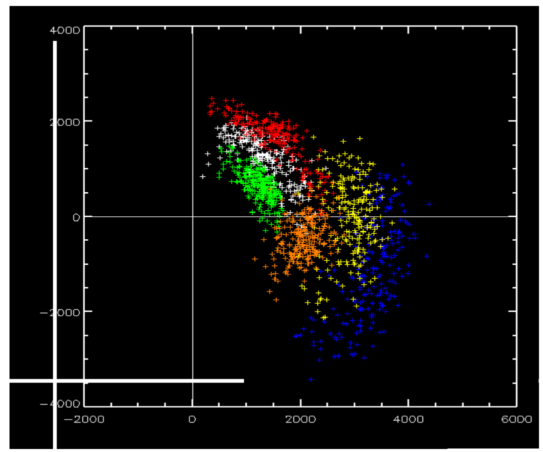
$$\begin{aligned}\langle V_{i,j}(t)\, V_{j,k}(t)\, V_{k,i}(t)\rangle &= \hat{I}(\boldsymbol{\nu}_{i,j})\,\hat{I}(\boldsymbol{\nu}_{j,k})\,\hat{I}(\boldsymbol{\nu}_{k,i})\,\langle \mathrm{e}^{\mathrm{i}[\phi_j(t)-\phi_i(t)]+\mathrm{i}[\phi_k(t)-\phi_j(t)]+\mathrm{i}[\phi_i(t)-\phi_k(t)]}\rangle \\ &= \hat{I}(\boldsymbol{\nu}_{i,j})\,\hat{I}(\boldsymbol{\nu}_{j,k})\,\hat{I}^*(\boldsymbol{\nu}_{i,k}+\boldsymbol{\nu}_{j,k})\end{aligned}$$

Statistics of Real Data

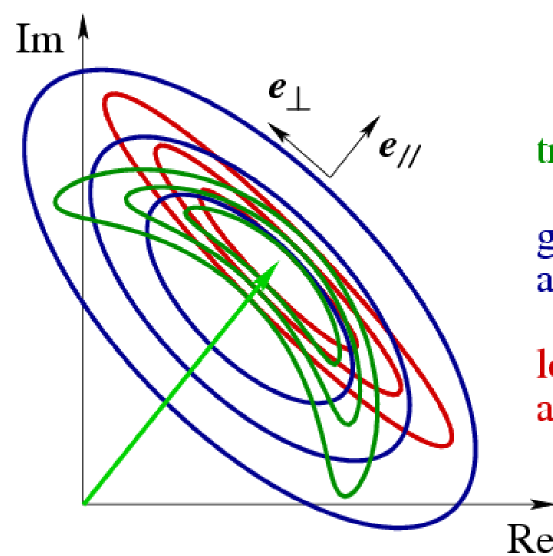
low SNR



high SNR



(triple product of FKV0509, C. Hummel *et al.*)



(source: S. Meimon, 2006)

Data Penalty Term

radio-astronomy

for complex visibility data:

$$f_{\mathrm{data}}(\boldsymbol{x}) = (\mathbf{A}\cdot\boldsymbol{x}-\boldsymbol{y})^{\mathrm{T}}\cdot\mathbf{W}\cdot(\mathbf{A}\cdot\boldsymbol{x}-\boldsymbol{y}) = \frac{1}{\sigma^2}\,\|\mathbf{A}\cdot\boldsymbol{x}-\boldsymbol{y}\|^2$$

hypothesis: Gaussian noise,
independent measurements

optical/IR interferometry

for bispectral data:

$$f_{\mathrm{data}}(\boldsymbol{x}) = \sum_k \frac{1}{\sigma_k^2}\left|\hat{x}_{j_{1,k}}\,\hat{x}_{j_{2,k}}\,\hat{x}^*_{j_{3,k}} - d_k\right|^2$$

with $\hat{\boldsymbol{x}} \equiv \mathbf{A}\cdot\boldsymbol{x}$

independent amplitude and phase data (***phase wrapping***)

non-convex

Algorithms

CLEAN

CLEAN (J. A. Högbom, 1974)

- point-like sources
- data: complex visibilities
- *matching pursuit* algorithm
- objective: find the N most significant point-like sources

$$f_{\text{prior}}(\boldsymbol{x}) = \mu \, \|\boldsymbol{x}\|_0 = \mu \, \text{Card}(\{ j \,; x_j \neq 0 \})$$

- can be approximated by (***Compressed Sensing***, E. Candès et al., 2006) :

$$f_{\text{prior}}(\boldsymbol{x}) = \mu \, \|\boldsymbol{x}\|_1 = \mu \sum_j |x_j|$$

Clean algorithm:
- initialize the residual dirty map to be the dirty map
- match the residual dirty map with the dirty beam
- remove a fraction of the peak intensity
- repeat until convergence
- convolve the spiky image with the clean beam and add the final residual dirty map

Maximum Entropy Methods

- **MEM** (review by R. Narayan & R. Nityananda, 1986)
  - model image: pixels
  - data: complex visibilities
  - régularization (neg-entropie, insures positivity)

$$f_{\text{prior}}(\boldsymbol{x}) = \mu \sum_j \left[\bar{x}_j - x_j + x_j \log(x_j/\bar{x}_j) \right]$$

  - reconstruction by non-linear optimisation in a local sub-space of search directions with μ tuned on the fly (J. Skilling & R. K. Bryan, 1984)

- **BSMEM** (D. Buscher, 1994)
  - idem for bispectrum data

Quadratic Priors

- quadratic regularization: $$f_{\text{prior}}(\boldsymbol{x}) = \mu\,(\mathbf{B}\cdot\boldsymbol{x}-\boldsymbol{c})^{\mathrm{T}}\cdot\mathbf{Q}\cdot(\mathbf{B}\cdot\boldsymbol{x}-\boldsymbol{c})$$
- least norm: $$f_{\text{prior}}(\boldsymbol{x}) = \mu\,\|\boldsymbol{x}\|_2^2 = \mu\sum_j x_j^2$$
- smoothness: $$f_{\text{prior}}(\boldsymbol{x}) = \mu\,\|\mathbf{D}\cdot\boldsymbol{x}\|_2^2$$
- Gaussian prior: $$f_{\text{prior}}(\boldsymbol{x}) = \mu\,(\boldsymbol{x}-\bar{\boldsymbol{x}})^{\mathrm{T}}\cdot\mathrm{Cov}(\boldsymbol{x})^{-1}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})$$
- **WIPE** (A. Lannes *et al.*, 1994): damping of data (complex visibilities) + avoid high frequencies $$f_{\text{prior}}(\boldsymbol{x}) = \sum_{k,\,\|\boldsymbol{\nu}_k\|>\nu_{\text{cut}}} |\hat{x}_k|^2$$

Other Algorithms

- **Building-Blocks** (K.-H. Hofmann & G. Weigelt, 1993)
  - image model: building blocks
  - data: bispectre
  - iterative reconstruction (linear approximation of the criterion and steepest descent)
  - objective: find the *N* most significant components

  ~ CLEAN for bispectrum

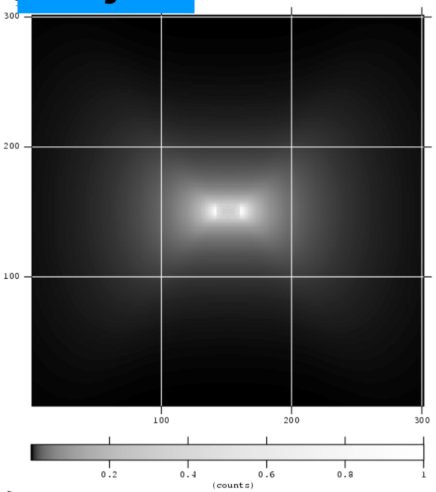
- multi-résolution (J.-F. Giovannelli & A. Coulais, 2005)
  - quadratiqc (quasi-L1 for point-like components) + positivity + support

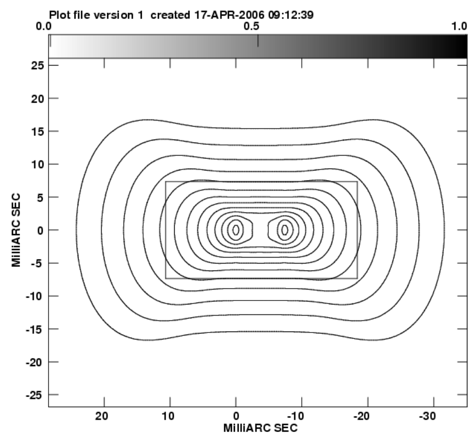
- MArkov Chain IMager (**MACIM**, M. Ireland *et al.*, 2006)

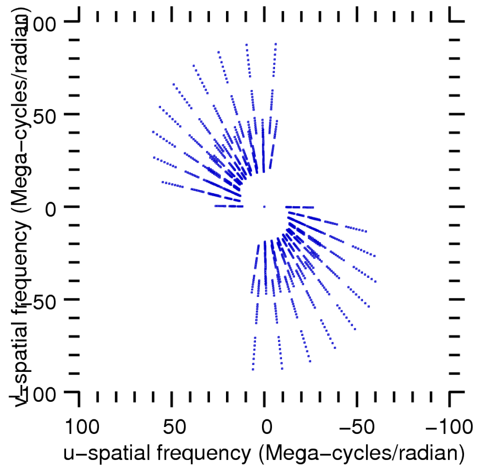
Imaging Beauty Contest 2006

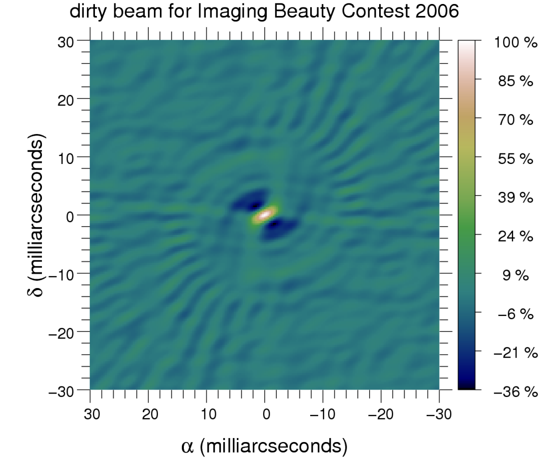
(Lawson *et al.*, 200

objet

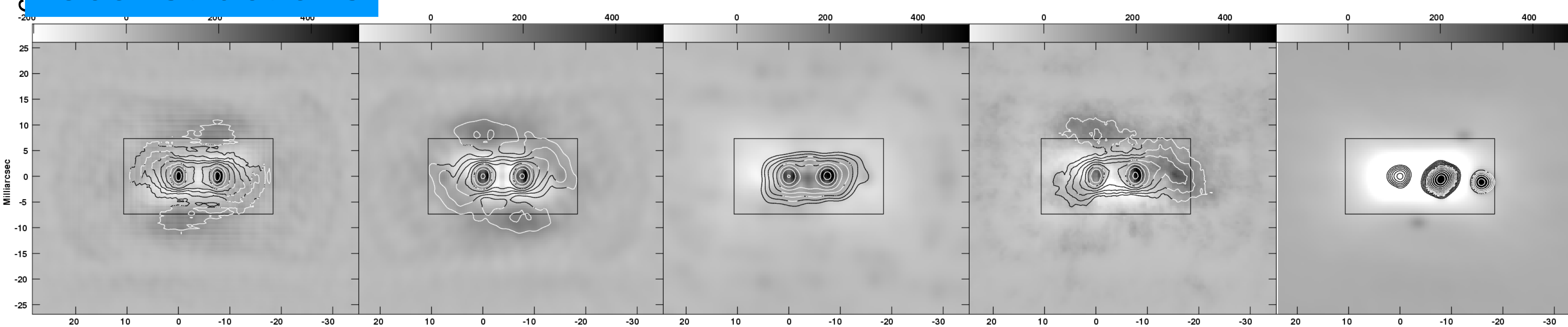








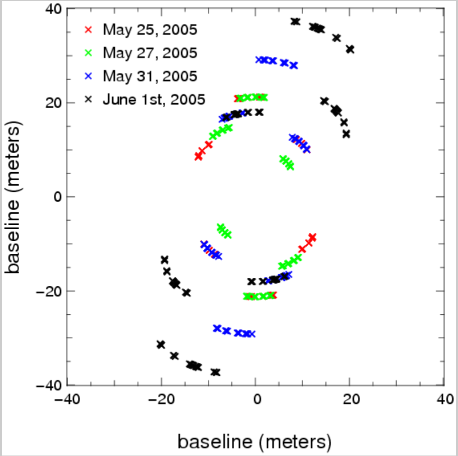
reconstructions

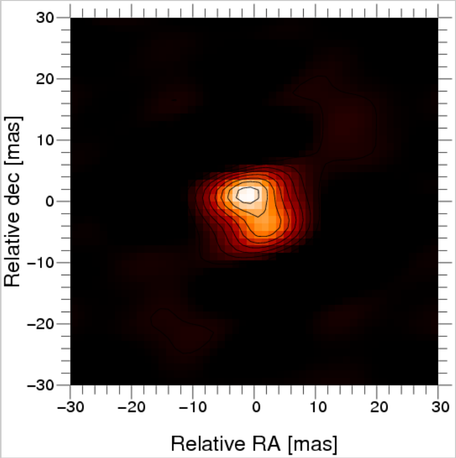


BSMEM | MIRA | building-blocks | MACIM | recursive phase reconstruction

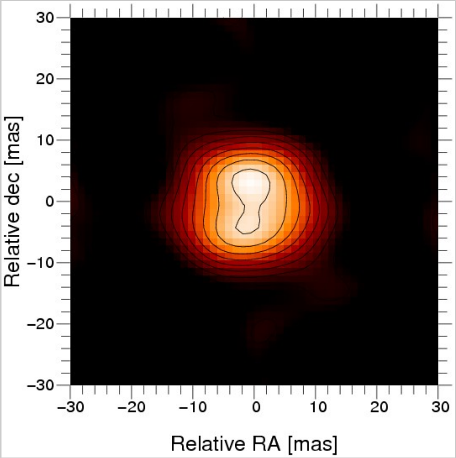
Application à des données *réelles*

Chi-Cygni



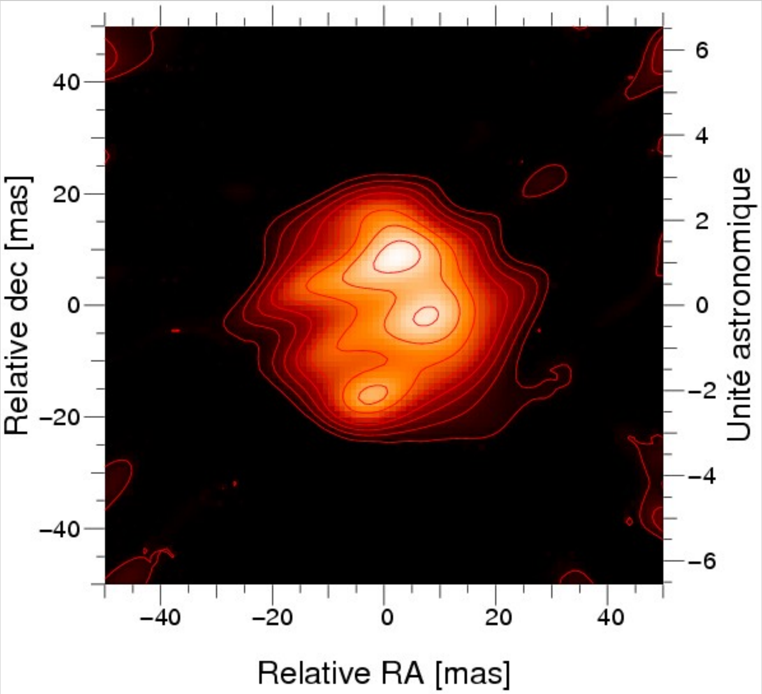


(mai-juin 2005)



(octobre 2005)

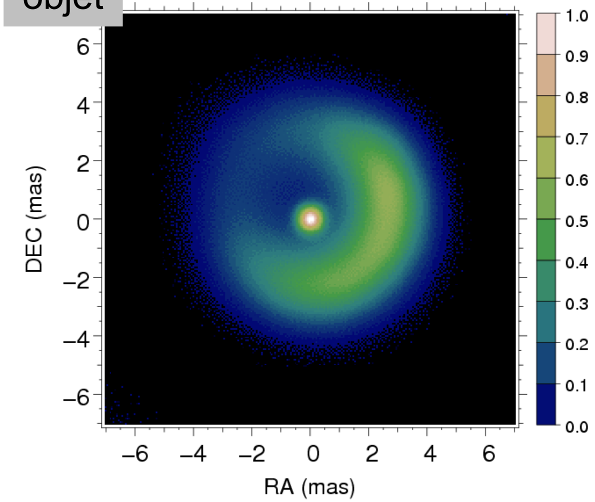
Bételgeuse



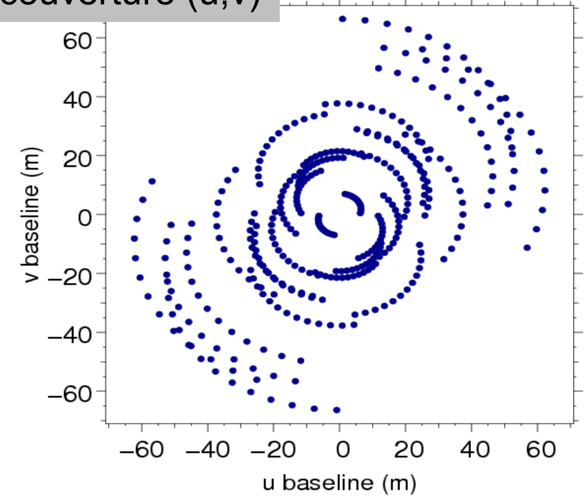
Source : **S. Lacour** *et al.*
Instrument : **IOTA** (H band 1,65µm)
Algorithme : **MIRA**

Reconstruction avec/sans information de phase

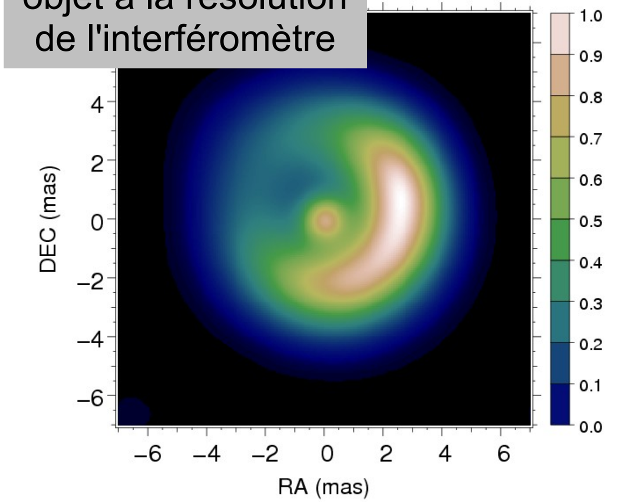
objet

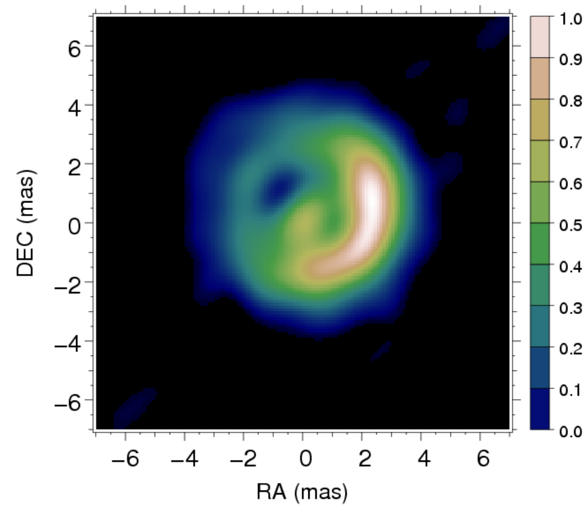


couverture (u,v)

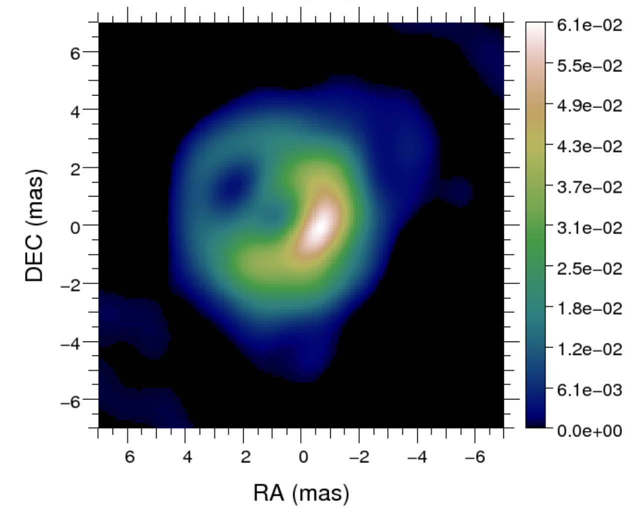


objet à la résolution de l'interféromètre





reconstruction (spectre de puissance + clôtures)



reconstruction (spectre de puissance)

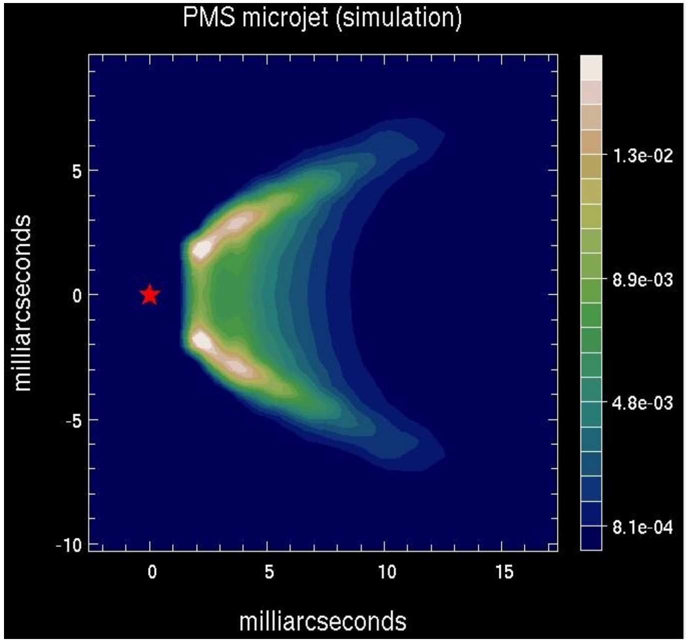
Difficulties

- non-linear
- sparse *u-v* couverage
- incomplete Fourier phase information
  (*e.g.* 1 phase closure out of 3 amplitudes)
- constraints
  - positivity
  - normalization (calibration)
- Fourier transform with irregular spectral sampling
- multi-modal optimization (needs ***global*** optimization)

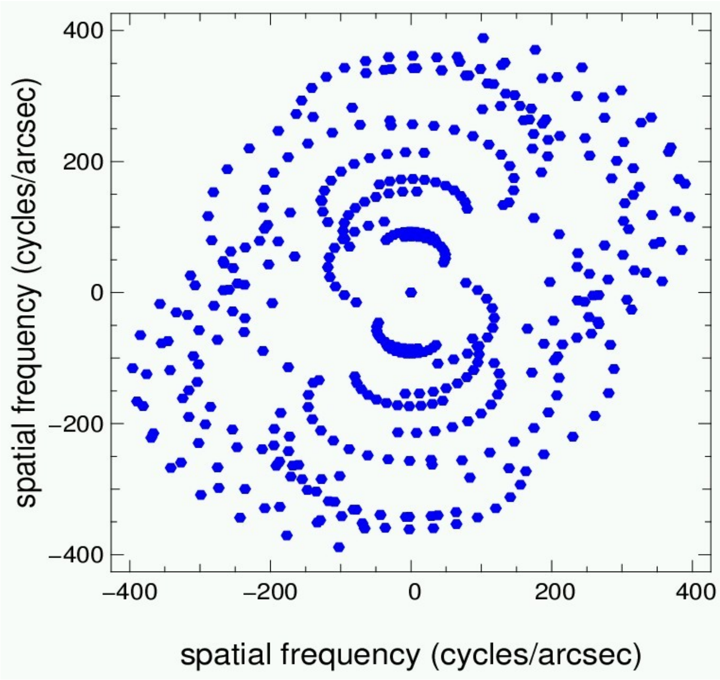
Regularization

Pre-Main Sequence Star Simulation

micro-jet emitted a PMS star
(model by P. Garcia et al.)

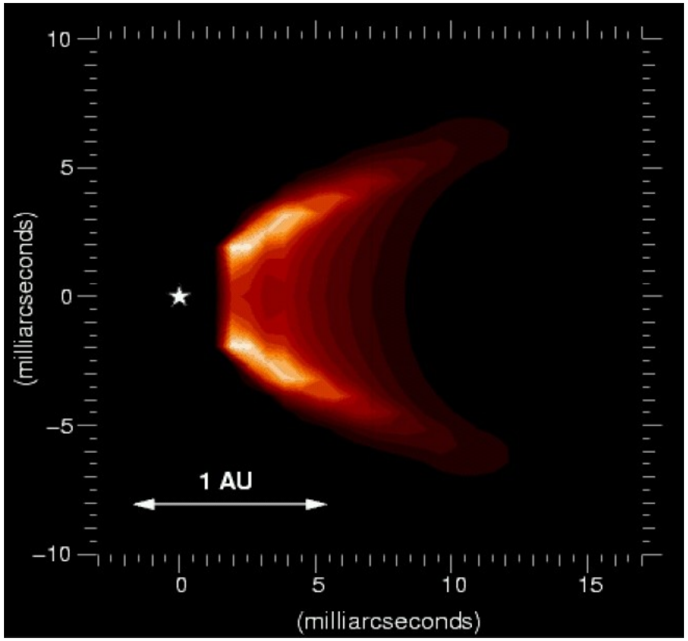


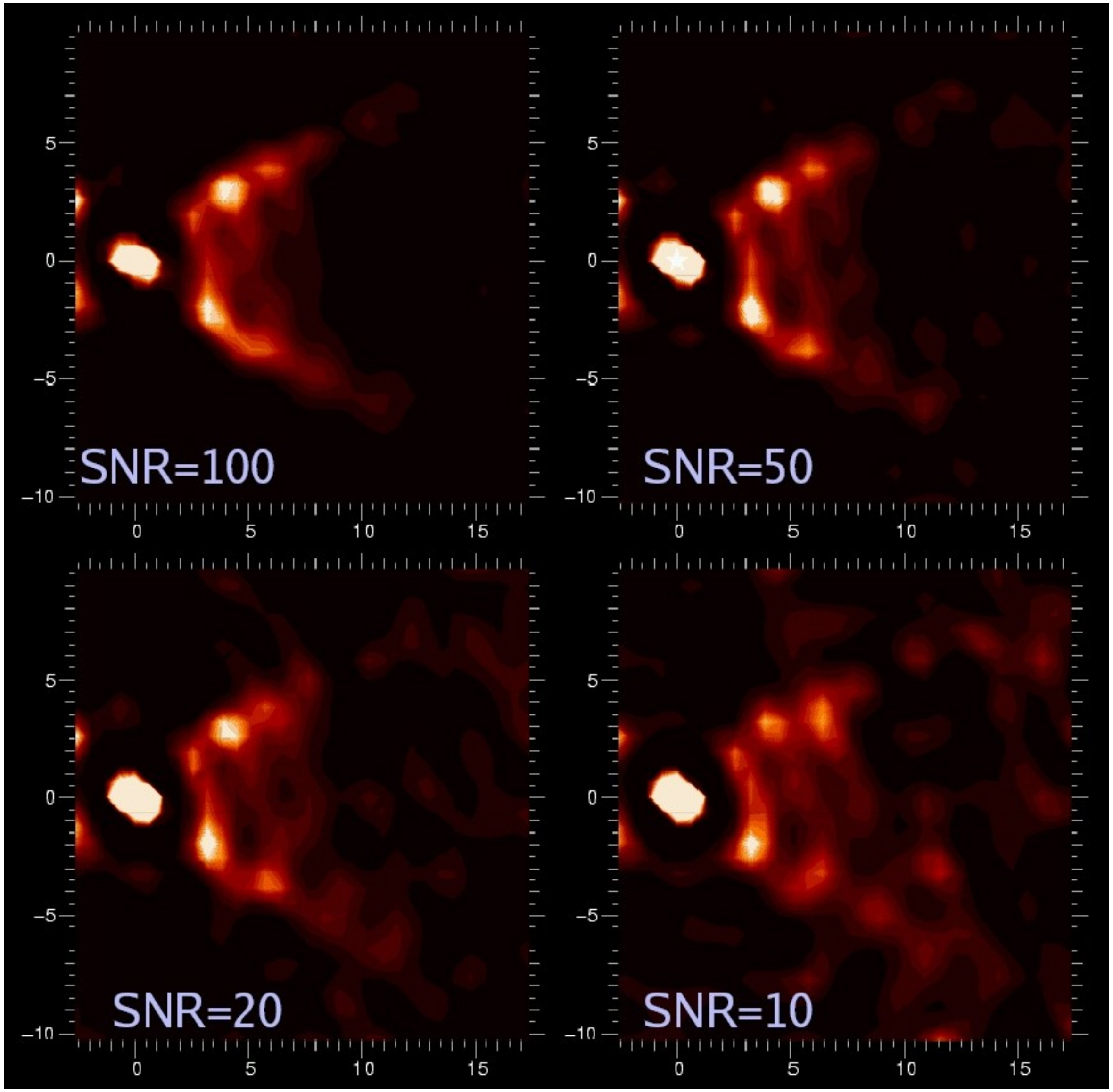
u-v coverage



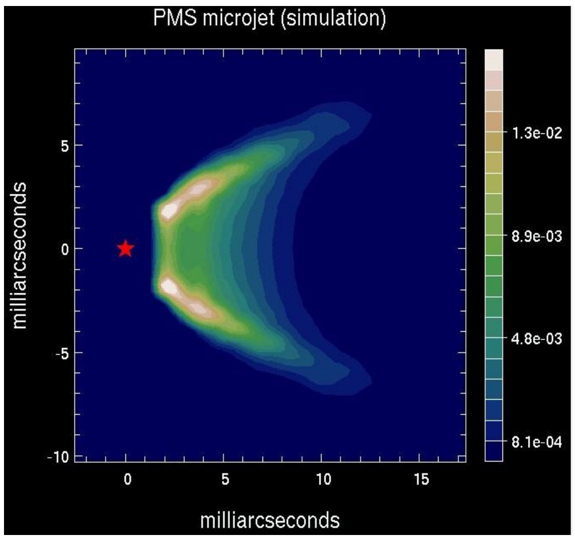
- 6 observing nights
- 6 configurations with 3 AT's
- ➔ 190 powerspectrum data
- ➔ 63 phase closures
- ➔ 1024 unknowns

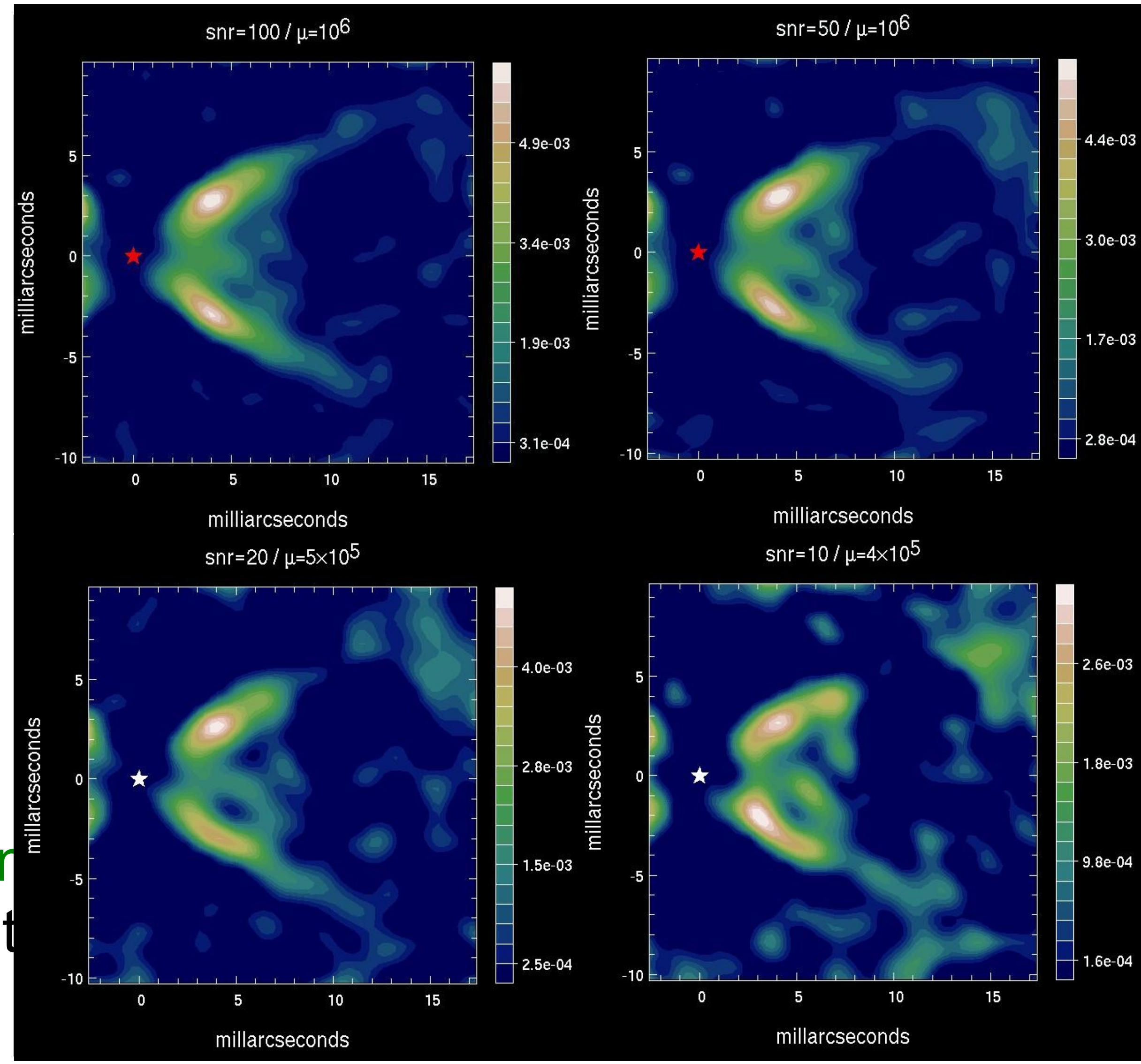
Bad Regularization Type





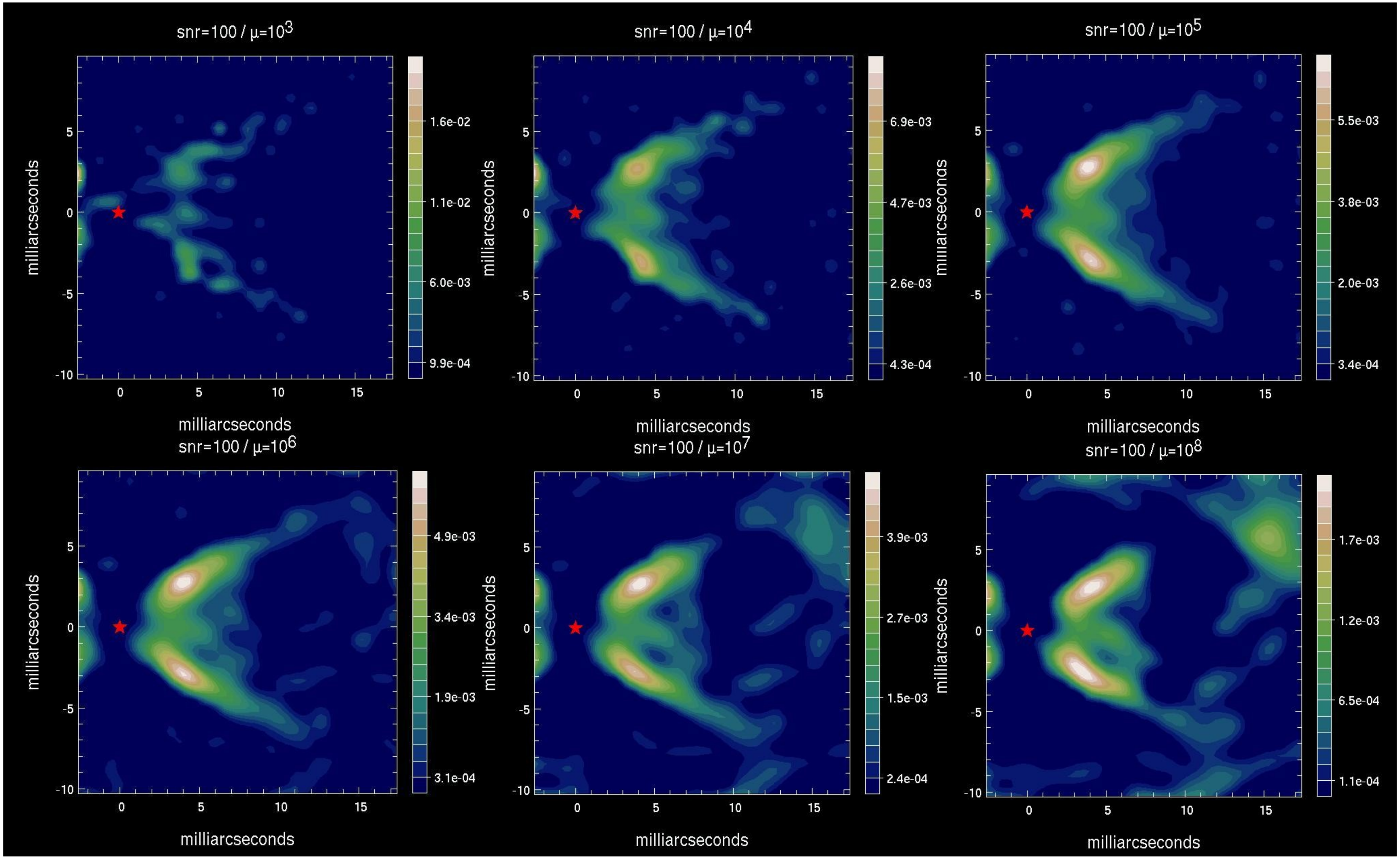
Effects of the SNR





- ✔ micro-jet identifiable
- ✔ opening angle can be estin
- ✔ only local optimization (not
- ✔ super-resolution (×4)

Varying the Regularization Level



Mira: a Multi-telescope Image Reconstruction Algorithm

Image reconstruction problem

direct model:

$$I(\boldsymbol{\xi}) = \sum_{k=1}^{k=N} x_k \, b_k(\boldsymbol{\xi}) \xrightarrow{\text{T.F.}} \hat{I}(\boldsymbol{\nu}_j) = \sum_{k=1}^{k=N} A_{j,k} \, x_k$$

spectral interpolation

parameters

function basis

image reconstruction re-stated as a ***constrained optimization problem***:

$$\boldsymbol{x}_{\text{best}} = \arg\min_{\boldsymbol{x}} \{ f_{\text{data}}(\boldsymbol{x}) + f_{\text{prior}}(\boldsymbol{x}) \} \quad \text{s.t.:} \quad x_k \geq 0, \forall k \quad \text{and} \quad \sum_{k=1}^{k=N} x_k = 1$$

data penalty (model must be *compatible* with the data)

regularization (accounts for a priori knowledge, *e.g.* compactness)

positivity

normalization

Mira algorithm in a nutshell

- Mira aims at minimizing the penalty

$$f(\boldsymbol{x};\mu) = f_{\mathrm{data}}(\boldsymbol{x}) + \mu\, f_{\mathrm{prior}}(\boldsymbol{x})$$

  by local minimization
- requirements:
  - input data (complex visibility, powerspectrum, bispectrum)
  - regularization type
  - regularization level
  - initial solution

you can get 0.9 version of MIRA at:

`http://www-obs.univ-lyon1.fr/labo/perso/eric.thiebaut/files/mira-0.9.tar.bz2`

Using Mira algorithm... simple image reconstruction

0. Load Mira software:
```
include, “mira.i”;
```

1. Load input data into opaque object db:
```
db = mira_new(MIRA_HOME+"data/data1.oifits");
```

2. Configure for image reconstruction:
```
mira_config, db, xform="exact", dim=100,
   pixelsize=0.4*MIRA_MILLIARCSECOND;
```

3. Choose a regularization method:
```
rgl = rgl_new("smoothness");
```

4. Attempt an image reconstruction (from scratch):
```
dim = mira_get_dim(db); img0 = array(double, dim, dim);
img0(dim/2, dim/2) = 1.0;
img1 = mira_solve(db, img0, maxeval=500, verb=1, xmin=0.0,
   normalization=1, regul=rgl, mu=1e6);
```

5. Continue reconstruction with recentered image:
```
img1 = mira_solve(db, mira_recenter(img1), maxeval=500,
   verb=1, xmin=0.0, normalization=1, regul=rgl, mu=1e6);
```

Using Mira algorithm... changing the regularization

0. Load Mira software.
1. Load input data into opaque object db.
2. Configure for image reconstruction.

3. Choose a regularization method:

```
dim = mira_get_dim(db);
img0 = array(double, dim, dim);
img0(dim/2, dim/2) = 1.0;
rgl = rgl_new("xsmooth");
rgl_config, rgl, "cost","cost_l2l1", "threshold",2e-5,
   "dimlist",dimsof(img0);
```

4. Attempt an image reconstruction (from scratch):

```
img1 = mira_solve(db, img0, maxeval=500, verb=1,
   xmin=0.0, normalization=1, regul=rgl, mu=1e6);
```

5. Continue reconstruction with recentered image:

```
img1 = mira_solve(db, mira_recenter(img1), maxeval=500,
   verb=1, xmin=0.0, normalization=1,
   regul=rgl, mu=1e7);
```