

Spots
Rotation
Magnetism
Practice Session



Stellar rotation

Hydrostatic equilibrium in a non rotating star.

$$\nabla P = \rho g \quad \text{and} \quad \frac{1}{\rho} \frac{dP}{dr} = -\frac{GM_r}{r^2}$$

Spherical star
 Force from gas and
 radiation pressure
 balanced by gravity.

**But, (fortunately) the universe
 is not that simple.**



LISTS FOUR WONDERS OF THE UNIVERSE

REVISING the wonders of the universe, Sir J. Arthur Thomson, British zoologist, suggests there are four of them. The first, he says, is the power that keeps stars and planets spinning on their axes. Immensity of space is the second. Third, the delicate mechanisms needed for the life of even the smallest of insects. The orderliness of Nature is fourth.

Stellar rotation: flattening

Roche model

Solid (rigid body) rotation and mass mainly concentrated in the center of the star:

Angular velocity (rotation)

$$\frac{1}{\rho} \nabla P = -\nabla \Phi + \frac{1}{2} \Omega^2 \nabla (r \sin \vartheta)^2 = -\nabla \Psi = g_{\text{eff}}$$

Gravitational potential

$$g = -\nabla \Phi = -\frac{GM_r}{r^2} \frac{r}{r}$$

Gravitational acceleration (gravity)

rotation

gravity

Rotation tend to flatten the star

Stellar rotation: flattening

Roche model

Solid (rigid body) rotation and mass mainly concentrated in the center of the star:

Stellar surface defined by the equipotentials $\Psi = \text{const.}$

$$\Psi(r, \vartheta) = -\frac{GM_r}{r} - \frac{1}{2} \Omega^2 r^2 \sin^2 \vartheta$$

$$\frac{GM}{R} + \frac{1}{2} \Omega^2 R^2 \sin^2 \vartheta = \frac{GM}{R_p}$$

➔

Effective gravity
(function of colatitude)

$$-\nabla \Psi = g_{\text{eff}}$$

Break-up or critical velocity is reached when $F_{\text{grav}} = -F_{\text{cent}}$
(NOTE: this limit changes for very massive stars where radiation pressure is important)

$$v_{\text{crit},1}^2 = \Omega_{\text{crit}}^2 R_{e,\text{crit}}^2 = \frac{GM}{R_{e,\text{crit}}} = \frac{2GM}{3R_{p,\text{crit}}}$$

At the v_{crit} the equatorial to polar ratio (flattening) is $R_e/R_p=1.5$

Stellar rotation: gravity darkening

From radiative transfer in LTE the flux within the star is:

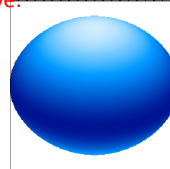
$$F = \frac{L_r}{4\pi r^2} = -\frac{4acT^3}{3\kappa\rho} \frac{dT}{dr}$$

In rigid rotation all surfaces coincide (equipotentials, isobars, temperature and density) so we have the von Zeipel relation for the **gravity darkening** effect (von Zeipel 1924):

$$F(\theta) \propto g_{\text{eff}}(\theta) \propto T_{\text{eff}}^4(\theta) \quad \Rightarrow \quad T_{\text{eff}}(\theta) \propto g_{\text{eff}}(\theta)^{0.25}$$

More generally, the **gravity darkening** coefficient is considered as a free parameter to account for more general energy transport mechanisms. Normalizing by the polar effective temperature and effective gravity we thus have:

$$T_{\text{eff}}(\theta) = T_p (g_{\text{eff}}(\theta)/g_p)^\beta$$



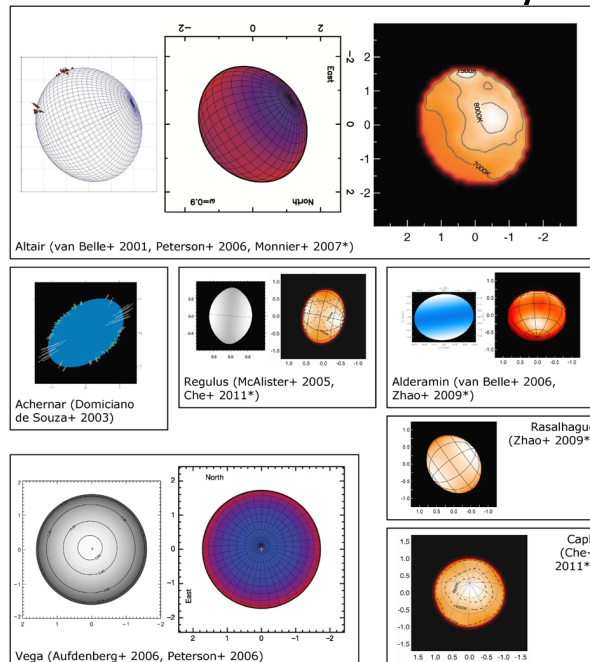
Poles brighter than the equator.

Rapid rotation from interferometry

Interferometry is very sensitive to the surface flattening and to the non-uniform flux distribution.

Several rapid rotators already observed by different interferometers.

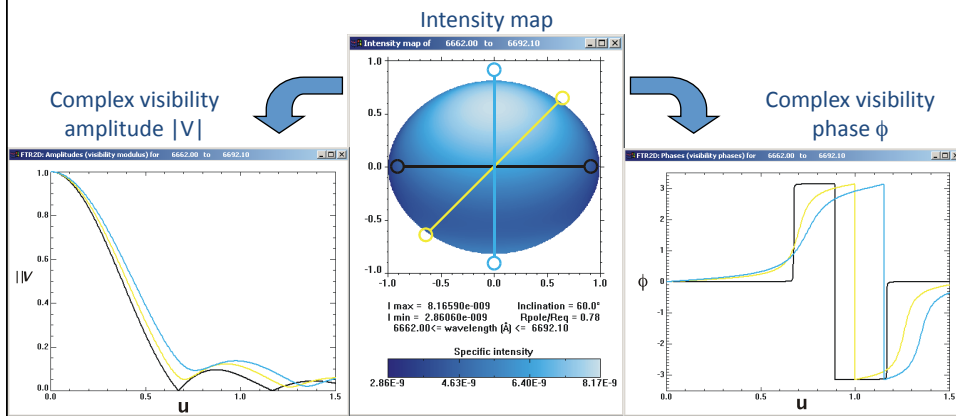
Image from van Belle 2012, A&A review



Rapid rotation from interferometry

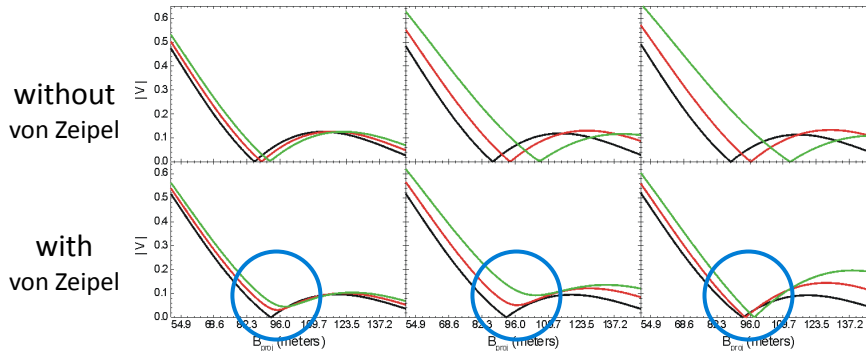
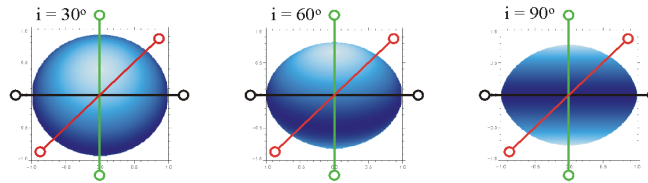
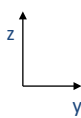
$$v_{eq} > 80\% v_{crit}$$

- Geometrical deformation (Roche approximation)
- Gravity darkening (von Zeipel effect)

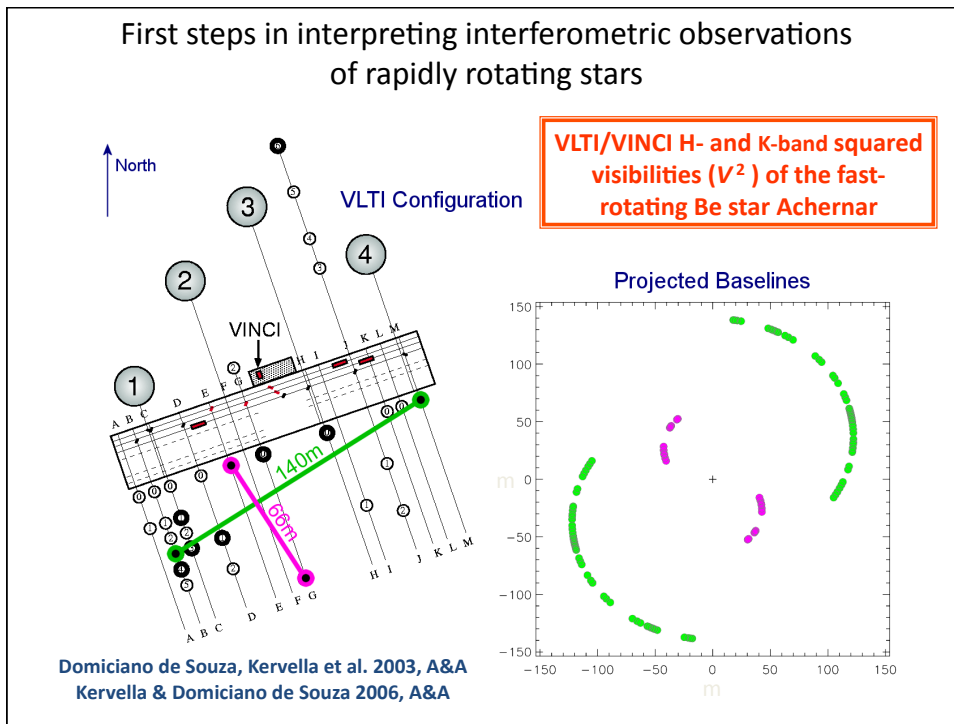


Visibility amplitude |V| curves

81% v_{crit}
 λ fixed



Domiciano de Souza et al. 2002, A&A



Be star Achernar (α Eridani)

Physical parameters:

- Spectral type B3Ve-B6Ve
- $T_{\text{eff}} \sim 12000$ K (equator); 18000K (poles)
- Magnitude V = 0.46 (brightest Be)
- Distance $d = 44$ pc = 143.5 AL (Hipparcos; Perryman et al. 1997)
- Mean angular diameter ~ 2.0 mas
- Mass $\sim 6M_{\text{sun}}$
- $R_{\text{eq}} \sim 9-11R_{\text{sun}}$; $R_{\text{eq}}/R_{\text{pole}} \sim 1.4-1.5$
- $v \sin i \sim 220-290$ km/s (fast rotator)
- Be \leftrightarrow B at time scales of a few years

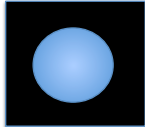
Achernar

Artist view

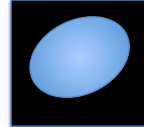
Limited data sets and/or measuring physical parameters → pre-defined models

Choose and test different models based on physics, angular resolution, and data quality

Uniform/limb darkened disk



Uniform/limb darkened ellipse



Rotational flattening, limb darkening, gravity darkening



Even closer to reality



Stellar rotation rate estimation

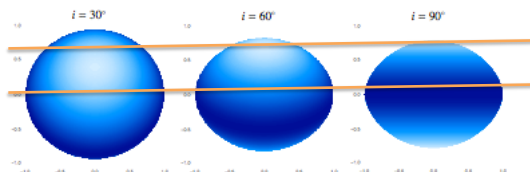
Roche model

- (a) uniform rotation with angular velocity Ω ;
- (b) all mass M is concentrated in a point at the center of the star.

The stellar equipotential surfaces are then given by:

$$\Psi(\theta) = \frac{\Omega^2 R^2(\theta) \sin^2 \theta}{2} + \frac{GM}{R(\theta)} = \frac{GM}{R_p}$$

The apparent flattening ratio can be considered a lower limit (real $i < 90^\circ$) to the actual ratio R_{eq}/R_{pole} :



Rotation velocity at the equator

$$V_{eq}/V_{crit} = (3(1 - R_{pole}/R_{eq}))^{0.5}$$

Where V_{crit} is the critical velocity

$$V_{crit} = (GM/(1.5R_{pole}))^{0.5}$$

(1)

$$V_{eq}/V_{crit} > (3(1 - 1/\text{flatten_ratio}))^{0.5}$$

Assuming that:

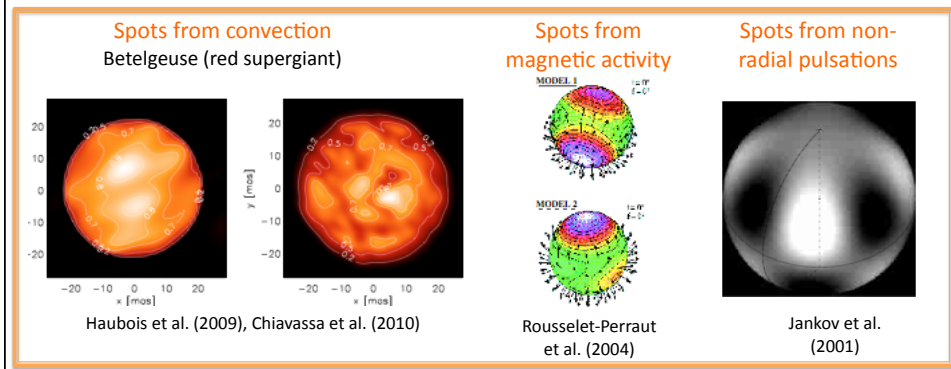
Ellipse ~ Roche model, with gravity and limb darkening

Stellar photospheric spots

Stellar spots can have different physical origins, such as **convection, magnetism, non-radial pulsations**

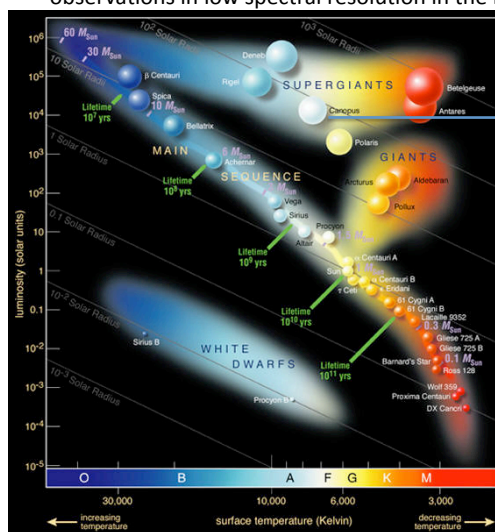
Whatever their origin, stellar interferometry is an ideal tool to study stellar spots, because it is sensitive to the detailed intensity distribution over the stellar photosphere.

The signature of spots can be detected on visibility amplitudes, differential phases, and phase closures.



Stellar photospheric spots

As an example of interferometric study of stellar spots, let us interpret spectro-interferometric observations of the blue supergiant Canopus from VLT/AMBER observations in low spectral resolution in the H and K bands.

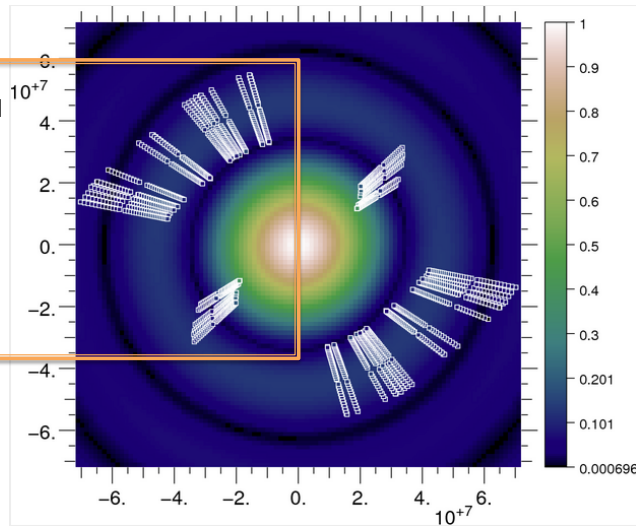


Canopus (α Carinae)

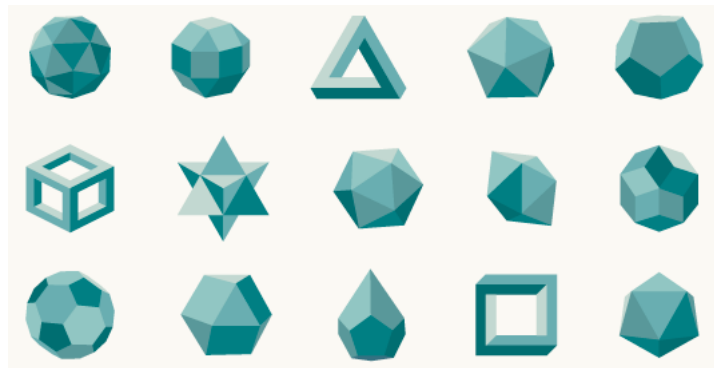
- Apparent visual magnitude -0.7
(2nd brightest star in the sky)
- Spectral type F0Ib -F0II
- Radius = 71.4 Rsun
- Mass = 9 Msun
- L ~ 13000 – 15000 Lsun
- Teff ~ 7300-7600 K
- Distance = 96 pc

Load H- and K-band OIFITS data files into LITpro

Squared visibilities and phase closures at spatial frequencies spanning the whole second Airy disk lobe, and parts of the first and third lobe.



Let us use LITpro to determine good geometrical models for Achernar and Canopus and to measure some of their physical parameters



*It is your turn to play now !
And may the fringe be with you...*