Interferometry and Asteroseismology: Determining precise stellar parameters

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Contents

- Introduction/objectives
- Waves and Power Spectra
- Stellar Pulsations across the HR diagram
- Solar-like Oscillations: theory and observations
- Interferometric diameters

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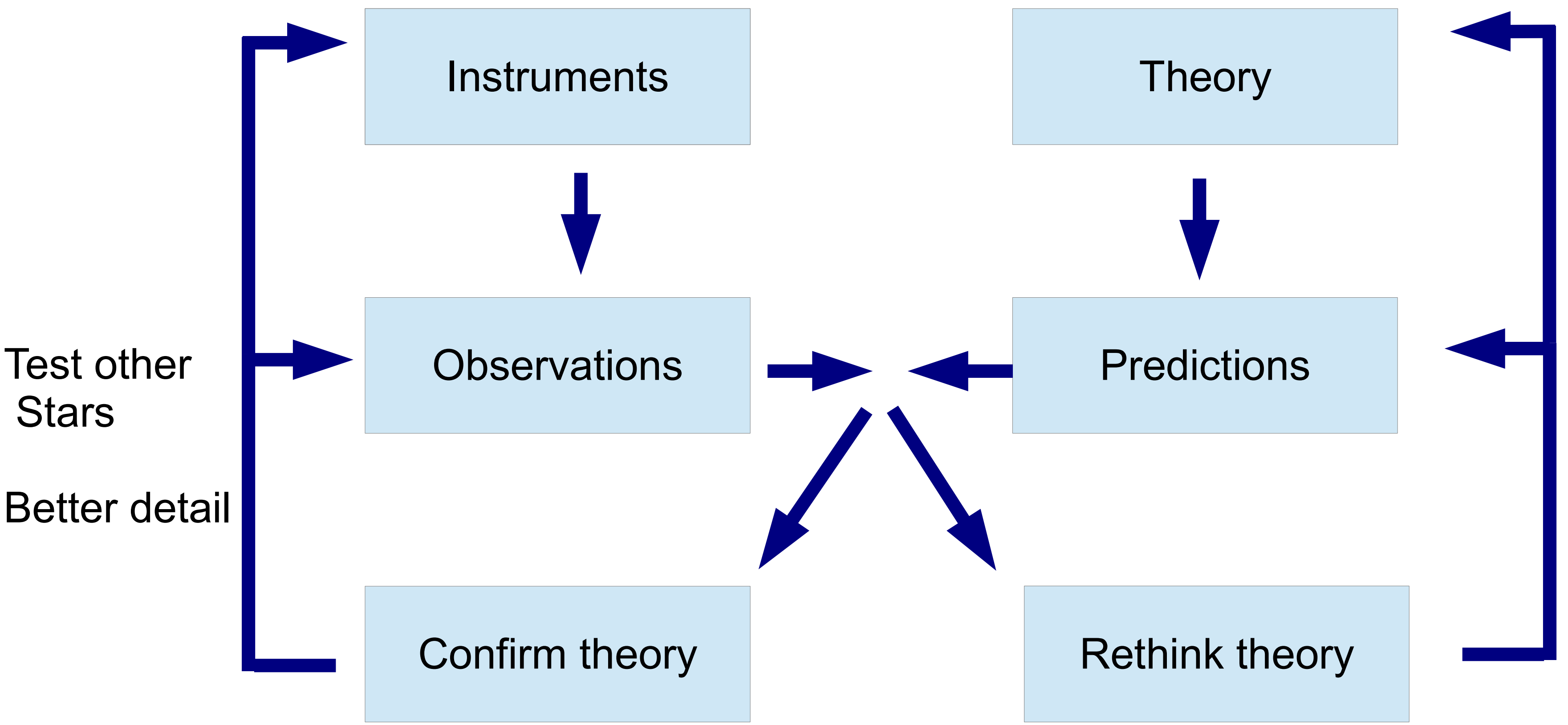
Goal of astrophysics



Theories

- Theory of stellar structure and evolution
- Theory of planet-formation and evolution
- Theory of galaxy evolution
- Cosmological theories
- .....

Observations

Effective temperatures (T_{eff})	(200 K)
Surface gravity (g)	(0.1 dex)
Distance (d)	(1-20%)
Metallicity [Fe/H]/[M/H]	(0.1 dex)
Magnitudes (BVRI)	(0.1 mag)
Colours (B-V)	(0.005 mag)

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Oscillation frequencies (v)	**(<0.1%)**
Angular diameter (θ)	**(<3%)**

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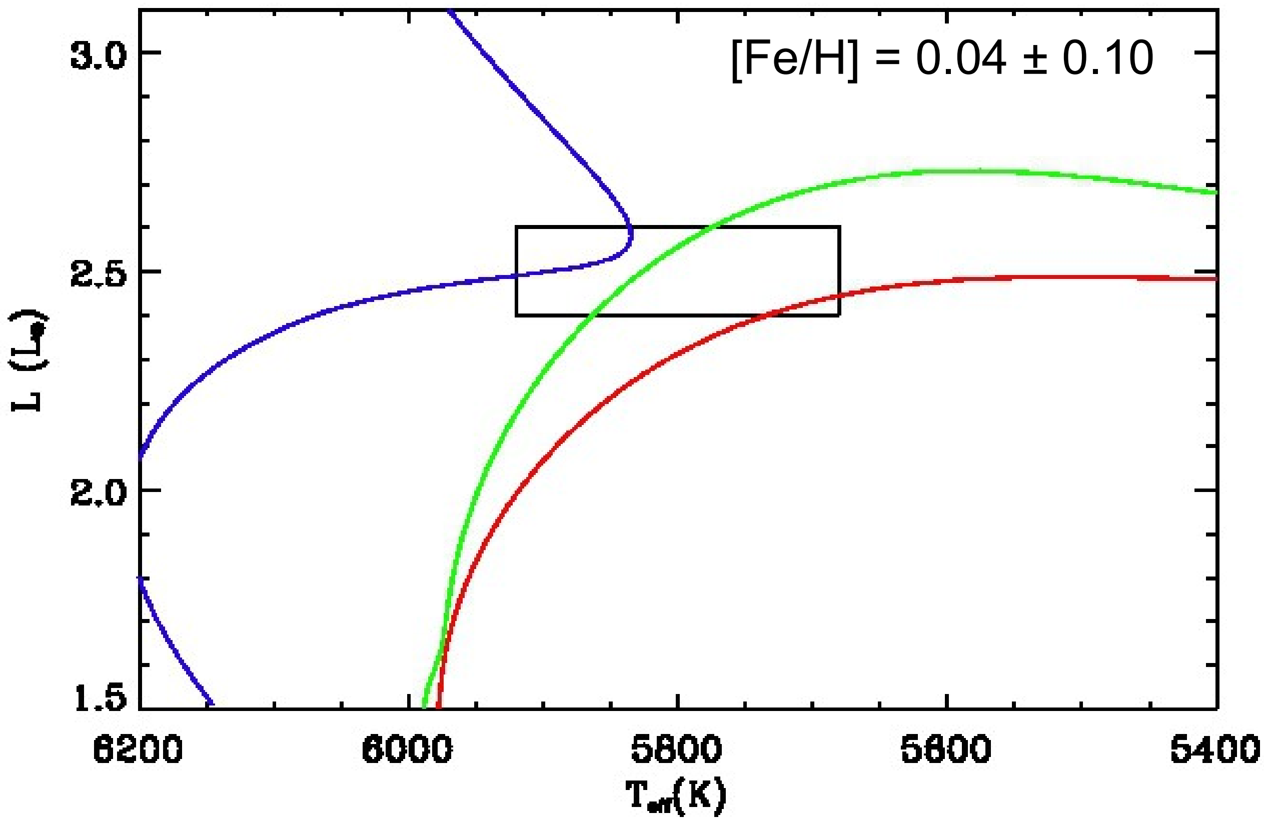
Colours (B-V)	(0.005 mag)	
Oscillation frequencies (v)	**(<0.1%)**	**ASTEROSEISMOLOGY**
Angular diameter (θ)	**(<3%)**	**INTERFEROMETRY**

Theories of stellar structure and evolution



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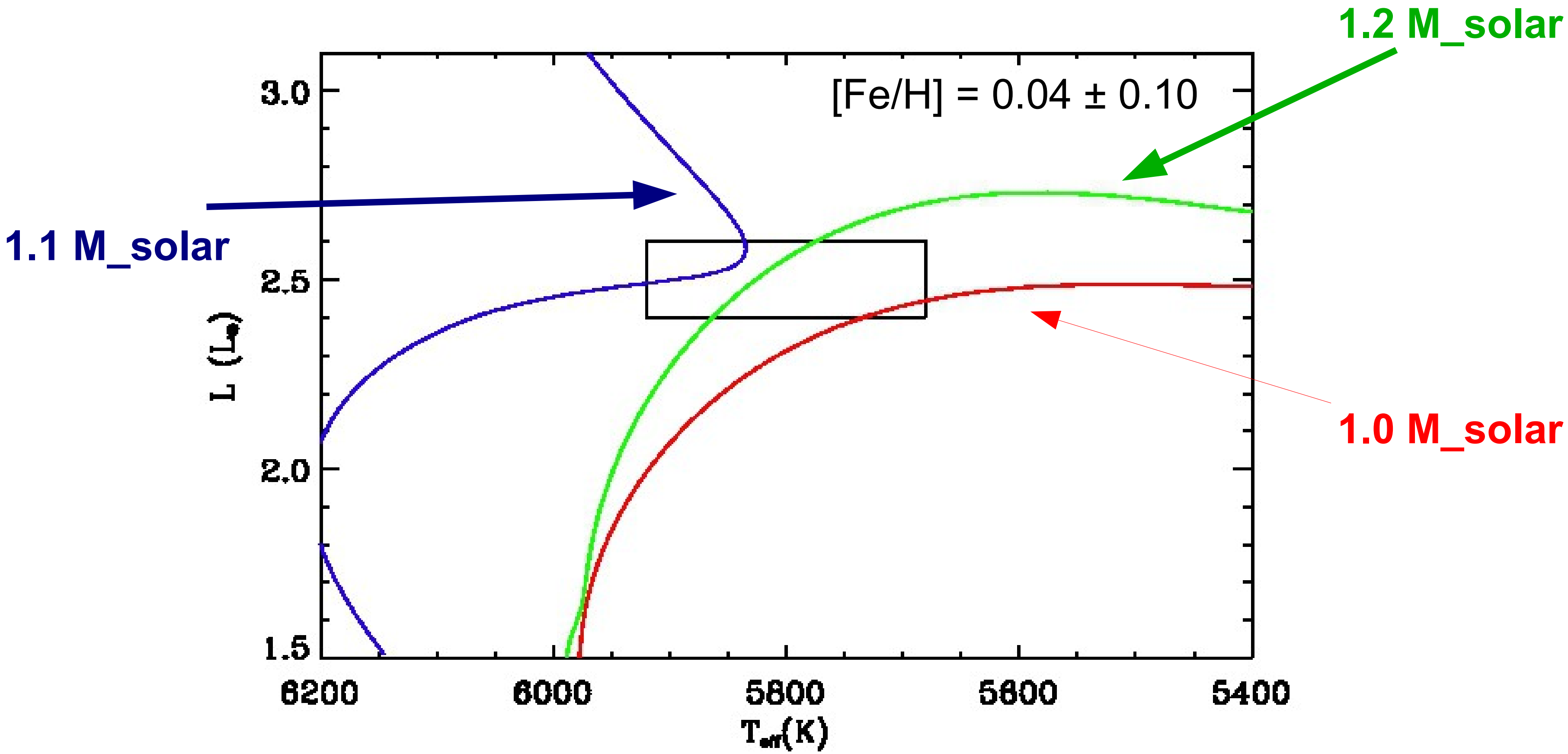

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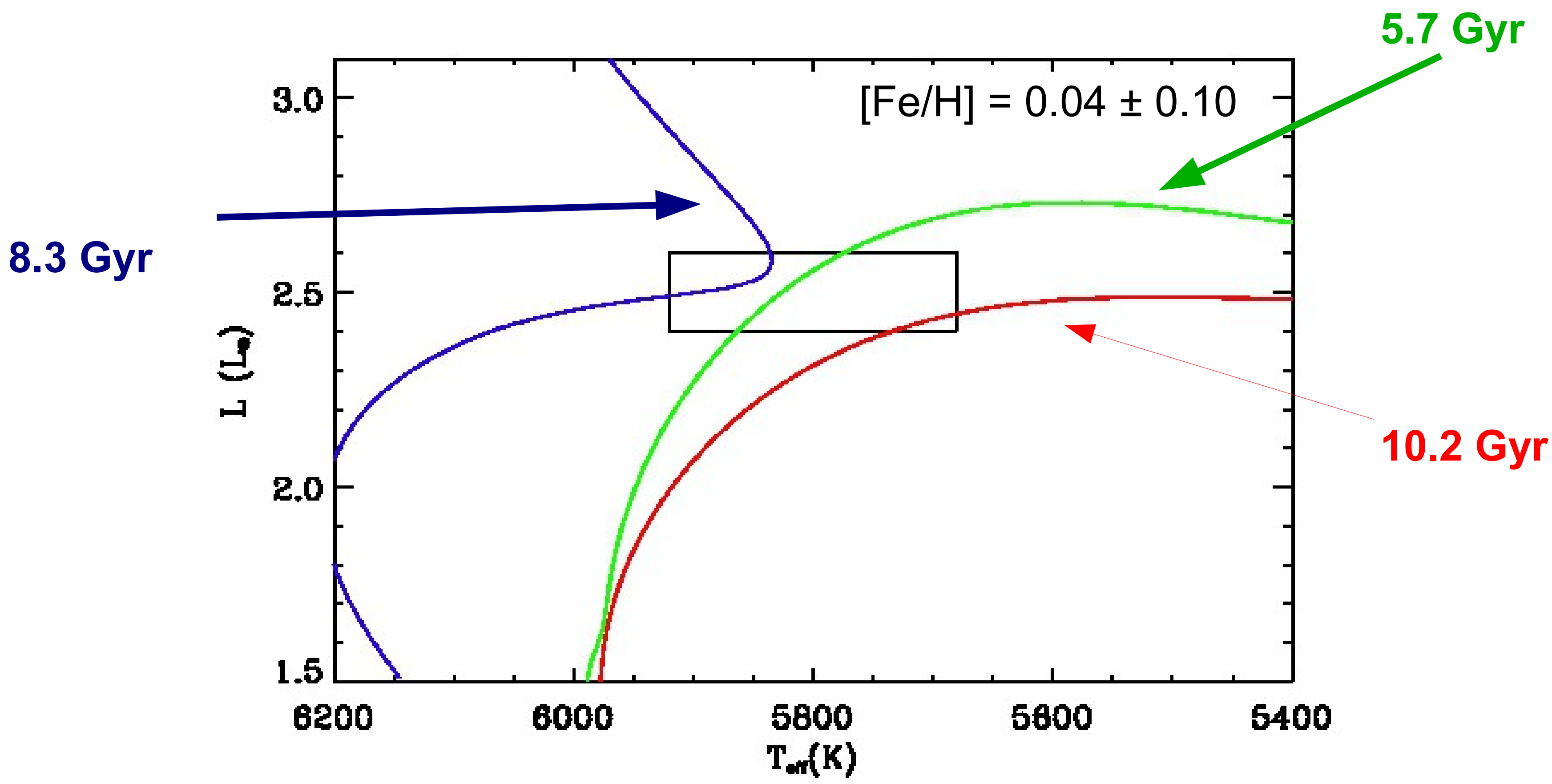

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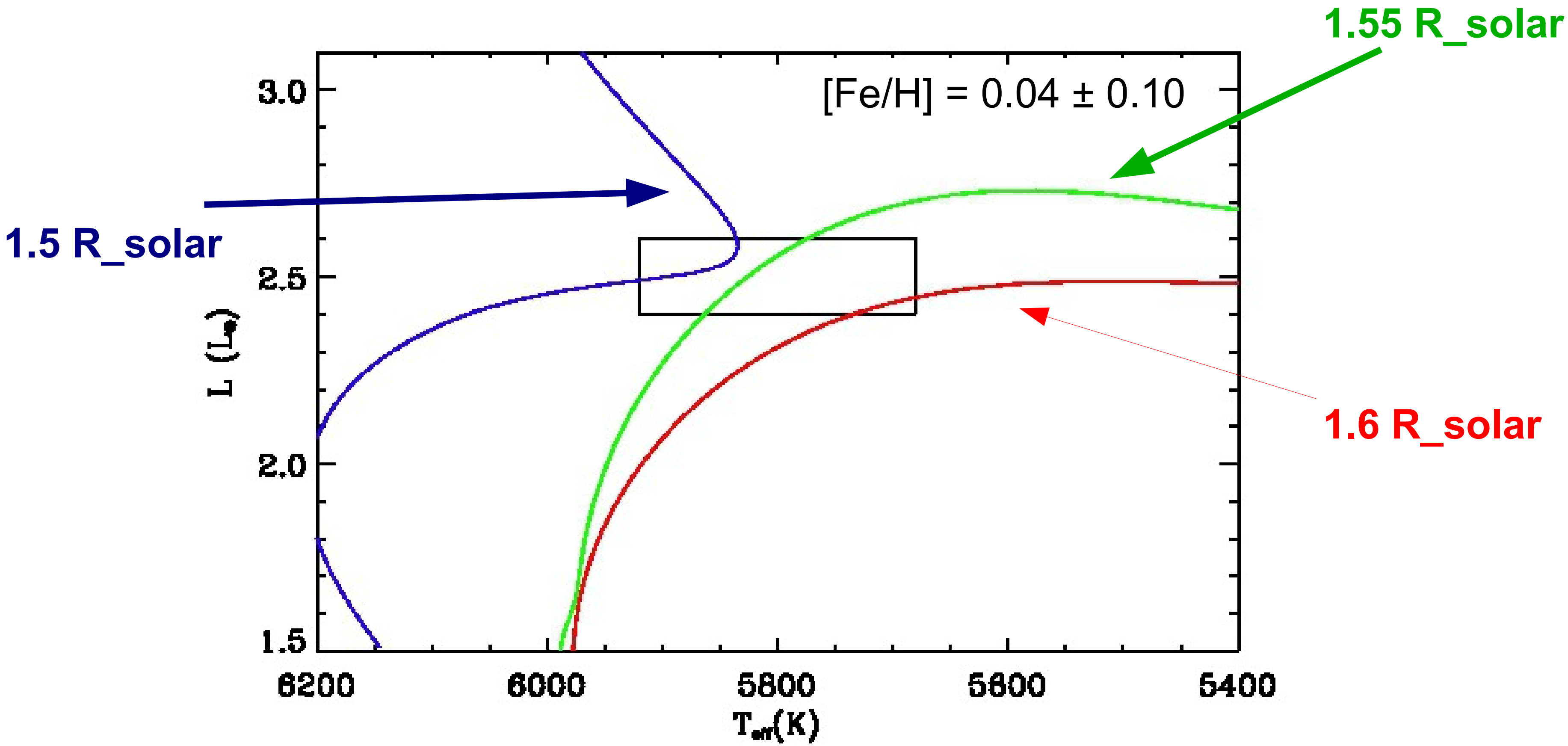
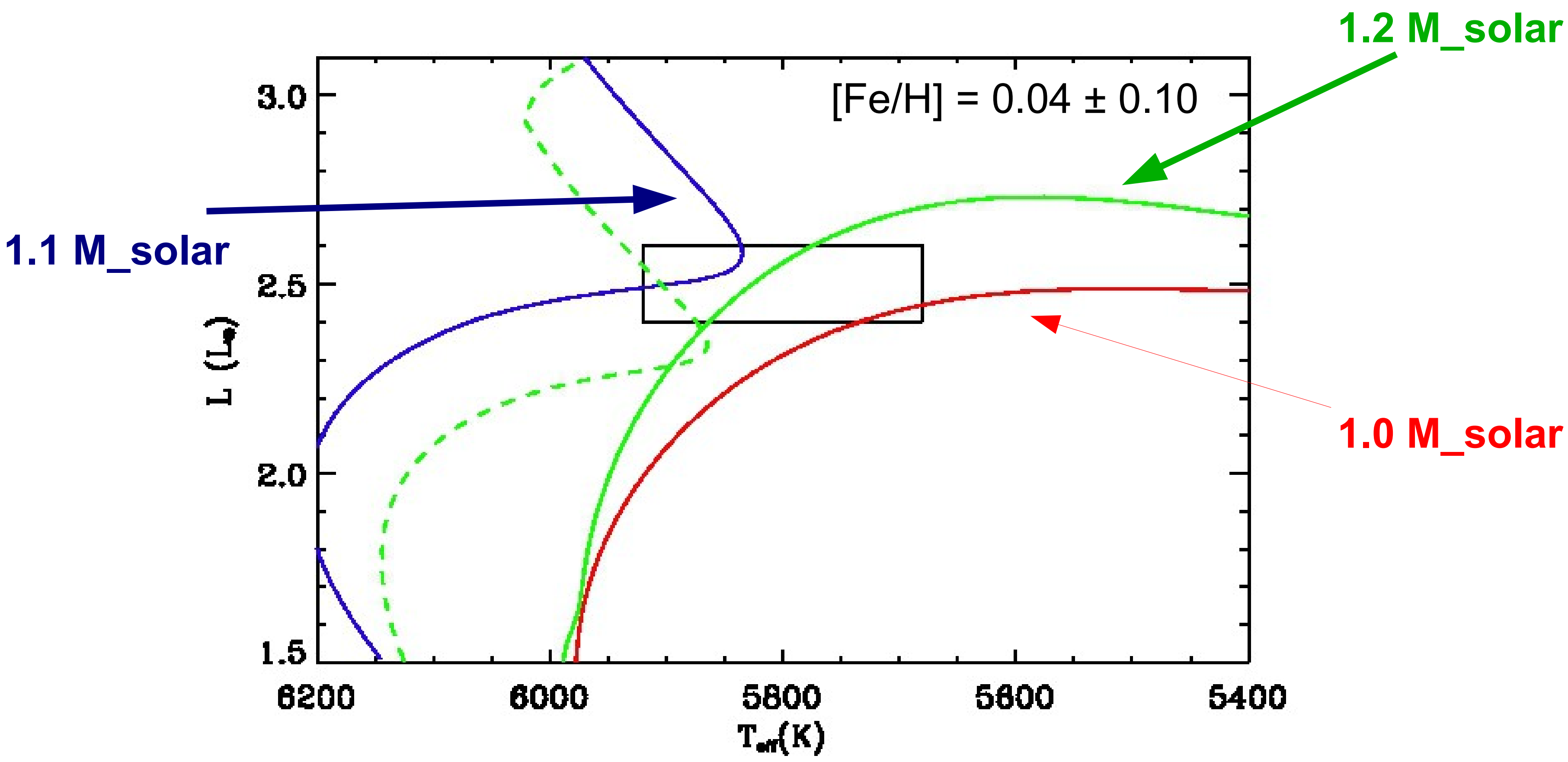


Theories of stellar structure and evolution



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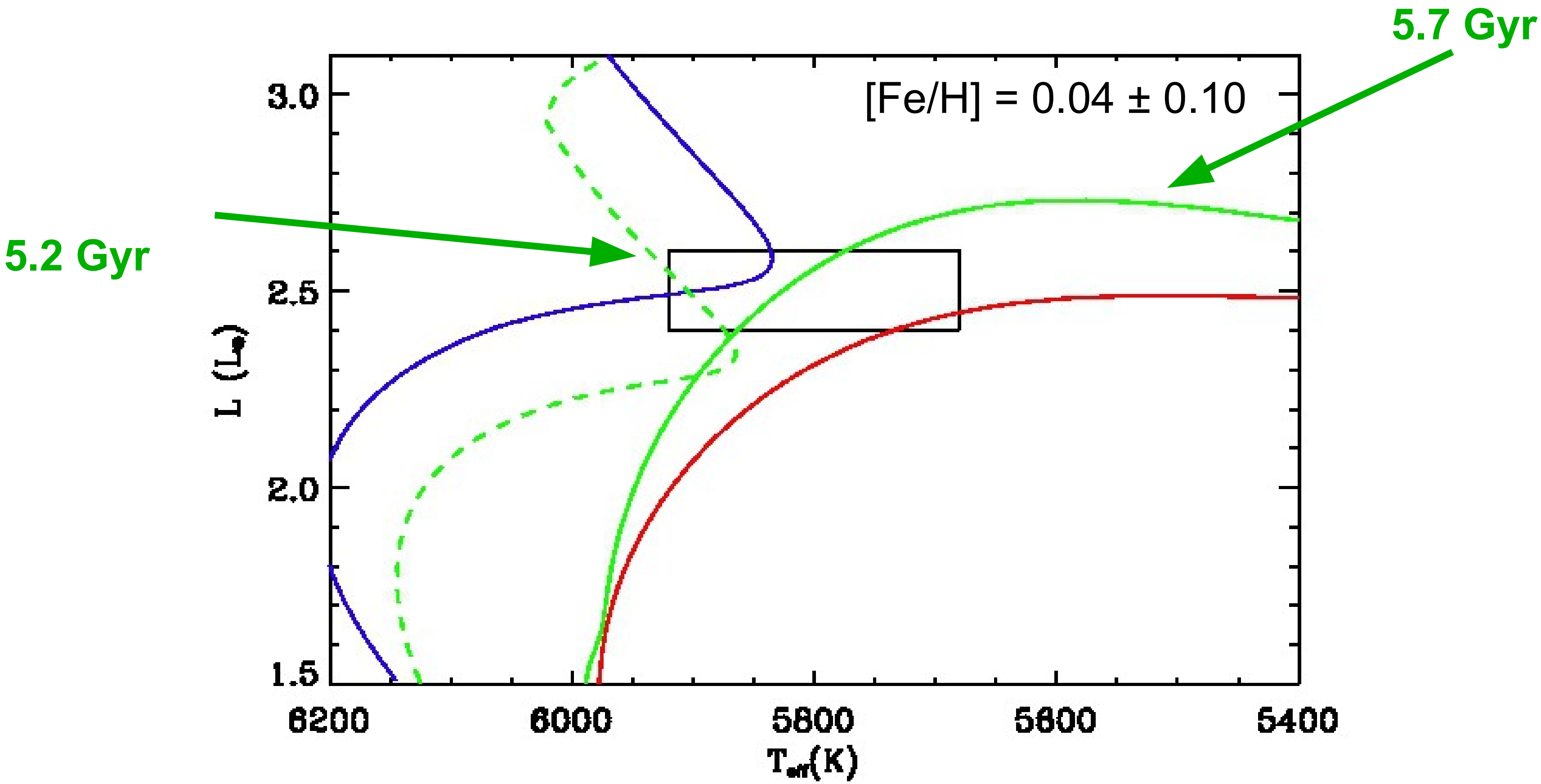
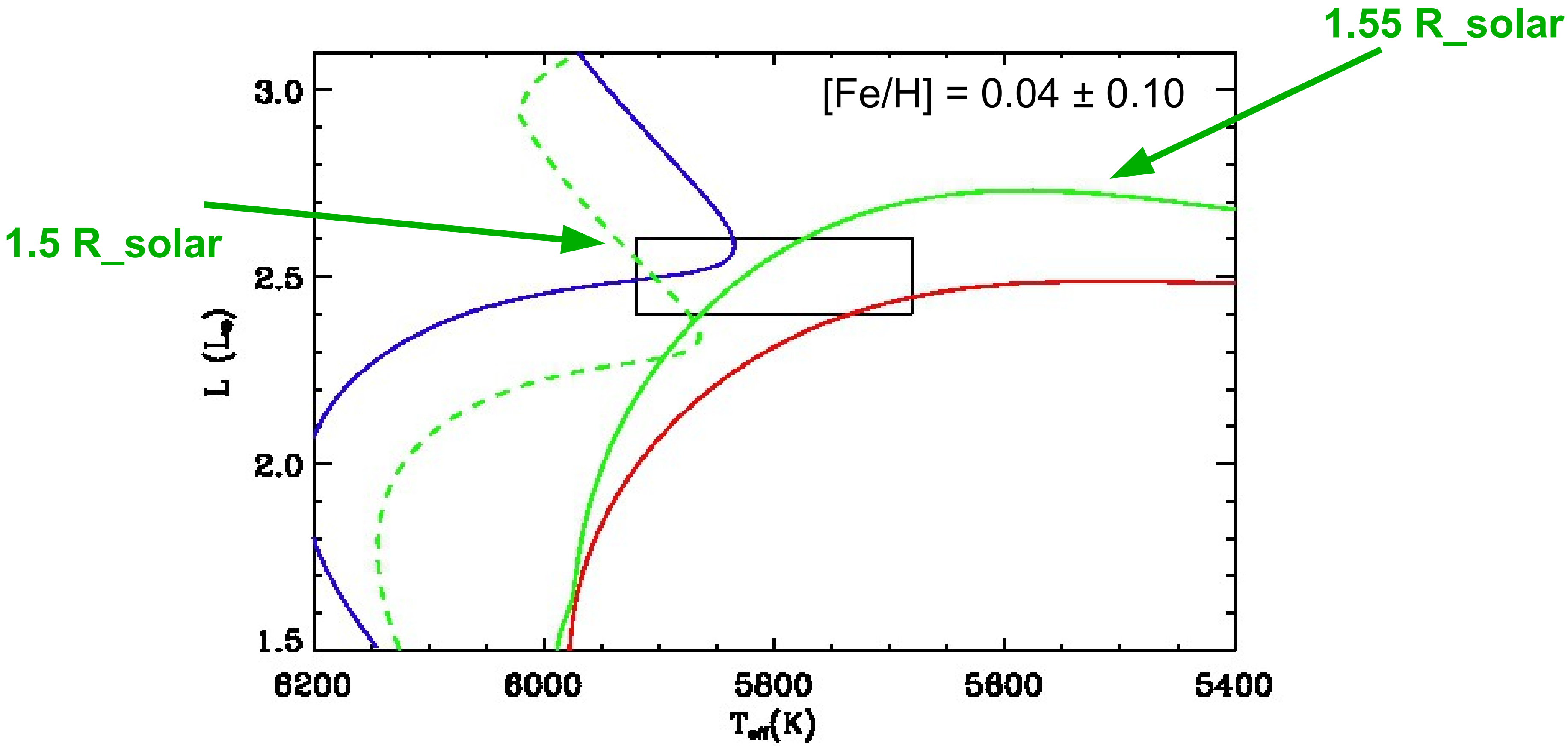


Theories of stellar structure and evolution



How asteroseismology helps (a lot)

The period of a sound wave Π is the time it takes to traverse the star

$$\Pi = 2\frac{R}{c_s}$$

c_s = speed of sound and R = stellar radius. We also know that $c_s^2 \sim \frac{GM}{R^2}$

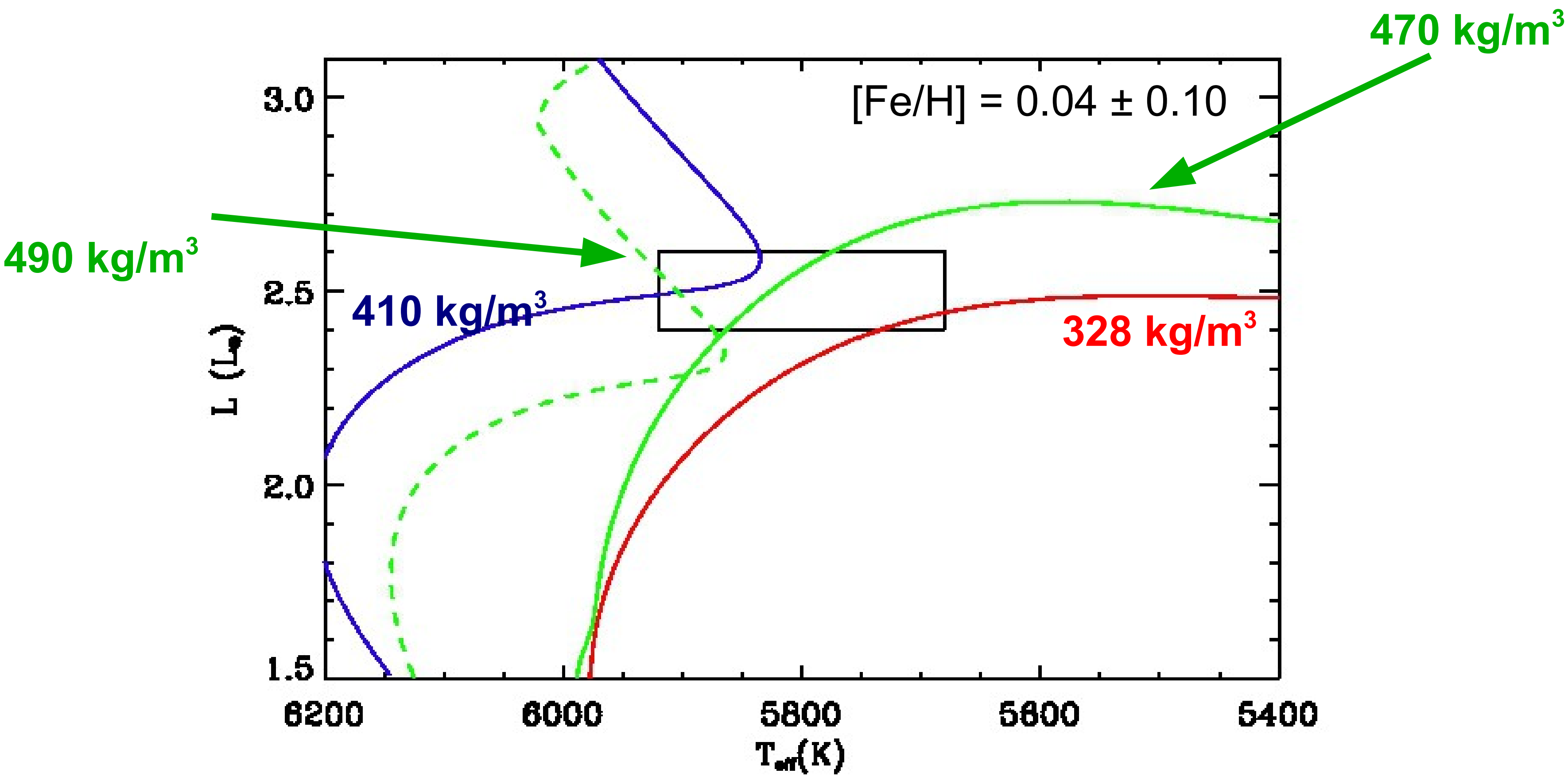
So substituting in, we get $\Pi \propto \sqrt{\frac{R^3}{M}}$ or if we invert the equation we

get that the fundamental oscillation frequency is proportional to the mean density of the star.

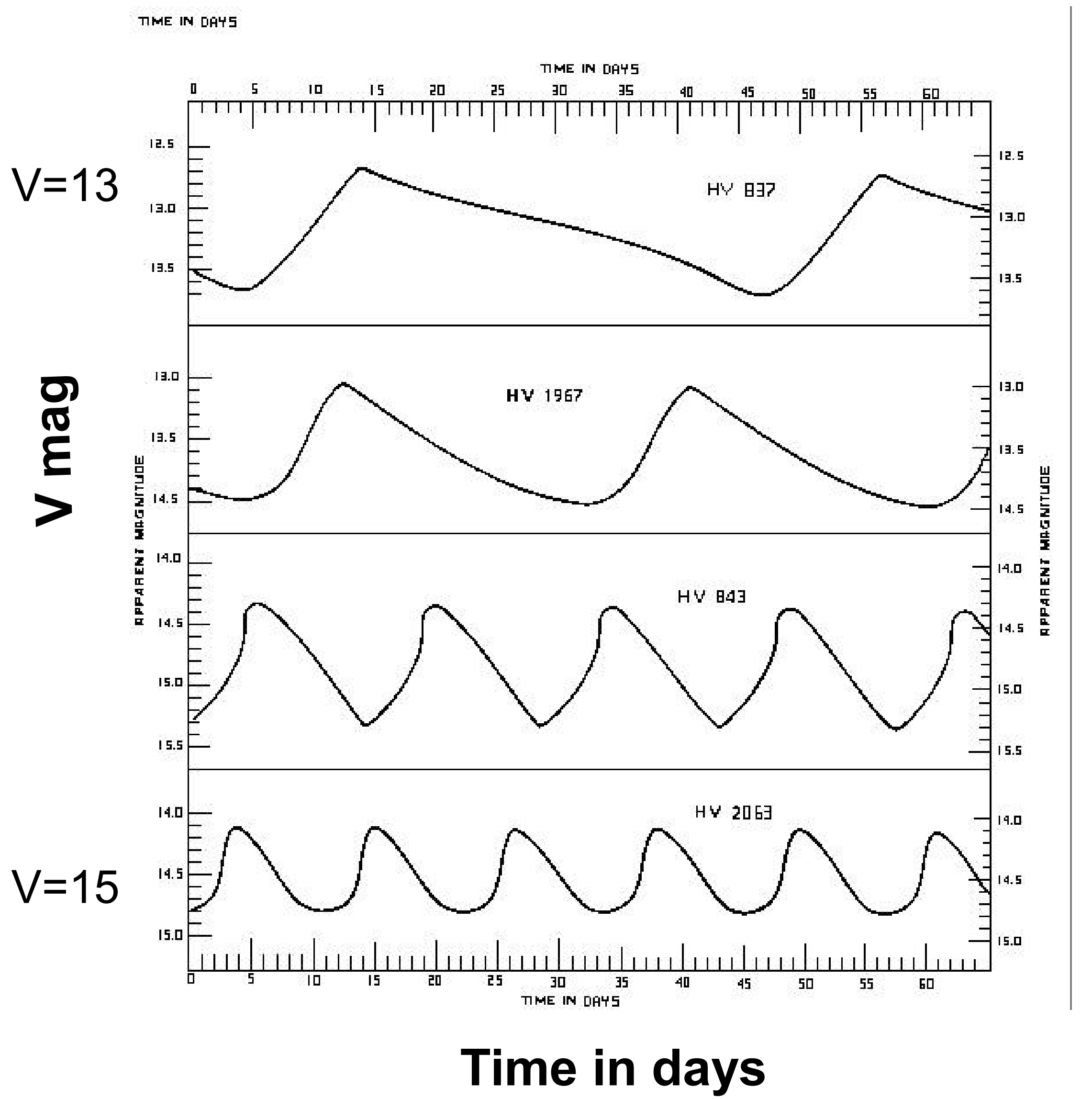
R is measured from interferometry and **Π** is measured from asteroseismology,

-> direct access to the mass of the star (and age)

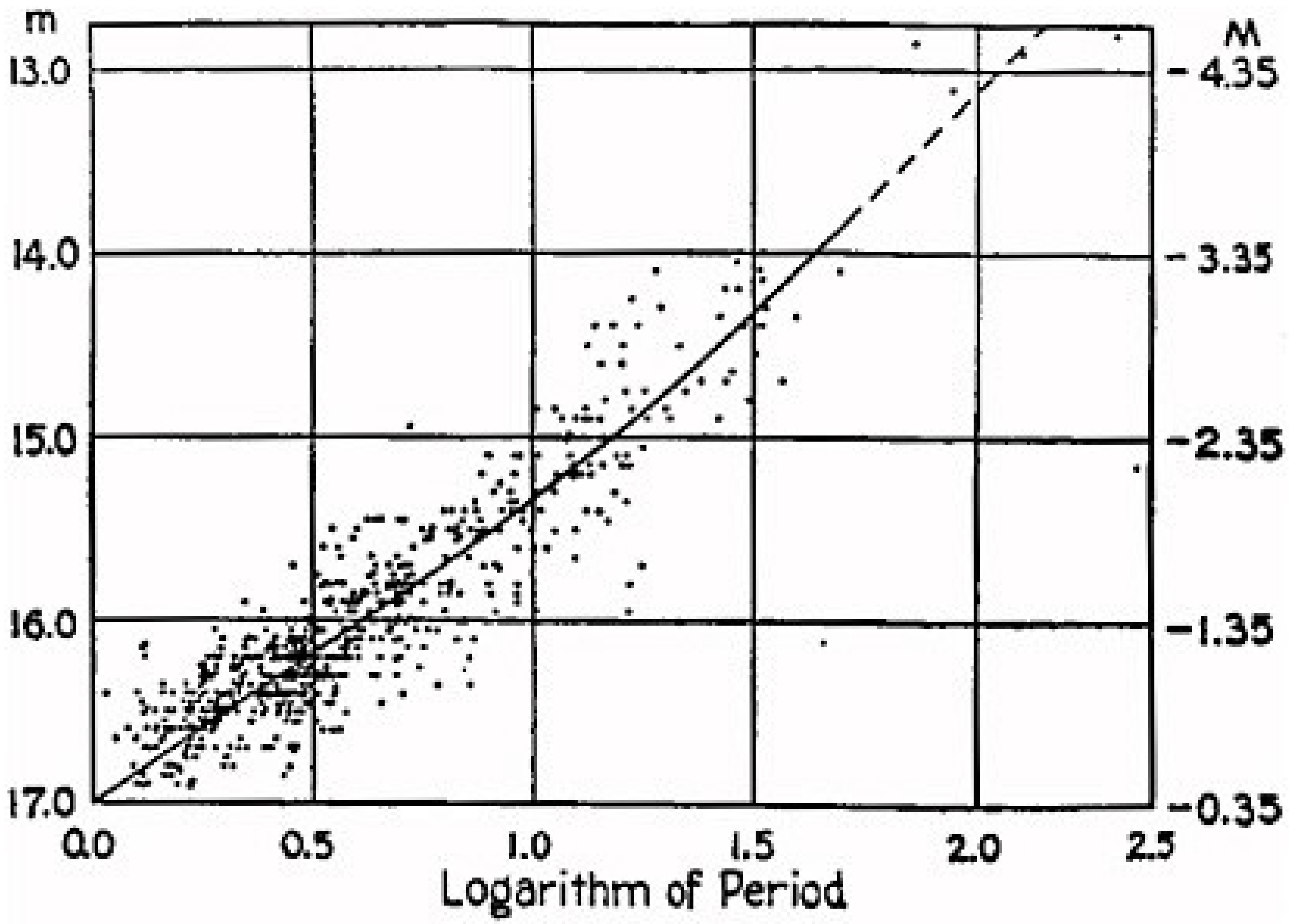
Theories of stellar structure and evolution



Cepheid pulsations in the SMC



Period-Luminosity Relation for SMC



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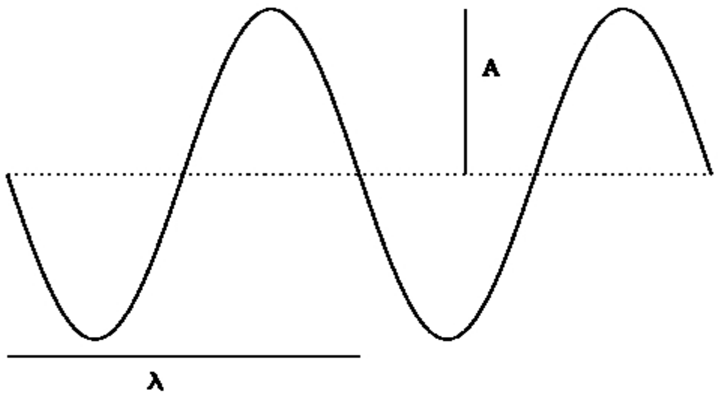
- Introduction/objectives
- Waves and Power Spectra
  - 1D waves, frequencies, period, amplitude
  - Representation of signals by sinusoids
  - Fourier Analysis, reconstruction of signal
  - Frequency resolution, time sampling
  - 1D, 2D, 3D 'spatial' waves
  - Oscillation modes

Waves and Power Spectra

A wave is described by several characteristics



A = amplitude
λ = wavelength
Φ = phase
n = node or 'zero' or 'null'

If the wave covers a distance → λ (m, km, ...)
If the wave covers a time → period, Π (cycles/sec)..

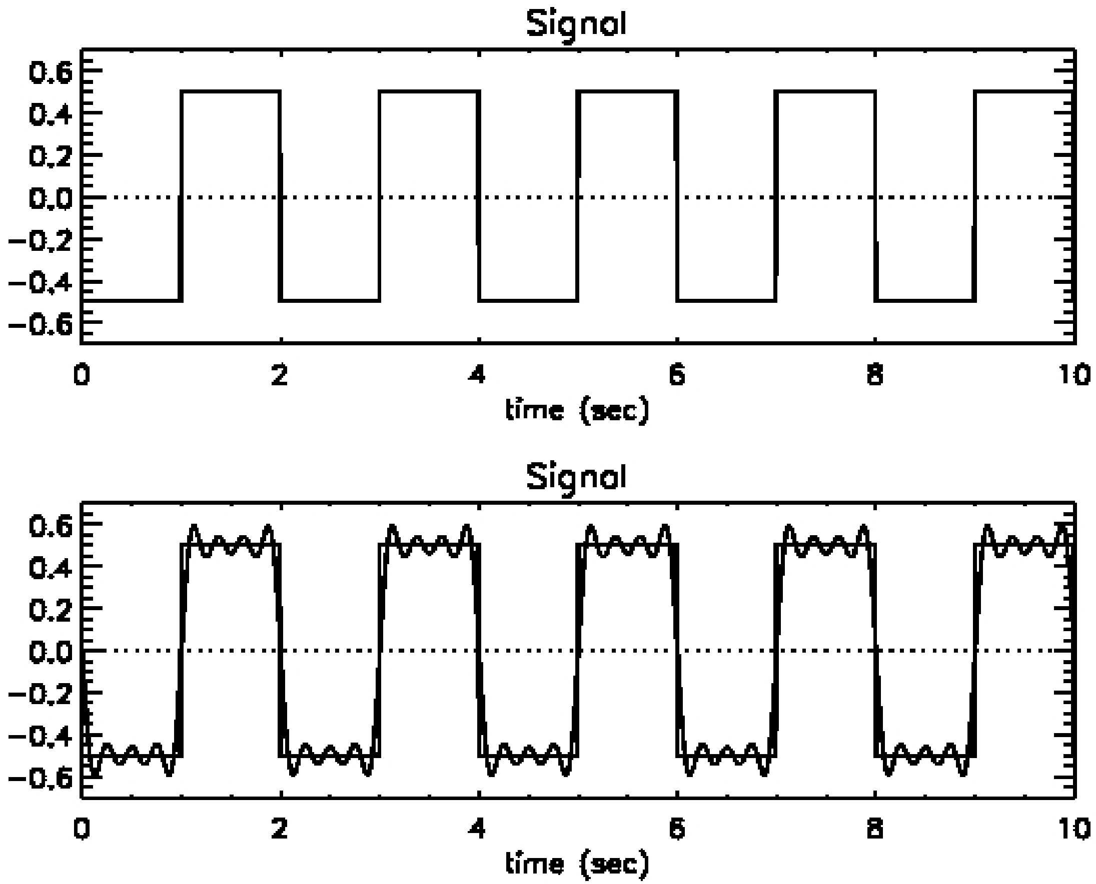
We can relate this to a function of time, $f(t) = A \sin(t/\lambda) = A \sin(t\omega)$
where ω is the cyclic frequency

Most physical processes can be represented by such cyclic behaviour.

Can you think of any examples?

Waves and Power Spectra

Even signals that do not look sinusoidal can still be represented by sinusoids or sums of sinusoids with different amplitudes.



Waves and Power Spectra

Joseph Fourier showed that by representing signals as a sum of trigonemtric functions, their analysis can be greatly simplified. This gave way to Fourier Analysis.

The Fourier Transform is a function that allows us to represent the components in signals in a simple manner, namely as frequencies, amplitudes and phases.

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-2\pi i t\omega}dt$$

where $$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

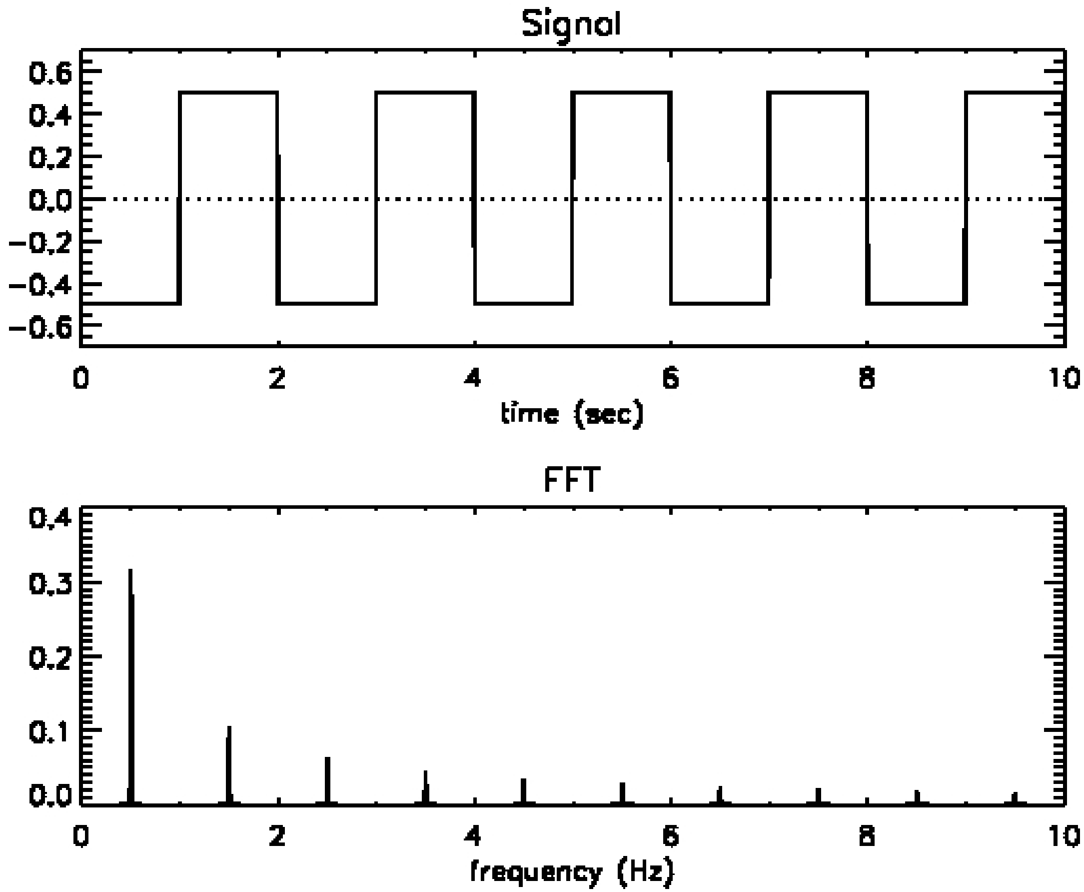
For an infinite continuous function f(t) we can calculate the Fourier coefficients f^ω by simply integrating the function times a wave of a frequency ω.

However, our signals s(t) are not continuous nor infinite, and so the Discrete Fourier Transform allows us to calculate the coefficients using the following formula:

$$A_\nu = \sum_{t=0}^{T-1} s(t)e^{-2\pi i t\nu/T}$$

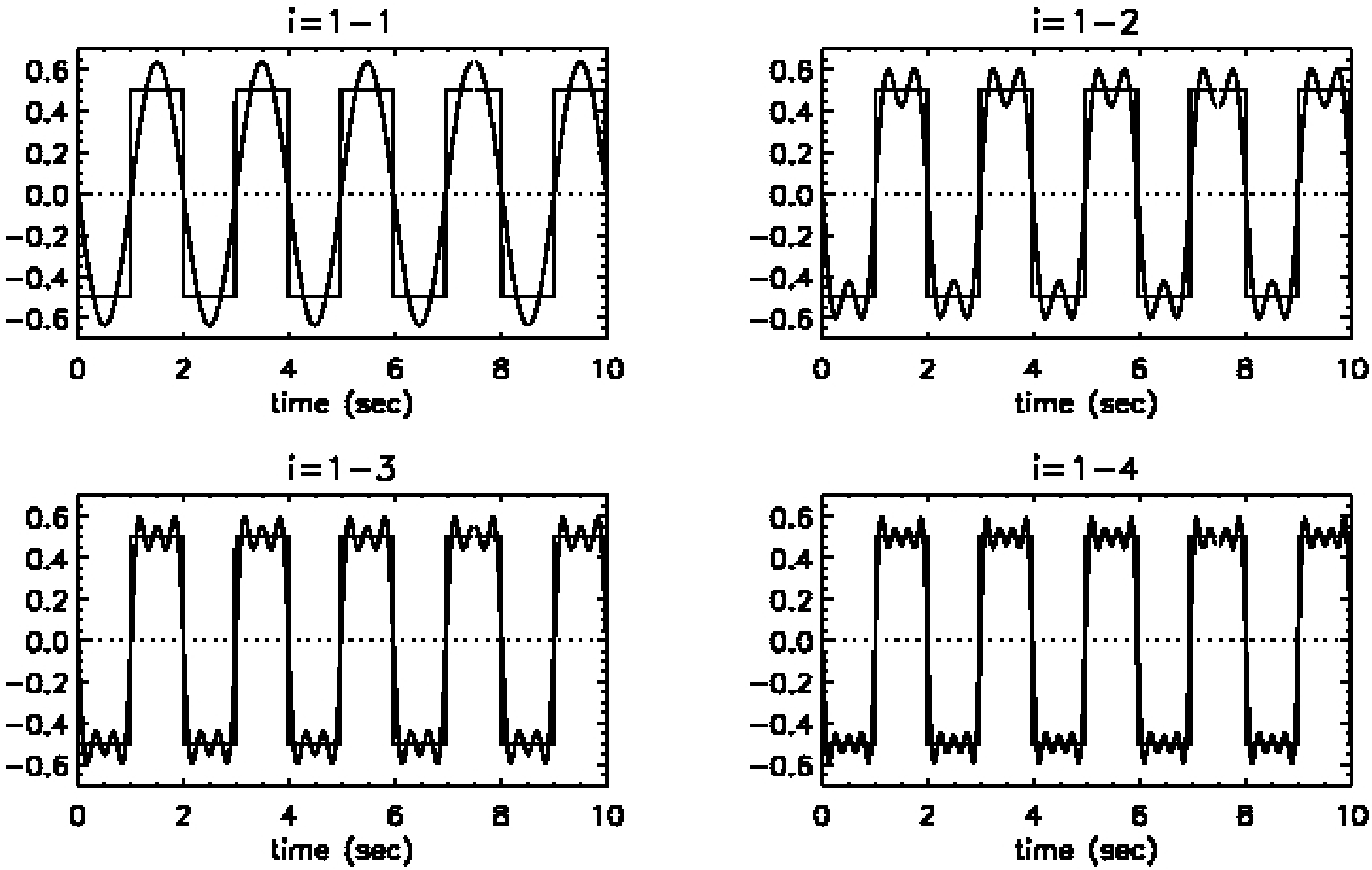
where A_v is the coefficient for the frequency v, and T is the number of points in s(t)

Waves and Power Spectra



Here we show the Fourier Transform (lower panel) of the boxcar signal (top).
Each peak shows the coefficient associated to a wave of a certain frequency

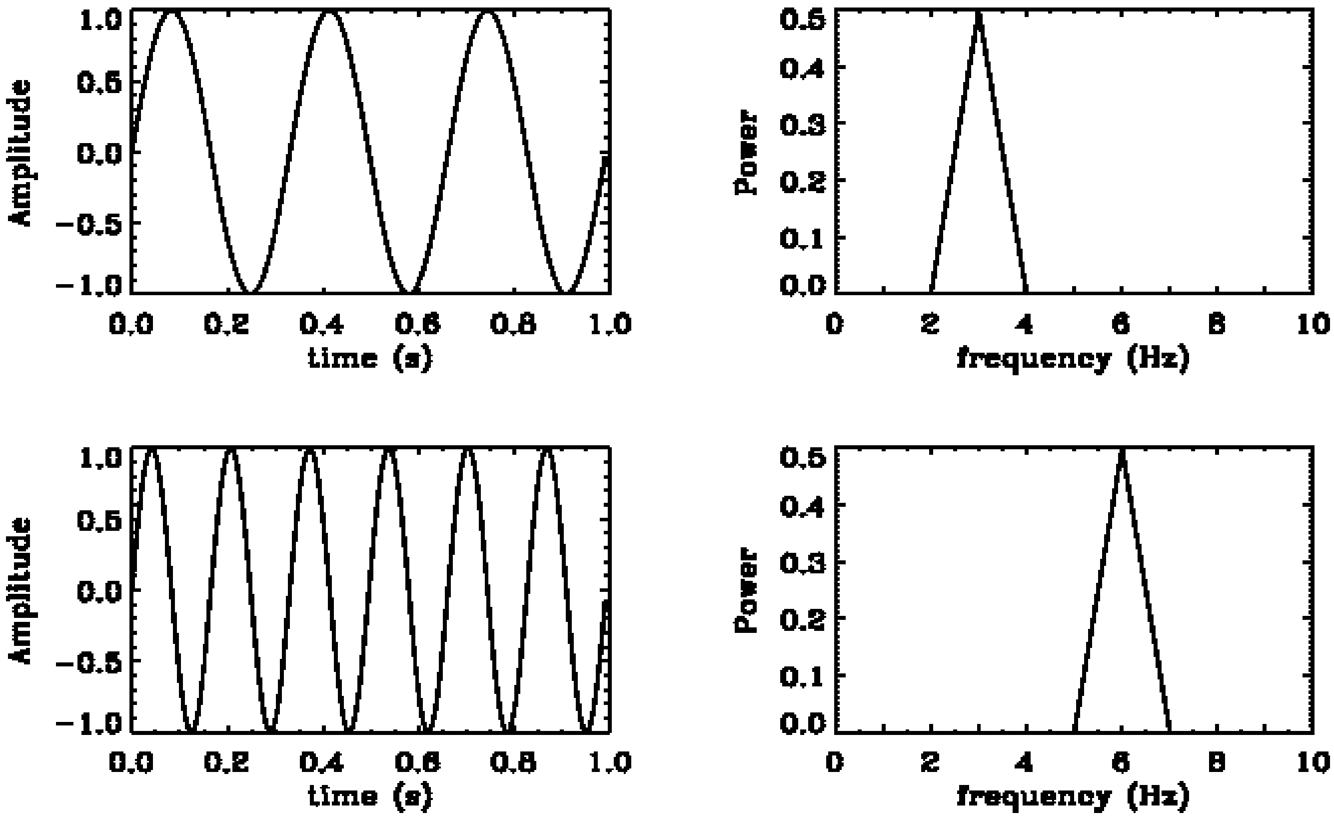
Waves and Power Spectra



Reconstructing the time signal using the first few coefficients of the DFT.

Waves and Power Spectra

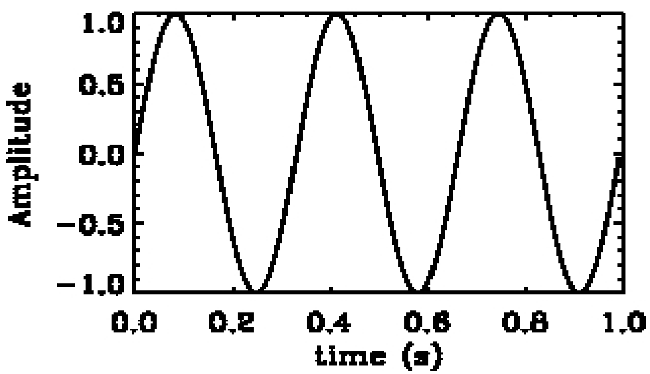
A signal usually has units of time. The natural unit is the second (s). **Frequency is the number of cycles per second c/s and is denoted by hertz (Hz).**
The FT is shown in Hz. It tells us the frequencies of the components of the signal

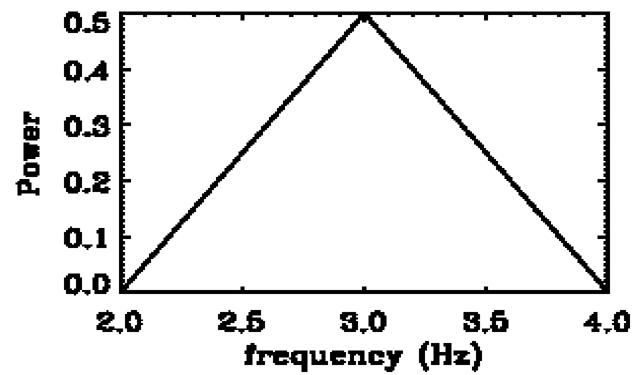


Waves and Power Spectra

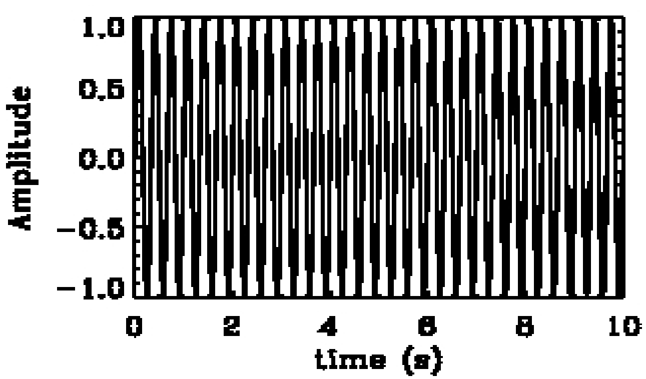
Increasing the length of the signal gives better **frequency resolution**

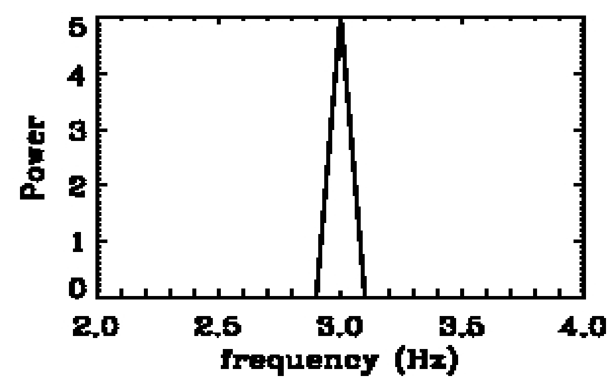
$\Delta f = 1/T$ – or a sharper peak in the FT





$\Delta f = 1/1 = 1$ c/s = 1 Hz



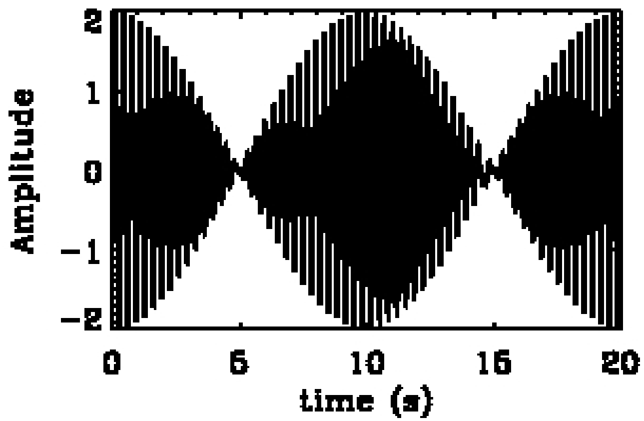


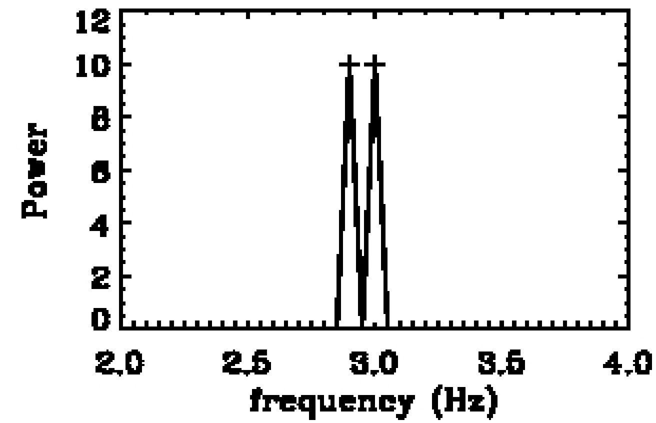
$\Delta f = 1/10 = 0.1$ c/s = 0.1 Hz

Waves and Power Spectra

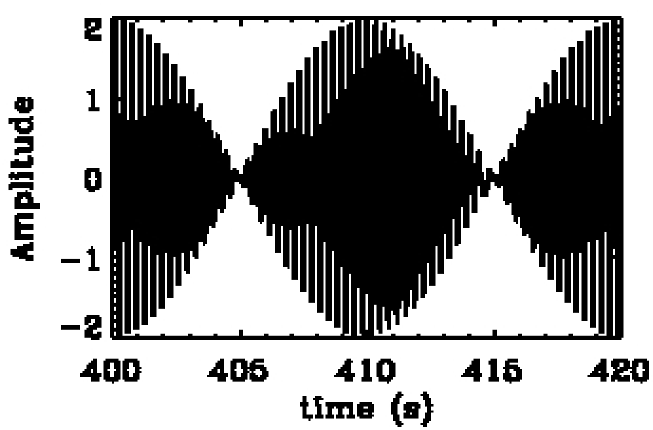
The frequency resolution is very important if we wish to resolve frequencies which are similar.

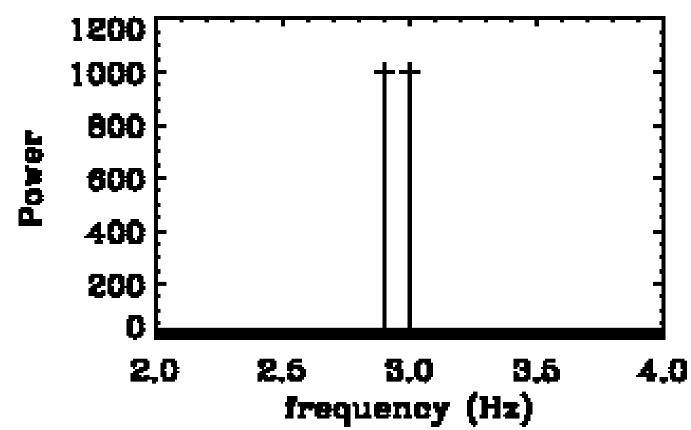
T = 20s





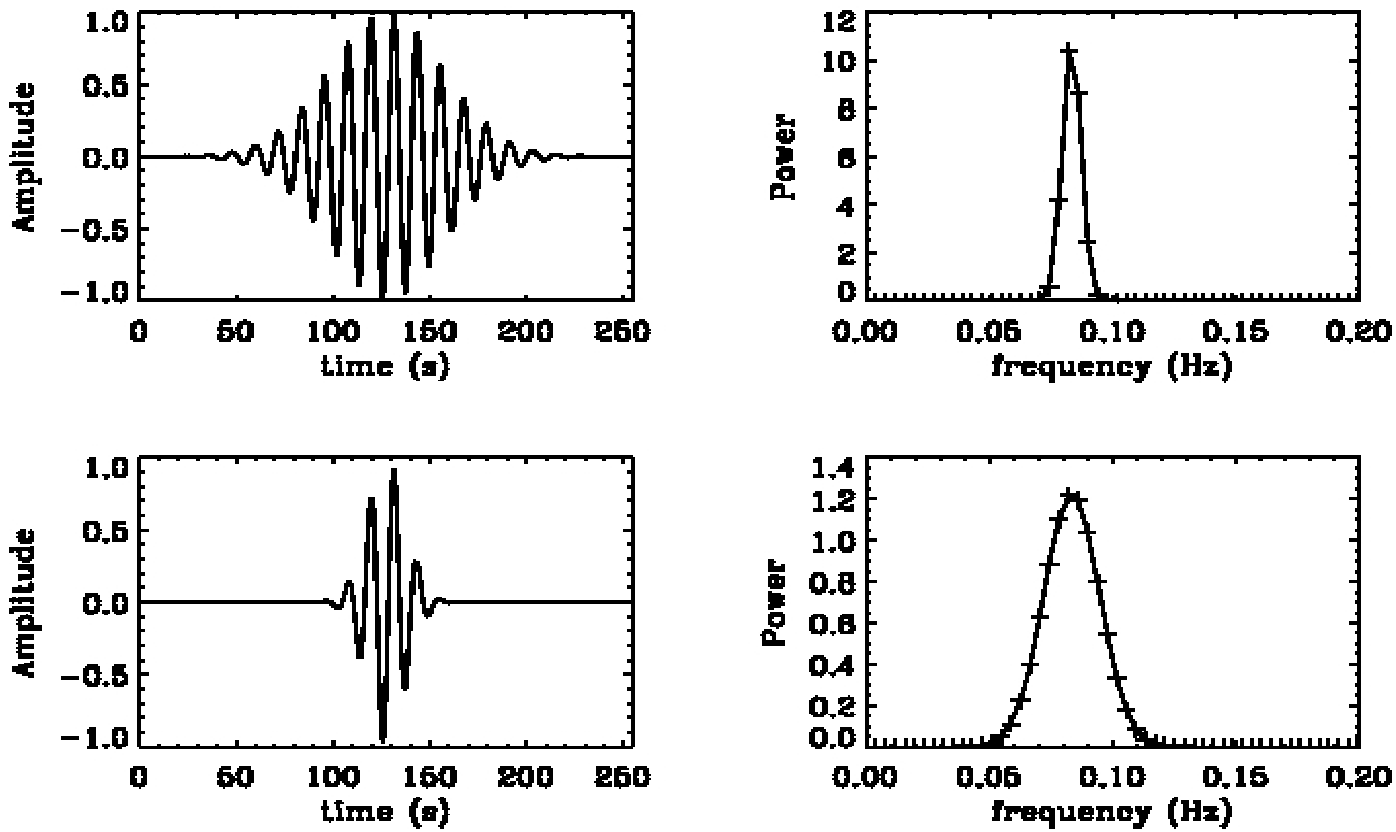
T = 500s





Waves and Power Spectra

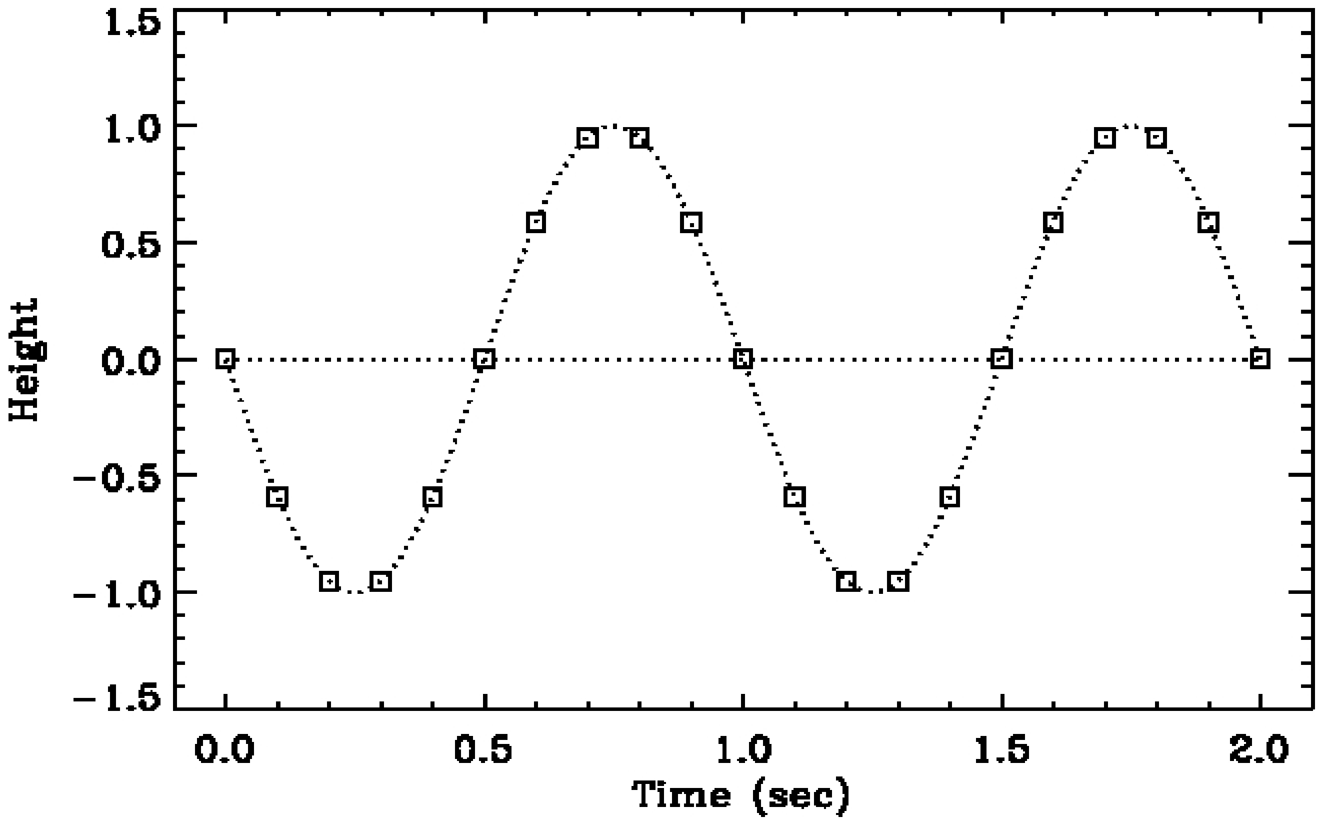
A damped oscillation (left) is represented as a sum of similar frequencies (right).
Its damping exponent (how quickly it decays) determines the width of the signal in the FT.



Waves and Power Spectra

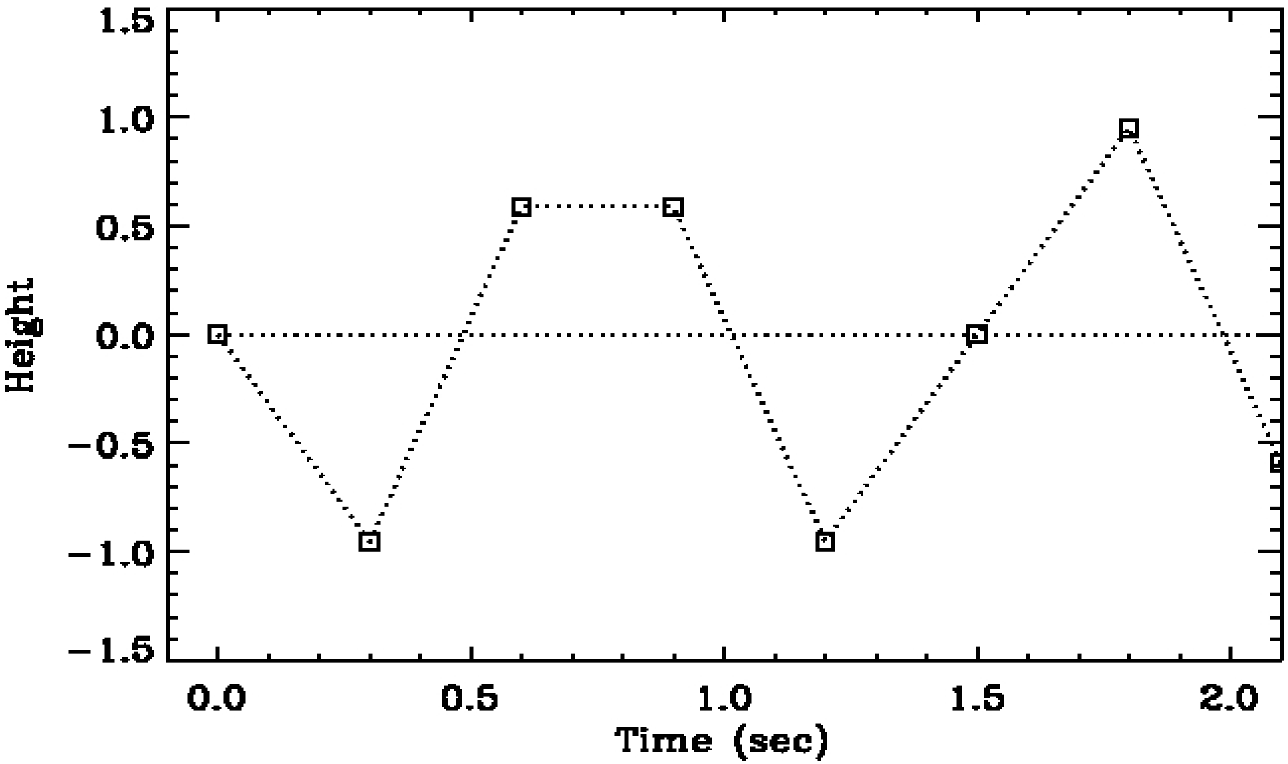
The sampling time **dt** determines the length of the shortest periodicity or highest frequency present.
In this figure we can easily resolve the frequency of 1Hz.
The highest frequency **v0 = 2 dt.**

What is dt?
What is v0?

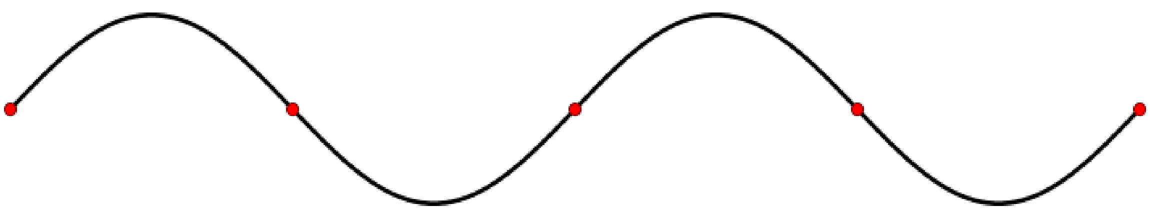


Waves and Power Spectra

Poorer sampling time. This sinusoidal signal is no longer like a sinusoid. However, we can still detect the oscillation frequency.



Waves in space



A wave on a string. The red dots show the nodes or 'zeroes'.
These are points where the medium does not displace.

Waves in space

f_0 f_1

f_2 f_3

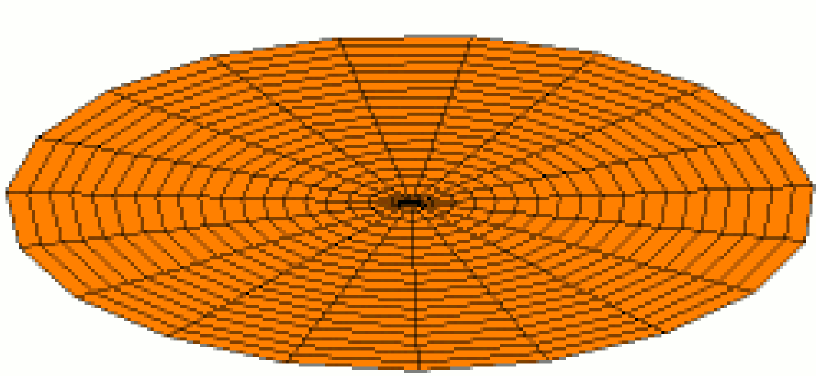
f_4 f_5

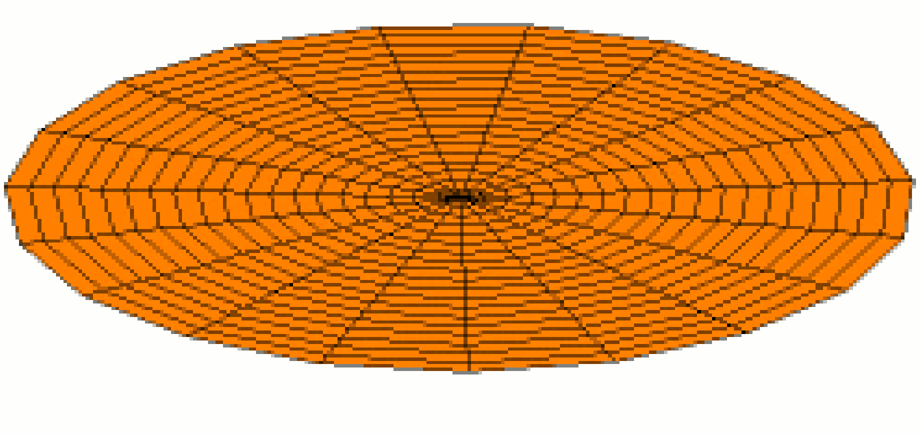
Different frequencies can be observed even for the same length of string.

The different frequencies are characterised by the **number of nodes** on the string, and they are an integer number times the fundamental frequency.

If v is the speed of the wave, then f_0 = v/2L, where L is the length, then the nth harmonic f_n = nf_0 = nv/2L (v=sqrt(T/rho))

2D Waves: Drum





A 2-D example of oscillations: the fundamental and first overtone

3D Waves: Star

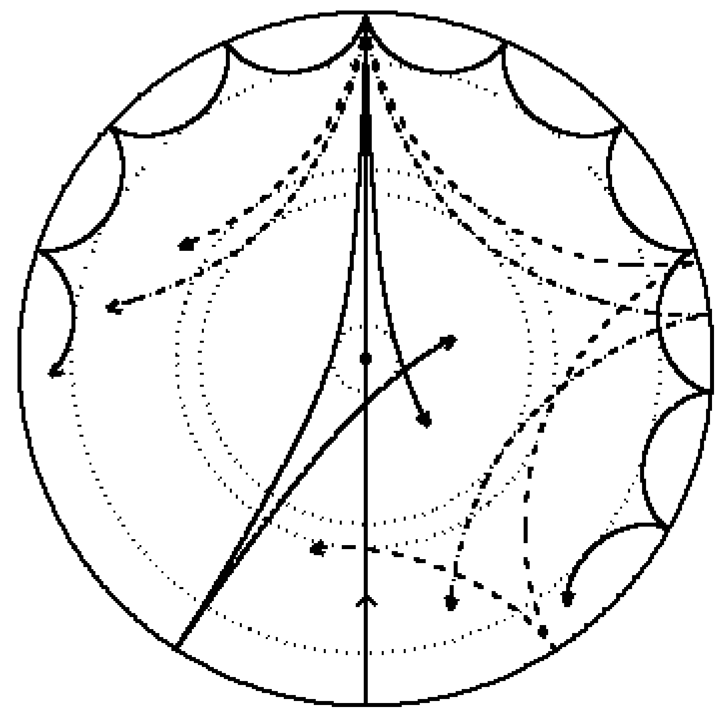
Each frequency corresponds
To a MODE (think of TYPE).

A mode is described by 3 numbers

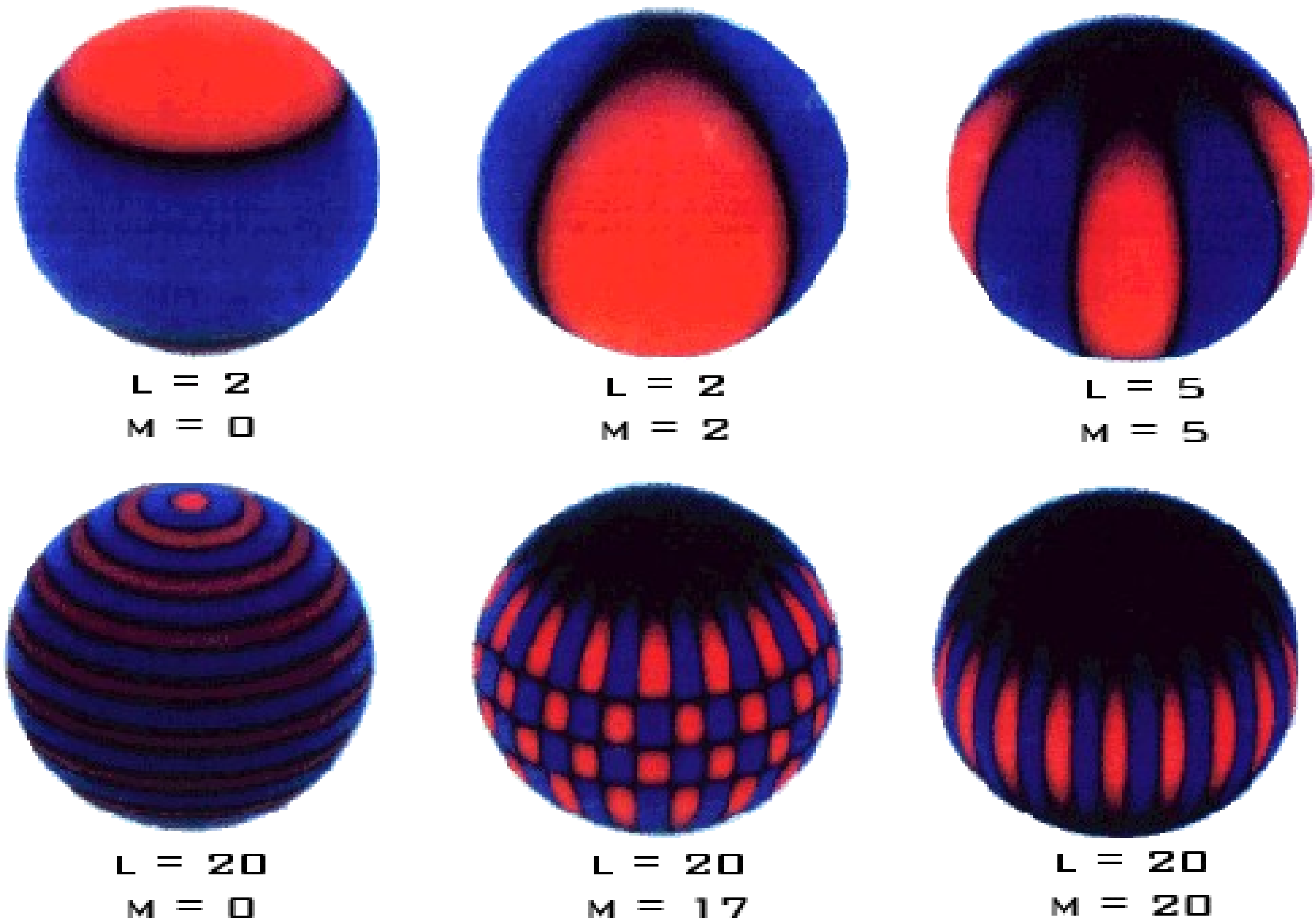
L = angular degree
N = radial order
M = azimuthal order

L, M describe the surface geometry of the mode, while N shows the number of zeroes between the surface and the center

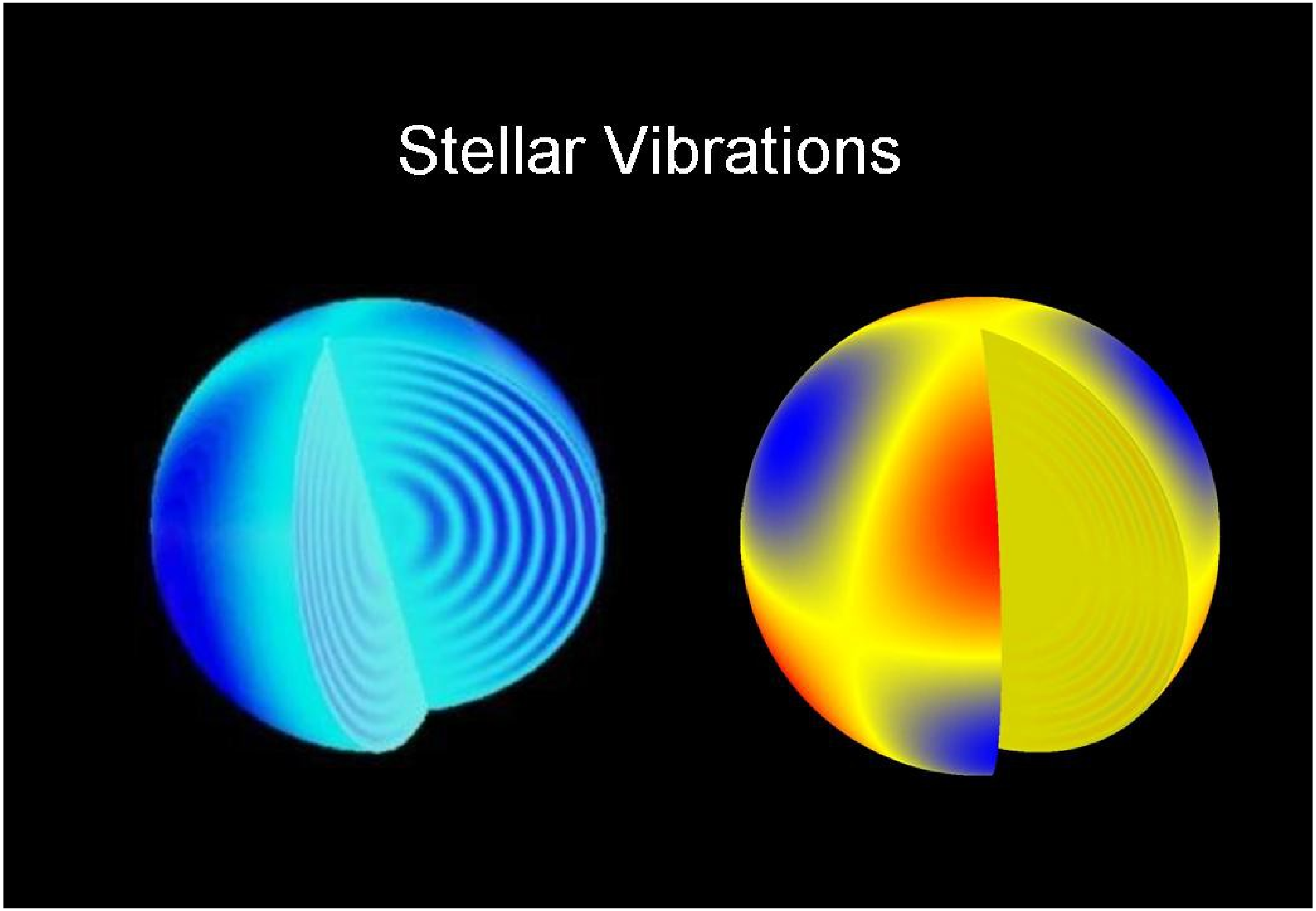
In this figure we can understand the angular degree, L.



3D Waves: Star

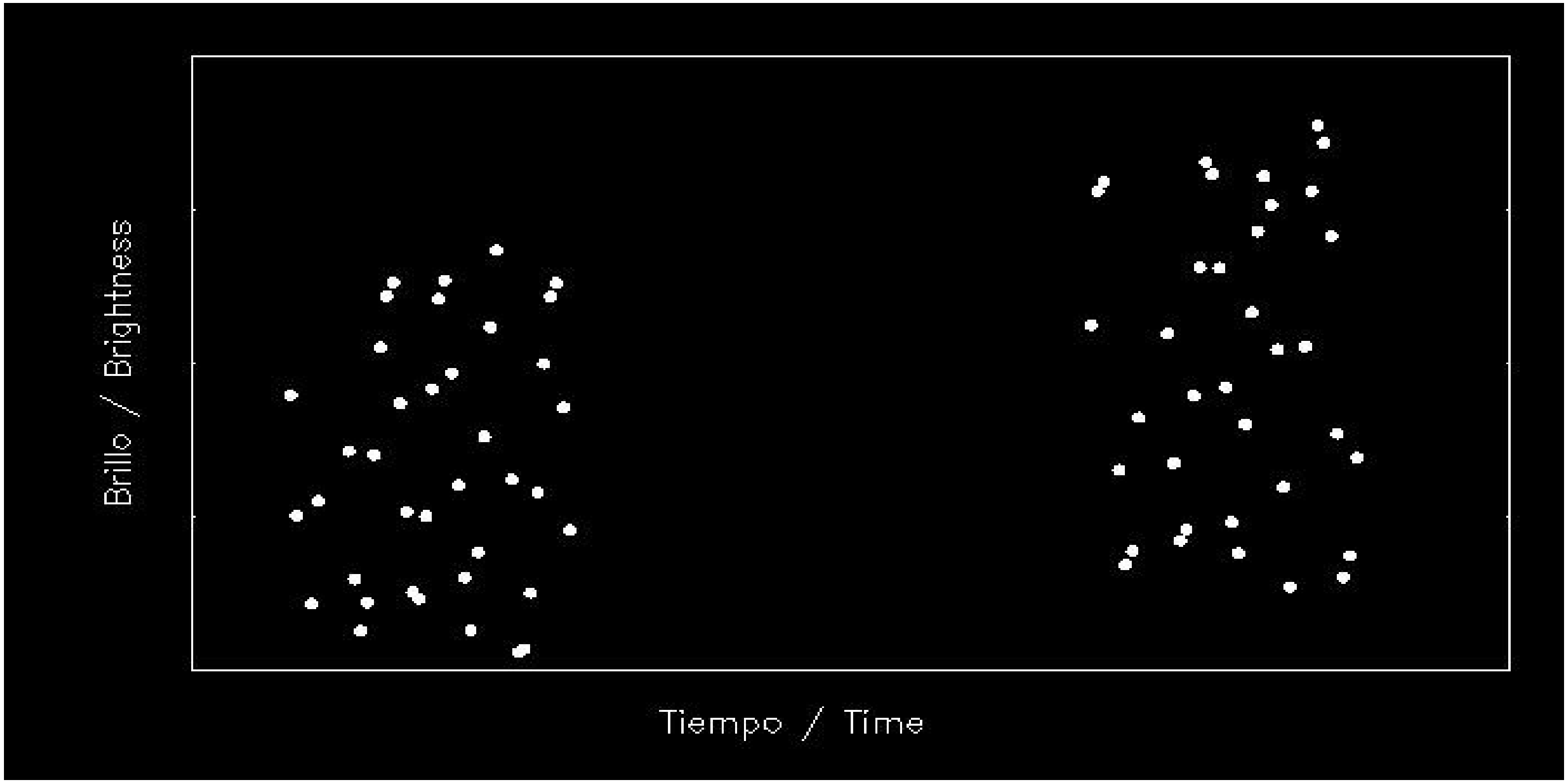


Waves in a star: modes



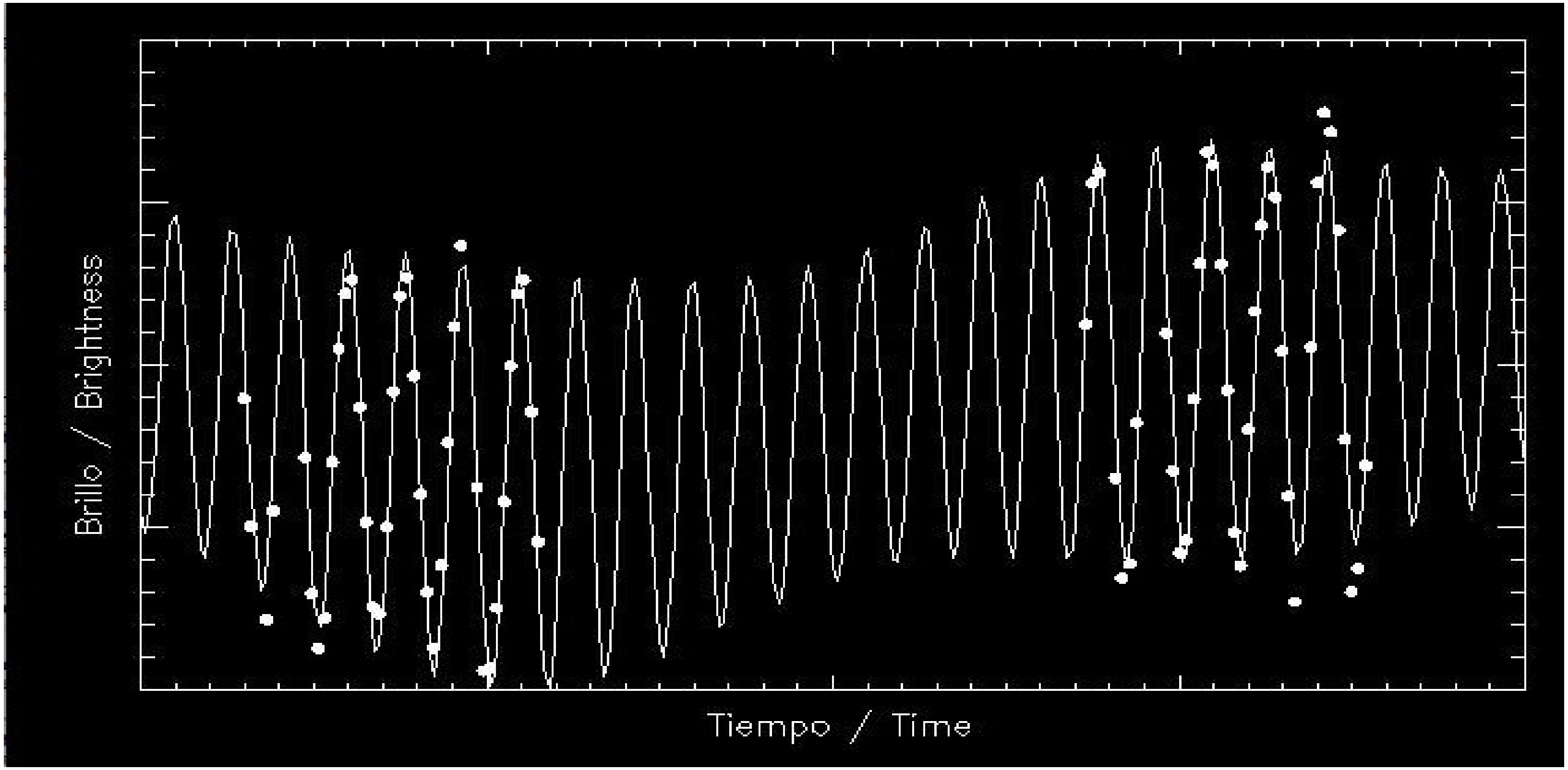
Observing pulsations

Unfortunately we can not resolve the spatial scales of oscillations of most stars.
We observe 'integrated light' (see below).
For the data analysis, this makes the problem simpler, we just think of simple oscillations in time.



Observing pulsations

Even if we can resolve the frequencies, do we know which MODE of oscillation we are observing? How do we assign l,m,n to each observed frequency?
It depends on the star and the excitation and damping mechanisms.



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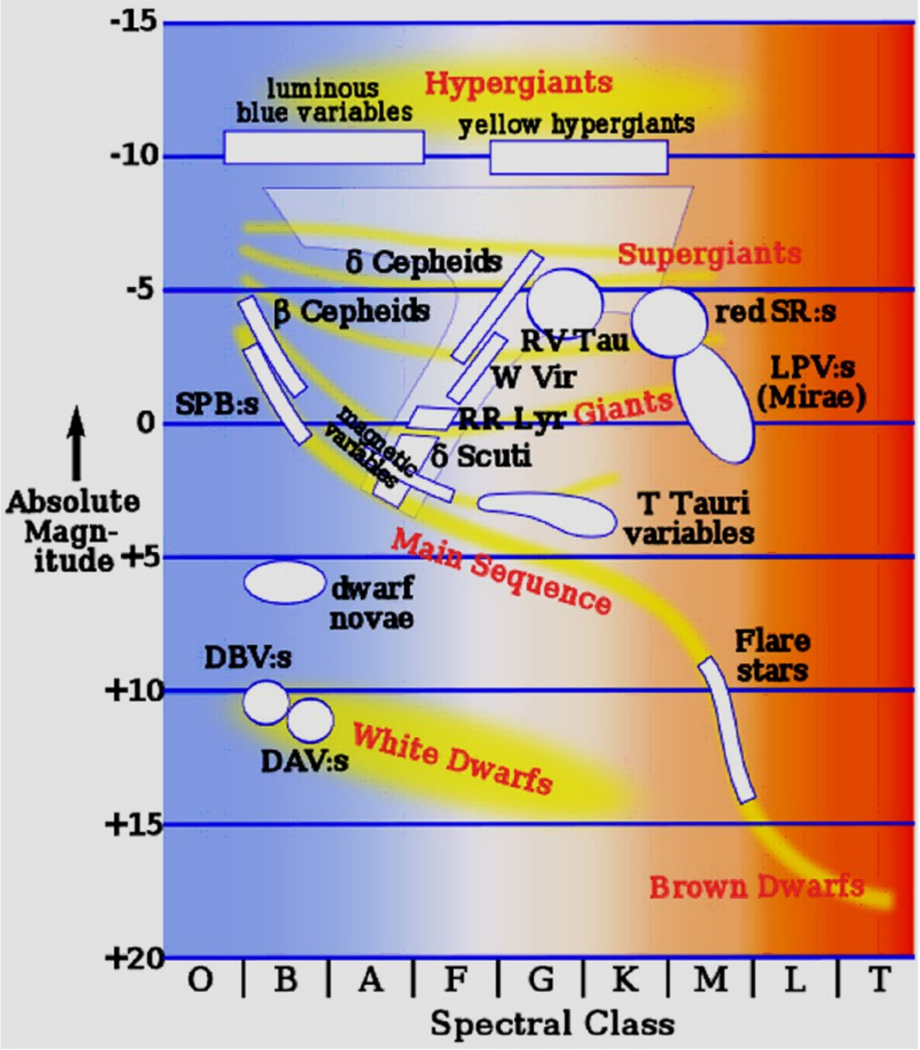
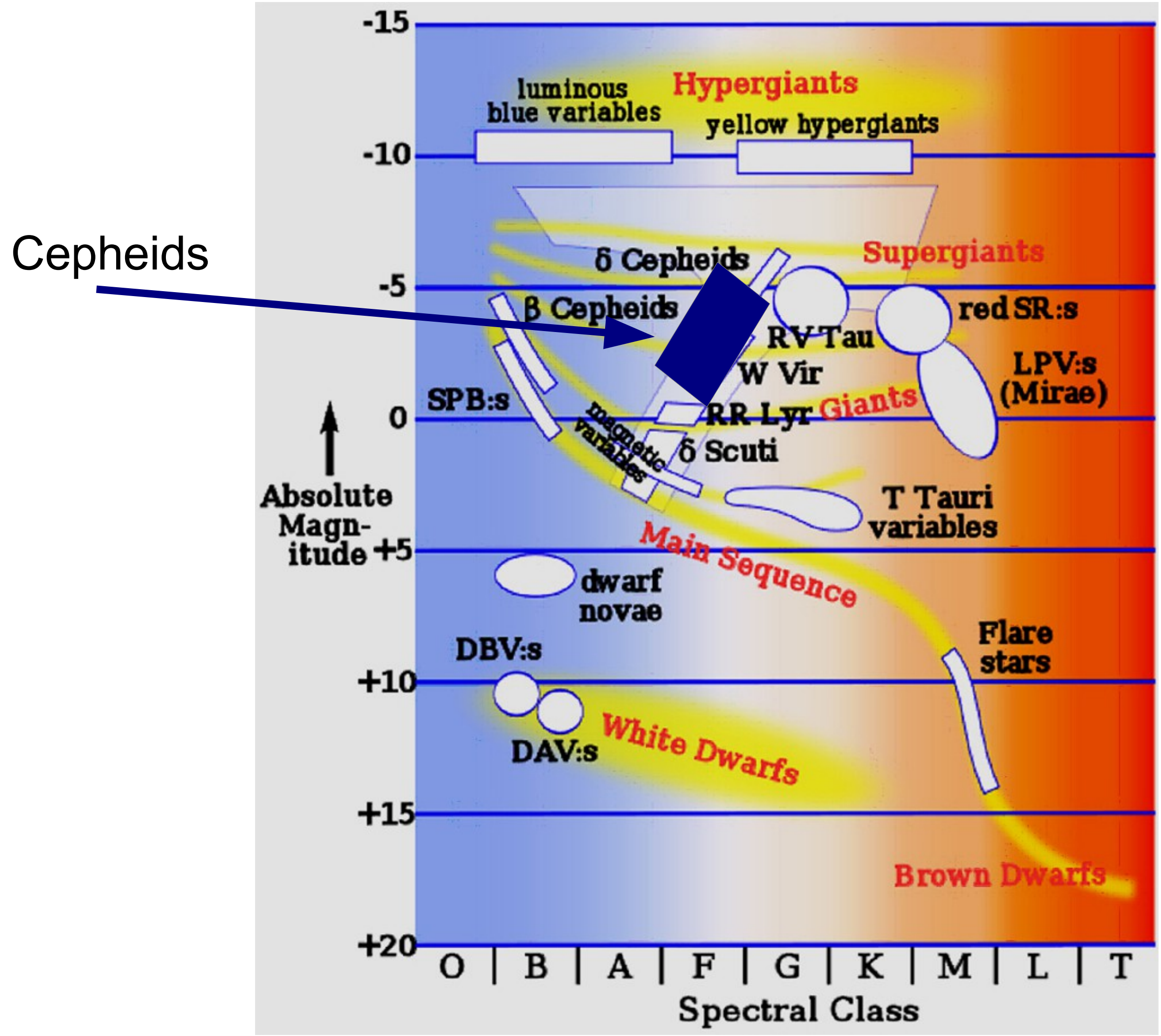
  - Types of pulsating stars
  - Power spectra of Cepheids/delta Scuti/solar-like
  - Excitation Mechanisms

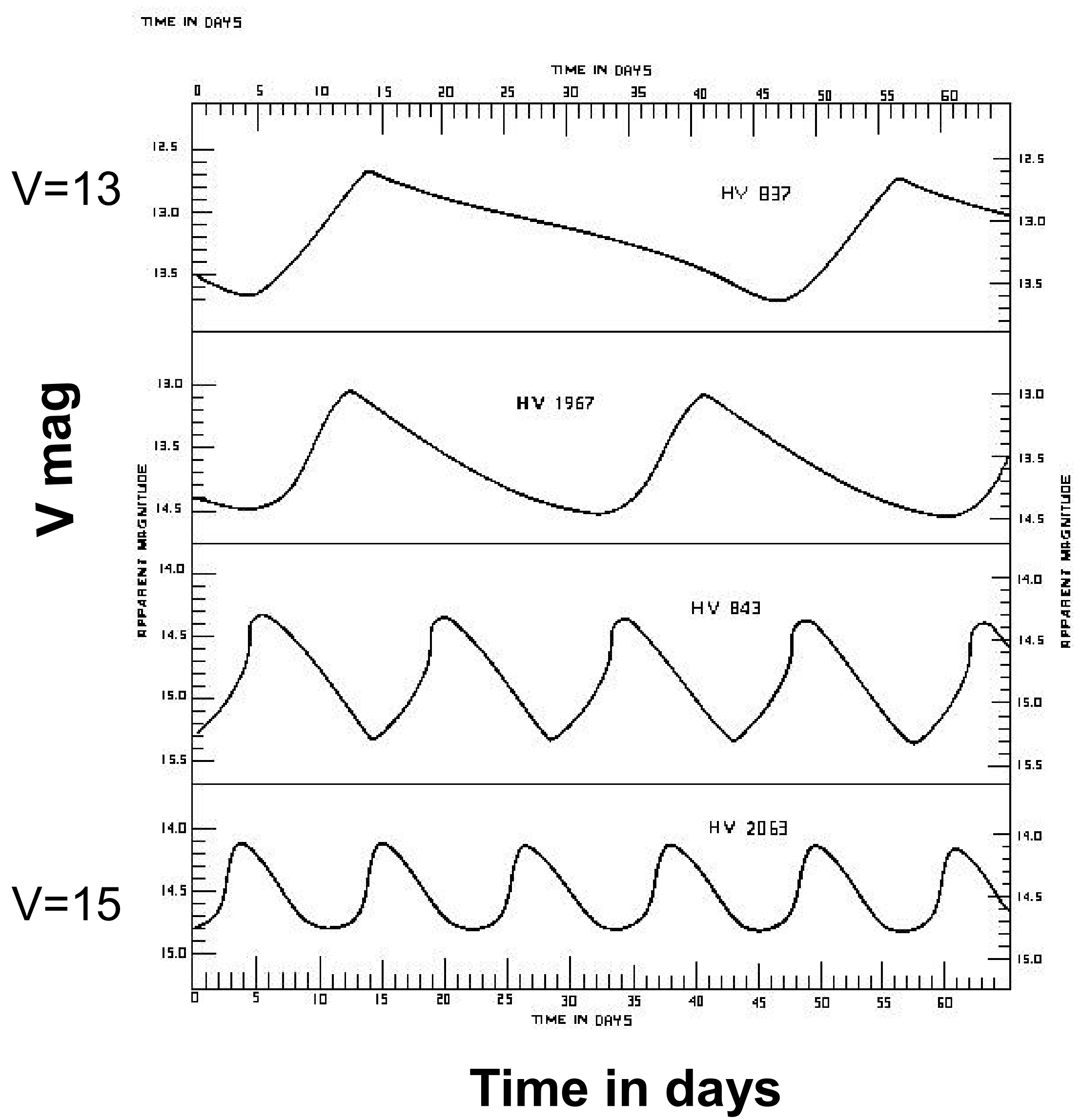
Oscillations across the HR diagram



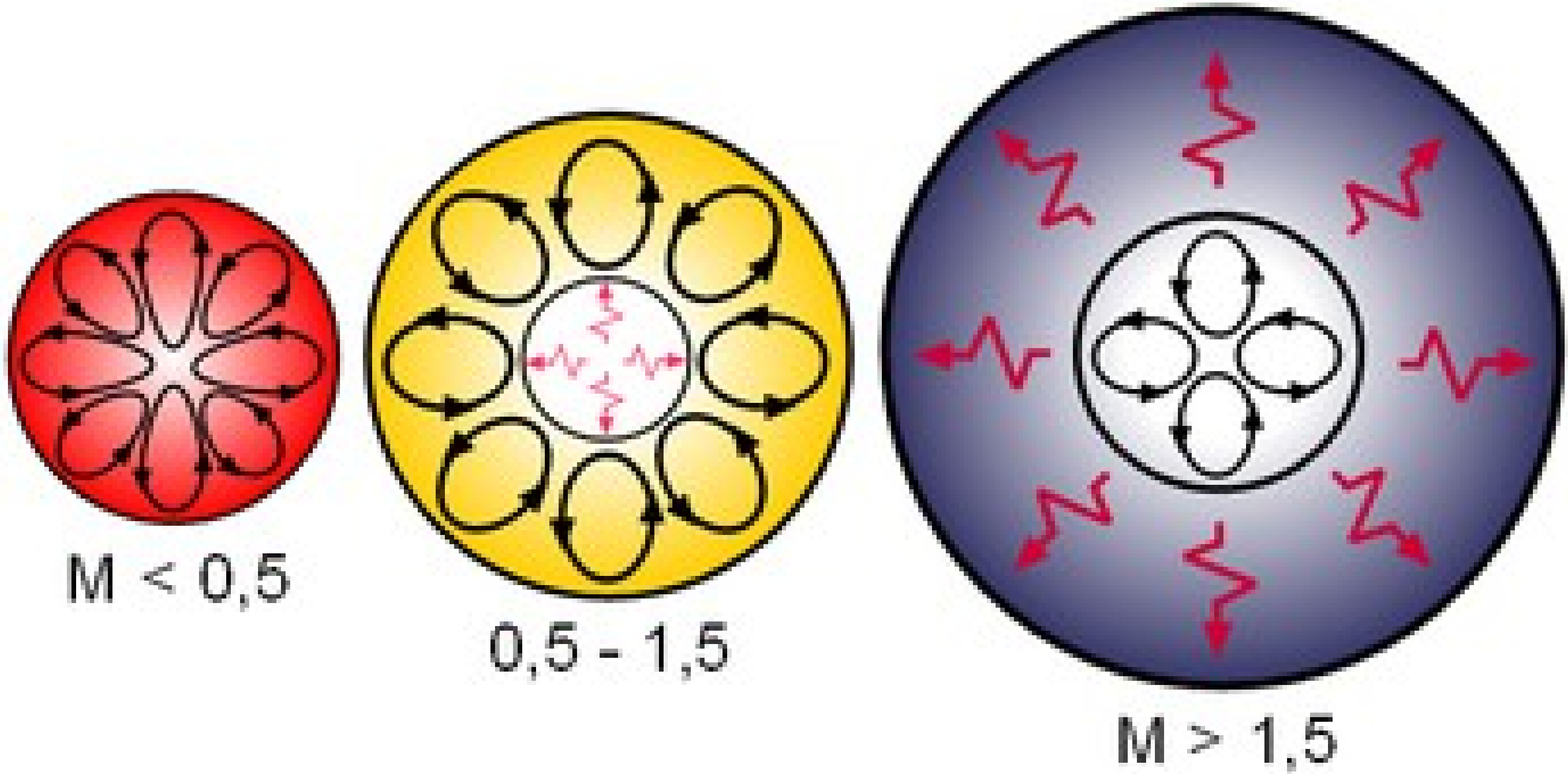
Oscillations across the HR diagram



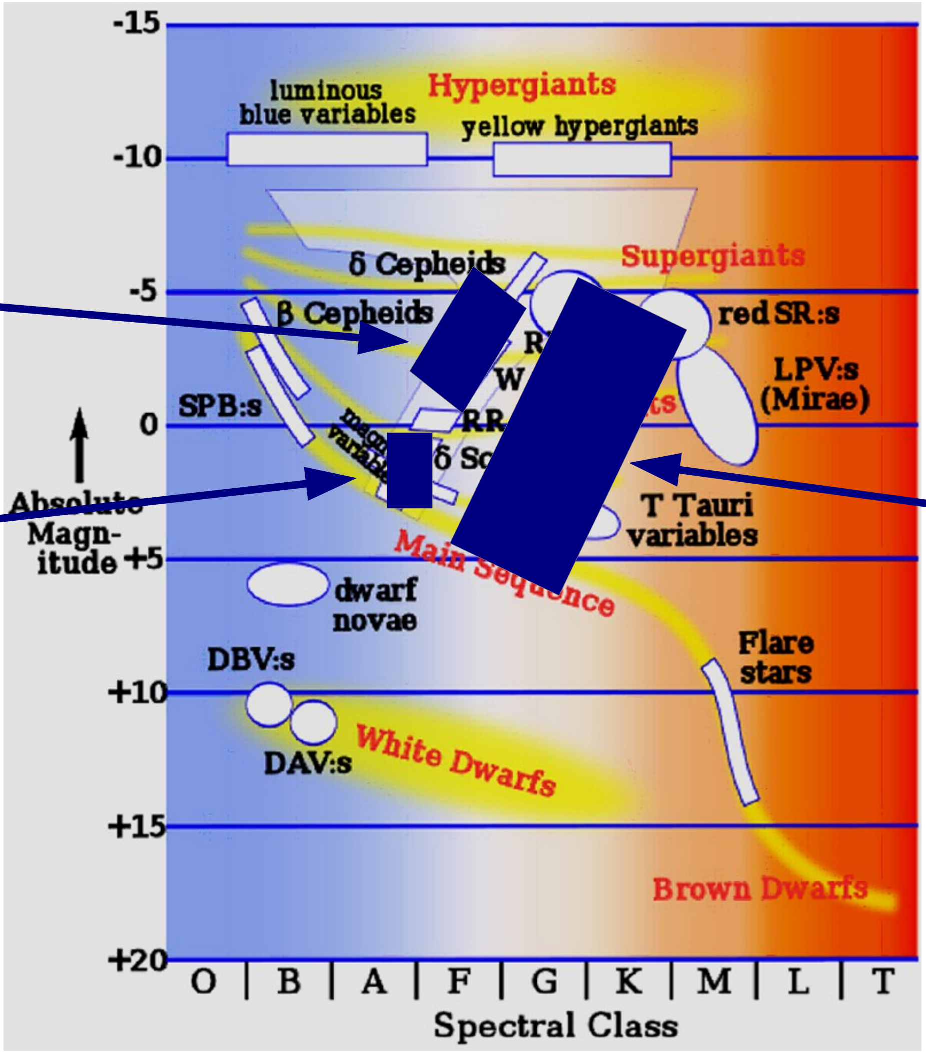
Brief History of Pulsations



Different stars



Oscillations across the HR diagram

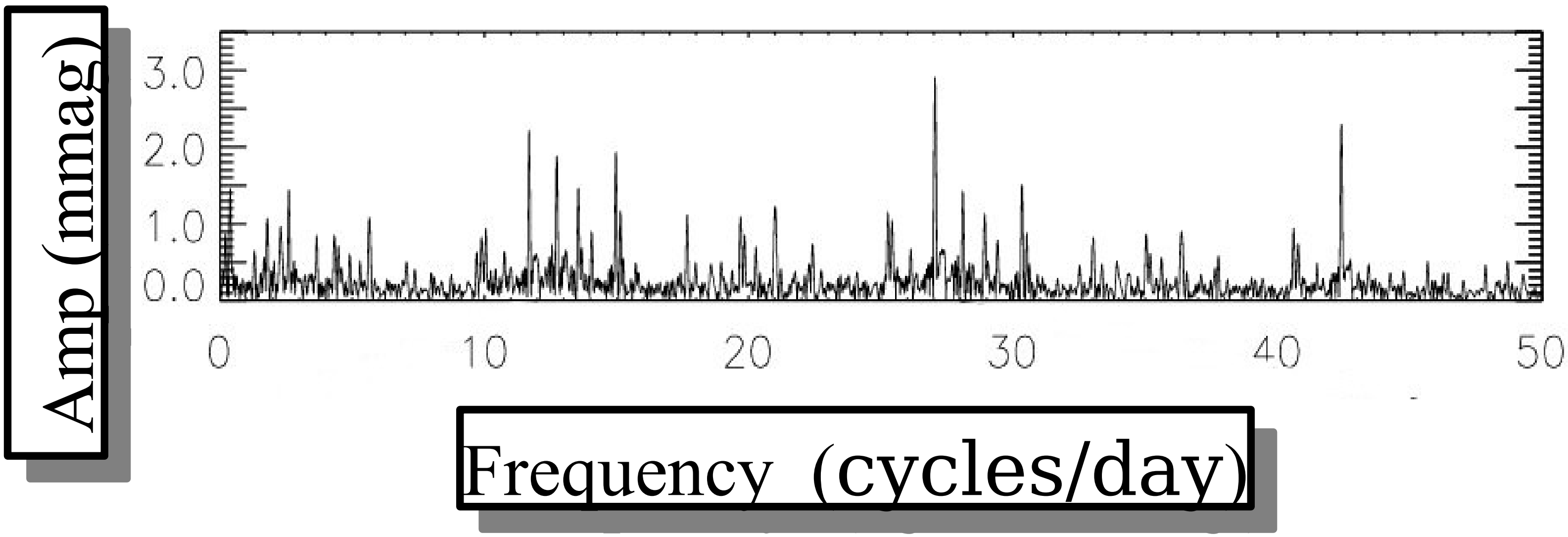


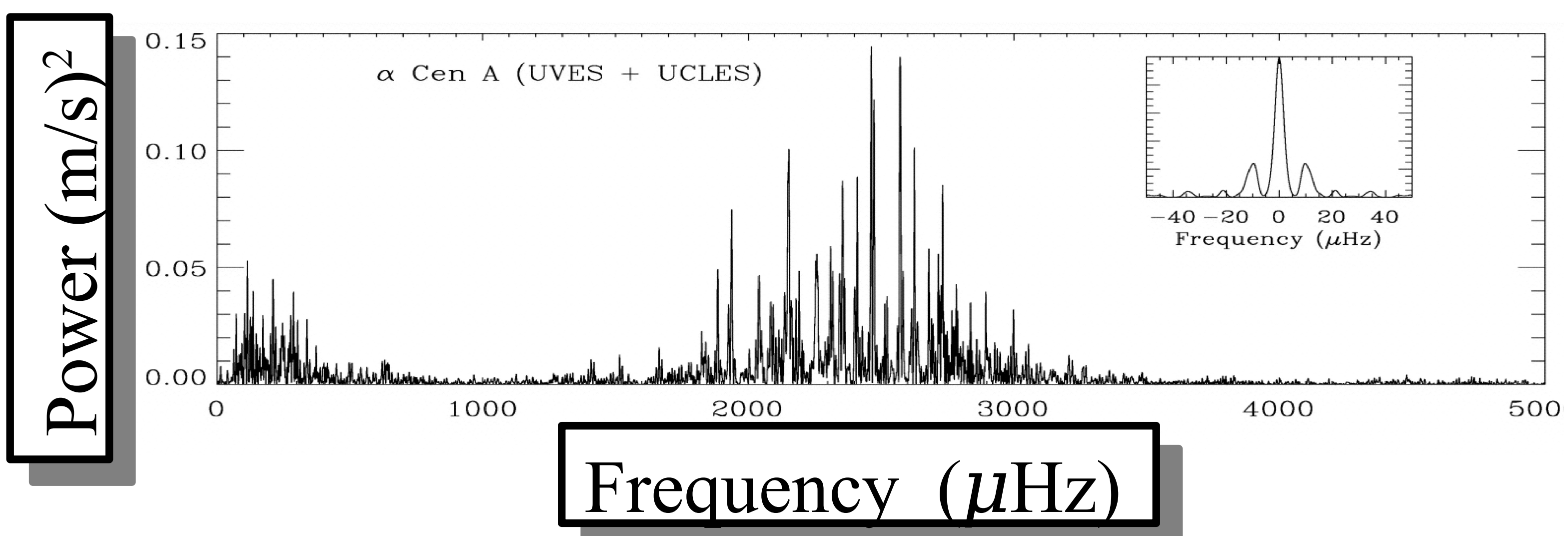
Cepheids

Delta Scuti stars

'Sun-like' stars

Power spectra showing oscillations in different stars





Why do they pulsate?

The **structure of the star determines** the **frequencies** of the oscillation modes, the reasons for pulsation, the amplitudes of pulsation (atmosphere), and the restoring force (p,g modes)

Stars located in different regions in the HR diagram have differing structures thus differing pulsation types.

Excitation mechanisms:

– **kappa mechanism:** Balance between radiation pressure and gas pressure: under certain conditions He+ atmosphere loses e- and makes gas more opaque. The radiation pressure increases and causes the star to expand. He++ becomes He+ and cools (more transparent) and star contracts.

– **stochastic oscillations:** convection zone's turbulent motions act as pistons to 'ring' the star and this oscillate in its resonant modes

Different stars will show different types of pulsation spectra

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- Interferometric diameters
  - Role of interferometry with seismic data
  - Precision in radius
  - Visibility curves
  - Determining the angular diameter
  - Visible and IR limits in angular resolution

Why Interferometry?

We have the following scaling relation

$$\langle \Delta \nu_{n\ell} \rangle \simeq \sqrt{\frac{M}{R^3}} \langle \Delta \nu_{n\ell} \rangle_\odot$$

With asteroseismology we can measure $\Delta \nu_{n,\ell}$ and with interferometry we can measure θ. When we know the distance, we get R. This means that we have direct access to M.

How do we measure the average value of $\nu_{n,\ell}$?

Calculating R from θ

The angular diameter θ of a star is the projected diameter onto the sky. R = radius of the star and d is the distance.

$$\theta = \frac{R}{d}$$

Rearranging the equation and converting the radius to solar radii we get

$$R(R_\odot) = 107.505\,\frac{\theta(\mathrm{mas})}{\pi(\mathrm{mas})}$$

where mas = milliarcsec and pi = parallax

Can you verify this equation?

Its error is given by
$$\sigma_R^2 = 107.505^2\left(\sigma_\theta^2\frac{1}{\pi^2} + \sigma_\pi^2\frac{\theta^4}{\pi^4}\right)$$

Measuring θ with Interferometry

The visibility curve of an object with an angular diameter θ is determined by the distance between the two (or more telescopes) called the baseline B and the wavelength of the interferometric instrument λ

.

$$V = \frac{2J_1(x)}{x}$$

where (pi = 3.14 here)

$$x = \pi B \theta \lambda^{-1}$$

Note: The following few slides are covered in the practical session

Interferometry: visibility curves

The visibility curve (V squared) of a star with angular diameter θ and measured with a visible interferometer which works in the wavelength λ for a *continuous* baseline coverage (unrealistic).

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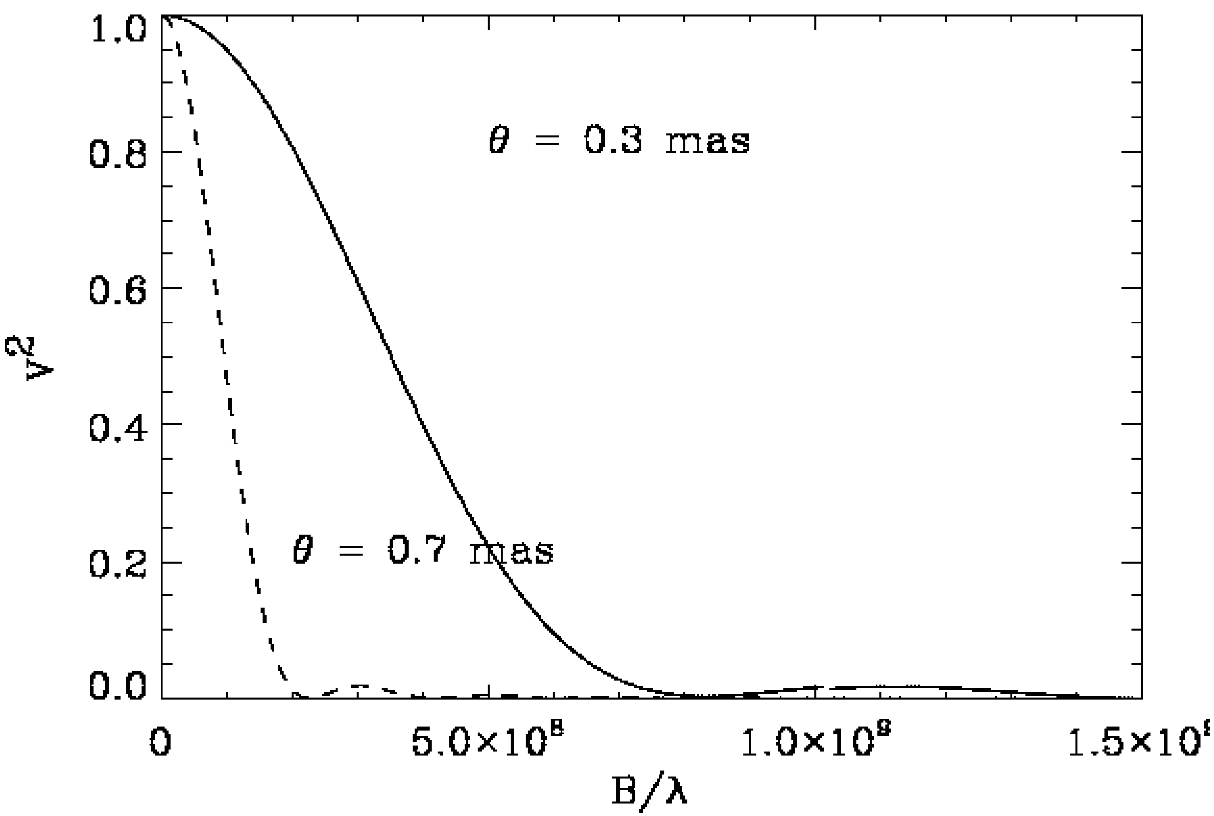

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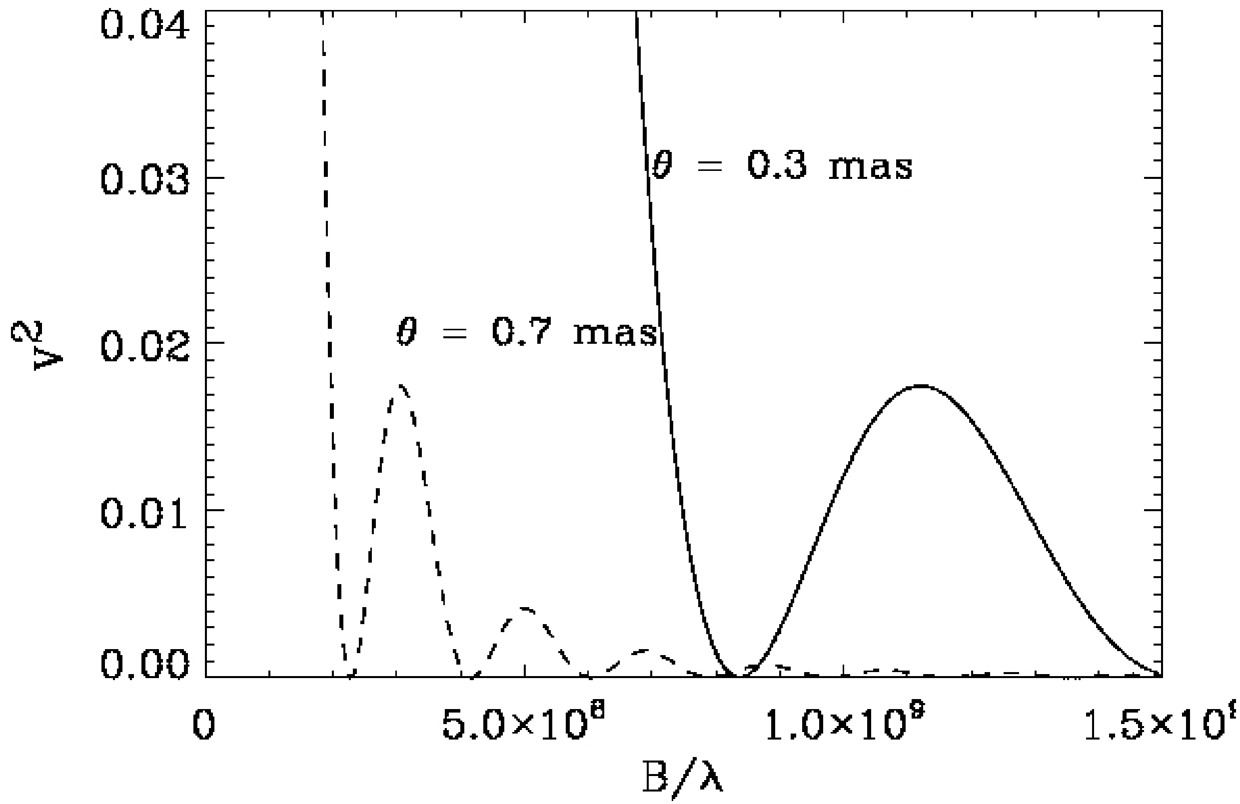

Interferometry: visibility curves

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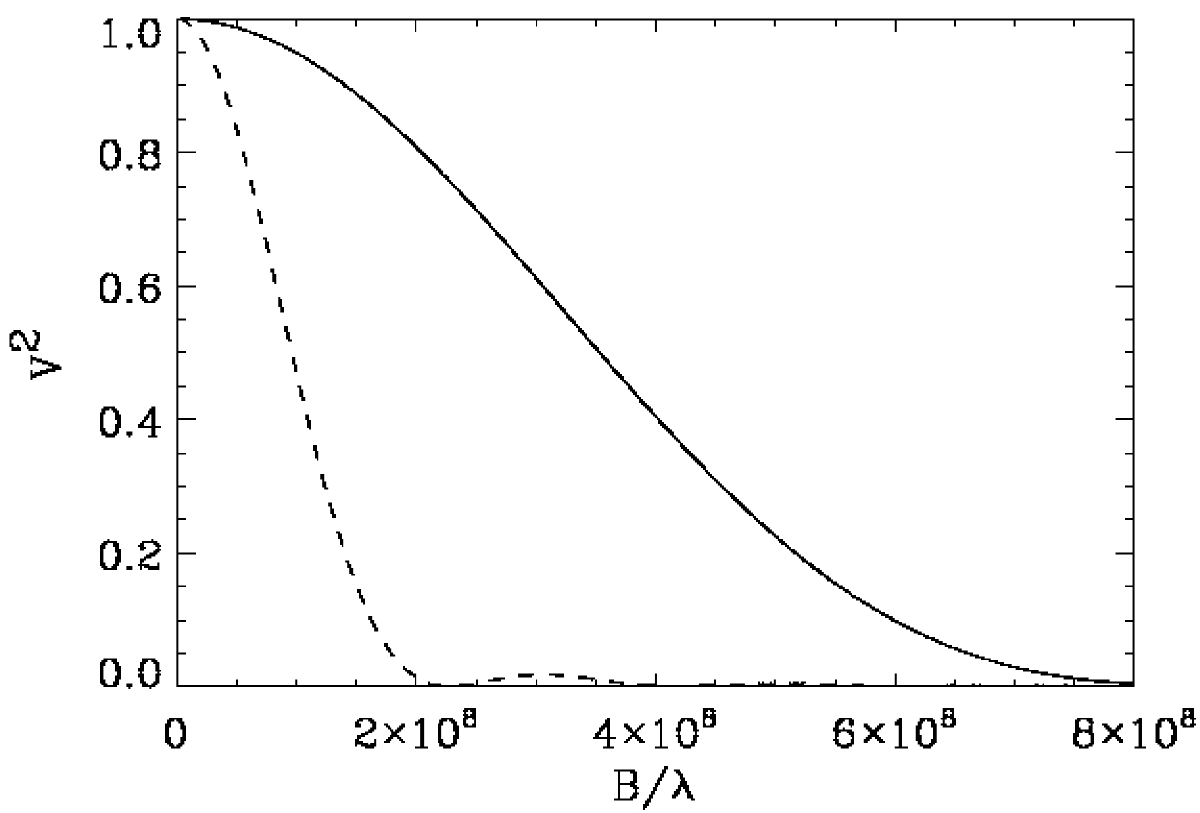

Interferometry: visibility curves

The visibility curve (V squared) of a star with angular diameter θ and measured with a visible interferometer which works in the wavelength λ for a *continuous* baseline coverage (unrealistic), with typical **LIMITS**

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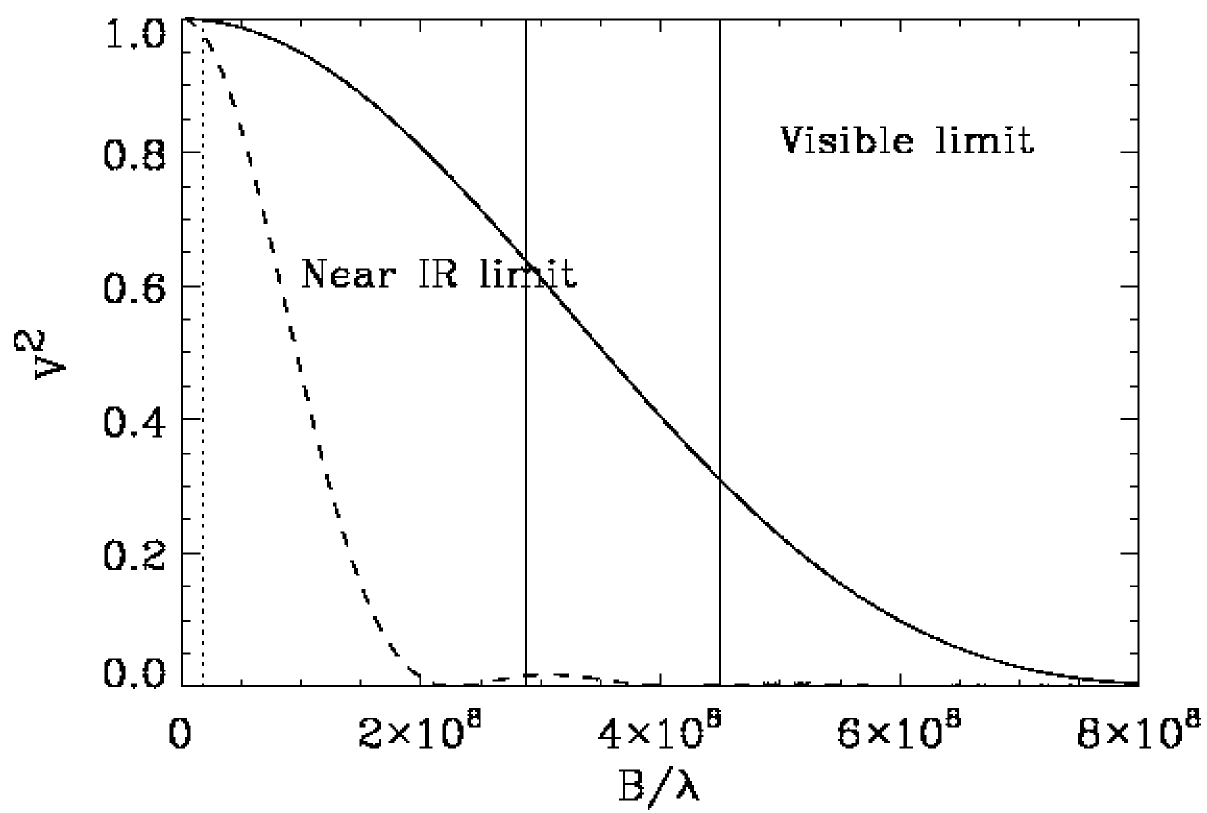

Interferometry: visibility curves

The visibility curve (V squared) of a star with angular diameter θ and measured with 10 equal-spaced baselines (30m - 270m) using a visible interferometer (blue) and a near IR interferometer (red)

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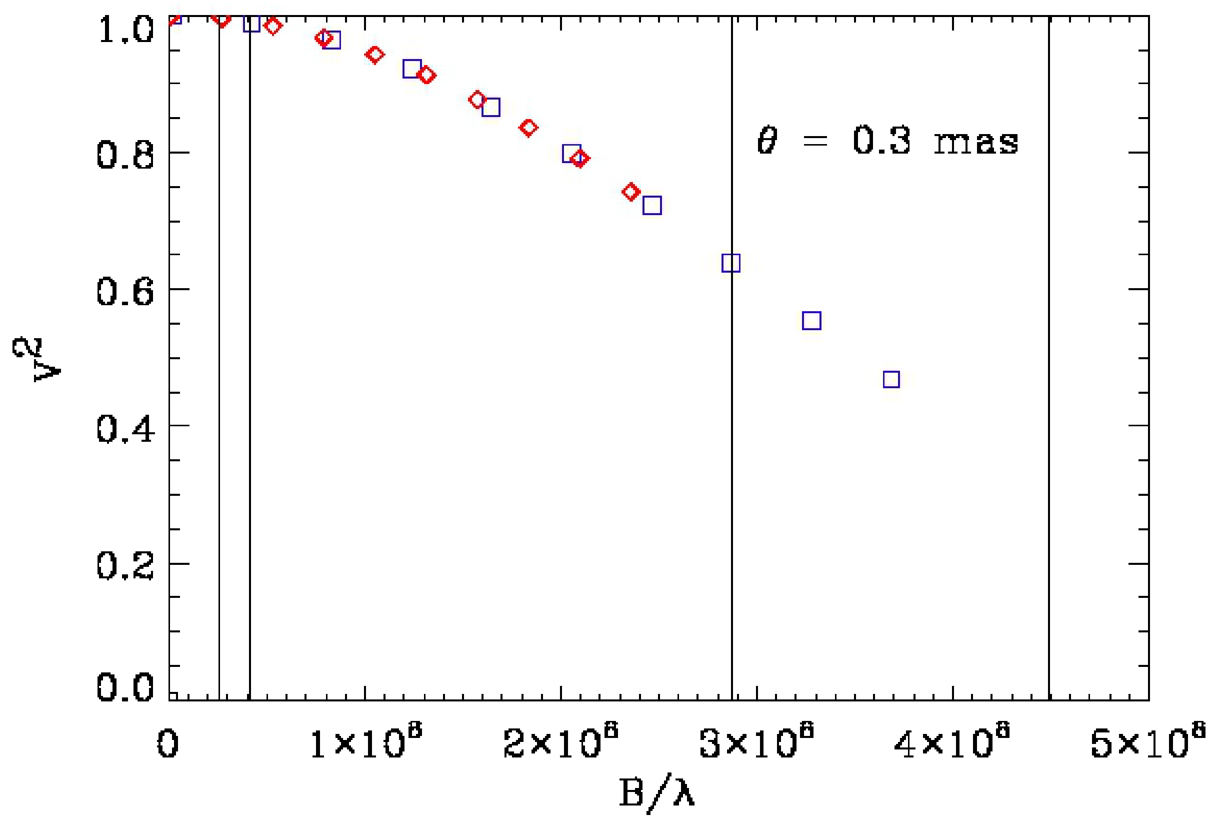

Interferometry: visibility curves

The visibility curve (V squared) of a star with angular diameter θ and measured with 10 equal-spaced baselines (30m - 270m) using a visible interferometer (blue) and a near IR interferometer (red)

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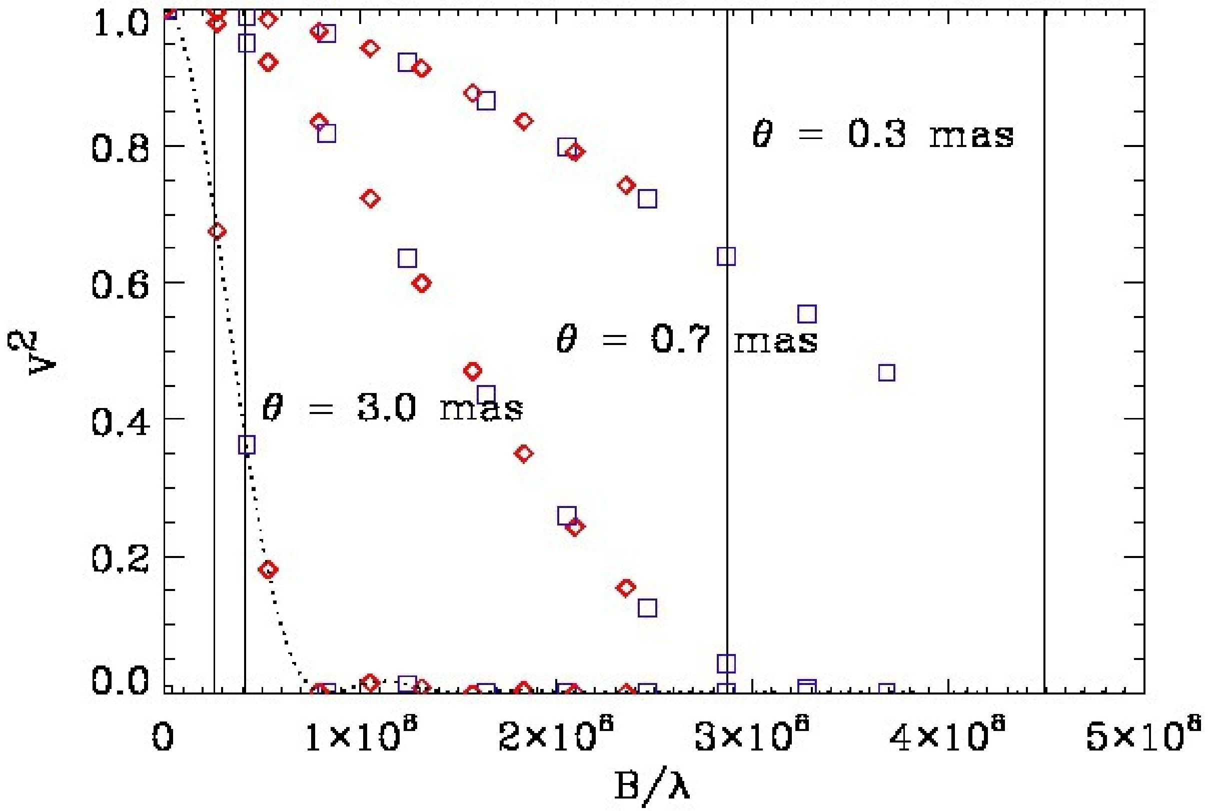

Interferometry: visibility curves

The visibility curve (V squared) of a star with angular diameter θ and measured with CHARA representative baselines (34m – 330 m) using a visible interferometer (blue) and a near IR interferometer (red)

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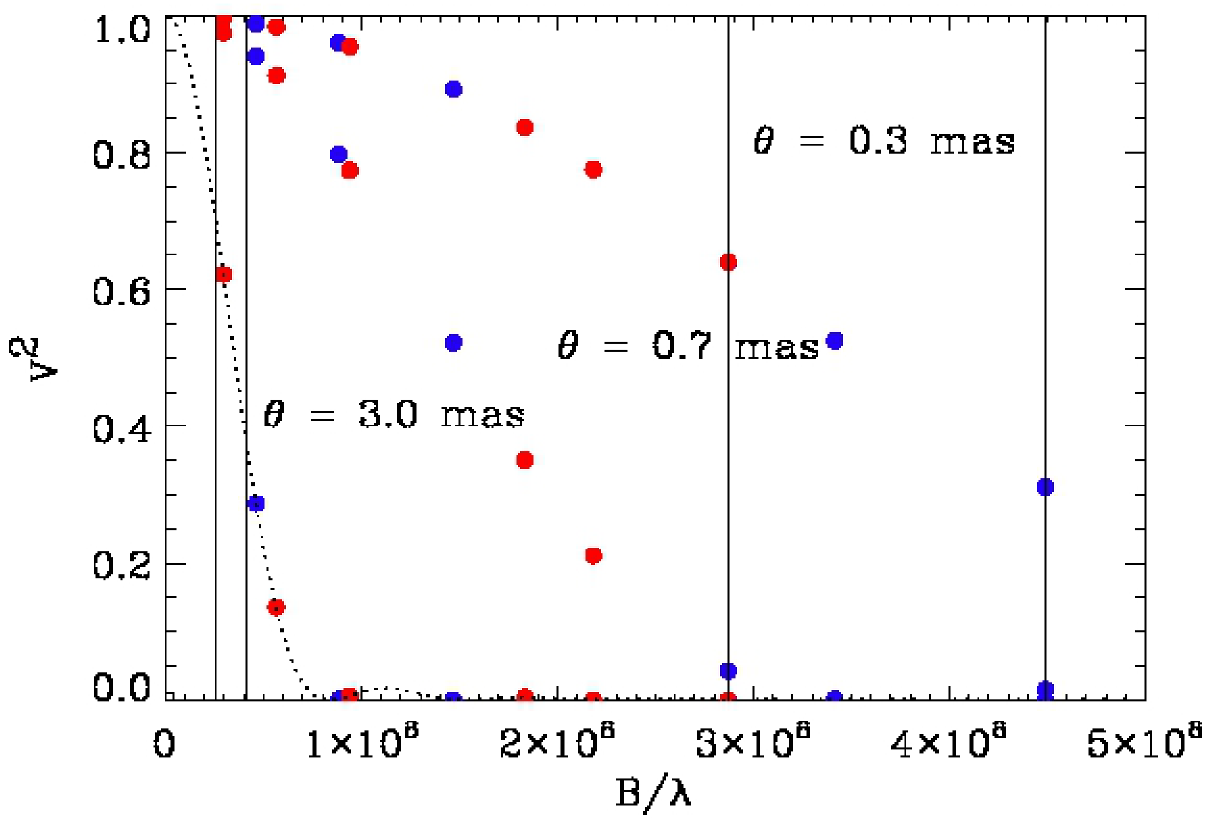
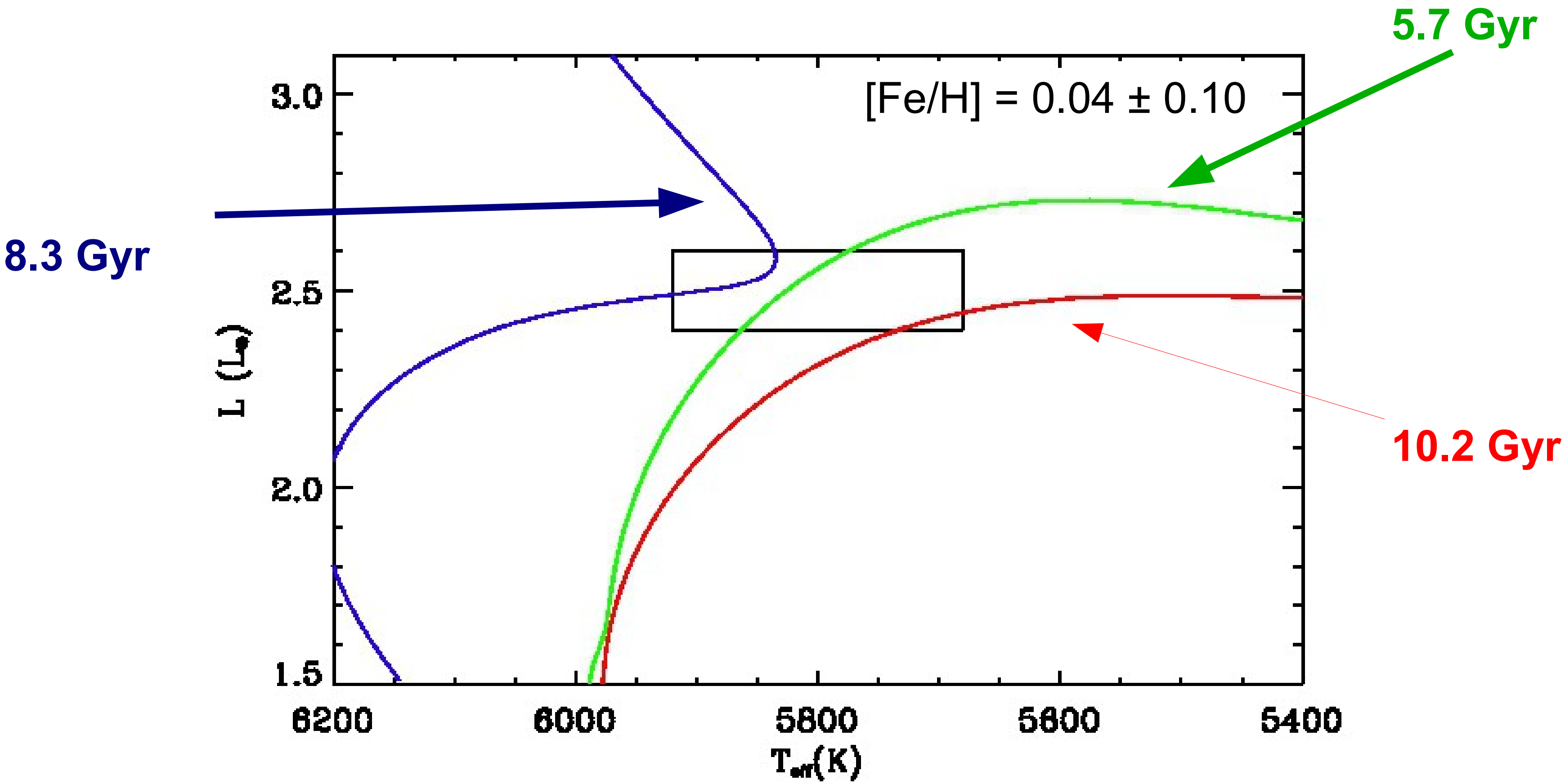


Contents

- Introduction/objectives
- Waves and Power Spectra
- Stellar Pulsations across the HR diagram
- Solar-like Oscillations: theory and observations
- Interferometric diameters

Theories of stellar structure and evolution



Contents

- Waves and Power Spectra: 1-3D waves, frequencies, representation of signals, Fourier analysis, frequency resolution, time sampling, fundamental frequency, oscillation modes (l,n,m)
- Stellar Pulsations across the HR diagram: types of pulsating stars, observations, excitation mechanisms
- Solar-like Oscillations: theory and observations: asymptotic expression, large and small frequency separations, freq of max amplitude, seismic quantities and fundamental properties, observations, combining interfero+astero
- Interferometric diameters: precision in radius, visibility curves, determining AD, visible/IR limits on angular resolution

Conclusion

We discussed
1) why we should combine interfero+astero

2) how to measure and analyse time series of pulsations and its Fourier Transform

2) that for solar-like stars the asteroseismic quantity **$\Delta\nu$** can be easily measured, and this gives us direct access to the mean density of the star.

3) how to calculate angular diameters, and radii from interferometric data

4) by combining R and **$\Delta\nu$** we can access directly M

5) how knowing these fundamental properties is a requirement if we want to determine the age of stars, or study stellar evolution and structure.