Systematics and biases in interferometric data

– advanced data analysis techniques –

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VLTI school – Porquerolles 2010

These slides do not contain any pretty pictures

Noise / Bias

Simple definitions

Noise

- unwanted signal
- unpredictable
- 0-average
- noise is different frame to frame, but has same characteristics

Averaging frames decreases the noise's contribution

Bias

- gives a result different to what is expected
- does NOT go away by averaging
- can be additive or multiplicative

Bias should be studied and removed

Precision / Accuracy

Precision / Accuracy

A **precise** result has a small error bar

An **accurate** result can be trusted

Collecting interferometric data requires to collect **a lot** of **noisy** data

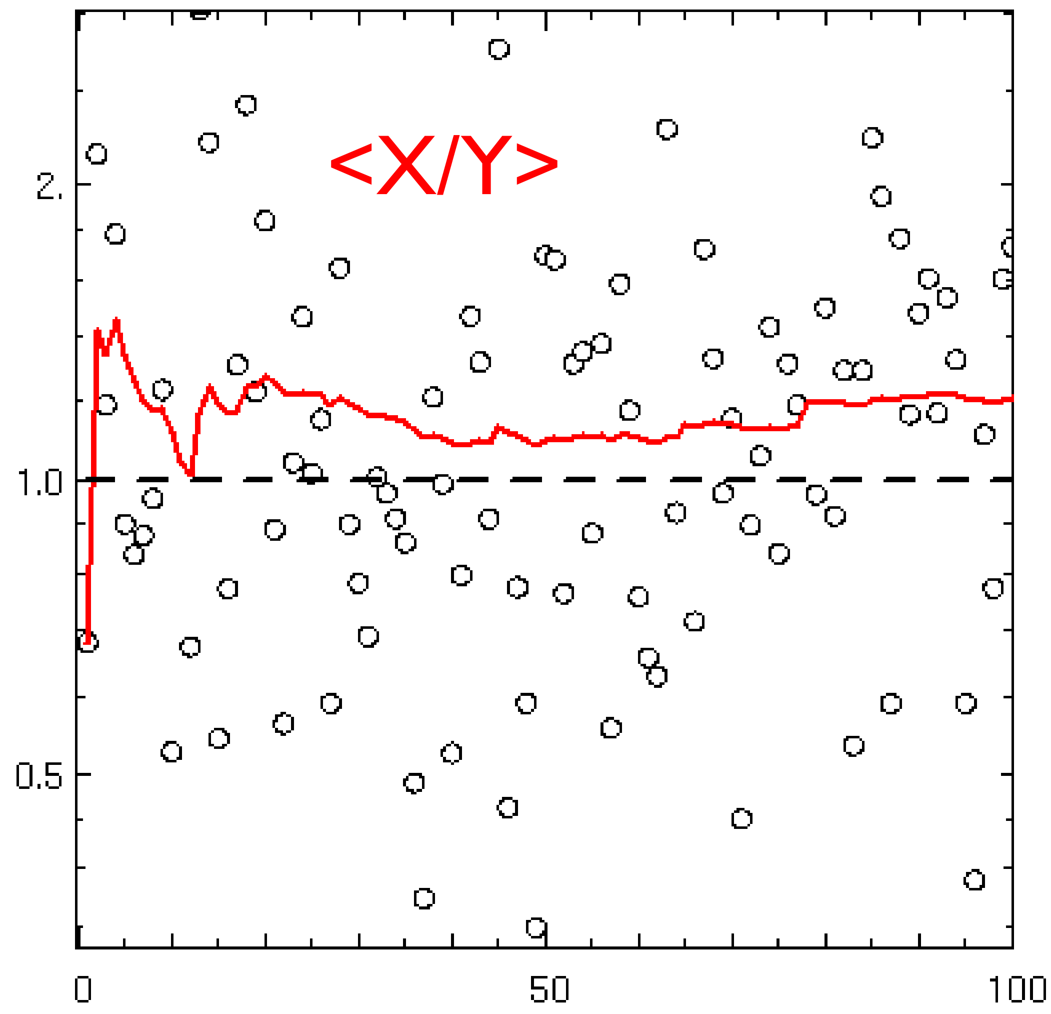
because we need to freeze the atmospheric turbulence

when you do statistics on noisy data, terrible things can happen...

Simple example

- two variables: X and Y
- one needs to estimate Z=X/Y
- simple idea: take all the "X/Y" and average them, <X/Y>
- using X = 3 ± 1 and Y = 3 ± 1

the noise introduces a bias!!!



Analysis

- two variables: X and Y with Gaussian distribution (μ_X, σ_X, μ_Y, σ_Y)
- In general, μ_z is not defined, but if $\sigma_Y << \mu_Y$, then with Z=X/Y

$$\mu_z \approx \frac{\mu_x}{\mu_y}\left(1+\frac{\sigma_y^2}{\mu_y^2}\right)$$

$$\sigma_z^2 \approx \frac{\mu_x^2}{\mu_y^2}\left(\frac{\sigma_x^2}{\mu_x^2}+\frac{\sigma_y^2}{\mu_y^2}-\frac{\sigma_y^4}{\mu_y^4}\right)$$

First order approximation

What should I do???

- If $Z=X/Y$ is my estimator, <X/Y> is biased
- This bias depends on the noise (!)
- The more important the noise, the more important the bias
- data on different targets (different SNR) will have different biases

I know my result should be μ_X/μ_Y… because it is what I should get if there was no noise

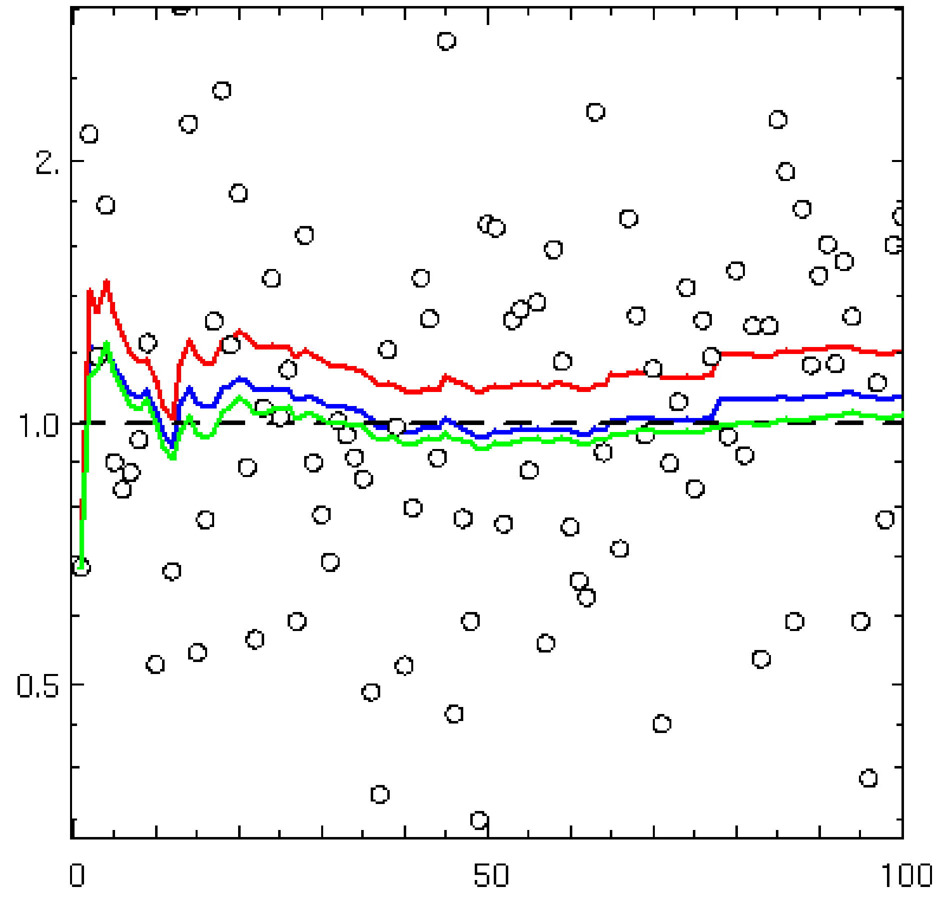
Unbiased estimators

one can **de-biased** the estimator:

$$< z_2 >= \left\langle \frac{x}{y} \right\rangle / \left(1 + \frac{var(y)}{< y >^2} \right)$$

or better, use an **unbiased** estimator:

$$< z_3 >= \frac{<x>}{<y>}$$



You used to fear dividing by 0

now, fear dividing by a noisy variable...

Real life example

Remember the normalized fringes?

$$I_1^c = \frac{I_1 - \kappa_{1\mathrm{A}} P_\mathrm{A} - \kappa_{1\mathrm{B}} P_\mathrm{B}}{2\left[\sqrt{\kappa_{1\mathrm{A}} P_\mathrm{A} \kappa_{1\mathrm{B}} P_\mathrm{B}}\right]_\mathrm{smoothed}}$$

If you estimate the visibility from that, it is biased...

You should average the correlated power and normalization factor separately:

$$\mu^2_\mathrm{biased} = \left\langle \mathrm{PSD}\left[\frac{I_1(t) - \kappa_{1A} P_A(t) - \kappa_{1B} P_B(t)}{2\sqrt{\kappa_{1A}\kappa_{1B} P_A P_B}}\right]\right\rangle$$

$$\mu^2_\mathrm{unbiased} = \frac{\langle \mathrm{PSD}[I_1(t) - \kappa_{1A} P_A(t) - \kappa_{1B} P_B(t)]\rangle}{2\sqrt{\kappa_{1A}\kappa_{1B}}\left\langle\sqrt{\overline{P_A}\ \overline{P_B}}\right\rangle}$$

average over # of frames

average in the frames

Real life example

$$\mu^2_{\text{biased}} = \left\langle \text{PSD}\left[\frac{I_1(t) - \kappa_{1A}P_A(t) - \kappa_{1B}P_B(t)}{2\sqrt{\kappa_{1A}\kappa_{1B}P_A P_B}}\right]\right\rangle$$

$$\mu^2_{\text{unbiased}} = \frac{\langle \text{PSD}[I_1(t) - \kappa_{1A}P_A(t) - \kappa_{1B}P_B(t)]\rangle}{2\sqrt{\kappa_{1A}\kappa_{1B}}\left\langle \sqrt{\overline{P_A}\;\overline{P_B}}\right\rangle}$$

- The first estimator overestimates V2 if $P_A P_B$ is noisy
- The noisier $P_A P_B$, the larger the bias
- The noise in $P_A P_B$ is the noise in the photometric signal, not in the fringes
- AMBER also uses an unbiased average:

$$\frac{\widetilde{|V^{ij}|^2}}{V_c^{ij\,2}} = \frac{\left\langle R^{ij^2} + I^{ij^2}\right\rangle - \text{Bias}\{R^{ij^2} + I^{ij^2}\}}{4\left\langle P^i P^j\right\rangle \sum_k v_k^i v_k^j}.$$

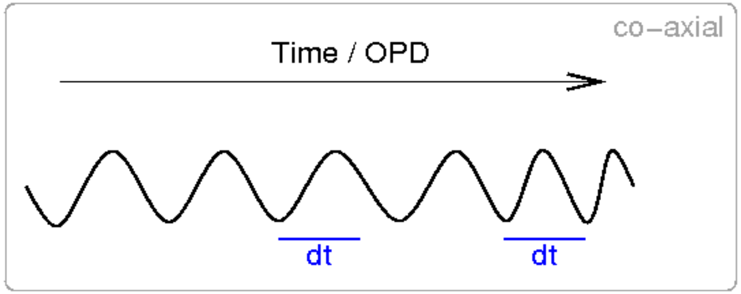
Tatulli et al. (2007)

Biases to the visibility

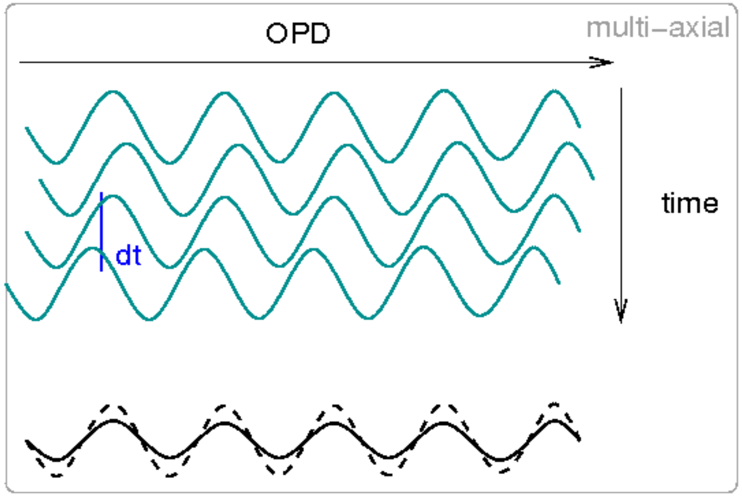
Jitter bias

Remember: $$I(\delta, t) \sim 1 + \mathrm{Re}\left(V e^{-2\pi i \left[\frac{\delta - \mathrm{OPD}_{\mathrm{jitter}}(\mathrm{t})}{\lambda} \right]} \right)$$

The effect is a reduction of the estimated fringe contrast:



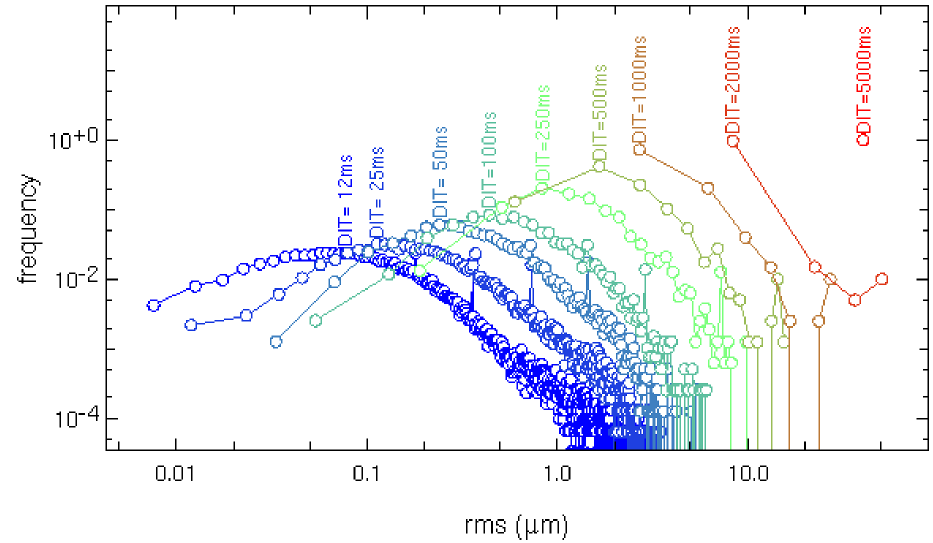
multiplicative bias

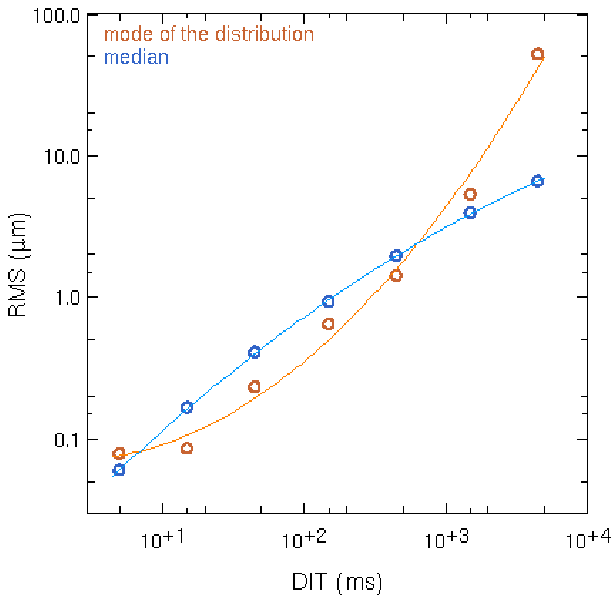


Yet, the effect is different in co-axial or multi axial...

Jitter caracteristics

- the jitter is a consequence of the turbulent atmosphere
- turbulent phenomena have more power at lower than higher frequencies.
- we can use FINITO (VLTI current fringe tracker) to measure it





When conditions get worse, you have less light in your beam combinator:

because the seeing gets bad and less light enters you spatial filters (single mode fibers)

because you have to decrease the exposure time to freeze the jitter

Interferometry sensitivity reacts very non linearly to degradations of atmospheric conditions

Other multiplicative biases

Uncorrelated fluxes

- if some part of the flux does not interfere, the visibility is biased by [$F_c / (F_c + F_{unc})$]
- could background or diffuse light in your instrument

Polarization

- if two beams have rotated polarization, the fringes' contrast is reduced

Calibrating multiplicative biases

– Stellar Calibrators –

Reminder

- One measures biased squared visibilities μ^2
- A calibrator is a target for which the instrumental response can be predicted
- Calibration is simple in theory:

$$V_{\rm sci}^2 = \mu_{\rm sci}^2 \times \left[\frac{V_{\rm cal}^2}{\mu_{\rm cal}^2} \right]$$

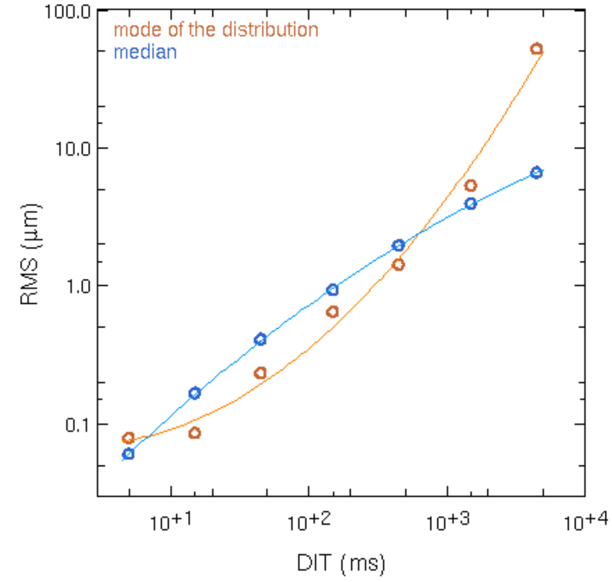
Transfer Function (TF)

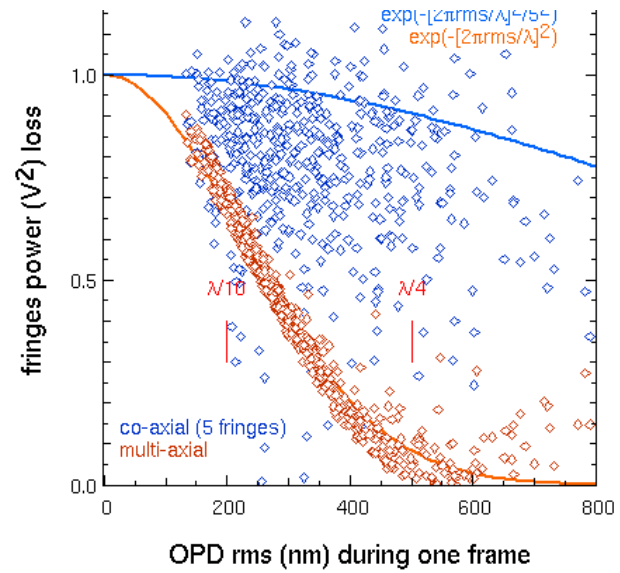
- The calibrator removes any **constant** (between SCI and CAL) multiplicative bias

basic rule 1

- You mainly want to correct for the jitter
- The median RMS of the atmospheric jitter over a frame changes with DIT
- The bias has a steep variation with the jitter

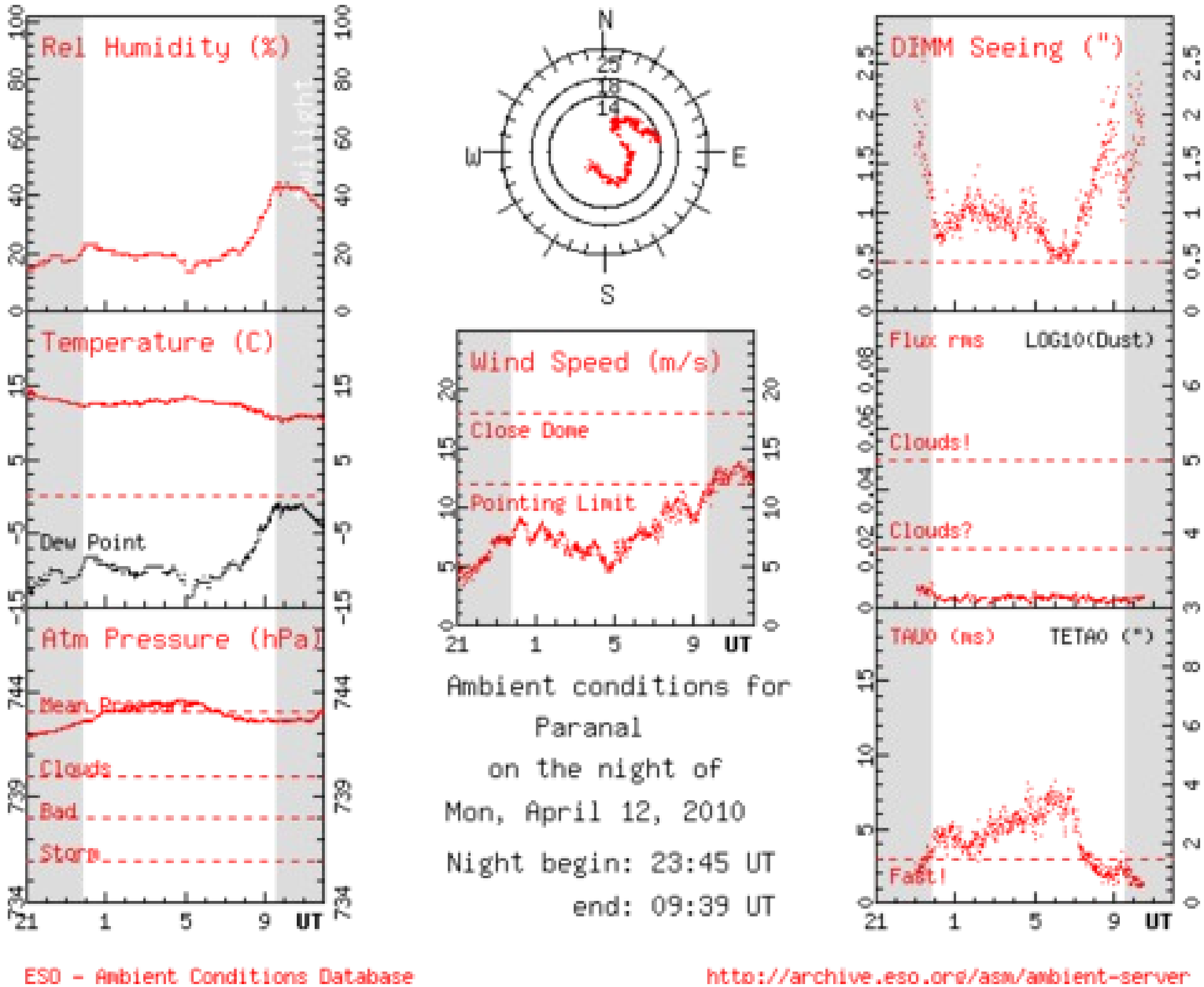
SCI and CAL must have the same DIT





basic rule 2

- Atmospheric conditions can change rapidly
- In practice, CAL-SCI should not be more than 30 min apart in time
- 10 or 20 degs apart on sky



basic rule 2bis

- The transfer function evolves with atmospheric conditions
- atmospheric conditions evolve with time
- to measure the 0-order TF : **CAL-SCI**
- to measure the 1-order TF : **CAL-SCI-CAL**

The two schemes exist for AMBER and MIDI in service mode...

- More complex calibration strategies? Visitor Mode

Calibrators

Most natural calibrators: Stars

Most stars look like disks (in projection on the sky)

Visibility is easy to predict

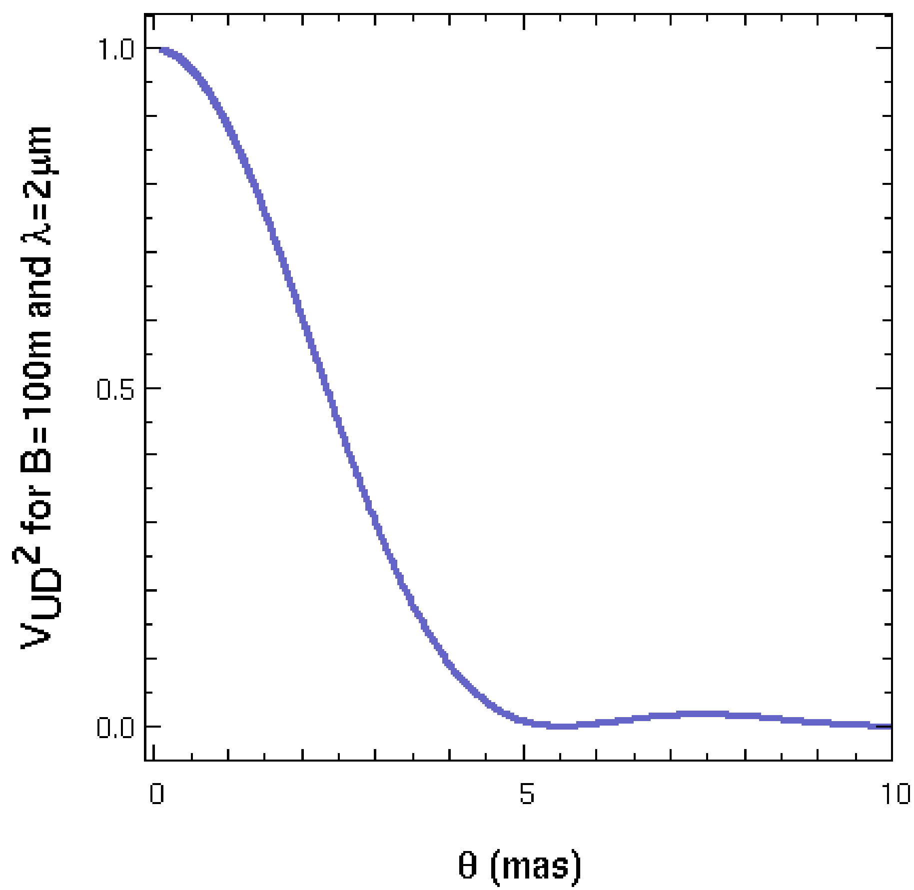
- interferometer: baseline B, and observation wavelength λ
- star: uniform disk angular diameter θ

$$V_{\mathrm{UD}}^2(B, \lambda, \theta) = 4 \left[\frac{J_1(\pi B\theta/\lambda)}{\pi B\theta/\lambda} \right]^2$$

Unresolved calibrators?

We want V^2 to be independent of the knowledge of θ

That is, we do not want the uncertainty on θ to contaminate V^2_{sci}



Unresolved calibrators?

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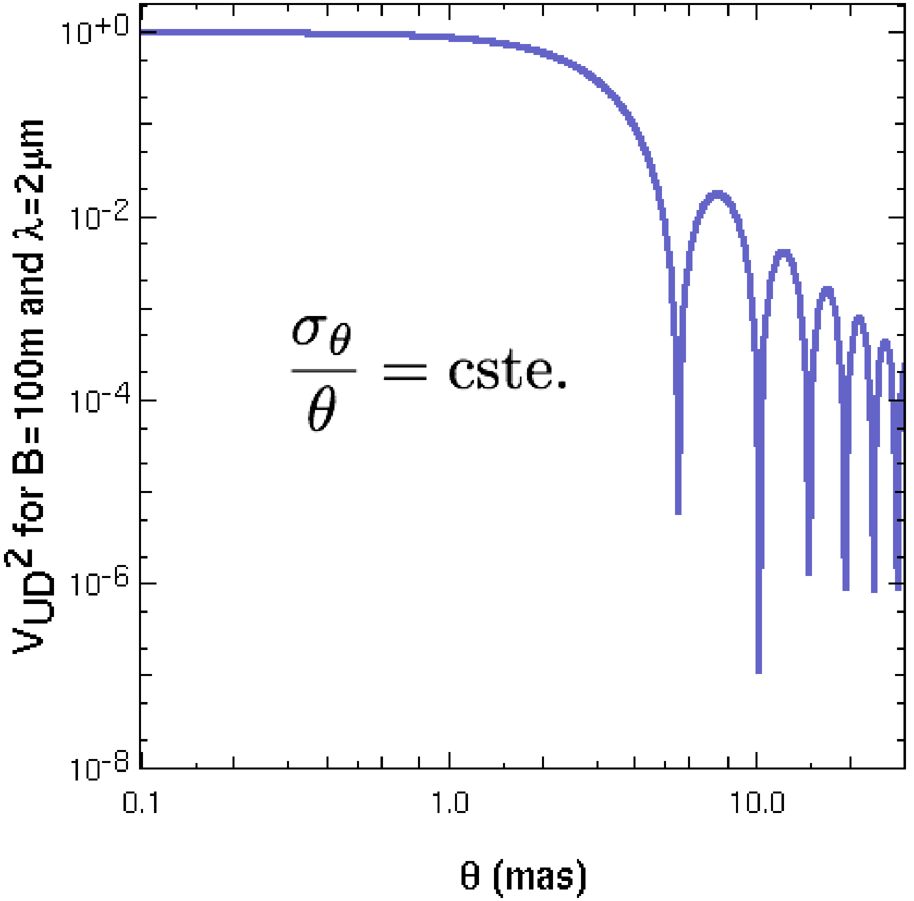
That is, we do not want the uncertainty on θ to contaminate V^2_{sci}

θ has to be small, ~0.1mas for B=100m and λ=2microns



There is a slight problem ...

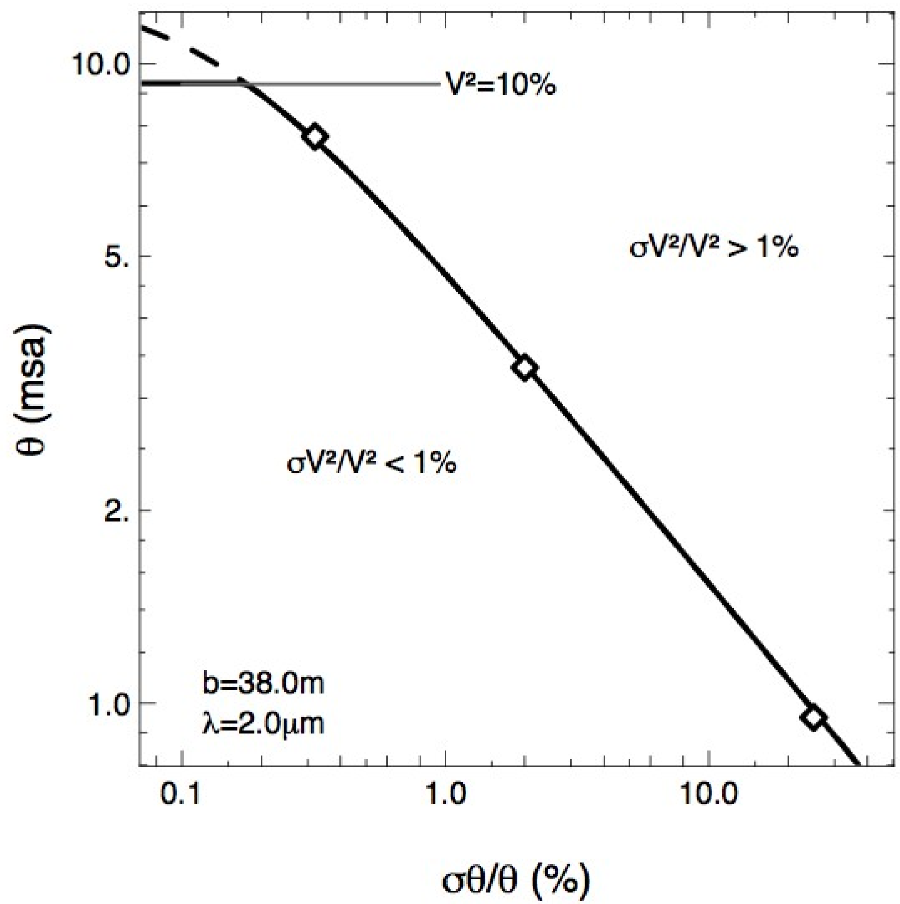
An infinitely small star is infinitely faint, unless it has an infinitely high surface brightness

More realistically, a 0.1mas star with stellar temperature of 10000K (A0) has a magnitude >7

It is hard to avoid the problem of knowing the angular diameter of the calibrator and its bias...

How to chose calibrators

- For IOTA (B=38m) in the K band
- 4% uncertainty of the cal diameter leads to less than 1% uncertainty on V^2_{cal} for θ<3mas
- for B=100m, the same is true for θ<1mas, i.e. V^2~0.9 *(100 times brighter than a 0.1mas star)*



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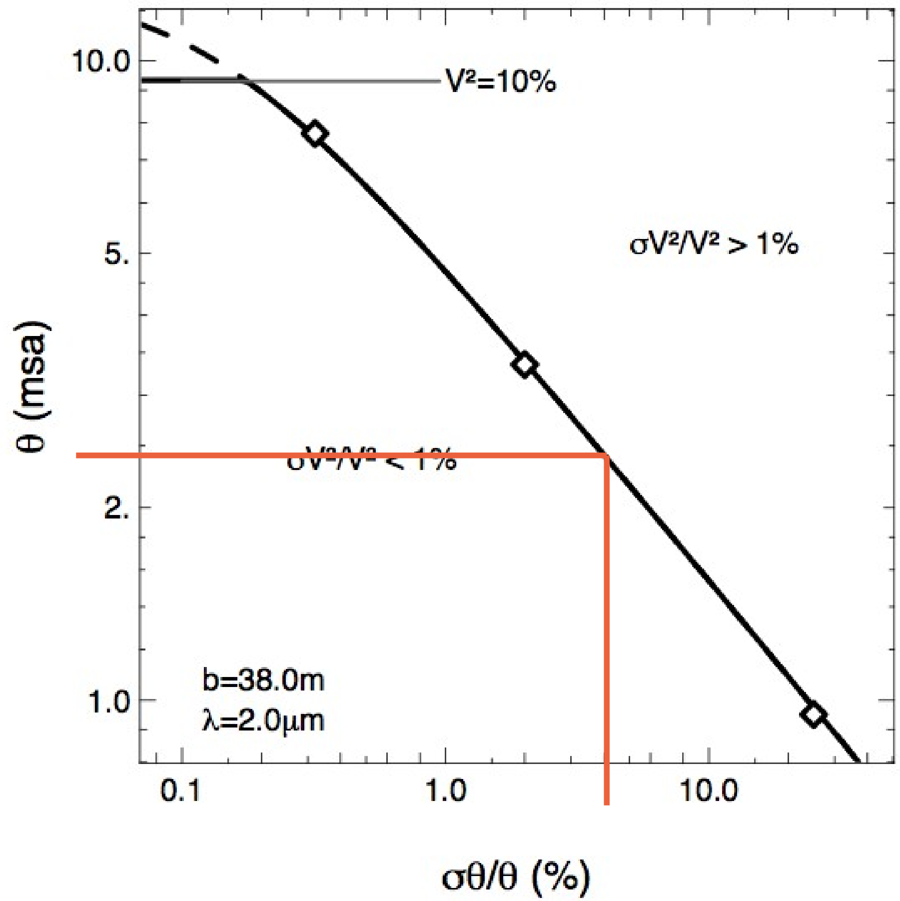


First order error propagation

$$V^2_{\mathrm{sci}} = \mu^2_{\mathrm{sci}} \times \left[\frac{V^2_{\mathrm{cal}}}{\mu^2_{\mathrm{cal}}}\right]$$

$$coT^2 = V^2_{\mathrm{cal}}/\mu^2_{\mathrm{cal}}$$

$$\frac{\sigma^2_{coT^2}}{(coT^2)^2} = \frac{\sigma^2_{\mu^2_{\mathrm{cal}}}}{(\mu^2_{\mathrm{cal}})^2} + \frac{\sigma^2_{V^2_{\mathrm{cal}}}}{(V^2_{\mathrm{cal}})^2}$$

The calibration is equally dominated by

- the uncertainty on the cal **estimated** visibility V^2
- the uncertainty on the cal **measured** visibility μ^2

Calibrators estimated diameters

- See for ex. recent review: Cruzalebes et al. (2010) *"Angular diameter estimation of interferometric calibrators - Example of lambda Gruis, calibrator for VLTI-AMBER"*
- Basically, the idea is to use the apparent luminosity and the surface brightness:
  - estimated from stellar templates (models)
  - calibrated from colors (e.g. V-K)
- Works well down to almost 1%, estimated from spectro-photometry (for boring stars)

Calibrators side effect

- I want to measure the diameter of a star SCI using CAL
- A single V_{sci}^2 has a given uncertainty due to the different noises
- The resulting measured diameter has a given uncertainty, too large to my science goal
- Just repeat the CAL-SCI sequence all night long to nail down that diameter!

By averaging all my $V_{sci}^2 \pm \sigma_{V2sci}$
I get a super precise visibility

I derive θ_{sci}=1.523±0.001mas

... compared to my calibrator
which has an estimated diameter
θ_{cal}=1.50±0.02mas

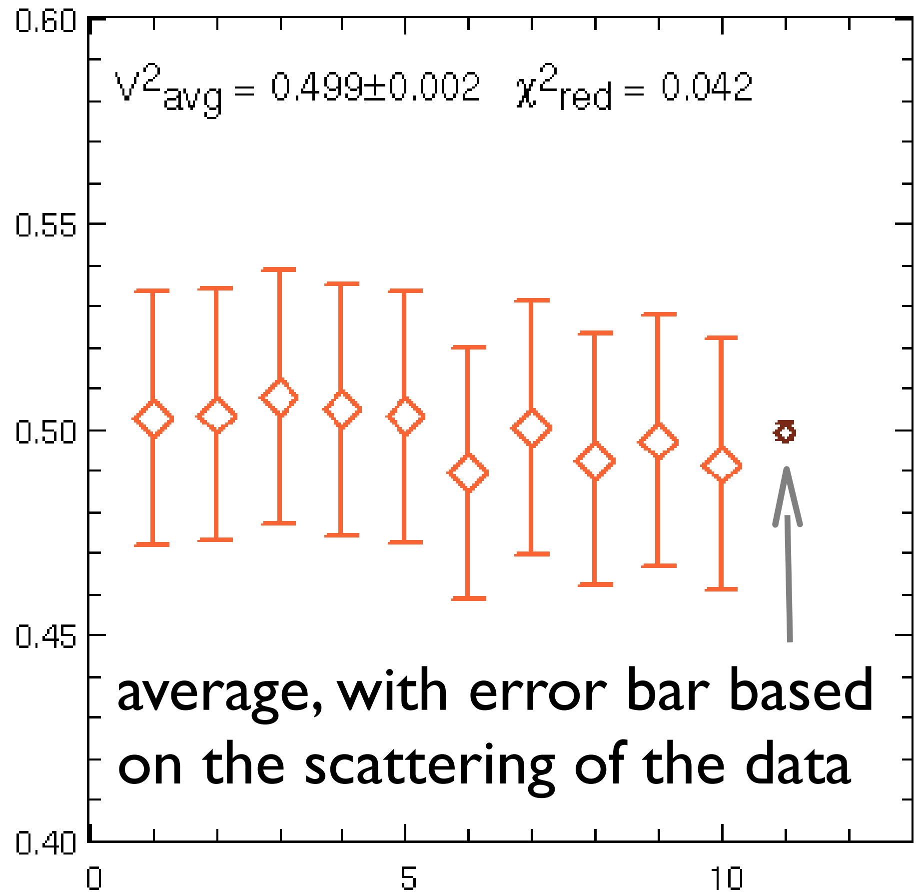
well...
CAL could also be 1.52mas, which leads to
θ_{sci}=1.543±0.001mas (20 sigmas...)

PRECISION ≠ ACCURACY

Simple case

- I take all my data
- I fit a constant
- I get a **precise** result
- But
  - look at the dispersion of the data points...
  - look at the reduced χ^2

  Uncertainties are overestimated?



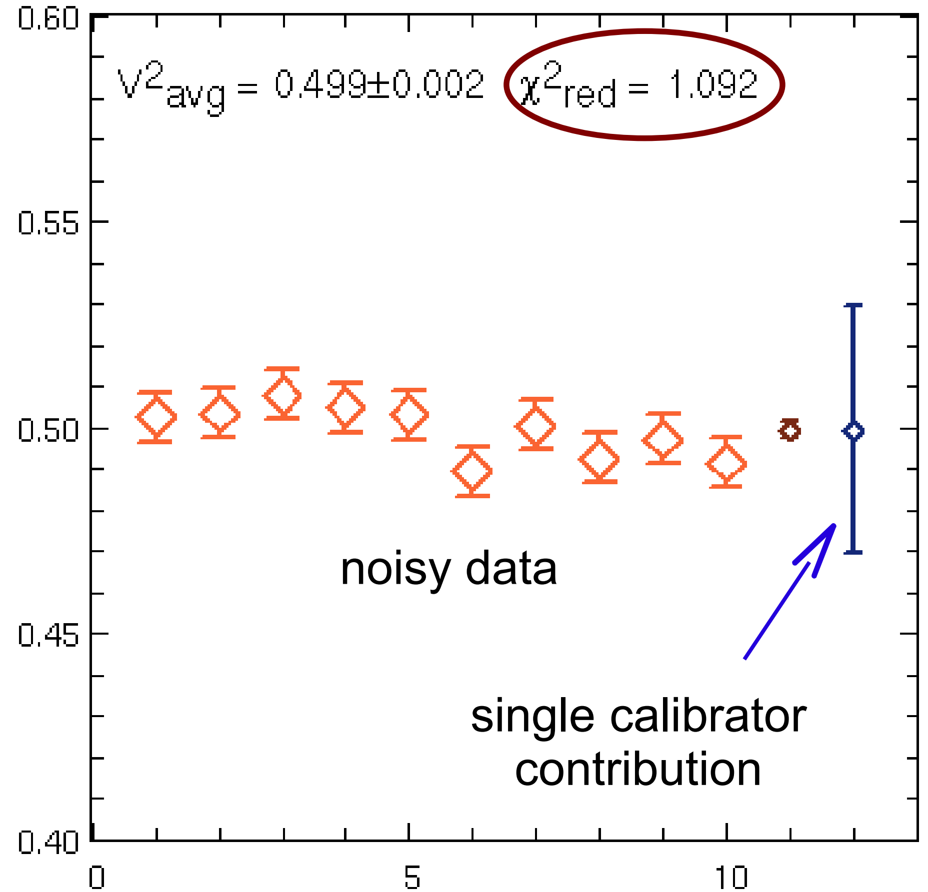
What **not** to do

- I see my data uncertainties are obviously overestimated
- I assume I probably made a mistake in my error propagation calculations...
- I take my data scatter as my uncertainty, because "data never lie"
- I reduce my computed error bar to get χ^2_{red} of ~1
- Go ahead with my initial analysis without giving it much thoughts...

Simple case (better) analysis

- Calibrator's contribution is not an uncertainty: it is **common** to all measurements
- It is a **systematic**
- If we separate the systematic, everything gets back to normal

$$V^2_{\mathrm{avg}} = 0.499$$
$$\pm 0.002_{\mathrm{stat}}$$
$$\pm 0.030_{\mathrm{cal}}$$



Systematics

Systematics are to **uncertainties**

what

biases are to **noises**

they just do not go away when you average

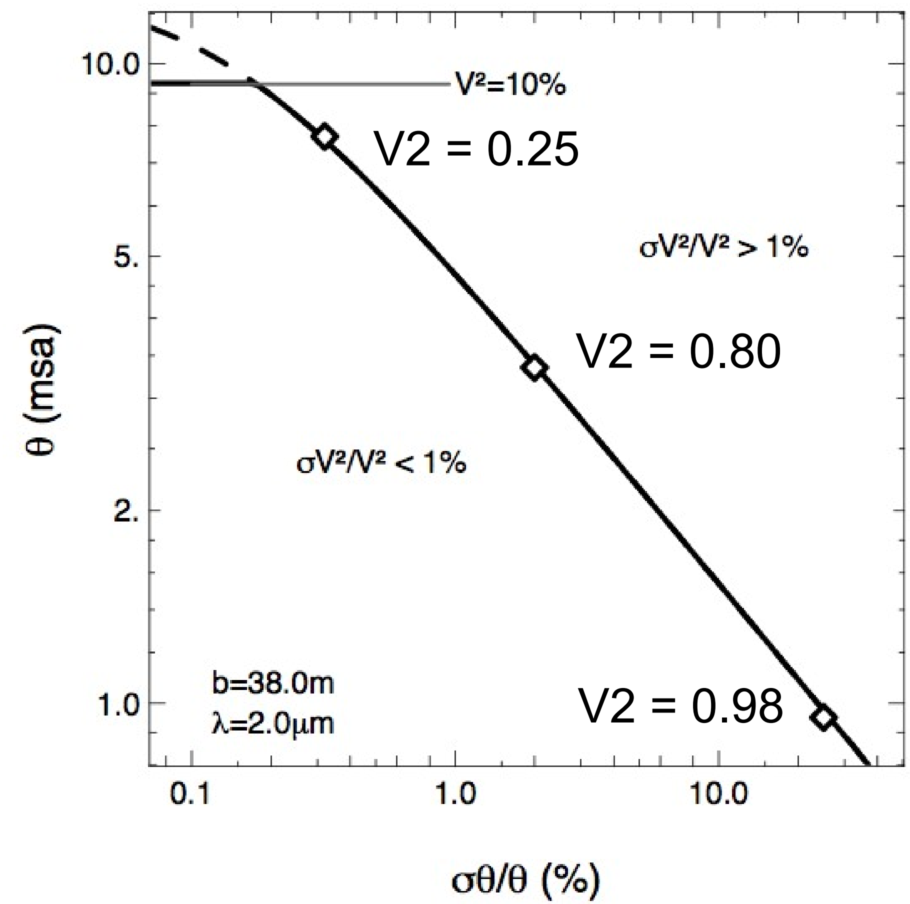
The easiest to do is to treat systematics and uncertainties separately, since uncertainties can be averaged but systematics cannot, since there are common to a given data set

Calibrators' systematics

Systematic: the uncertainty on the calibrator's **estimated** visibility. It is reduced for:

- an unresolved calibrator
- a calibrator with precise (and accurate) diameter's estimate

Compromise: the uncertainty on the calibrator's visibility **measurement** should also be under control (not too faint!)



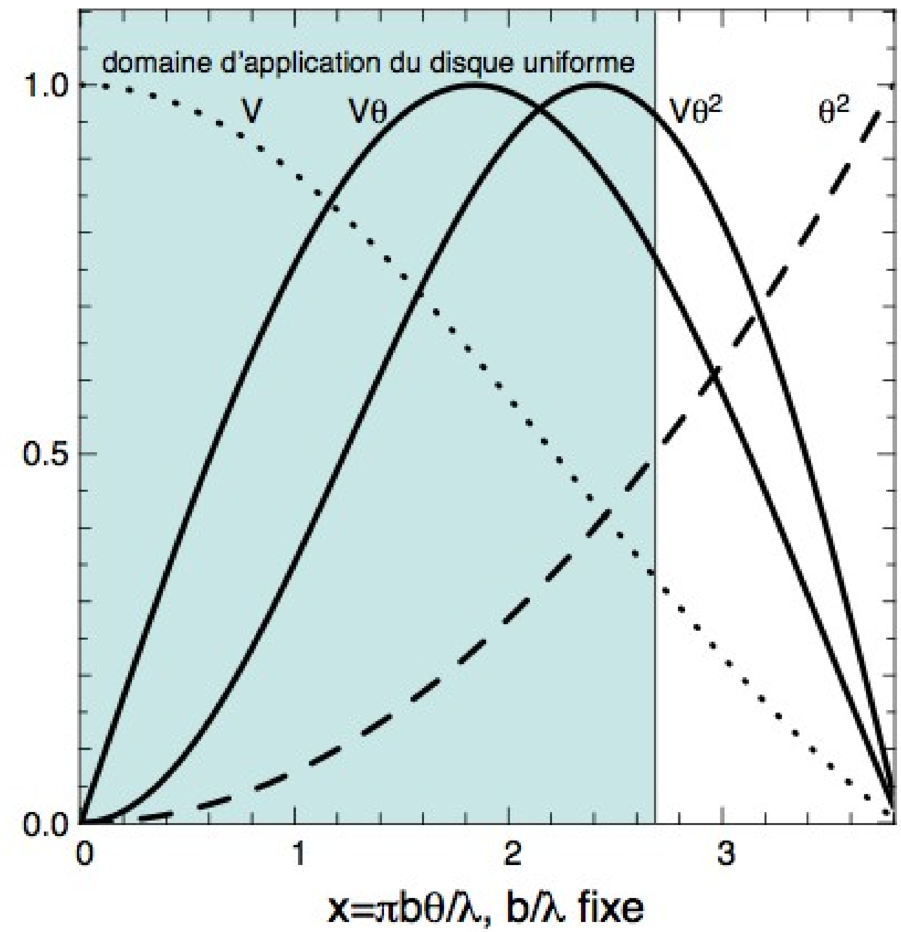
Optimized calibrators

the optimum diameter in term of signal to noise:

- flux goes as θ^2
- contrast goes as V

depending on the dominating noise (readout or photon), a slightly resolve calibrator is best

It corresponds to cal with diameter uncertainties of ~1%



Why it matters?

If you are not careful you might:

- publish inaccurate results (i.e. wrong) with ridiculously small error bars
- get in fights with colleagues because your results differ by 20 sigmas
- miss interesting results...

about systematics

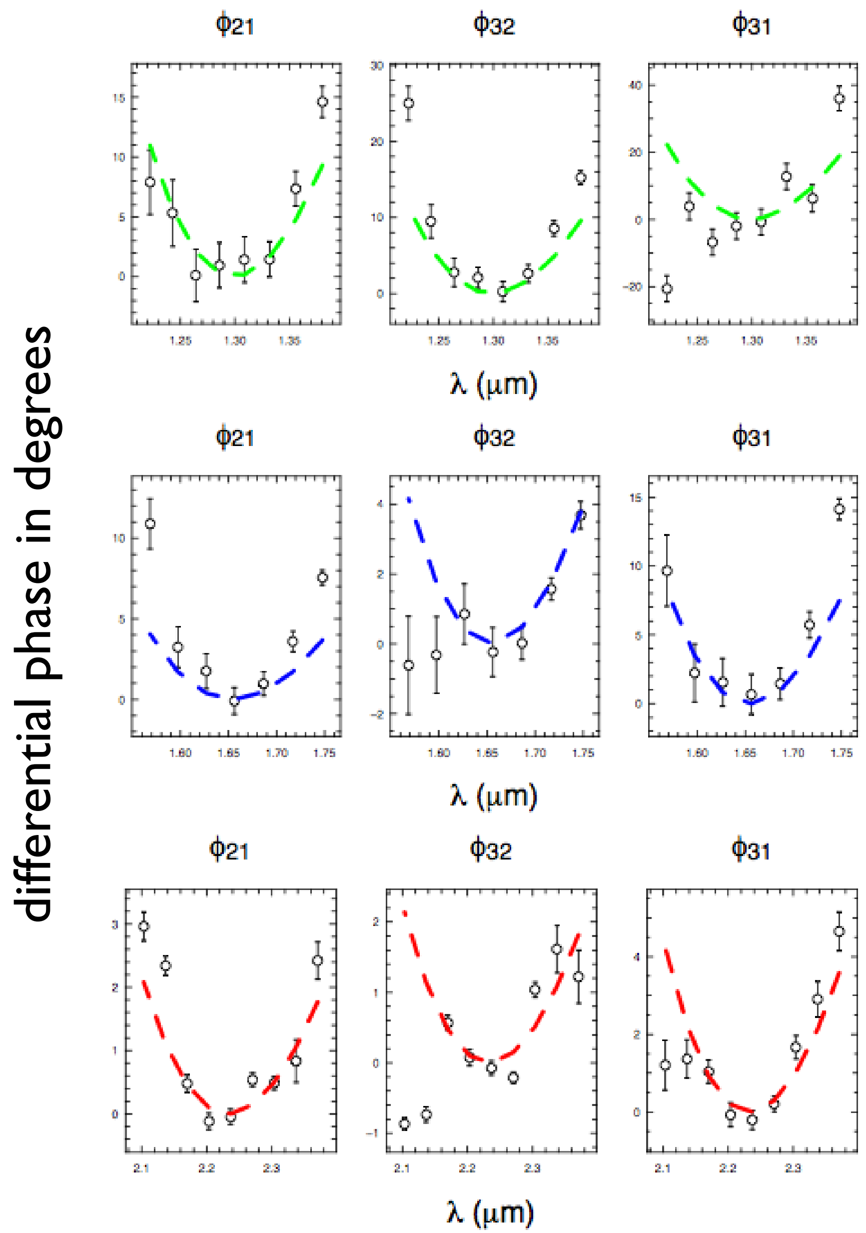
Some systematics can be estimated and controlled

- resolved calibrators
- dispersion in differential phase

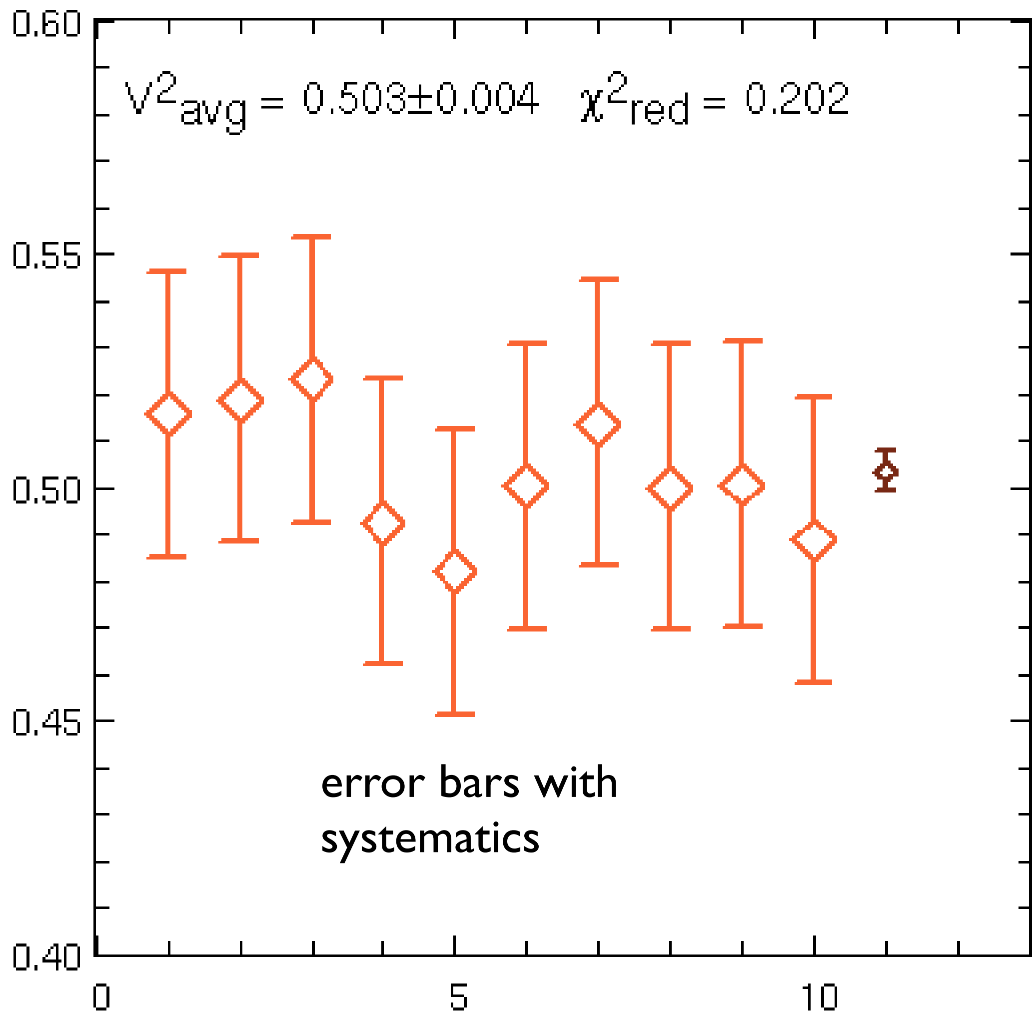
  OPD is [ΔLair * n(λ)] so it is not 0 for all wavelengths in your band

Some are not

- “frame selection” in AMBER is not robust

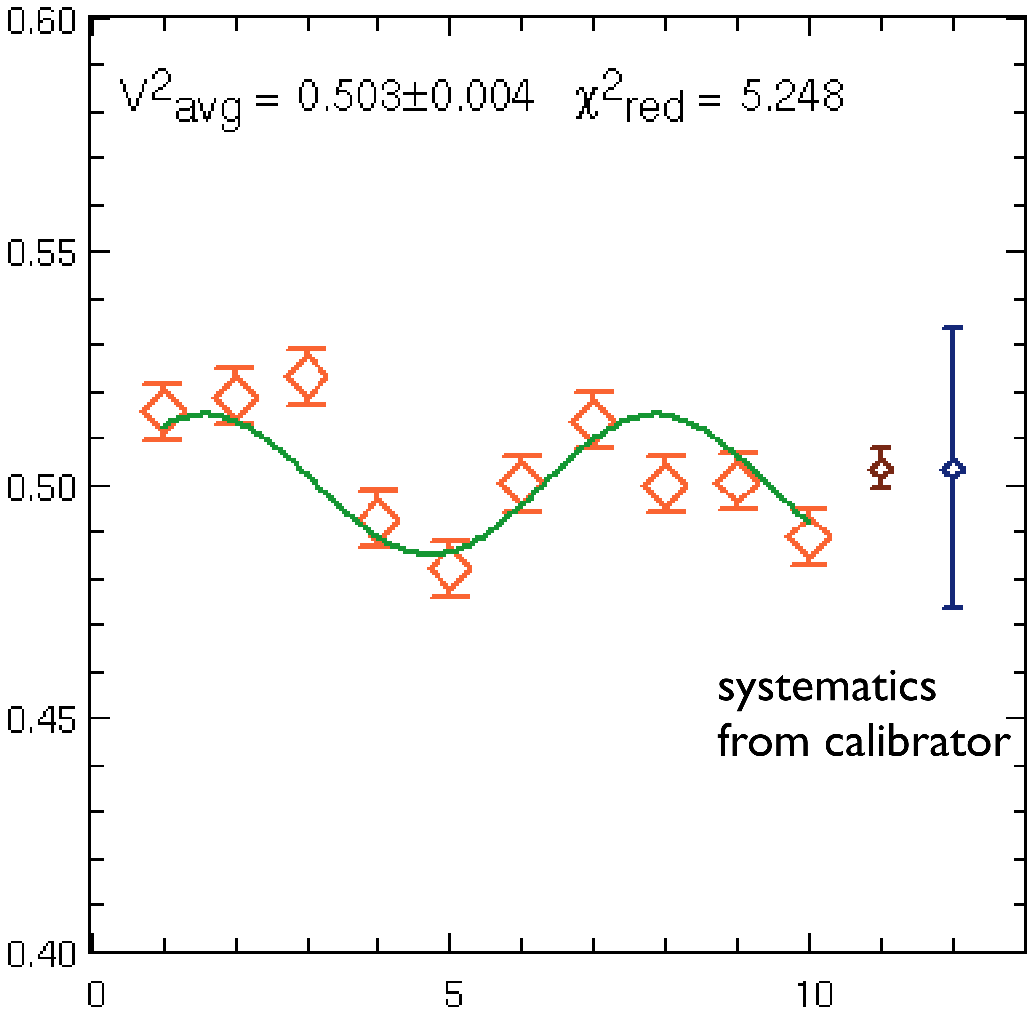


Do you see it?



0.60
0.55
0.50
0.45
0.40
0
5
10
V2avg = 0.503±0.004 χ2red = 0.202
error bars with
systematics

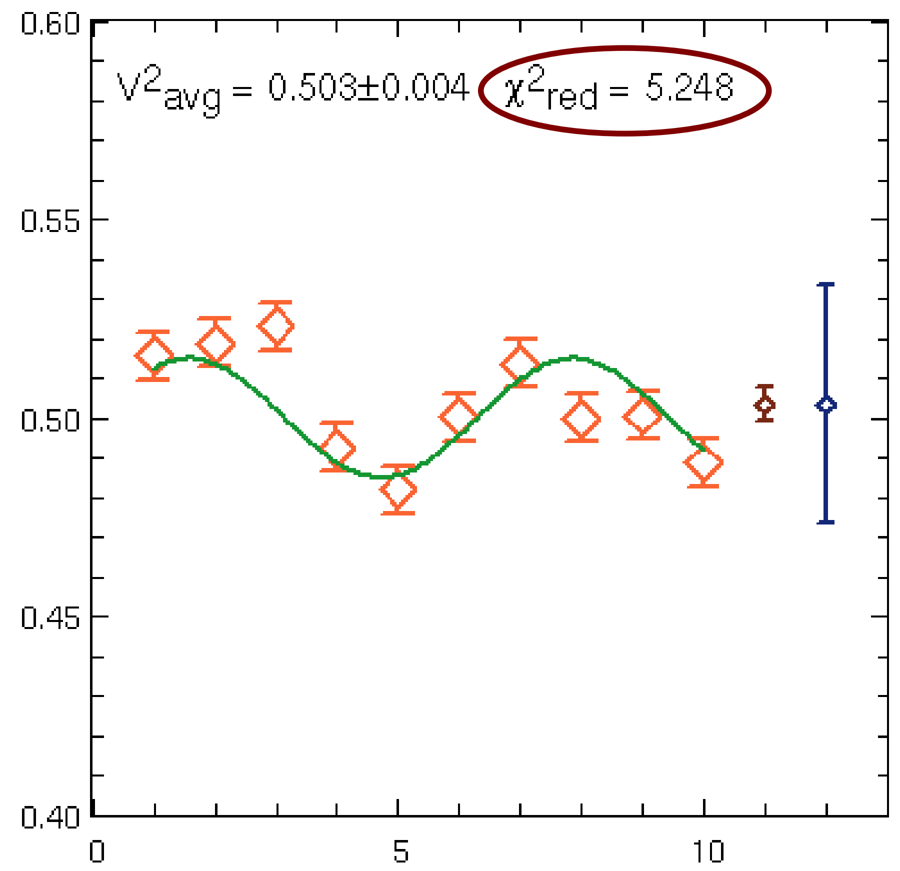
And now?



V2avg = 0.503±0.004 χ2red = 5.248
0.60
0.55
0.50
0.45
0.40
0
5
10
systematics
from calibrator

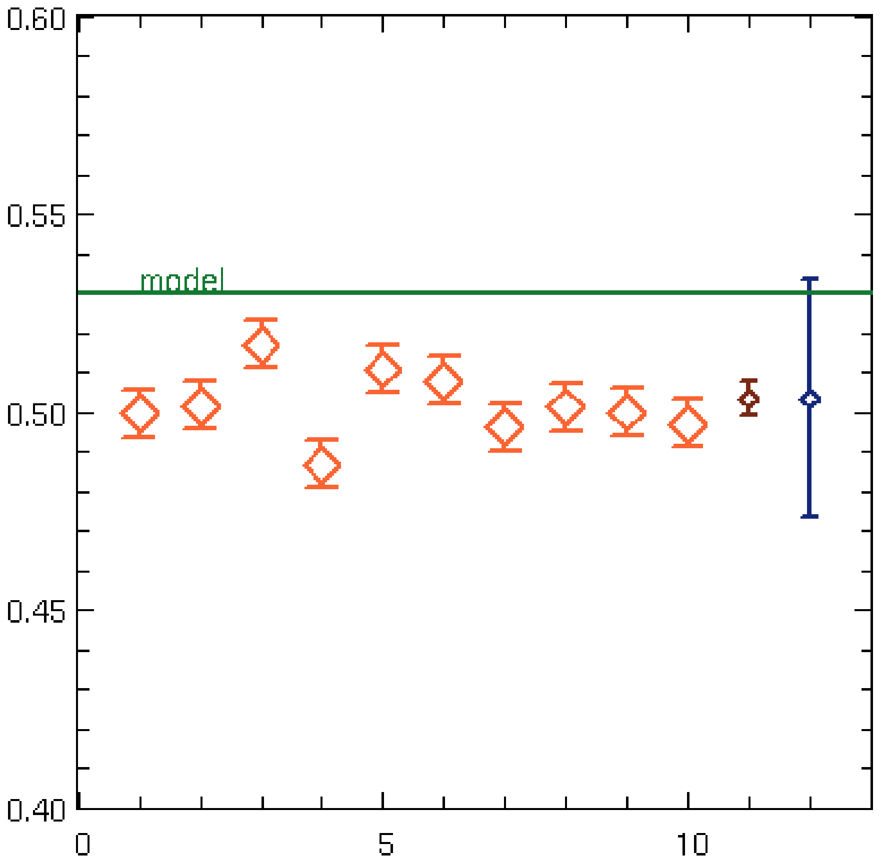
It is not about "seeing" the result

- By not taking into account systematics, the analysis fails to **detect** the modulation
- Comparing the χ^2 goes from *"over estimated error bars"* to *"a constant is inconsistent with my data"*



When you get limited by your calibrators...

- Our simple example is clearly dominated by the calibrator
- Our observations do not challenge the model
- but **they could have!** with smaller systematics
- The calibration strategy was poorly designed



Do not get limited by your calibrators

- Limit the contribution of the calibrator to the calibrated visibility
  - accurate estimation of V^2_{cal}
  - limited error bars on μ^2 measurement
- If you need to repeat your observations with one calibrator, be aware of the systematics
- Think of your calibration strategy long in advance

Multi calibrators: simple case

- Simple case: each observation uses a different single calibrator
- The contribution of the calibration is independent from one data point to the other
- In this case, and this case only, it is equivalent to not having systematics

General case

- In general, you want to use a few calibrators (e.g. 2 or 3) even if you have dozens of observations
- You can more easily detect bad calibrators among your set (e.g. binary)
- In a CAL1 – SCI – CAL2 – SCI – CAL3... sequence, you want to calibrate any SCI with the 2 adjacent calibrators
- There is a formalism for this...

Variance co-variance matrix formalism

Perrin 2003 A&A 400-1173

"The calibration of interferometric visibilities obtained with single-mode optical interferometers. Computation of error bars and correlations"

Ideas: error bars are correlated

- no longer N data and N error bars, but N data and NxN error matrix (VcV)
- diagonal elements of VcV are variances (σ^2)
- non-diagonal elements of VcV are variance co-variance ($\neq 0$ if calibrators in common)

Handling correlated error bars

- A generalized chi squared function needs to be minimized to fit the parameters "a" of your model "M"

$$\chi^2(a) = \sum_i \frac{(V_i^2 - M(a,i))^2}{\sigma_i^2}$$

$$S(a) = \left[V_i^2 - M(a,i)\right]^t C^{-1} \left[V_i^2 - M(a,i)\right]$$

$$C = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma 2 & \ldots & \rho_{1N}\sigma_1\sigma_N \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \ldots & \rho_{2N}\sigma_2\sigma_N \\ \ldots & \ldots & \ldots & \ldots \\ \rho_{N1}\sigma_N\sigma_1 & \rho_{N2}\sigma_N\sigma_2 & \ldots & \sigma_N^2 \end{bmatrix}$$

- "LIT pro" can minimize it for you, for example
- Or you can generalized the Levenberg-Marquardt

Unfortunately...

- All I explained to you work when you can **quantify** the systematics
- Calibrating does not guaranty that all systematics are gone
- For example, in AMBER the frame selection can introduce a systematic
  - if the atmo conditions were different and/or FINITO behaved differently between SCI/CAL
  - if SCI and CAL have different characteristics (SNR...)

Conclusions

Take home messages

Accurate / Precise

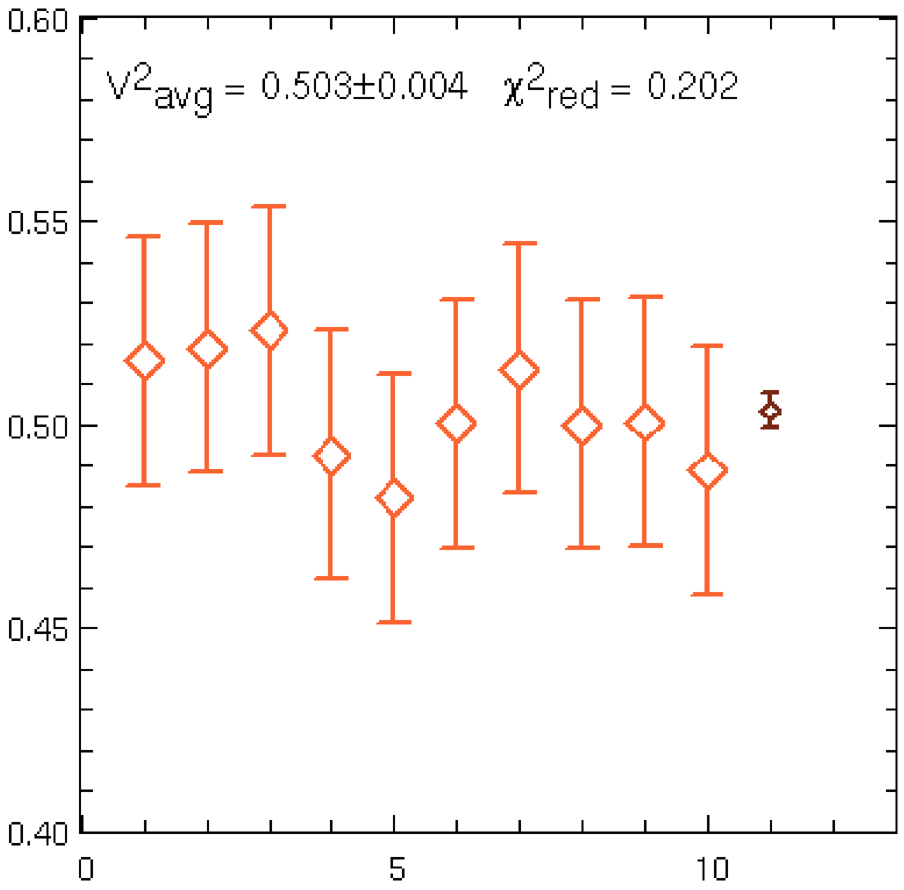
Accuracy matters: the smaller your precision, the better you should take care of the systematics

If you want to improve an observational result already published:

- you have to obtain a better precision
- you will probably encounter systematics that the previous author was not sensitive to

Accuracy and Accuracy

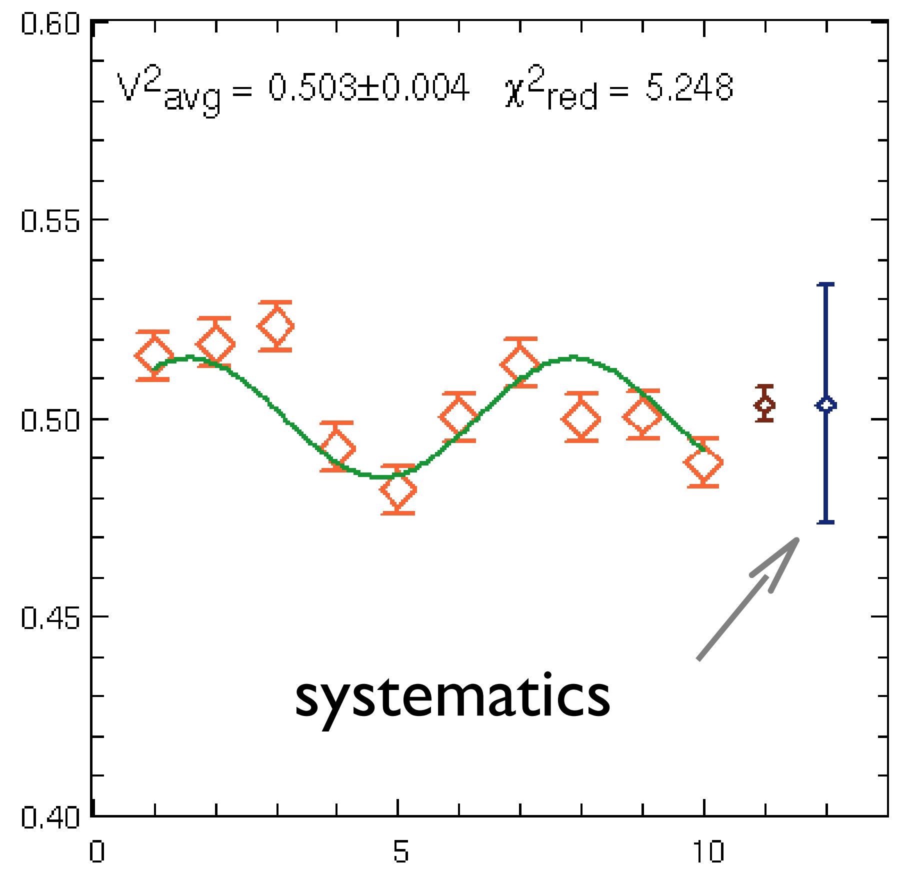
- In this example, the accuracy on the average is poor because of the systematics



Accuracy and Accuracy

- In this example, the accuracy on the average is poor because of the systematics
- By removing the systematics, I have a **differential** signal which contains information

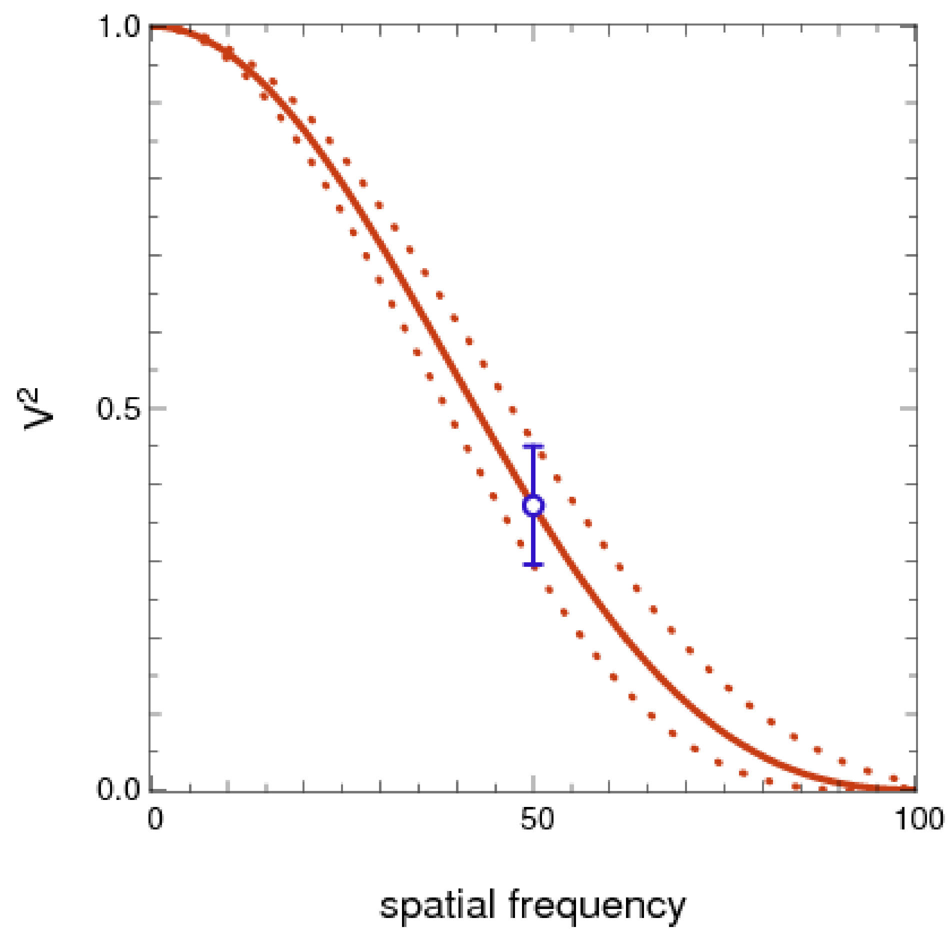
example: *calibrated visibility in AMBER are not accurate, try to use the differential signal*



Absolute / Differential

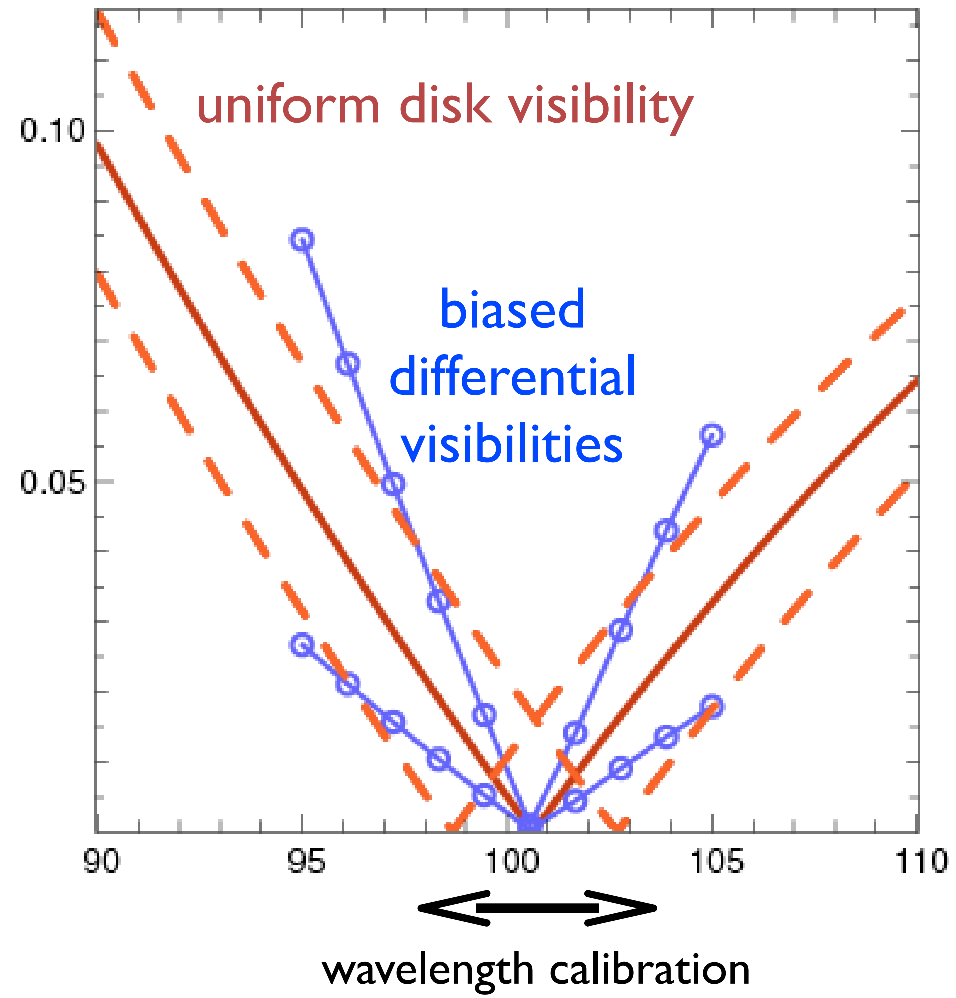
- Example: measure a diameter
- traditional: measure V2 and derive the diameter
- If your visibility is biased, the diameter is biased

Diameter requires accurate (absolute) visibilities



And what if the accuracy is bad?

- Use the 0 crossing of the **differential** visibility
- Insensitive to multiplicative biases
- very precise
- BUT! the accuracy on the diameter is limited by the wavelength accuracy



One or many?

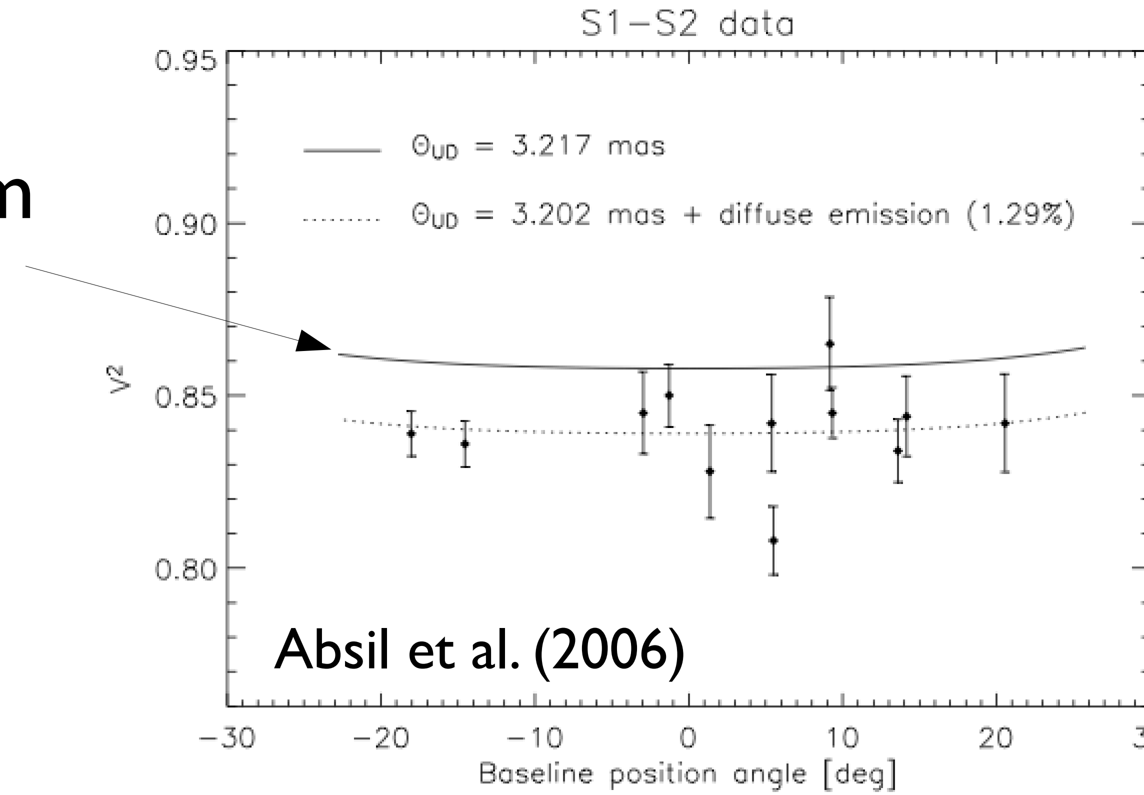
How many calibrators should I use?

- if you want only **differential** quantity: 1 (or 0)
  - use **CAL-SCI** in VLTI service mode
- if you want any **absolute** visibility information (or phase closure, to a certain extent)
  - many, especially if you are limited by systematics
  - but not as many as SCI, because you want to repeat calibrators to make sure they are good ones...
  - use **CAL-SCI-CAL** in VLTI service mode

When accuracy matters

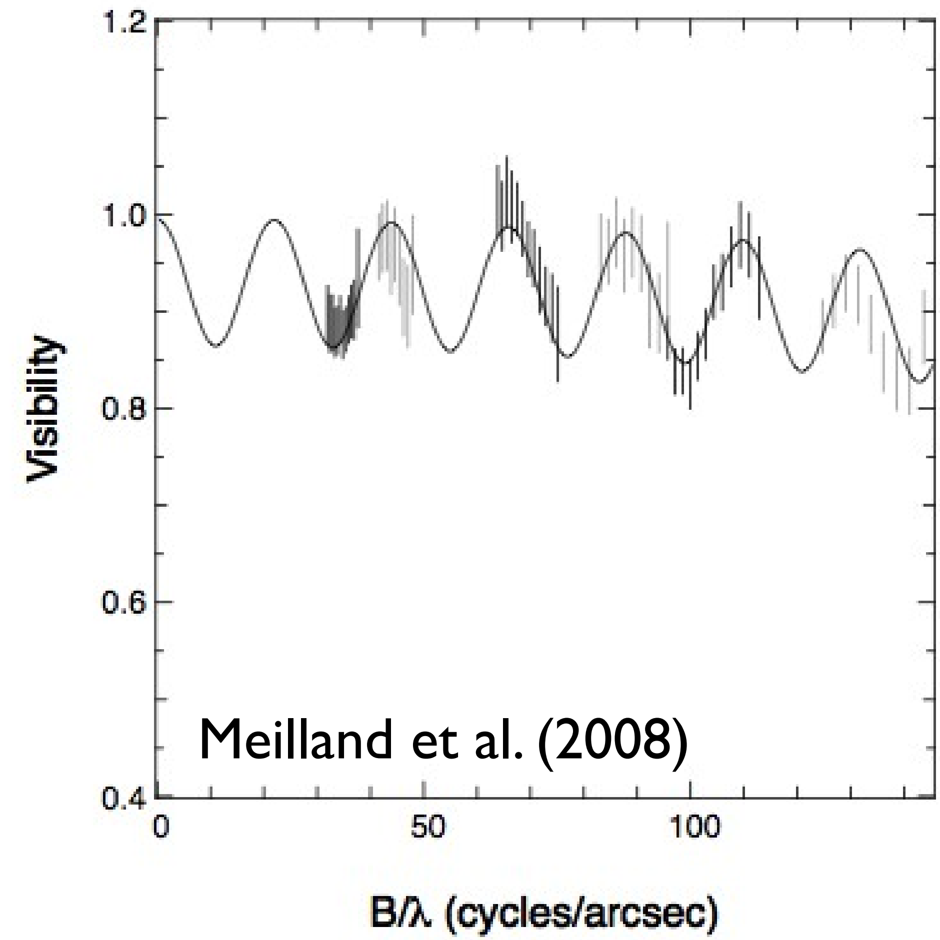
- Measure a photometric excess has visibility deficit
- Expected visibility from the diameter
- many calibrators used to reduce the systematics

$$V = \frac{F_\star V_\star + F_{\rm CSM} V_{\rm CSM}}{F_\star + F_{\rm CSM}} \approx \frac{F_\star}{F_\star + F_{\rm CSM}}$$



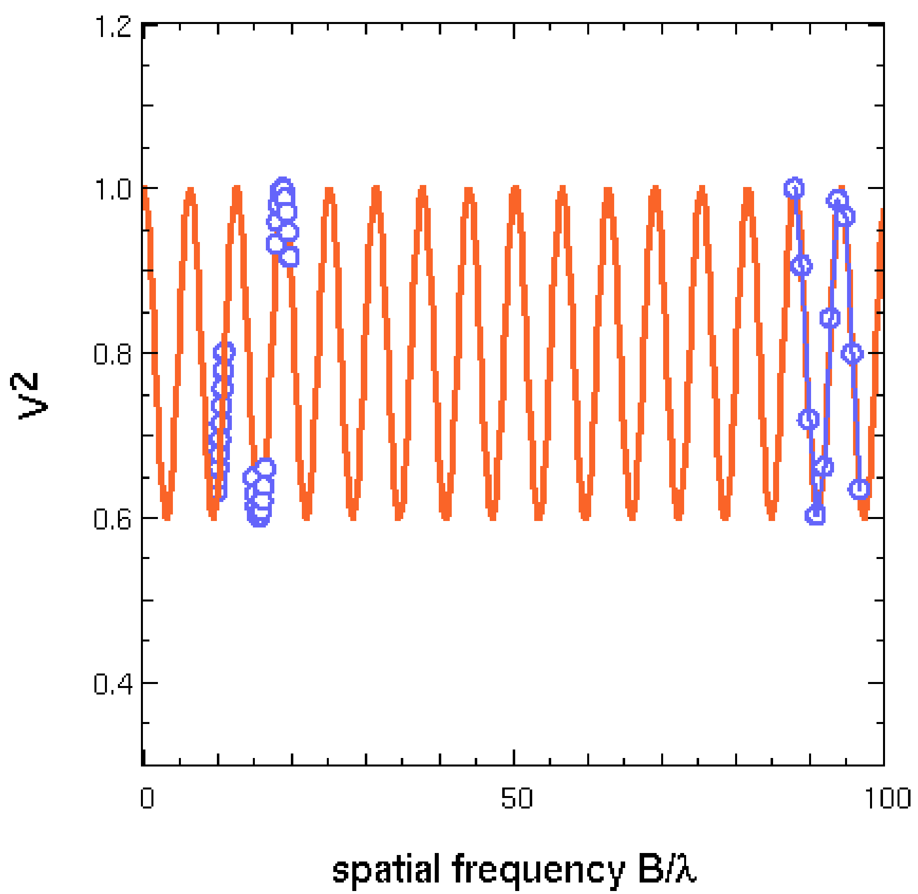
When it is not required...

- Sample the visibility modulation due to a binary
- Simultaneous observations in many spectral channels (using AMBER)
- Channels have strong systematics, but they are the same
- Binary contrasts not well constrained...



Not exclusive

- A binary can be characterized using absolute or differential measurements
- the 2 approaches need different setups
- use spectral dispersion for B/λ diversity



Conclusion

- Differential is usually more robust than absolute
- Differential and absolute "see" different things in your object
- Sometimes, you can use differential when you thought Absolute was required
- Understand the limitations (and there are many)
- Think ahead: you will need to put your strategy in your proposal.
- Even if you think you thought about everything, stay critical