Stellar Oscillations

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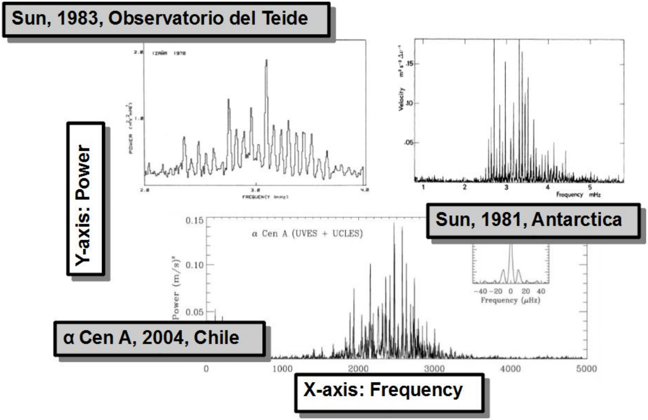
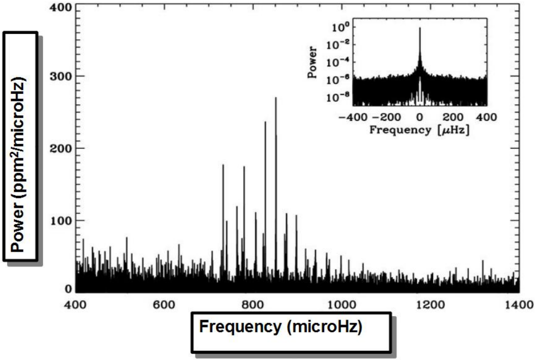
- Asymptotic expression for stellar oscillations
- Large and small frequency separations
- CD diagram
- Frequency of maximum amplitude
- Relation between seismic quantities and ρ and g
- Determining M, R, and age from scaling relations
- Observations of solar-like oscillations
- Combining interfer + astero: precision in M

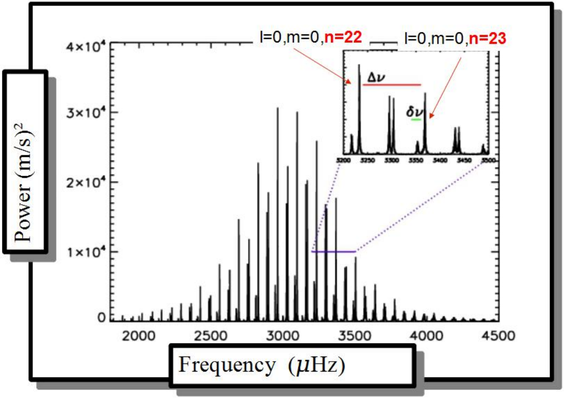
Observations of oscillations: Our closest solar-like stars



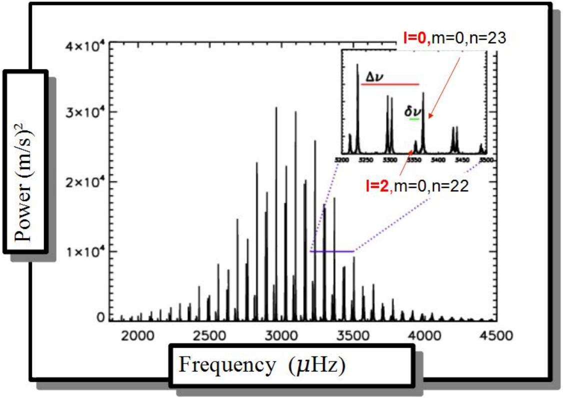
Space-based observations of giants: Kepler $V > 9$



Observations of solar-like oscillations



Observations of solar-like oscillations



Approximation for stellar oscillations

- Describing oscillations in stars can be simplified if we consider the asymtotic approximation, i.e. $\ell / n -> 0$.
- A second order approximation is given by the following

$$\nu_{n,l} = \Delta\nu_0 \left(n + \frac{\ell}{2} + \epsilon \right) - \Delta\nu_0^2 \left(\frac{A[\ell(\ell+1)] - B}{\nu_{n,\ell}} \right)$$

where $\Delta\nu_0 = \left(2 \int_0^R \frac{dr}{c} \right)^{-1}$

- $R =$ stellar radius, $\nu_{n,l}$ is a frequency ν of radial order n and degree ℓ, and c is the sound speed
- ϵ depends on boundary conditions (e.g. surface) and B depends on surface conditions.

Large frequency separation $\Delta\nu$

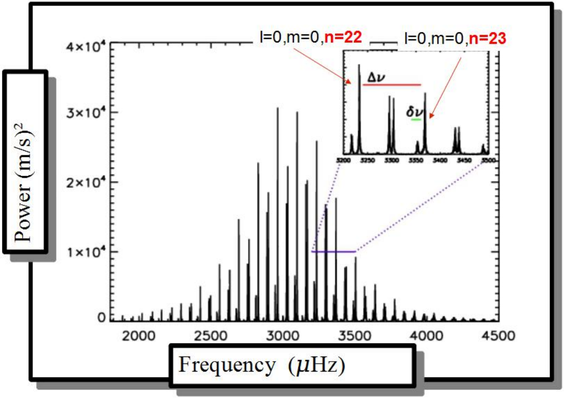
Considering just the first term of the RHS: $\nu_{n,l} = \Delta\nu_0 \left(n + \frac{\ell}{2} + \epsilon\right)$

- For a given degree ℓ, e.g. $\ell = 0$, $\nu_{n,l}$ and $\nu_{n+1,l}$ will be separated by $\Delta\nu_0$ $(\times 1 + 0 + \epsilon \sim 1)$
- $\nu_{n,0}$ and $\nu_{n,1}$ will be separated by approx. $\Delta\nu/2$
- The first term also implies that $\nu_{n,0} = \nu_{n-1,2}$
- If we define the *large frequency separation* as

$$\Delta\nu_{n,l} = \nu_{n,l} - \nu_{n-1,l}$$

then we have that $\Delta\nu_{n,l} \sim \Delta\nu_0 \sim \langle\Delta\nu\rangle$

Observations of solar-like oscillations: $\Delta\nu$



Large frequency separation $\Delta\nu$

- $\Delta\nu_0$ is defined as $\left(2\int_0^R \frac{dr}{c}\right)^{-1}$, which is the inverse of the time it takes for a sound wave to travel through the star
- We can equate this with $\Delta\nu_0 = \frac{c}{2R}$
- The speed of sound through a star can be written as $c^2 = \frac{dP}{d\rho}$ and in an isothermal gas we have $P = \rho^\gamma$ where γ is the adiabatic exponent, so $c^2 = \frac{\gamma P}{\rho}$
- We also have $\frac{GM^2}{R} = 3M\frac{kT}{\mu}$ and the IGL states $\frac{P}{\rho} = \frac{kT}{\mu}$
- By combining these equations and substituting for the sound speed we get $\frac{c}{2R} \propto \sqrt{\frac{M}{R^3}}$
- Thus we have

$$\Delta\nu_{n,l} \propto \sqrt{\rho}$$

Large frequency separation $\Delta\nu$

- From the asymptotic approximation we have

$$\Delta\nu_0 = \left(2\int_0^R \frac{dr}{c}\right)^{-1}$$

- Or in other words:

$$\Delta\nu_{n,l} \sim \langle\Delta\nu\rangle \propto \sqrt{\rho}$$

- By scaling with the solar values we have a *scaling relation*

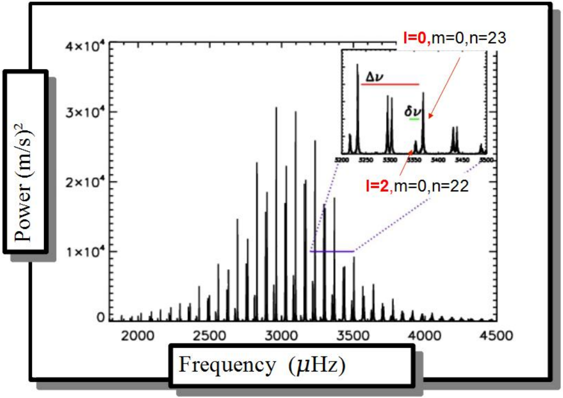
$$\frac{\langle\Delta\nu\rangle}{\langle\Delta\nu\rangle_\odot} \simeq \sqrt{\frac{\rho}{\rho_\odot}}$$

Small frequency separations $\delta\nu$

$$\nu_{n,l} = \Delta\nu_0 \left(n + \frac{\ell}{2} + \epsilon \right) - \Delta\nu_0^2 \left(\frac{A[\ell(\ell+1)] - B}{\nu_{n,\ell}} \right)$$

- We showed by considering just the first term of the RHS of the asymptotic relation that $\nu_{n,l} = \nu_{n-1,l+2}$, but considering the two terms, this is no longer true
- Here $A = \left(4\pi^2\Delta\nu_0\right)^{-1} \left(\frac{c(R)}{R} - \int_0^R \frac{dc}{dr}\frac{dr}{r} \right)$
- If we define the *small frequency separations* as $\delta_{l,l+2}(n) = \nu_{n,l} - \nu_{n-1,l+2}$, then we have $\delta_{l,l+2}(n) \simeq -(4l+6)\frac{\Delta\nu_0}{4\pi^2\nu_{nl}} \int_0^R \frac{dc}{dr}\frac{dr}{r}$
- We can see that $\delta_{n,02}$ is sensitive to the sound speed gradient, and in particular where r is close to 0, i.e. in the core
- $\delta_{n,02}$ or $\langle\delta\nu\rangle$ is sensitive to evolution state

Observations of solar-like oscillations: $\delta\nu$



CD diagram

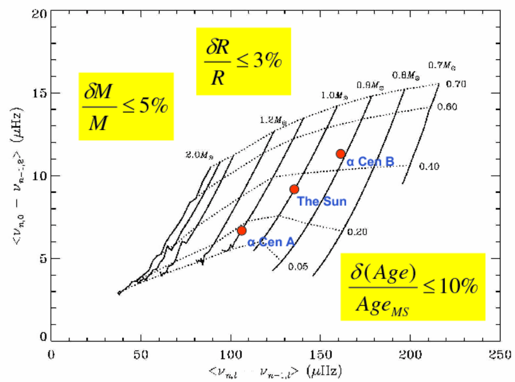


Figure : The CD diagram[2]. This figure shows how the small and large frequency separations evolve for stars of different masses as the star burns up hydrogen in its core (main sequence).

CD diagram not so simple

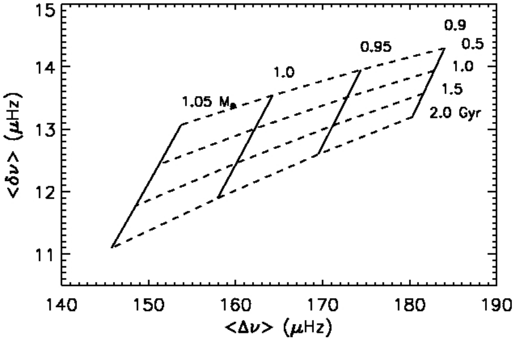


Figure : The CD diagram assuming a certain metallicity and input physics.

CD diagram not so simple

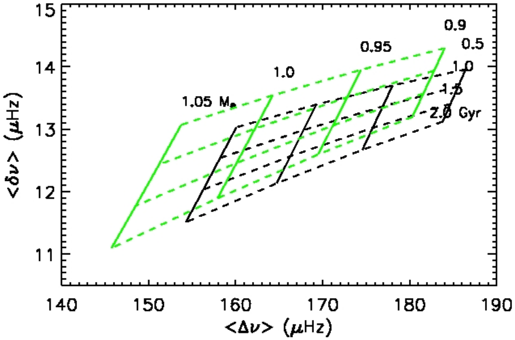


Figure : The CD diagram assuming a certain metallicity and input physics.

Observations of solar-like oscillations

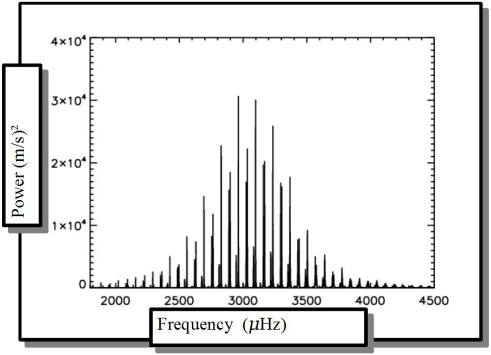
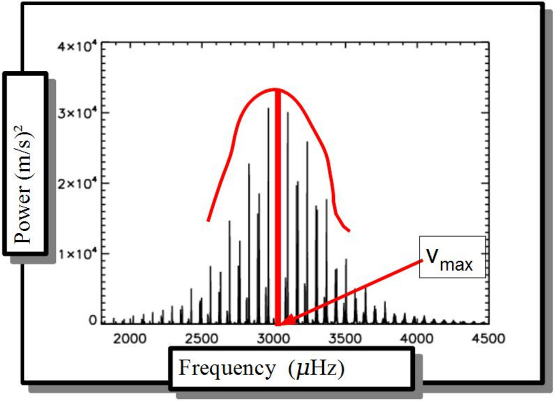


Figure : Observations of solar-like oscillations.

Observations of solar-like oscillations: $\nu_{\max}$



Frequency of maximum amplitude $\nu_{\max}$

- The observed power is modulated by a Gaussian-like envelope
- The frequency of maximum power/amplitude is called $\nu_{\max}$ and it is related to the damping and excitation mechanism, which happens in the near surface layers of the star
- Waves near the surface are strongly influenced by the acoustic cut-off frequency $\nu_{\rm ac}$, and so it has been suggested that $\nu_{ac} \propto \nu_{\max}$
- The acoustic cut-off frequency is given by $\nu_{\rm ac}^2 = \left(\frac{c}{4\pi H}\right)^2 \left(1 - 2\frac{dH}{dr}\right)$ where $H = -(d\ln\rho/dr)^{-1}$ is the density scale height
- An isothermal approximation gives $\nu_{\rm ac} = \frac{c}{4\pi H}$ and thus

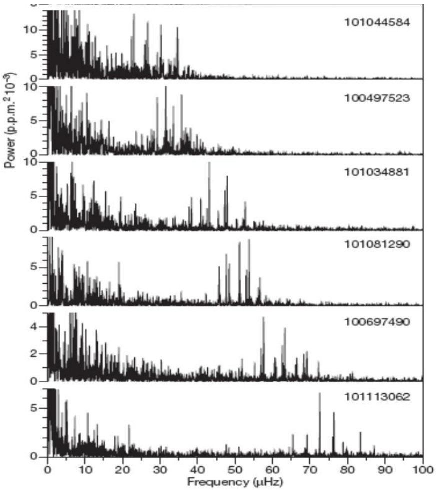
$$\nu_{\max} \propto \frac{c}{H} \propto g T_{\rm eff}^{-1/2}$$

Frequency of maximum amplitude $\nu_{\max}$

- We have $\nu_{\max} \propto g T_{\rm eff}^{-1/2}$
- Writing this in terms of solar values we have the following *scaling relation*

$$\frac{\nu_{\max}}{\nu_{\max,\odot}} \simeq \frac{g}{g_\odot}\sqrt{\frac{5777}{T_{\rm eff}}}$$

Space-based observations of giants: CoRoT



Solving for M and R

- The two scaling relations for $\nu_{\max}$ and $\Delta\nu$ can be written in terms of M and R where M and R are the stellar mass and radius in solar units:

$$\langle \Delta\nu \rangle \simeq \sqrt{\frac{M}{R^3}} \langle \Delta\nu \rangle_\odot$$

$$\nu_{\max} \simeq \frac{M}{R^2 T_{\mathrm{eff}}} \sqrt{T_{\mathrm{eff},\odot}} \nu_{\max,\odot}$$

- There are two equations with two unknowns if we can measure T_{eff}, and so we can solve for M and R.
- Typical precisions are 4-6% for R and 10-18% for M.
- If we could measure one of them independently, very high accuracy could be obtained on the other. Any ideas?

Using individual frequencies, $\nu_{n,l}$

- Even without considering the global seismic quantities $\langle\Delta\nu\rangle$ and $\nu_{\rm max}$, the oscillation frequencies themselves $\nu_{n,l}$ are of course extremely sensitive to the mean density of the star.
- They can be measured with much higher precision and thus the density can be determined to even higher precision than using just the global seismic quantities.
- Interpretation of the frequencies requires also the use of stellar models. Using the scaling relations provides a model-independent determination.
- We can not always access the individual frequencies. As the brightness decreases, the S/N also decreases, and in some cases only $\langle\Delta\nu\rangle$ and $\nu_{\rm max}$ can be determined.

Combining interferometry and asteroseismology

$$\langle \Delta\nu \rangle \simeq \sqrt{\frac{M}{R^3}} \langle \Delta\nu \rangle_\odot$$

- Considering the scaling law above, if we measure $\langle \Delta\nu \rangle$ with asteroseismic data, and R from interferometry, we gain access to M (hence age).
- M is very difficult to determine for cool stars and generally relies on the use of stellar models *if* the star is 'single'
- But how well we can determine these quantities depends on several factors
  - Precision in the radius
  - Other available observations
  - Precision in these other observations

Precision in mass

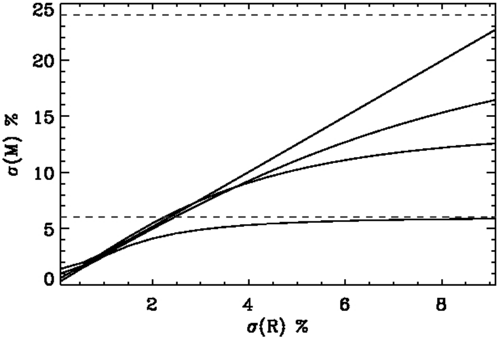


Figure : How the uncertainty in the mass improves as the radius is measured more precisely for a 1.0 $M_\odot$ model.