The radius and mass of stars using interferometric and asteroseismic data

Orlagh Creevey

1 Introduction

The objective of this practical session is to give students a feel for working with some interferometric visibilities in order to determine the angular diameter of a star and its uncertainty. In combination with the distance to the star and some asteroseismic information we will subsequently determine the mass of a star. Then we will apply what we have learnt to α Cen A. If there is time at the end we will use stellar evolution tracks to estimate the age of a star (Section 5.2). There is also an optional section on the analysis of time series signals if you prefer to do this (Section 7).

This practical is done using the computing language `IDL`. If you are unfamiliar with `IDL`, all of the commands that you will need to successfully complete this practical are given in this handout. However, it may take you a bit longer to complete the practical.

To begin, type the following at the command prompt:

```
idl
```

Once `IDL` has started, then do the following command to load in some variables and to print out some basic information:

```
@start
```

If at any stage you forget the commands to use in IDL, you can either look through this handout, or type `@start` and this will print out information to the screen.

2 Drawing visibility curves

If you have done `@start` in IDL, then you will have loaded two arrays: one called `xx` and the other called `ll`. These correspond to the baselines and the effective wavelengths of the interferometric observations. To illustrate them you can do the following:

```
plot, xx
plot, ll
```

In these two arrays we have defined a continuous baseline coverage, and this will allow us to play around with looking at how the visibility curves change for different angular diameters.

In order to determine the visibility curves (squared visibilties) corresponding to an angular diameter and projected baseline, we have an `IDL` function, called `udangdiam`. This function calculates the uniform disk angular diameter, given some parameters. Do the following commands, and explain what you are plotting each time.

```
plot, xx/ll, udangdiam(0.5,xx,ll)
plot, xx/ll*1e-8, udangdiam(0.5,xx,ll)
plot, xx/ll*1e-8, udangdiam(0.5,xx,ll), xran=[0,6]
oplot, xx/ll*1e-8, udangdiam(0.75,xx,ll), col = 200
oplot, xx/ll*1e-8, udangdiam(0.3,xx,ll), lines= 2
xar = xx/ll*1e-8
yar = udangdiam(0.26,xx,ll)
oplot, xar, yar, col=200, line=3
```


Here we have used a function called 'udangdiam', meaning *uniform disk angular diameter*. This function requires a set of 3 inputs and returns a variable which is calculated using the 3 input variables. In `udangdiam` the first argument is the angular diameter in milliarcseconds (mas) and this is a constant. The second argument (xx) defines the baselines (in metres) and this argument is as long as we wish it to be. For observations, we have one baseline for each visibility point. The third argument (ll) is the wavelength in which the instrument works. This is also given in metres and is necessarily the same length as xx. The function calculates the *squared visibilities*. In the last example, `yar` are the squared visibilities corresponding to all of the input 'baselines/lambda' for a uniform disk of angular diameter 0.26 mas.

In the examples above we sampled the visibilty curve at 1000 points. However, normally we can not do this, and we must be able to select the optimal telescopes and instruments in order to sample the curve as best as possible.

The following illustrates how we can sample the visibility curve by using different baselines (different telescope configurations) and different wavelengths (different instruments, blue, red, and IR).

```
plot,xx/ll*1e-8, udangdiam(0.5,xx,ll)
```

```
print,x2,l2
oplot,x2/l2*1e-8,udangdiam(0.5,x2,l2),ps=6
print,x3,l3
oplot,x3/l3*1e-8,udangdiam(0.5,x3,l3),ps=5
```

In the *observations* for x2 and l2, you can see that the observations were sampled using two different baselines, in other words, different telescope configurations. The baselines are 100m and 160m, and we use the same wavelength (or instrument).

In the second example, we are using the same baseline (200m) but we have obtained visibility points using instruments that operate in the blue, red, and IR (at λ = 550nm, 735nm, and 1150nm).

1. Can you determine what wavelength we'd need to work with if we want to sample in the first lobe? (using a 200m baseline).

2. Is this feasible? Why?

3. If the angular diameter is expected to be 0.9 mas and we work with the visible interferometer at 7.35e-7 m, what baselines would we need to obtain squared visibilities around 0.6 and 0.1?

Use the following example to help you:

```
plot,xx/ll*1e-8, udangdiam(0.9,xx,ll)
plot,xx/ll*1e-8, udangdiam(0.9,xx,ll),xran=[0,5]
x2=[20,200]  ; try these baselines
oplot,x2/l2*1e-8,udangdiam(0.9,x2,l2),ps=6
```

4. The following are two observation points using an instrument operating at 7.35e-7m. The two baselines are 55m and 120m, and their observed squared visibilities are 0.60 and 0.05. Can you determine the uniform disk angular diameter?

If you would like to create a postscript figure you can do the following commands *before* your plotting commands

```
set_plot,'ps'
device,filename='myplot.ps'
```

And then after you have finished plotting your figure, do:

```
device,/close
set_plot,'x'
```

2.1 Limb-darkened angular diameters

Now we will repeat some of the above exercises but using the function `ldangdiam` which means the *limb-darkened angular diameter*. In reality stars are not uniform disks, unfortunately, and we observe a profile of brightness from the center to the limb. The amount of limb-darkening changes the center-limb contrast and is characterised by a coefficient μ. To call the `ldangdiam` function, do the following:

```
ldc = 0.5
plot,xx/ll*1e-8, ldangdiam(0.9,xx,ll,ldc),xran=[0,5], linestyle=2
```

1. Compare the visibility curves using the same angular diameter but the different functions. At what projected baseline is the difference largest? And by how much?

2. At what limb-darkening coefficient is the largest difference between the uniform disk and limb-darkened diameter less than 0.01?

2. Compare different visibility curves using different limb-darkening coefficients (e.g. μ = 0.4 - 0.7)[1].

[1] In ADS, look for Claret (2000), on-line data, and select 'atlasco' catalogue. Here you can specify the logg, Teff and [M/H] of the star (or specify one or none of these) or in 'Coeff' write 'u' and you will get a table with the linear limb-darkening coefficients 'u'

3. What is the largest difference in visibilities (using the last example) when we use a coefficient of 0.45 and 0.65?

---

4. The following are two observation points using an instrument operating at 7.35e-7m. The two baselines are 55m and 120m, and their observed squared visibilities are 0.94 and 0.74. Can you determine the limb-darkened angular diameter? Use a limb-darkening coefficient of 0.65

---

5. **Read this question, but skip it if you are going slowly, and come back to it at the end.**
If you have access to ADS on-line, go to the Claret (2000) on-line catalogue of limb-darkening coefficients. If you have never used ADS, you can go directly to the following webpage to browse `www.cdsads.u-strasbg.fr/abstract_service.html`*. If you would like help to get to the on-line data, just ask. Choose a star of* $\log g$ *= 4.0,* $T_{\rm eff}$ *= 5500 K, [M/H] = 0.0 dex, and the coefficient 'u' (this coefficient is for the linear limb-darkening models). How does the coefficient change if we observe in the blue, red, and IR? At which bands is it more important?*
How do the coefficients change if $\log g$ is 4.5 dex? if $T_{\rm eff}$ = 6000 K? if [M/H] = 0.5dex?
In terms of precisions in observations (0.1 dex for $\log g$ and [M/H], and 200 K for $T_{\rm eff}$)?
Which is the most important parameter here?

---

3 Fitting angular diameters from interferometric data

In the following section we will read in some visibility data and determine their angular diameters.

3.1 Easy data

Using the files data0a.txt, data0b.txt and data0c.txt in the directory 'data', find the angular diameter that best fits the data. Use the command `readcol` to read in the columns of data in each file:

```
readcol, 'data/data0a.txt', vis2, err2, base, lambda
```

1. Use the coefficients 0.6 for files a and b, and the coefficient 0.4 for c. Plot the visibility data for each set and estimate the limb-darkened angular diameter. Use the following commands to help you:

```
plot, base/lambda*1e-8, vis2, ps=2, xran=[0,10]
oploterr, base/lambda*1e-8, vis2, err2
oplot, xx/ll*1e-8, ldangdiam(0.3, xx,ll,0.6),col=200
```

2. Can you give a reason why the the coefficients change? even though we use the same wavelength?

3.2 Data with errors

Now we will work with some data with errors. The following files contain visibility data: data1.txt, data2.txt, data3.txt and data4.txt. Begin with data1.txt.

1. Read in the data, plot them, and try to estimate the limb-darkened angular diameters. You can use the limb-darkening coefficient 0.6. There is a useful IDL command to plot the yrange of interest; `yran = [0,1]`. *Don't forget to plot the error bars.*
2. For each set of data, estimate the angular diameter and the error (by eye).

3. What is the difference between data1.txt, data2.txt, and data3.txt?

---

4. What is the difference between data3.txt and data4.txt?

3.3 Estimating error bars

Now we will look at a way to estimate the true error bars. Let's begin with data1.txt. *1. Read and plot these data and overplot the visibility curve with the best angular diameter.*

There is a way to quantify how well the data match the theoretical data and that is by comparing them directly. In the following example, `AD` needs to be replaced with the value that you obtain for the angular diameter.

```
plot, base/lambda*1e-8, vis2, ps=2, xran=[0,10], yran = [0,0.2]
oplot, base/lambda*1e-8, ldangdiam(AD, base, lambda, 0.6), ps=2, col=200

plot, base/lambda*1e-8, vis2 - ldangdiam(AD, base, lambda, 0.6), $
    ps = 6, xran = [0, 10], ytitle = 'Data - Model', $
    xtitle='Projected Baseline (x 1e-8 m)'
oplot,[0,10],[0,0],linestyle=2

plot, base/lambda*1e-8, (vis2 - ldangdiam(AD, base, lambda, 0.6))/err2, $
    ps = 6, xran = [0,10]
oplot,[0,10],[0,0],linestyle=2
```

2. What is the difference between the last two plots?

To quantify how alike the data and the model are, we calculate its $\chi^2 = \frac{(y_i - y_m)^2}{\sigma_i^2}$ where y_i are the squared visibilities, σ_i are the errors, and y_m are the *theoretical* or *model* values.

```
print, total ( (vis2 - ldangdiam(AD, base, lambda, 0.6))^2 / err2^2)
```

```
dof = n_elements(vis2) - 1
reducedchisq = total ( (vis2 - ldangdiam(AD, base, lambda, 0.6))^2 / err2^2)/ dof
print,reducedchisq
```

Now you can calculate the reduced χ^2 for a range of angular diameters, and plot the results as a function of angular diameter. I have a created a function called `chisq` which allows us to do this easily. Follow the example below:

```
angdiamrange = findgen(20)*0.01+0.35 ;create an array of angular diam.
plot, angdiamrange, ps = 1 ;to see the values we are sampling

rchisq = chisq(vis2, err2, base, lambda, 0.6, angdiamrange)
plot,angdiamrange,rchisq, ytit = '!7v!6!a2!n!iR!6', xtit='angular diameter (mas)'
```

3. Find the angular diameter corresponding to the minimum χ^2_R. You can change the sampling of the angular diameters as you wish, e.g. `angdiamrange = findgen(100)*0.005+0.30`.

For a function with *one degree of freedom* we can define the uncertainties on the fitted parameter i.e. the angular diameter, as the range of angular diameters for which its χ^2_R value is less than the minimum value of $\chi^2_R + 1$. Do
`oplot, [0.2,1.0], replicate(min(rchisq)+1, 2), lines=2`
to see the values of the angular diameter that satisfy this criteria. Now you can define the angular diameter and its 1σ uncertainty.

4. Calculate the angular diameter and its uncertainties for the data in data2.txt, data3.txt, and data4.txt. What are the differences and why?

---

4 Calculating the radius and its error

Once we have the angular diameter, denoted θ, and the parallax (or distance) to the star, then we can calculate the star's radius:

$$R = 107.5 \times \frac{\theta}{\pi} \tag{1}$$

where π is the parallax in mas, and θ is also given in mas. The result is the radius R in solar radii ($R_\odot$).

1. Calculate the radius of the star using the angular diameter that you have determined using the data from the file data3.txt. Its parallax is 51.55 ± 0.09 mas.
2. Use a standard propagation of errors formula[2] to determine the uncertainty on the radius.
3. Compare the radii of the star using data3.txt and data4.txt.

[2] if $f = f(a, b)$ then $\sigma_f^2 = \sigma_a^2(\frac{\partial f}{\partial a})^2 + \sigma_b^2(\frac{\partial f}{\partial b})^2$ where σ denotes uncertainty/error.

5 The mass of the star

5.1 The asteroseismic mass

As we have learnt in the lesson, the asteroseismic quantity known as the *mean large frequency separation* $\langle \Delta\nu \rangle$ has been shown to scale with the mean density $\langle \rho \rangle$ of the star:

$$\frac{\langle \Delta\nu \rangle}{\langle \Delta\nu \rangle_\odot} \approx \sqrt{\frac{\rho}{\rho_\odot}} = \sqrt{(M/M_\odot)/(R/R_\odot)^3} \tag{2}$$

where $\langle \Delta\nu \rangle_\odot$ is the solar value ($\langle \Delta\nu \rangle_\odot = 134.9$ μHz.) From an asteroseismic analysis of our target star, the following value has been determined: $\langle \Delta\nu \rangle = 149.5 \pm 0.9$ μHz.

1. Rearrange the equation above to calculate the mass of the star (in solar masses) by using your previously determined radius.
2. Use a standard propagation of error formula to determine the uncertainty in the mass.
3. If I want to have half the uncertainty on the mass, how small do the errors in the seismic data and/or the radius have to be? How can these errors be achieved?

5.2 Models

If you have still about 30 minutes left, you can do this section, otherwise skip to question 6.1 and then come back to it, or continue to Section 7 for something different!.
If asteroseismic data are not available, we must rely on stellar models to estimate the mass of the star. Here we will work with some stellar models to try to determine the mass. The files in the directory `models` contain theoretical data showing the evolution of a stellar model. There are tracks for different masses, e.g. m090 is a 0.90 $M_\odot$ star, and different metallicities, e.g. z016 is a star with an metallicity fraction of 0.016. The three metallicities satisfy the observed metallicity constraint: [M/H] = 0.06 ± 0.10 dex.

For each *.dat file there are three columns: age (in Gyr), luminosity (in solar luminosity), and effective temperature (in K). You can read in the data as follows:

```
readcol, 'models/m100_z024.dat', age, lum, teff
```

1. Plot the evolution track in a HR diagram.
2. Now plot the luminosity of the star as it evolves.
3. Repeat for the effective temperature. (at the end of your plot command use `,/ynozero`*)*
4. Read in a new evolution track and overplot its evolving effective temperature.
5. Using the effective temperature only (5578 $\pm$ 100 K) can you determine the mass of the star.? Remember that we have tracks of certain masses, and you may need to interpolate between tracks to have a better estimate of the mass.

Plot a HR diagram but using radius on the y-axis. You need to calculate R from L and $T_{\rm eff}$ first[3]. You can overplot the error box using the following command:

```
tt = [5578, 100]
rr = [AD, ERROR] ;where AD = your angular diameter and ERROR = the uncertainty
oplot, tt(0)+tt(1)*[-1,-1,1,1,-1], rr(0)+rr(1)*[-1,1,1,-1,-1]
```

1. Can the mass of the star be as low as 0.8 $M_\odot$? Can it be as high as 1.1 $M_\odot$? Why?
2. Can you determine more or less the limiting range of mass?

[3]The Stefan-Boltzmann constant is 5.67037e-5 erg cm^{-2} s^{-1} K^{-4}, $L_\odot$ = 3.846e33 erg s^{-1} and $R_\odot$ = 6.96e5 km.

6 The mass and radius of α Cen A

In the directory `alphacena` you will find two data files for the star α Cen A. One file contains interferometric data and the other contains asteroseismic frequencies, with some information about the units (be careful!) The distance to the α Cen system is 1.34 ± 0.01 pc.

1. *Determine the interferometric radius and the asteroseismic mass of α Cen A.*
2. *Using the models provided, can you estimate an age for this star?*

7 Determining oscillation frequencies from time series data

In this section we will work with time series signals, in particular, understanding frequencies, periodicities, and calculating the Fourier Transform. All of the data files are in the directory 'data'.

1. Open the file 'ts1a.dat' and read in the time series data. The first column is time in seconds, and the second column is height in metres.

What is the total length of time of the data?
How many data points are there? `print, n_elements(x)`
What is the time sampling? (the time between each data point)?
*What is the smallest periodicity (highest frequency) that we can determine with this time sampling? (smallest period = 2*time sampling)*
What is the oscillation period in seconds? (How long does it take to complete an oscillation period?)
What is the frequency of this signal? (Frequency = 1/Period)

2. Repeat for ts1b.dat, again time is given in seconds.

Using the program *myfft.pro* calculate the Fourier Transform of both of the time signals. To do this you need to do the following command: `myfft, time, height, freq, amp`. Here `time` and `height` are the independent and dependent variables, and `time` has to be in seconds (Remember this!). To plot the FT, you do `plot,freq,amp`.

3. What are the frequencies present in the signals?
Are they the same as those that you calculated?

If you would like to determine the values of points on the graph, in IDL you can do `cursor, xpt, ypt` and then click on the graph. xpt and ypt give you the x- and y-coordinates of where you clicked.

4. The file ts2.dat contains differential magnitudes of a star observed over one night. The observations were obtained every two minutes. Plot the time series signal.
Can you determine the oscillation periods present? (zooming in may help)
Calculate its Fourier Transform and determine the frequencies present in the signal.
What are the frequencies?
What are the differences in the frequencies?

5. Repeat for file ts3.dat (noise in this data), calculate $\Delta\nu$ and estimate $\nu_{\rm max}$.
If $T_{\rm eff} = 5700 \pm 150$ K, can you solve for the mass and radius? (without knowing the radius).
If the radius is what you obtained earlier, what does this tell you about the mass?