An introduction to optical/IR interferometry

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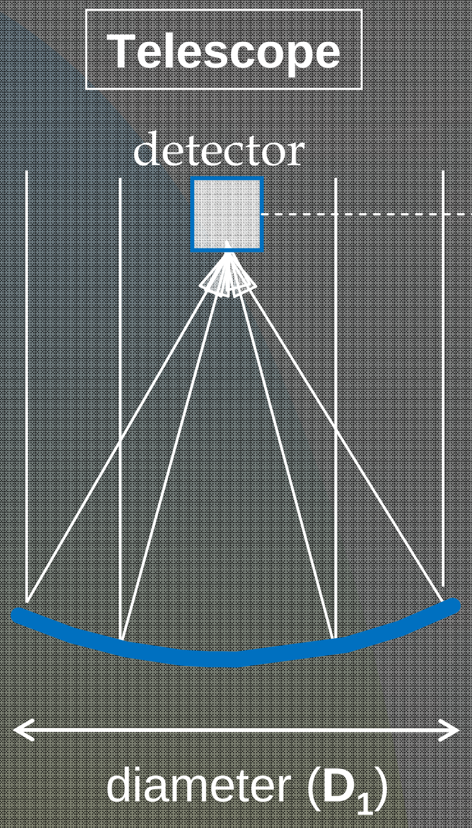
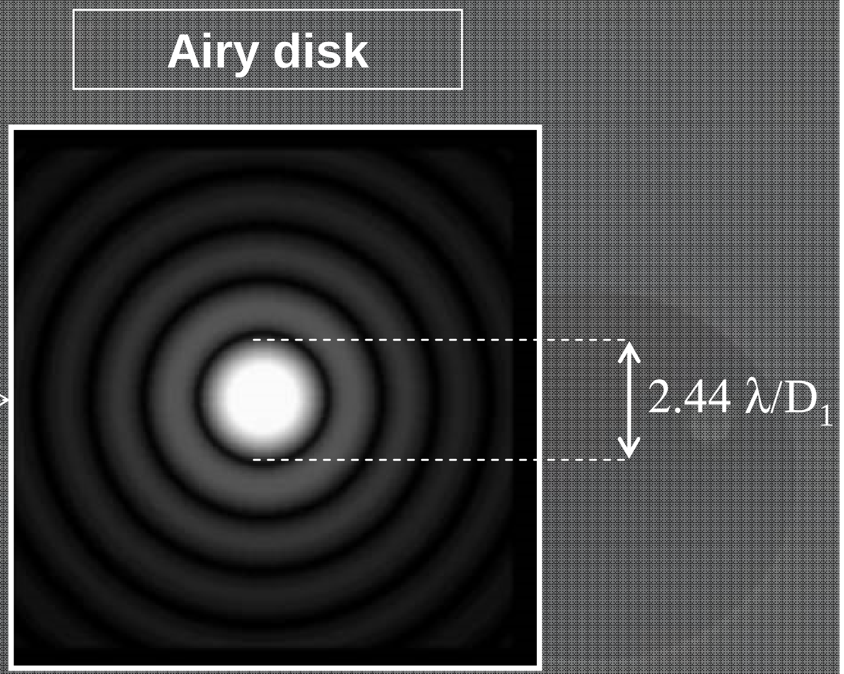
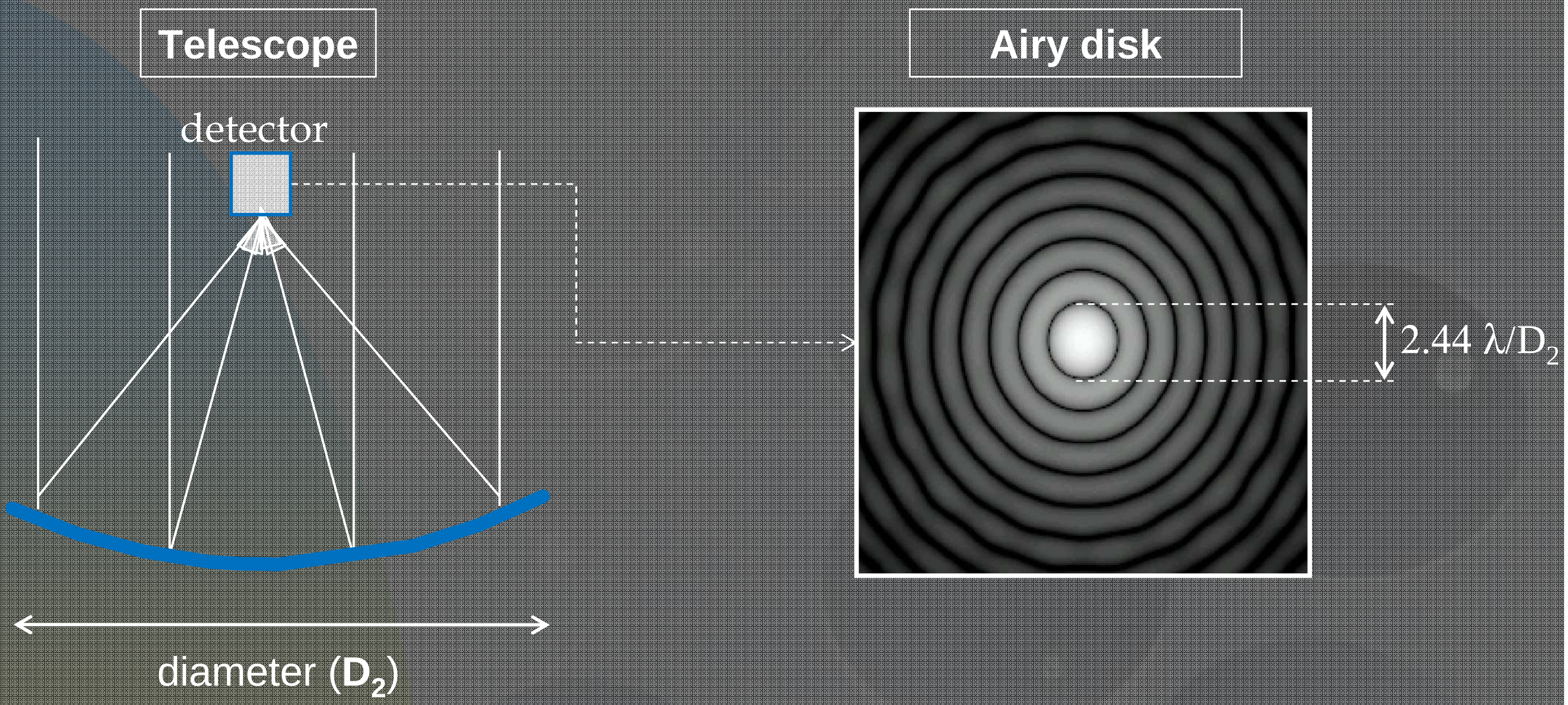
http://hdl.handle.net/2268/30144
http://orbi.ulg.ac.be/bitstream/2268/30144/1/EII_VLTI_School_J_Surdej.pdf



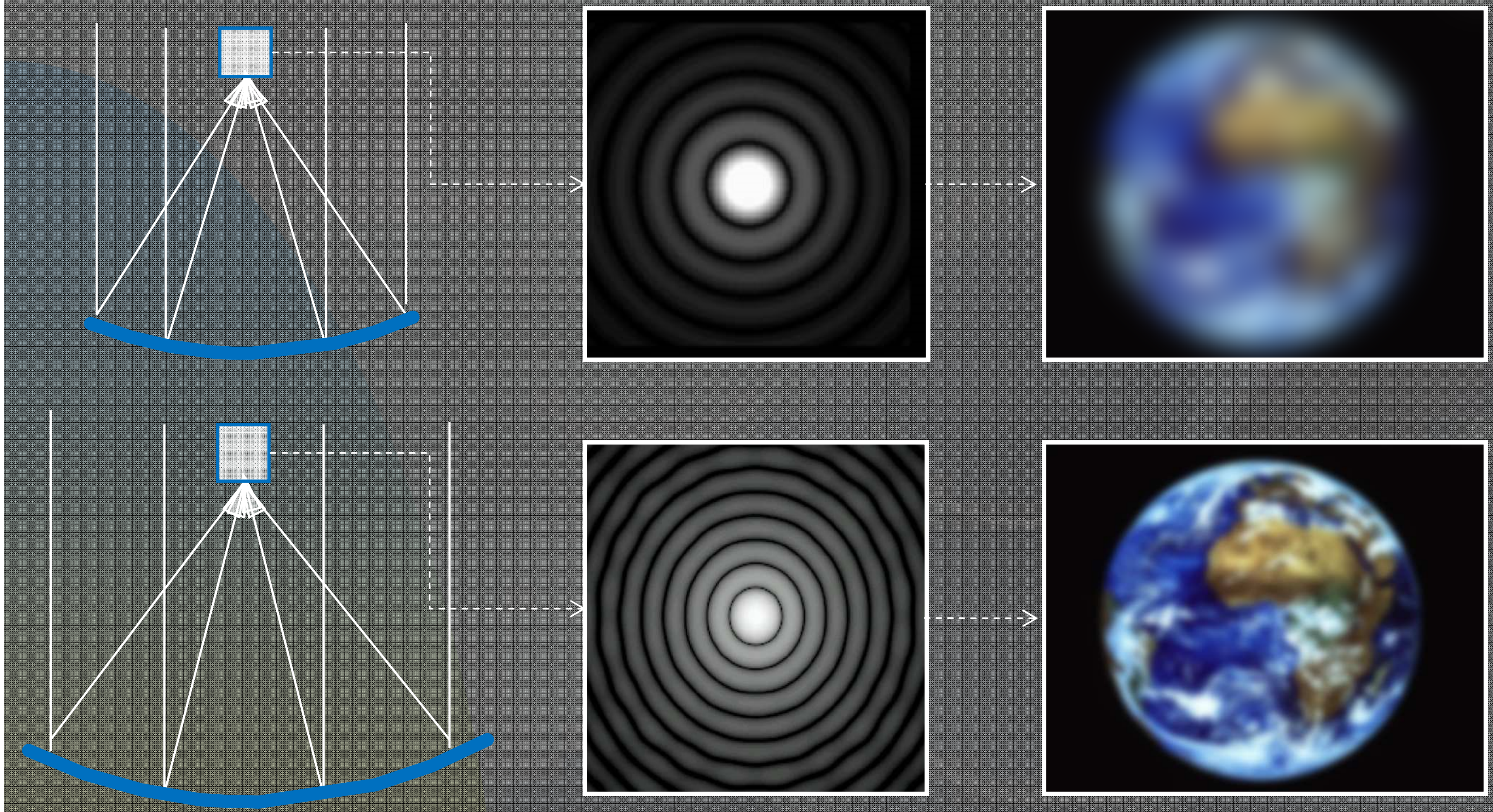


- The image of a star is like a dot!

c/o Denis Defrère



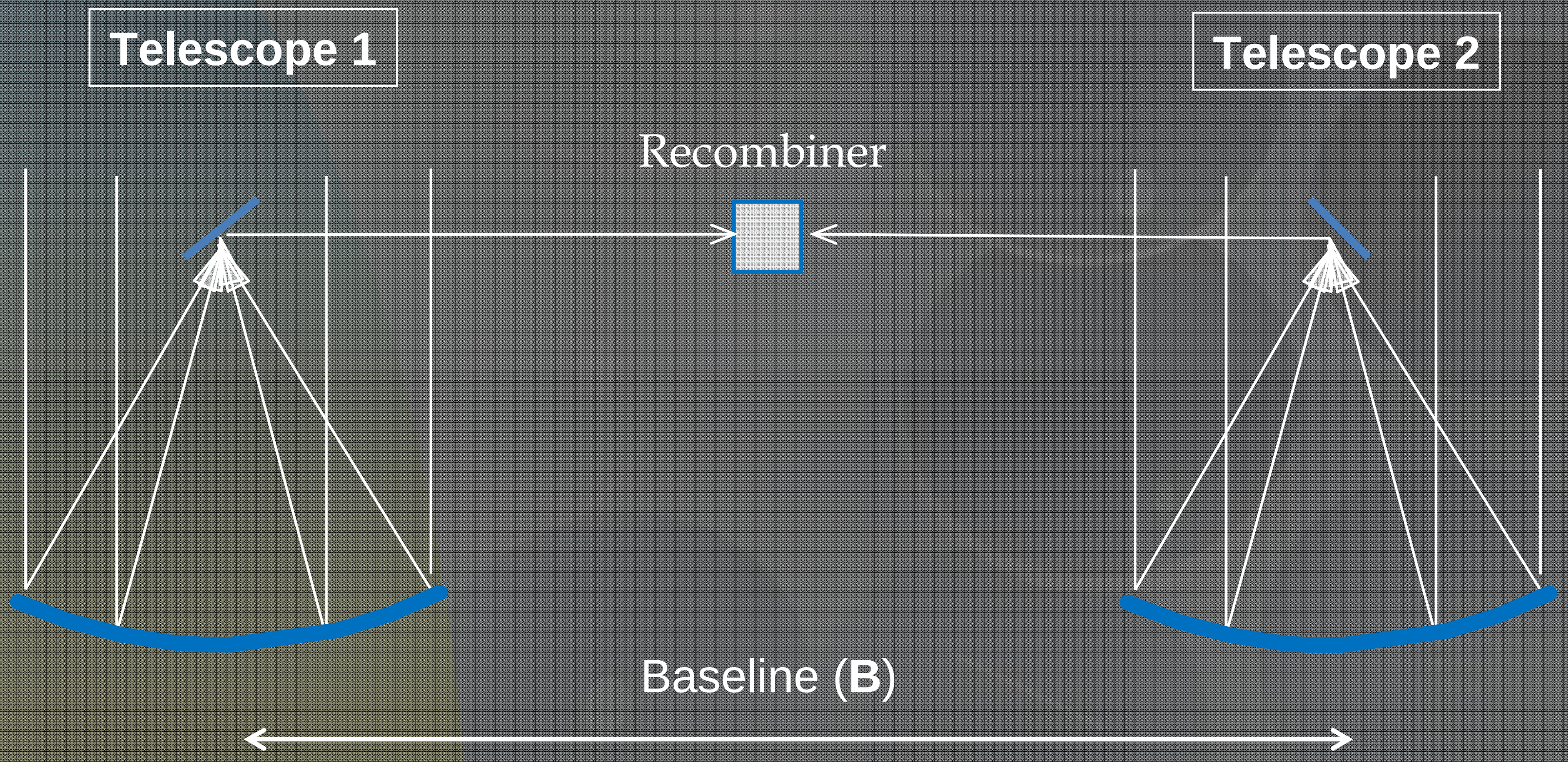
- The image of a star is still like a dot!

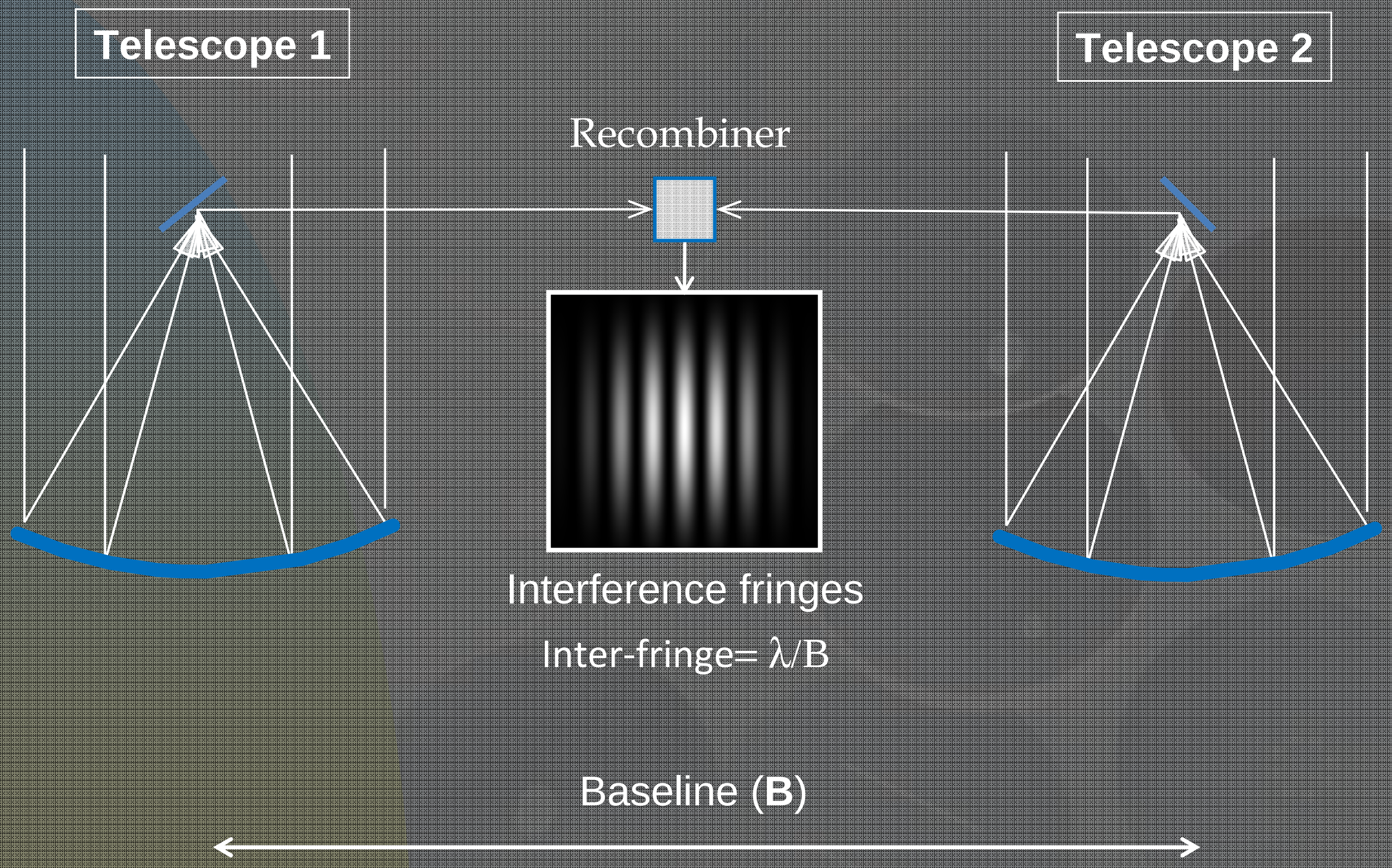


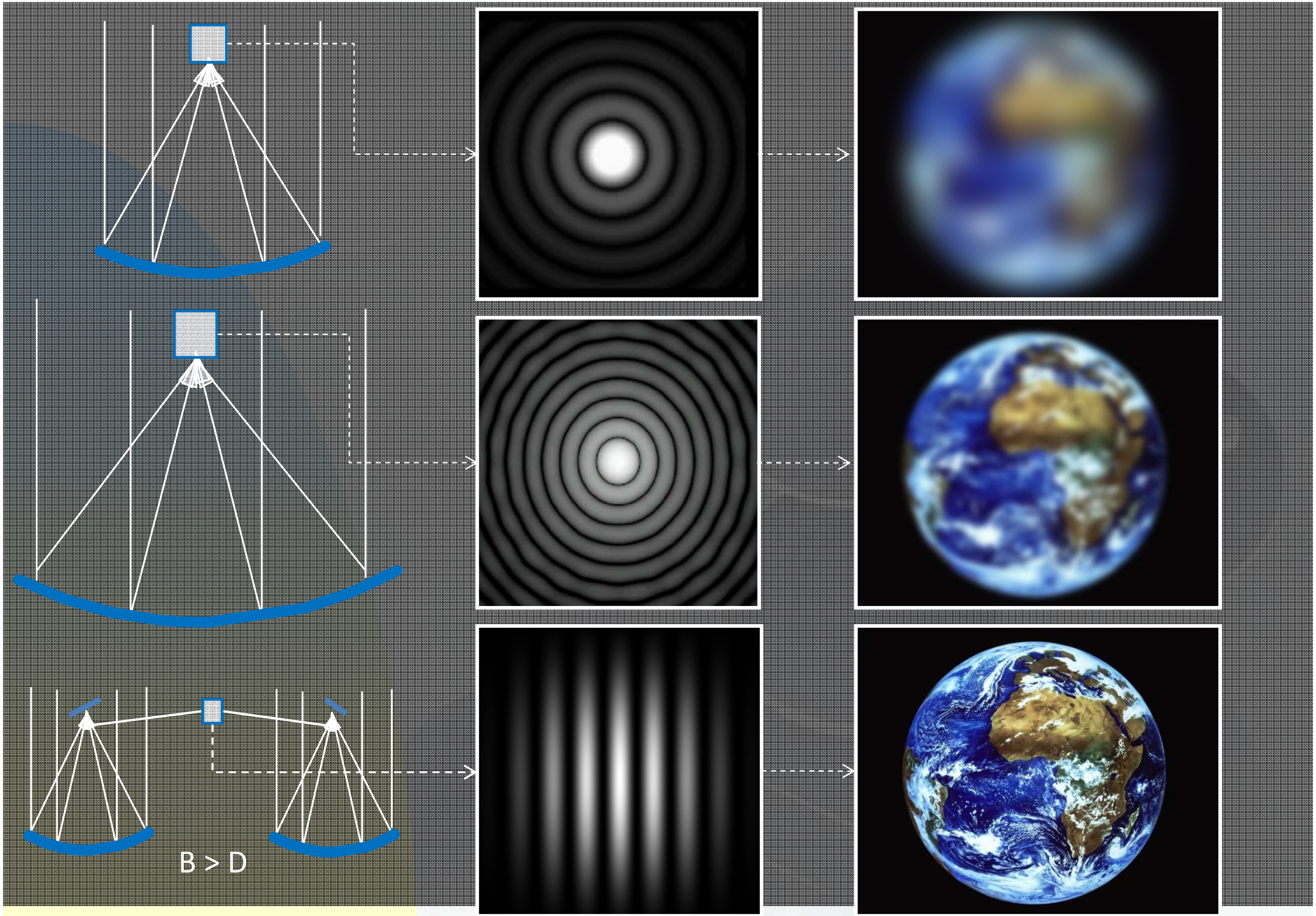
- Need for very large telescopes !!!

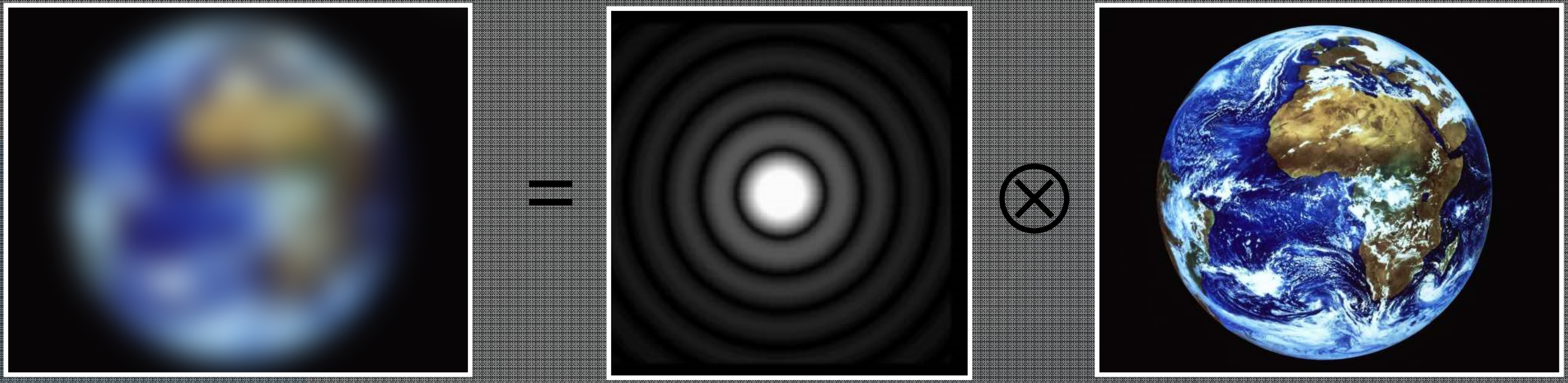
- H. Fizeau and E. Stephan (1868-1870):

  “In terms of angular resolution, two small apertures distant of B are equivalent to a single large aperture of diameter B”




Telescope 1
Telescope 2
Recombiner
Interference fringes
Inter-fringe= λ/B
Baseline (B)


B > D



$$I(\zeta,\eta) = \iint PSF(\zeta-\zeta',\eta-\eta')\, O(\zeta',\eta')\, d\zeta'\, d\eta' = PSF(\zeta,\eta) \otimes O(\zeta,\eta)$$

$$TF(I(\zeta,\eta))(u,v) = TF(PSF(\zeta,\eta))(u,v)\ TF(O(\zeta,\eta))(u,v)$$

$B_u = u\ .\ \lambda\ ,\ B_v = v\ \lambda$

$$TF(PSF(\zeta,\eta))(u,v) = \iint A^{*}(x,y)\ A\ (x+u, y+v)\, dx\, dy$$

An introduction to optical/IR interferometry

- 1 Introduction
- 2 Reminders
- 3 Brief history of stellar diameter measurements
- 4 Interferometry with two independent telescopes
- 5 Light coherence (Zernicke-van Cittert theorem)
- 6 Examples of optical interferometers
- 7 Results
- 8 Three important theorems (Fundamental, Convolution and Wiener-Khintchin theorems)!

An introduction to optical/IR interferometry

- 1 Introduction

$$R = \rho\, z \qquad (1.1)$$


$$\pi F = \pi f \,/\, \rho^2 \qquad (1.2)$$

$$T_{eff} = (\pi F/\sigma)^{1/4} \qquad (1.3)$$

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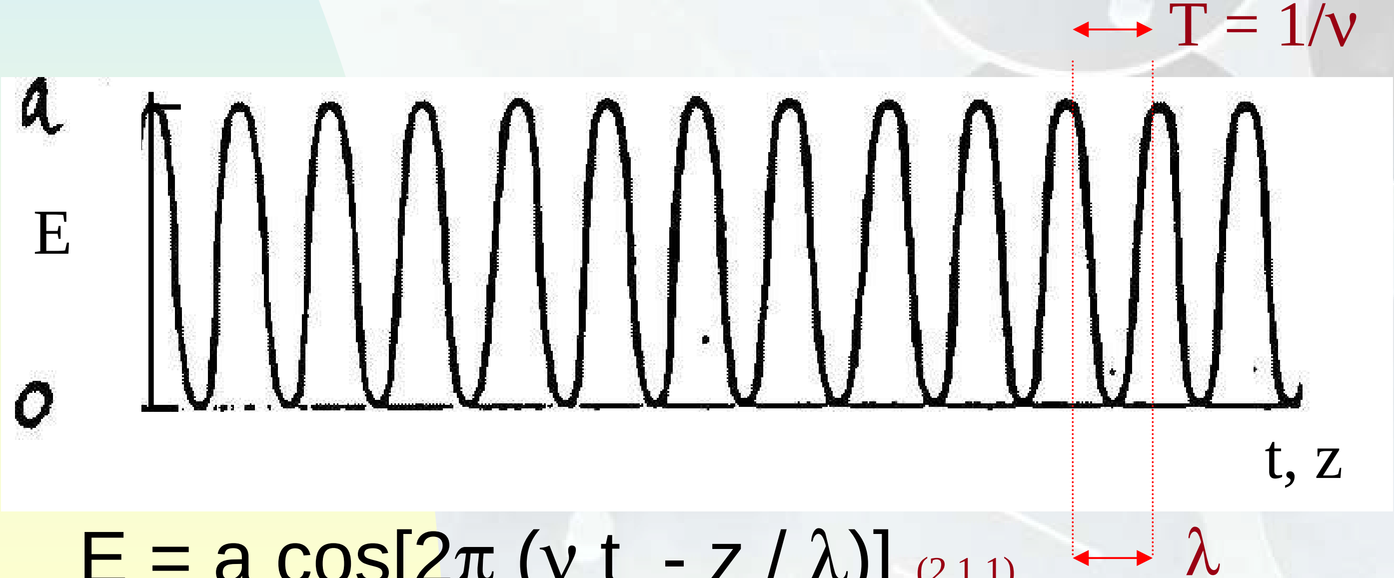
- 2 Reminders

- 2.1. Representation of an electromagnetic wave



$$E = a \cos[2\pi (\nu t - z / \lambda)] \quad (2.1.1)$$

where $\lambda = c / \nu = c\,T.$ (2.1.2)

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- 2.1. Representation of an electromagnetic wave

$$E = \mathrm{Re}\{ a \exp[i2\pi(\nu t - z / \lambda)]\} \quad (2.1.3)$$

$$E = \mathrm{Re}\{ a \exp[-i \phi] \exp[i2\pi\nu t]\} \quad (2.1.4)$$

where $\phi = 2\pi z / \lambda.$ (2.1.5)

$$E = a \exp[-i \phi] \exp[i2\pi\nu t] \quad (2.1.6)$$

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- 2.1. Representation of an electromagnetic wave

$$E = A \exp[i2\pi\nu t] \qquad (2.1.7)$$

with $$A = a \exp[-i\ \phi] \qquad (2.1.8)$$

$\nu \sim 6\ 10^{14}$ Hz for $\lambda = 5000$ Å

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- 2.1. Representation of an electromagnetic wave

$$\langle E^2 \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} E^2 dt \qquad (2.1.9)$$

$$\langle E^2 \rangle = a^2 / 2 \qquad (2.1.10)$$

$$I = A\,A^* = |A|^2 = a^2 . \qquad (2.1.11)$$

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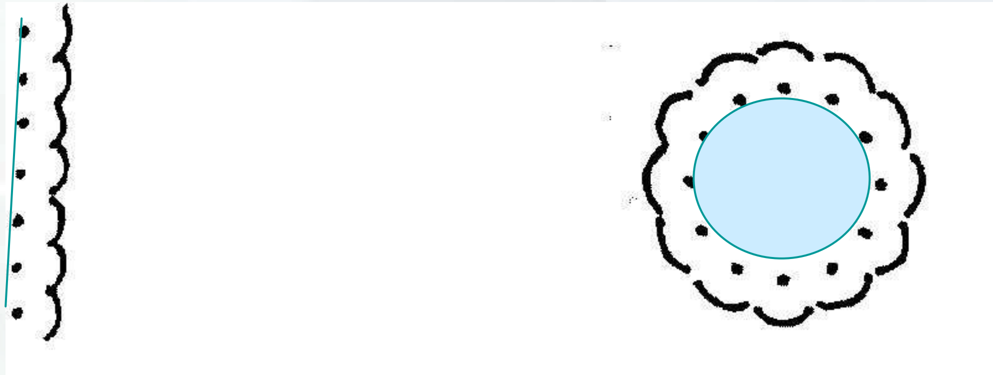
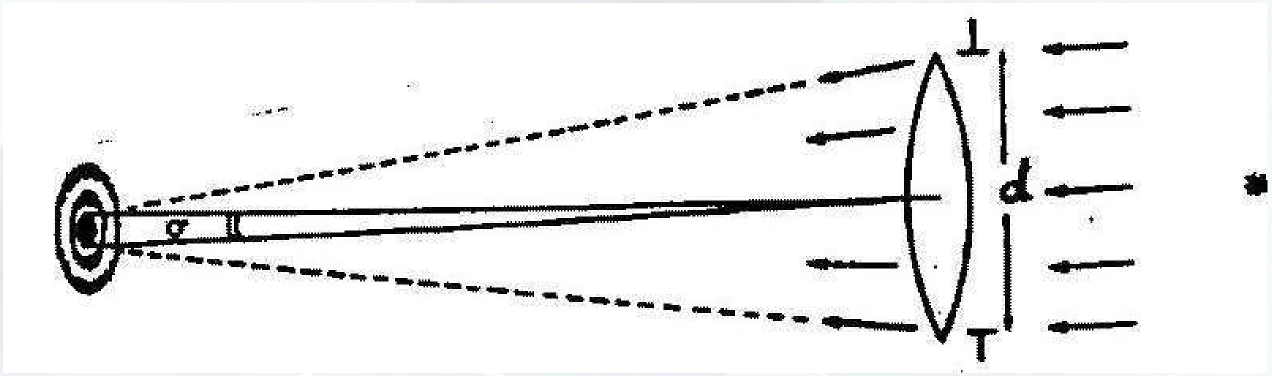
- 2.2. The Huygens-Fresnel principle





$$\sigma = 2.44\ \lambda / d$$

(2.2.1)

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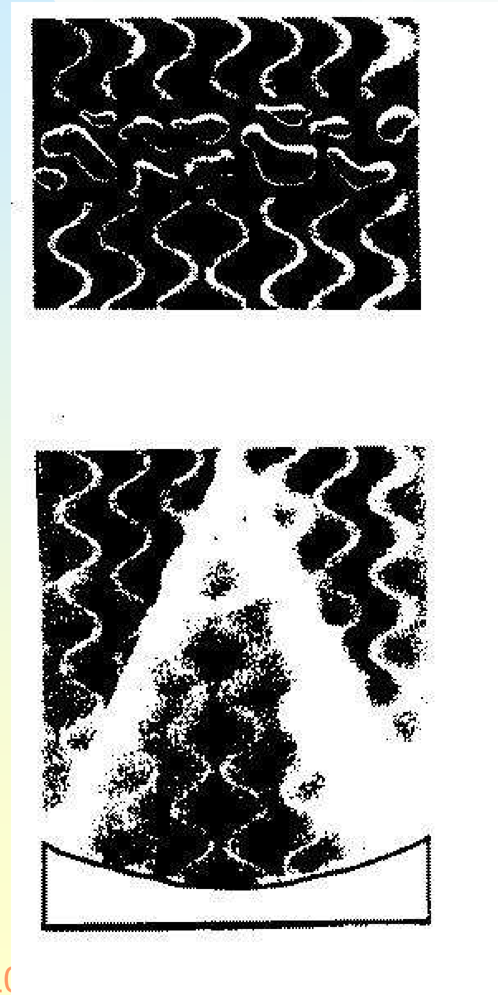
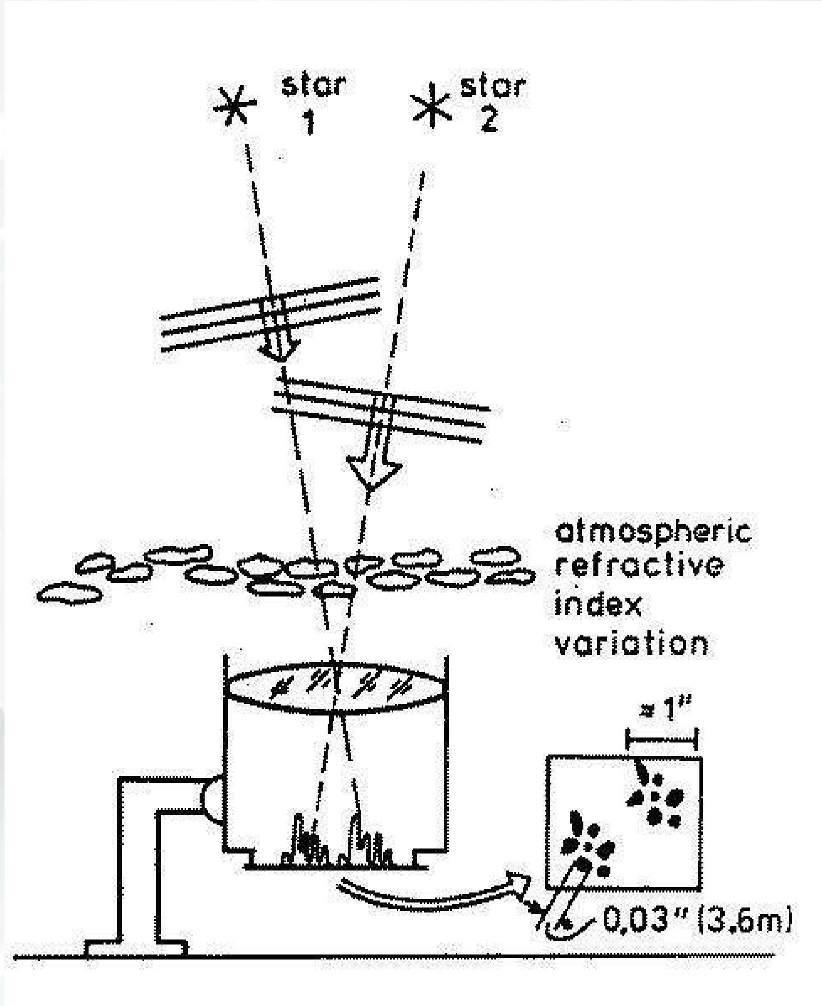
- 2.2. The Huygens-Fresnel principle





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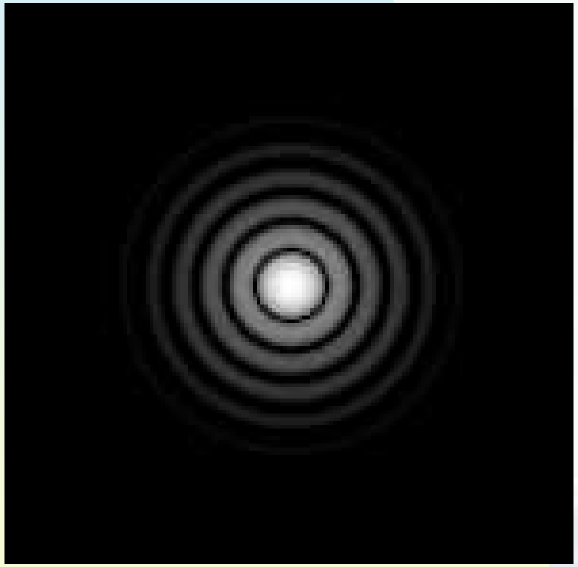
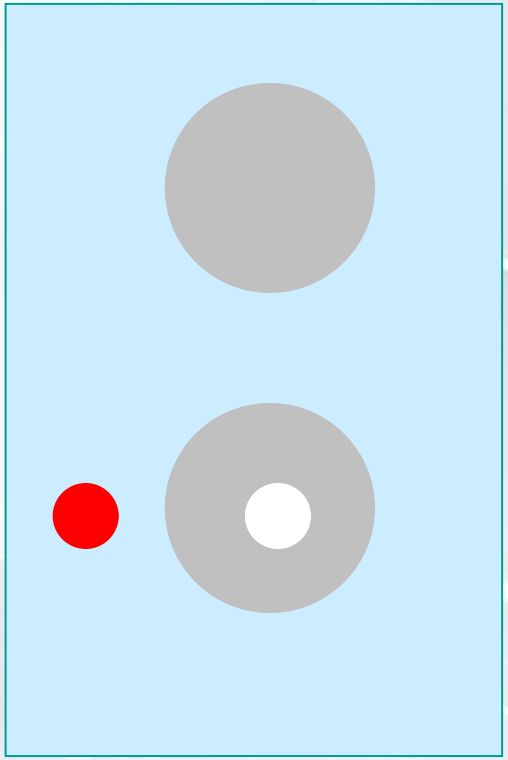
- 2.2. The Huygens-Fresnel principle





1st experiment!

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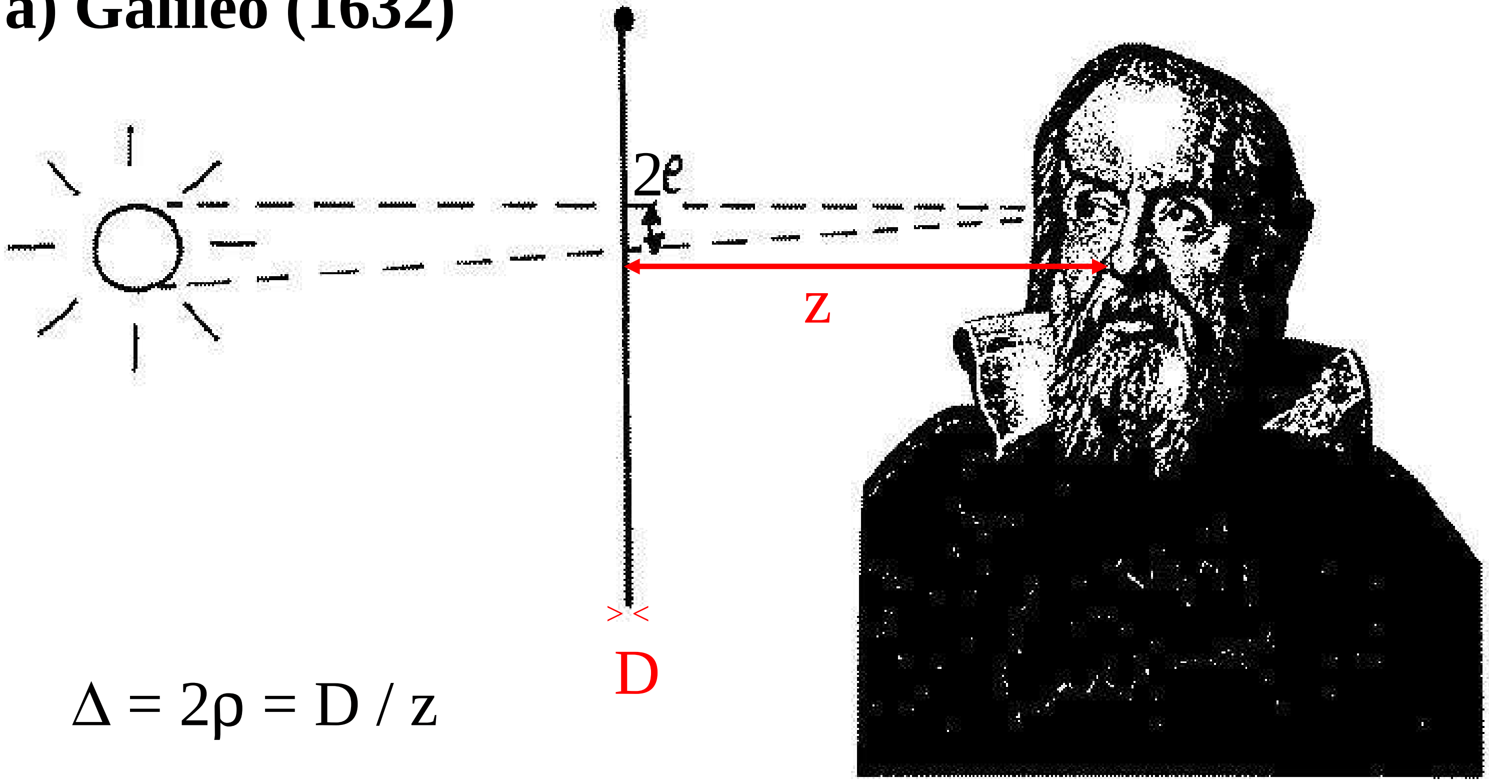
- 3 Brief history of stellar diameter measurements

a) Galileo (1632)



$\Delta = 2\rho = D / z$

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- 3 Brief history of stellar diameter measurements

b) Newton:

$$V_{\odot} - V = -5 \log (z / z_{\odot}) \,, \qquad (3.1)$$

$$\Delta = 2 R_{\odot} / z \,, \qquad (3.2)$$

$$\Delta \sim 2\ 10^{-3''}\ (8\ 10^{-3''})\ . \qquad (3.3)$$

c) Fizeau-type interferometry

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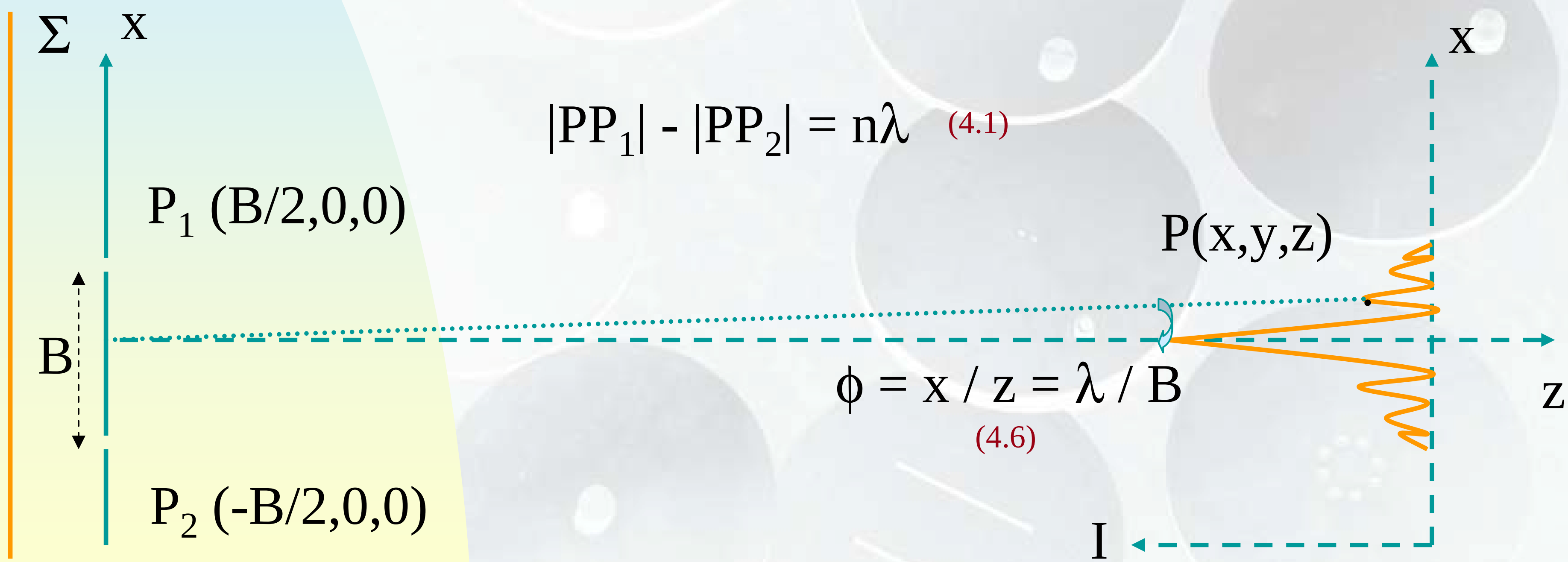
- 4 Interferometry with two independent telescopes

a) Young's double hole experiment (24-11-1803)



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- 4 Interferometry with two independent telescopes

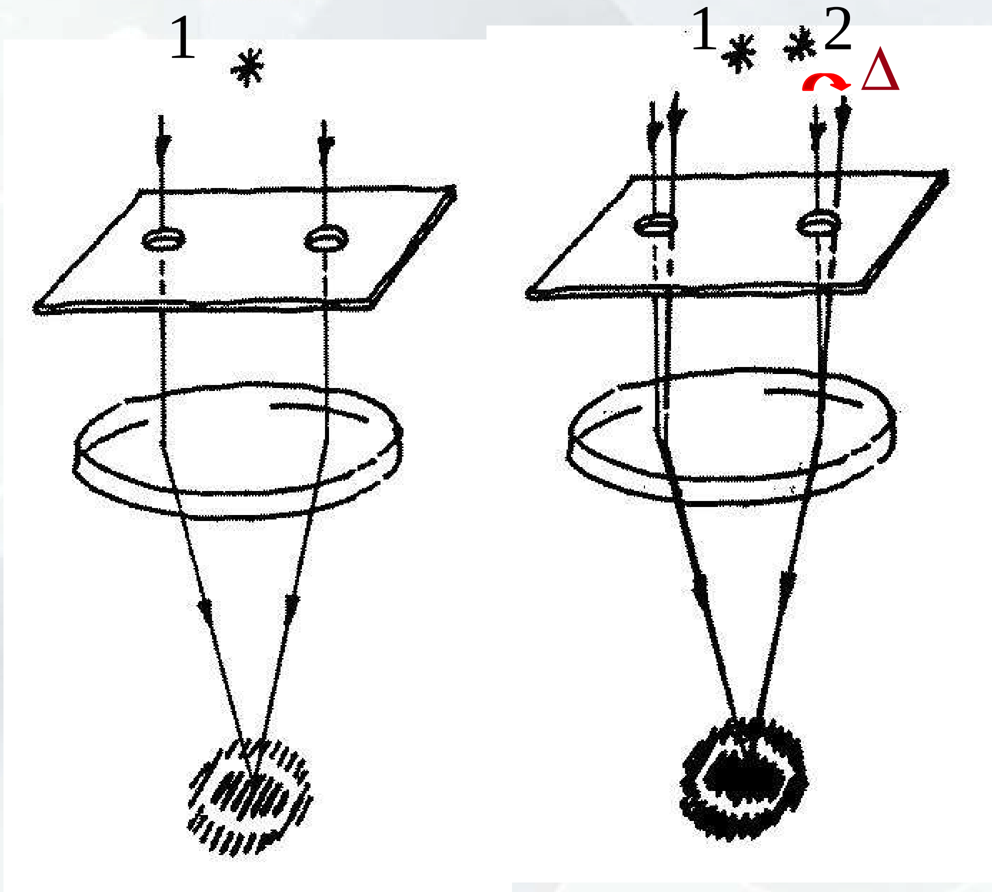
b) Fizeau … the father of stellar interferometry (1868)

If $\Delta \geq \phi/2 = \lambda / (2B)$, (4.7)

fringe disappearance!

Fringe visibility:

$$\upsilon = \left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \right)$$



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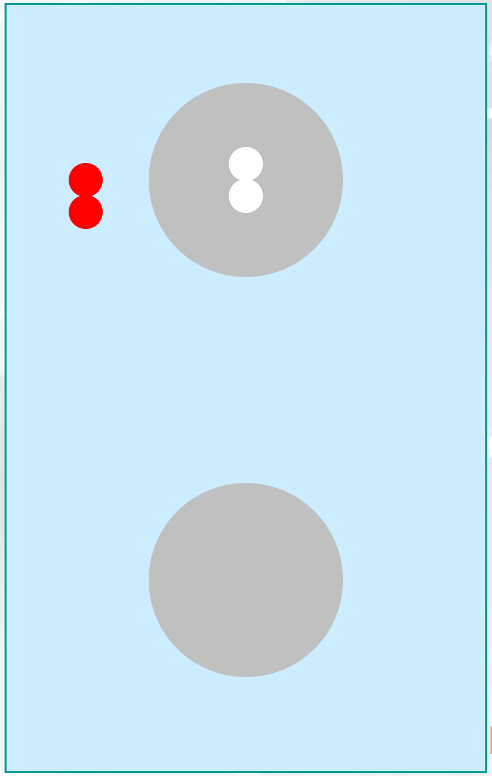
- 4 Interferometry with two independent telescopes

b) Fizeau … the father of stellar interferometry (1868)

2nd experiment!



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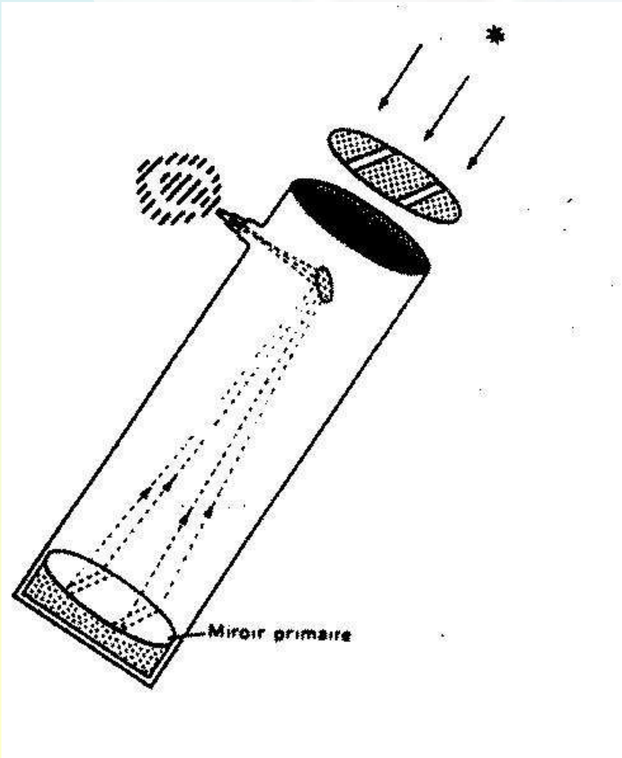
- 4 Interferometry with two independent telescopes

b) Fizeau … the father of stellar interferometry (1868)



Stéphan, 1873
Δ << 0,16''

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- Marseille 80 cm telescope


http://www-obs.cnrs-mrs.fr/dynamique/pap/compact.html

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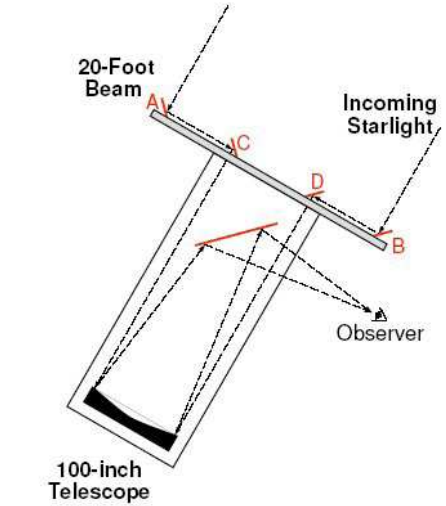
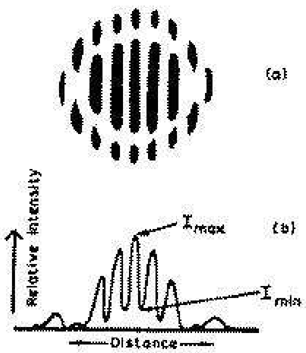
- 4 Interferometry with two independent telescopes

b) Fizeau … the father of stellar interferometry (1868)

• Michelson, 1890 (satellites of Jupiter)

• Michelson and Pease (1920)





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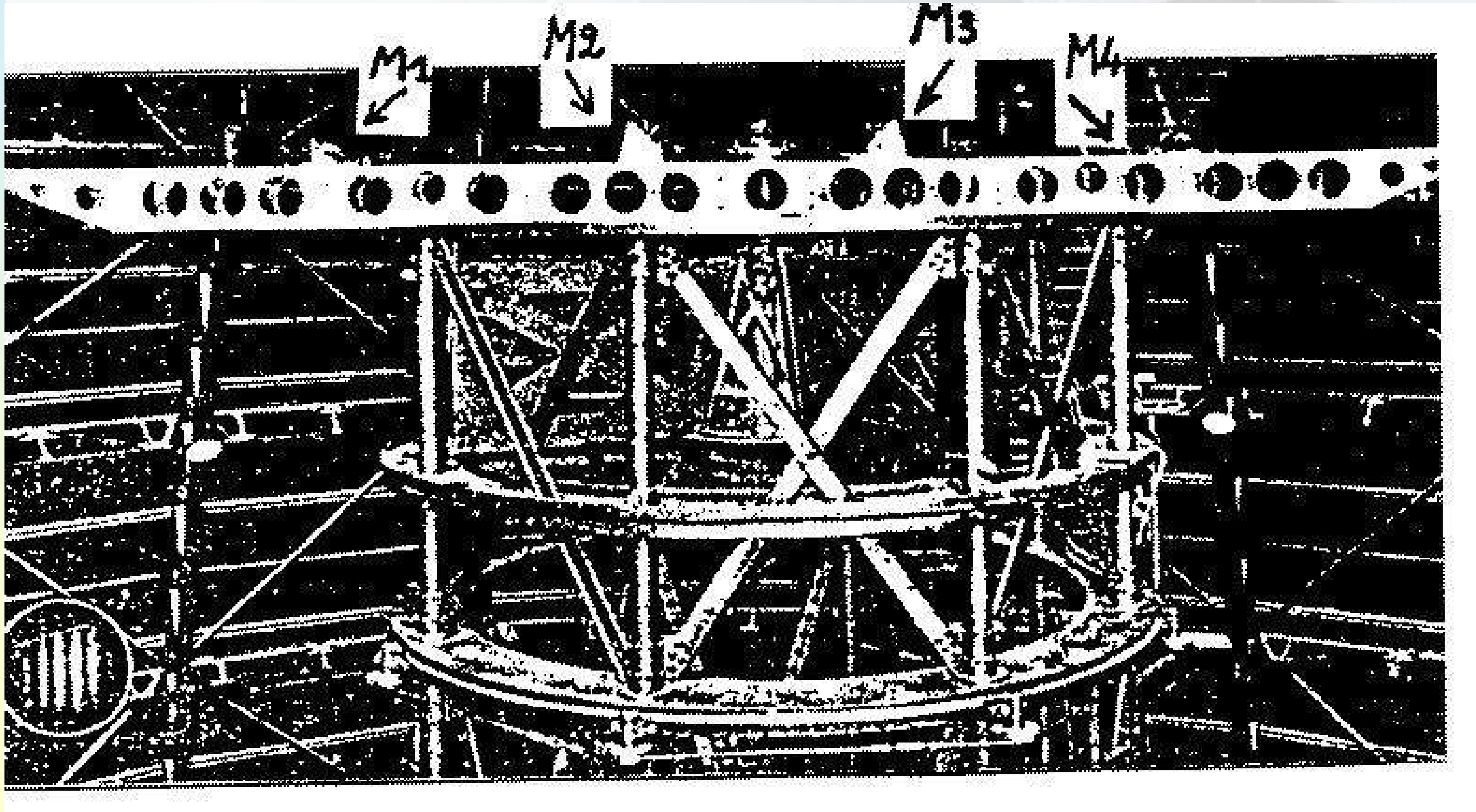
- 4 Interferometry with two independent telescopes

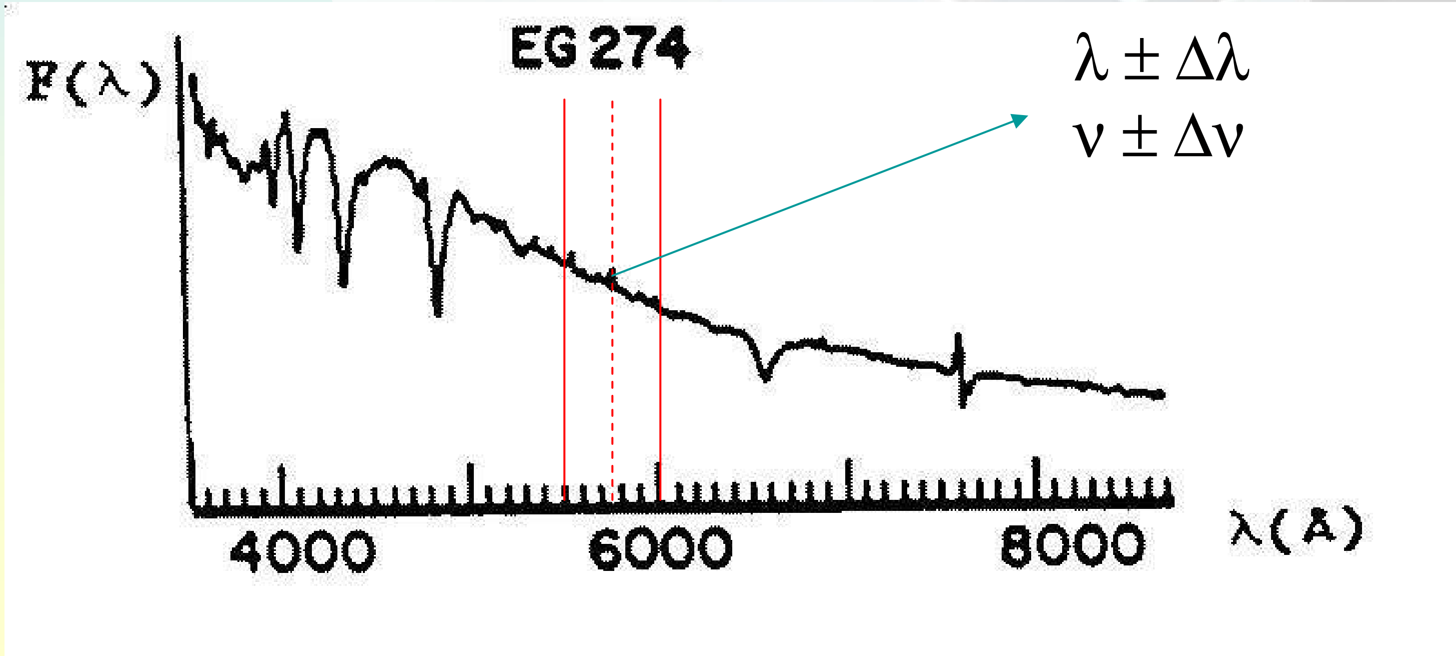
b) Fizeau … the father of stellar interferometry (1868)

- Anderson
- Brown and Twiss (1956)
- Radio Interferometry (1950)



An introduction to optical/IR interferometry

- 5 Light coherence
- **5.1 Quasi monochromatic light (waves and wave groups)**



An introduction to optical/IR interferometry

- 5 Light coherence
- **5.1 Quasi monochromatic light (waves and wave groups)**

$$V(z,t) = \int_{\nu-\Delta\nu}^{\nu+\Delta\nu} a(\nu')\exp(i2\Pi(\nu' t - z/\lambda'))d\nu' \quad (5.1.1)$$

exp(-i2Π(νt-z/λ)) exp(i2Π(νt-z/λ))

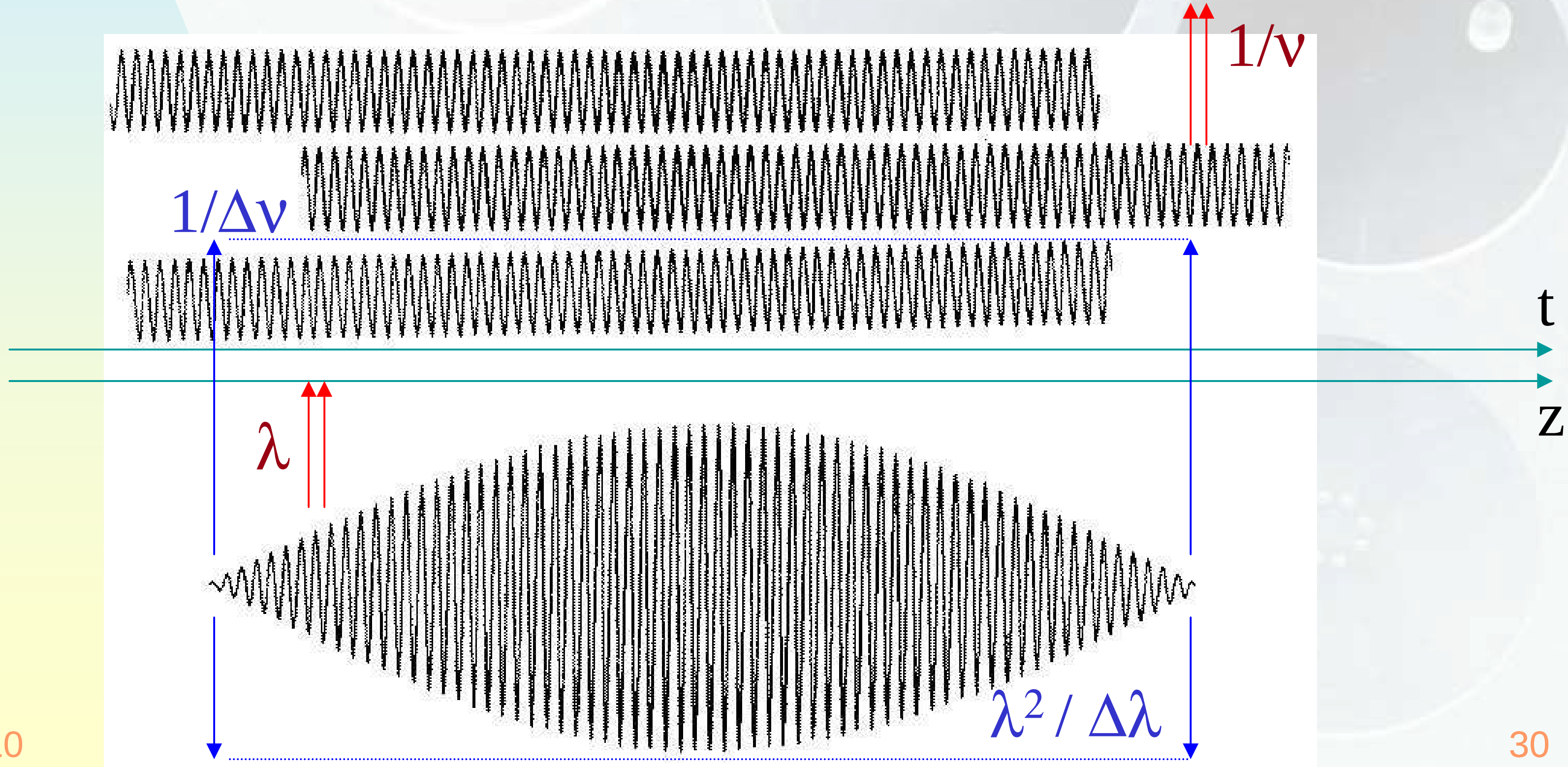
$$I = \langle V(t)V^*(t)\rangle \quad (5.1.2)$$

$$V(z,t) = A(z,t)\exp(i2\Pi(\nu t - z/\lambda)) \quad (5.1.3)$$

$$A(z,t) = \int_{\nu-\Delta\nu}^{\nu+\Delta\nu} a(\nu')\exp(i2\Pi((\nu'-\nu)t - z(1/\lambda' - 1/\lambda)))d\nu' \quad (5.1.4)$$

An introduction to optical/IR interferometry

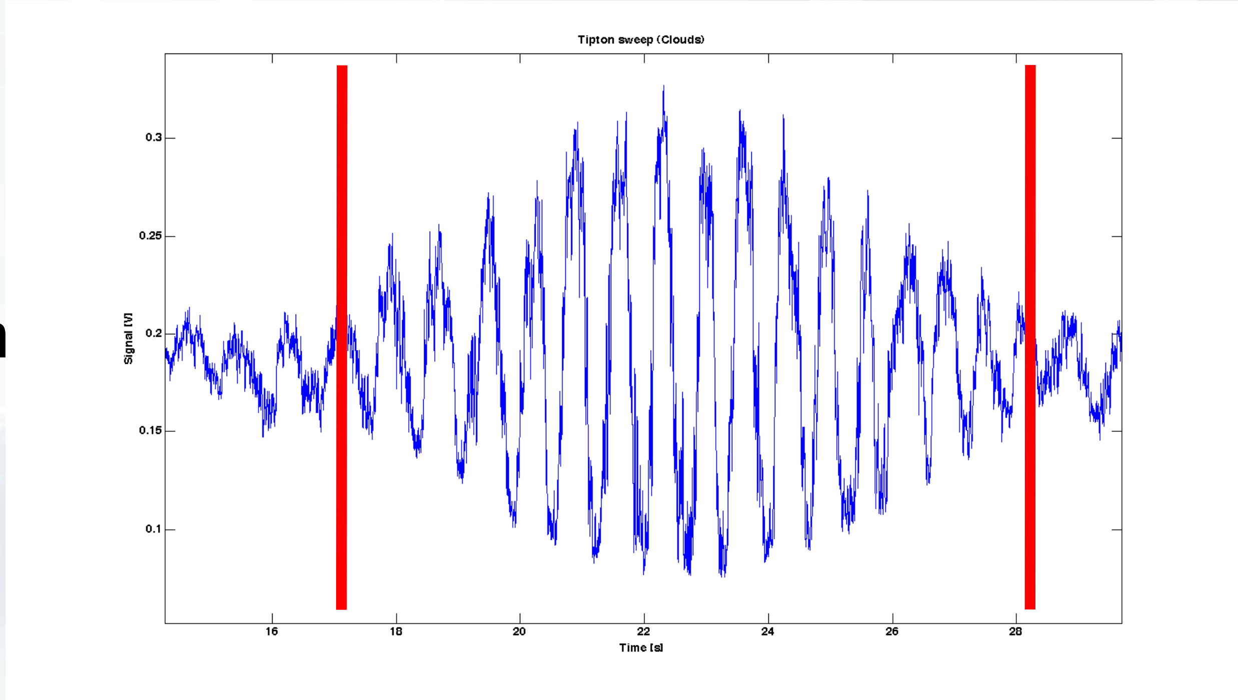
- 5 Light coherence
- **5.1 Quasi monochromatic light (waves and wave groups)**



An introduction to optical/IR interferometry

- 5 Light coherence
- **5.1 Quasi monochromatic light (waves and wave groups)**

$\lambda_0 = 2.2\mu m$
$\lambda \in [2.07 ; 2.33]\mu m$
$\Delta \lambda = 0.13\mu m$



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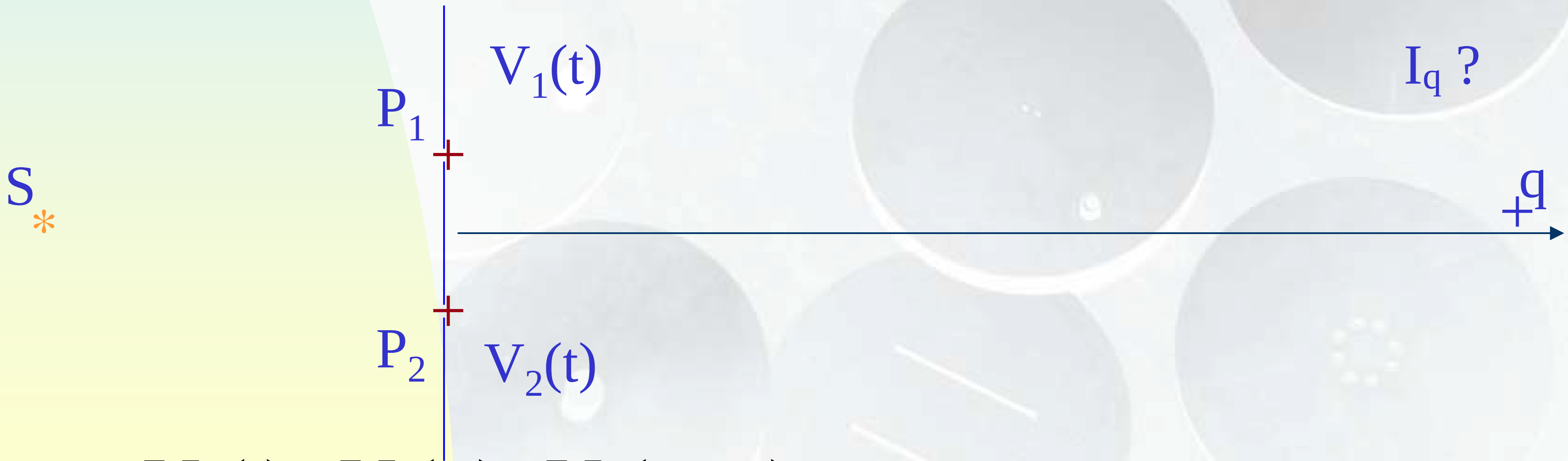
- 5 Light coherence
- **5.2 Fringe visibility**

$$I_q = \langle V_q^*(t) V_q(t) \rangle \quad (5.2.1)$$

$$V_q(t) = V_1(t - t_{1q}) + V_2(t - t_{2q}) \quad (5.2.2)$$



$$V_q(t) = V_1(t) + V_2(t - \tau) \quad (5.2.3)$$

$$\tau = t_{2q} - t_{1q} \quad (5.2.4)$$

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- 5 Light coherence
- **5.2 Fringe visibility**

$$I_q = I + I + 2I \ \mathrm{Re}\{\gamma_{12}(\tau)\}$$

(5.2.5)

$$\gamma_{12}(\tau) = \langle V_1^*(t) V_2(t-\tau)\rangle / I$$

(5.2.6)

$$\gamma_{12}(\tau) = \langle A_1^*(z,t)\, A_2(z,t-\tau)\rangle \exp(-i2\Pi\nu\tau) / I$$

(5.2.7)

If $\tau \langle\langle 1/\Delta\nu$

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau=0)| \exp(i\beta_{12} - i2\Pi\nu\tau)$$

(5.2.8)

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.2 Fringe visibility**

$$I_q = I + I + 2I\, |\gamma_{12}(0)| \cos\left(\beta_{12} - 2\Pi\nu\tau\right) \qquad (5.2.9)$$

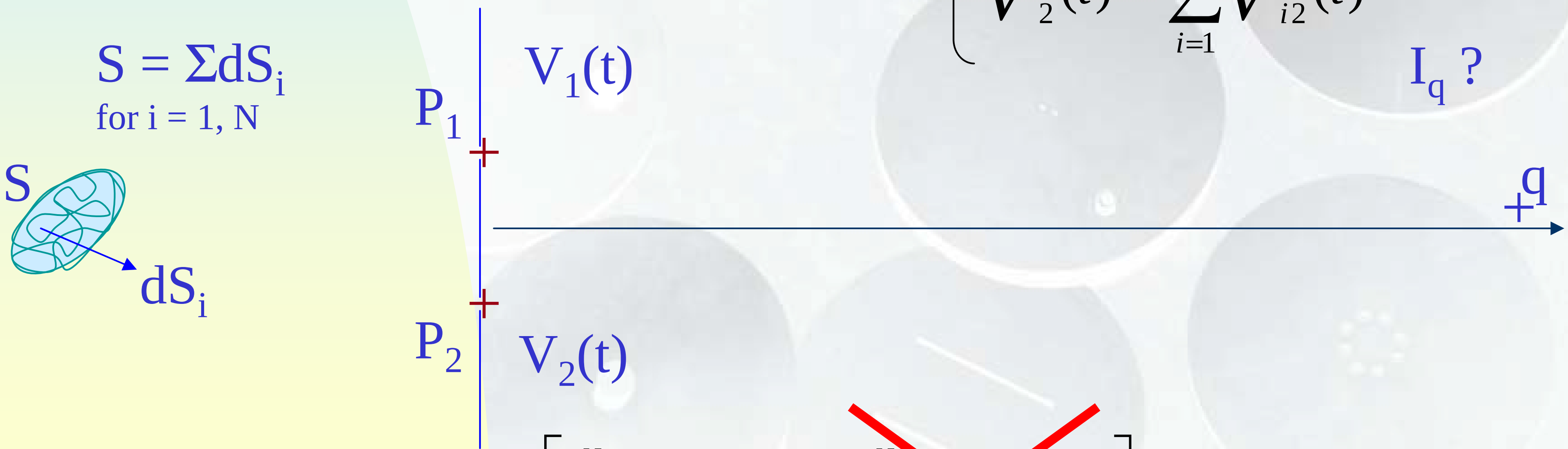
$$\upsilon = \left(\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}\right) = |\gamma_{12}(0)| \qquad (5.2.10)$$

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

?? $$\gamma_{12}(\tau = 0) = \langle V_1^*(t) V_2(t) \rangle / I \quad (5.3.1)$$

$$\begin{cases} V_1(t) = \sum_{i=1}^{N} V_{i1}(t) \\ V_2(t) = \sum_{i=1}^{N} V_{i2}(t) \end{cases} \quad (5.3.2)$$



$$\gamma_{12}(0) = \left[\sum_{i=1}^{N} \langle V_{i1}^* V_{i2} \rangle + \sum_{i \neq j}^{N} \langle V_{i1}^* V_{j2} \rangle \right] / I \quad (5.3.3)$$

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- 5 Light coherence
- **5.3 Spatial light coherence**

$$\gamma_{12}(0) = \left[\sum_{i=1}^{N}\langle V_{i1}^{*}V_{i2}\rangle + \sum_{i\neq j}^{N}\langle V_{i1}^{*}V_{j2}\rangle\right]/I \qquad (5.3.3)$$

$$\begin{cases} V_{i1}(t) = \left(a_i(t - r_{i1}/c)/r_{i1}\right)\exp\{i2\Pi\nu(t - r_{i1}/c)\} \\ V_{i2}(t) = \left(a_i(t - r_{i2}/c)/r_{i2}\right)\exp\{i2\Pi\nu(t - r_{i2}/c)\} \end{cases} \qquad (5.3.4)$$

$$V_{i1}^{*}(t)V_{i2}(t) = \left|a_i(t - r_{i1}/c)\right|^2/(r_{i1}r_{i2})\exp\{-i2\Pi\nu(r_{i2} - r_{i1})/c\} \qquad (5.3.5)$$

as long as:

$$\left|r_{i1} - r_{i2}\right| \leq c/\Delta\nu = \lambda^2/\Delta\lambda = \ell \qquad (5.3.6)$$

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- 5 Light coherence
- **5.3 Spatial light coherence**

$$I(s)ds = \left| a_i (t - r / c) \right|^2 \qquad (5.3.7)$$

$$\gamma_{12}(0) = \int_S \frac{I(s)}{r_1 r_2} \exp\left\{ -i2\Pi \left(r_2 - r_1 \right) / \lambda \right\} ds / I \qquad (5.3.8)$$

!!! Theorem of Zernicke-van Cittert !!!

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- 5 Light coherence
- **5.3 Spatial light coherence**

$$|r_2 - r_1| = |P_2P_i - P_1P_i| = |-(X^2 + Y^2) / 2\,Z' + (X\,\zeta + Y\,\eta)| \qquad (5.3.9)$$

where $\zeta = X' / Z'$ and $\eta = Y' / Z'$ (5.3.10)

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- 5 Light coherence
- **5.3 Spatial light coherence**

$$\gamma_{12}(0, X/\lambda, Y/\lambda) = \exp(-i\phi_{X,Y}) \frac{\iint_S I(\zeta,\eta)\exp\{-i2\Pi(X\zeta + Y\eta)/\lambda\}d\zeta d\eta}{\iint_S I(\zeta',\eta')d\zeta' d\eta'} \quad (5.3.11)$$

$$I'(\zeta,\eta) = I(\zeta,\eta) / \iint_S I(\zeta',\eta')d\zeta' d\eta' \quad (5.3.12)$$

Setting: u = X/ λ , v = Y/ λ :

$$\gamma_{12}(0,u,v) = \exp(-i\phi_{u,v})\iint_S I'(\zeta,\eta)\exp\{-i2\Pi(u\zeta + v\eta)\}d\zeta d\eta \quad (5.3.13)$$

$$I'(\zeta,\eta) = \iint \gamma_{12}(0,u,v)\exp(i\phi_{u,v})\exp\{i2\Pi(\zeta u + \eta v)\}d(u)d(v) \quad (5.3.14)$$

An introduction to optical/IR interferometry

- **5.4 Fourier transform (cf. Léna 1996)**

5.4.1 Definitions:

$$TF\_f(s) = \int_{-\infty}^{\infty} f(x)\, e^{-2i\pi sx}\, dx, \qquad (5.4.1)$$

$$f(x) = \int_{-\infty}^{\infty} TF\_f(s)\, e^{2i\pi sx}\, ds, \qquad (5.4.2)$$

$$\int_{-\infty}^{\infty} \left| f(x) \right|^2 dx. \qquad (5.4.3)$$

- **5.4 Fourier transform (cf. Léna 1996)**

5.4.1 Definitions: Generalisation:

$$TF\_f(\vec{w}) = \int_{-\infty}^{\infty} f(\vec{r})\, e^{-2i\pi \vec{r}\vec{w}} d\vec{r} \quad . \qquad (5.4.4)$$

5.4.2 Some properties:

a) Linearity:

TF_(af) = a TF_f, a $\in \Re$, a being a constant, (5.4.5)

TF_(f+g) = TF_f + TF_g. (5.4.6)

An introduction to optical/IR interferometry

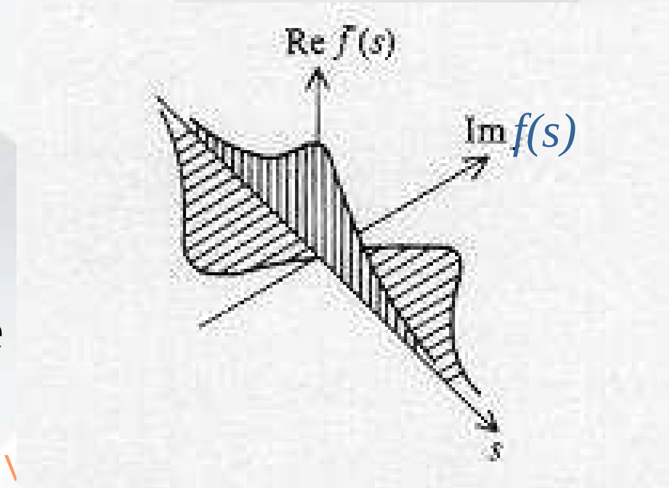
- **5.4 Fourier transform (cf. Léna 1996)**

5.4.2 Some properties: b) Symmetry & parity:

f(x) = P(x) + I(x), (5.4.7)

$$TF\_f(s) = 2\int_0^\infty P(x)\cos(2\pi xs)dx - 2i\int_0^\infty I(x)\sin(2\pi xs)dx \, . \qquad (5.4.8)$$

Illustration of TF_f(s): f(x) is a real fonction. The real and imaginary parts of TF_f(s) are shown.



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- **5.4 Fourier transform (cf. Léna 1996)**

c) Similitude:

$$TF\_(f(ax))(s) = |a|^{-1}\, TF\_(f(x))(s/a), \quad (5.4.9)$$

where $a \in \Re$, is a constant.

d) Translation:

$$TF\_(f(x - a))(s) = e^{-2i\pi as}\, TF\_(f(x))(s) \quad (5.4.10)$$

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- **5.4 Fourier transform (cf. Léna 1996)**

e) Derivation:

$TF\_(df/dx)(s) = 2i\pi s\ TF\_f(s)$, $TF\_(d^n f/dx^n)(s) = (2i\pi s)^n\ TF\_f(s)$. (5.4.11)

5.4.3 Some important cases (one dimension):

a) Door function:

$\Pi(x) = 1$ if $x \in\ ]-1/2, 1/2[$, (5.4.12)

$= 0$ if $x \in\ ]-\infty, -1/2]$ or $x \in [1/2, \infty[$.

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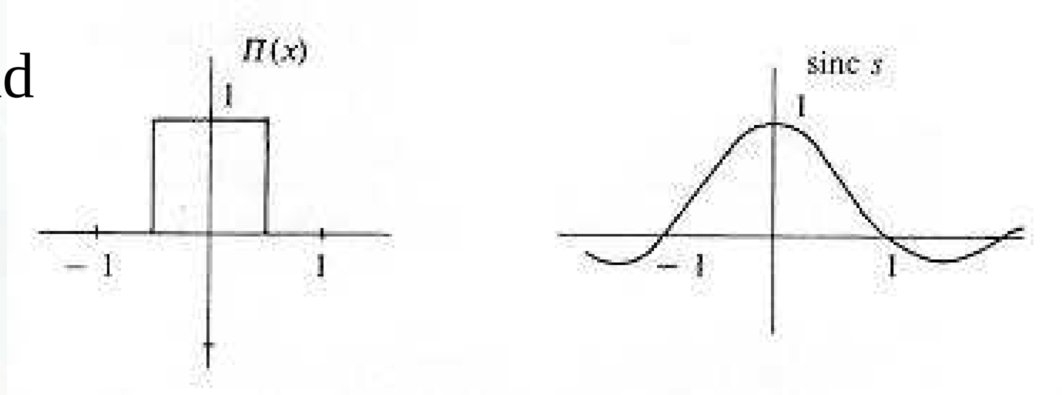
- **5.4 Fourier transform (cf. Léna 1996)**

The door function and its Fourier transform (cardinal sine)



$$TF\_ (\Pi(x))(s) = \mathrm{sinc}(s) = \sin(\pi s) / \pi s. \quad (5.4.13)$$

$$TF\_ (\Pi(x/a))(s) = |a|\ \mathrm{sinc}(as) = |a| \sin(\pi as) / \pi as. \quad (5.4.14)$$

An introduction to optical/IR interferometry

- **5.4 Fourier transform (cf. Léna 1996)**

b) Dirac distribution:

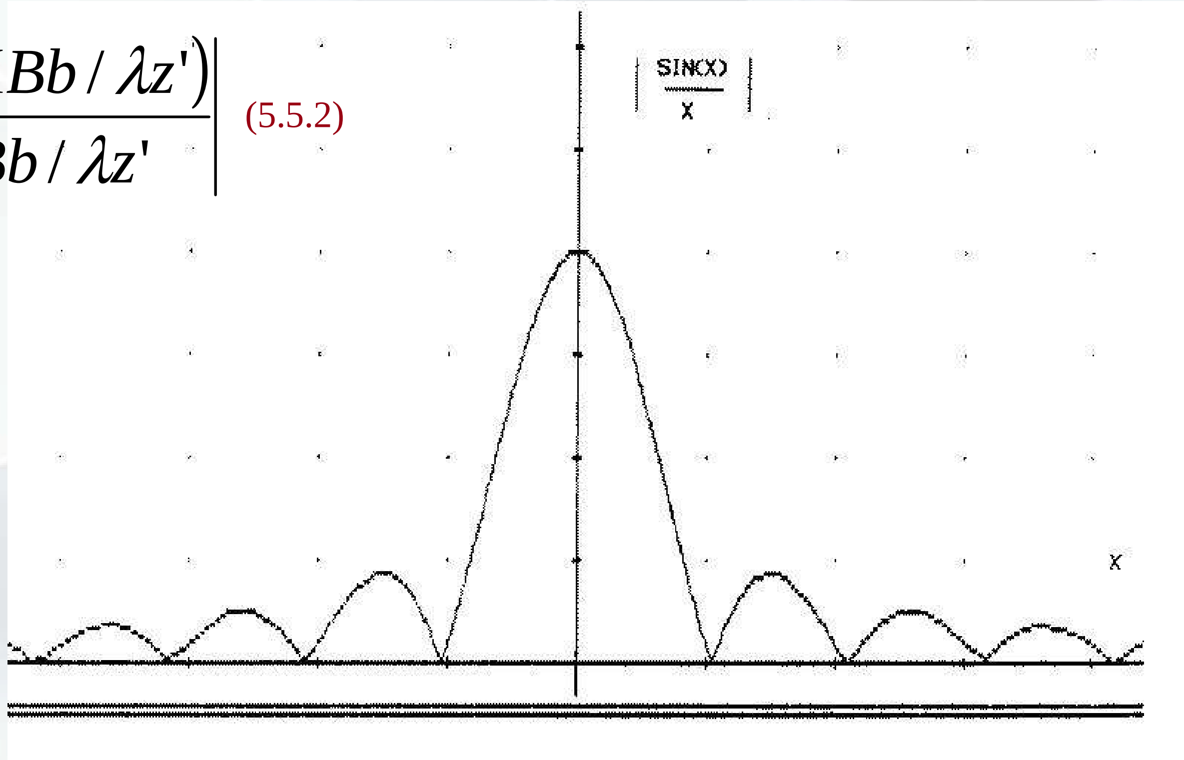
$$\delta(x) = \int_{-\infty}^{\infty} e^{2i\pi sx} ds \,. \qquad (5.4.15)$$

its Fourier transform is thus unity (= 1) in the interval]-∞, ∞[.

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.5 Aperture synthesis**

$$\upsilon = \left|\gamma_{12}(0, B/\lambda)\right| = \left|\frac{\sin(\Pi Bb/\lambda z')}{\Pi Bb/\lambda z'}\right| \quad (5.5.2)$$



$$\Pi Bb/\lambda z' = \Pi \quad (5.5.3)$$

$\Delta \sim \lambda$ / B, for a (5.5.4)
rectangular source.

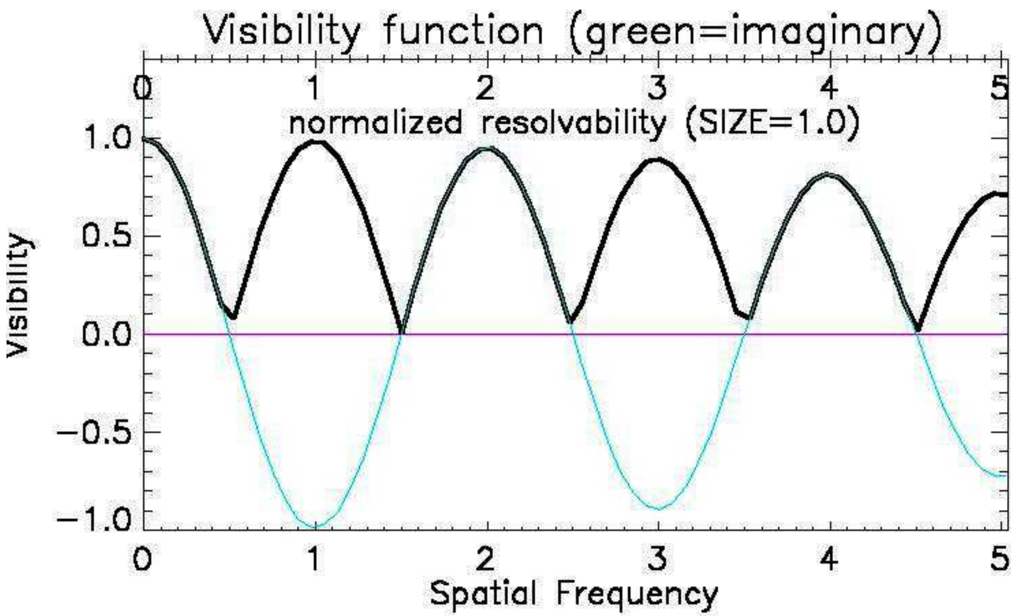
$\Delta \sim 1.22\ \lambda$ / B, for (5.5.5)
a circular source !

An introduction to optical/IR interferometry

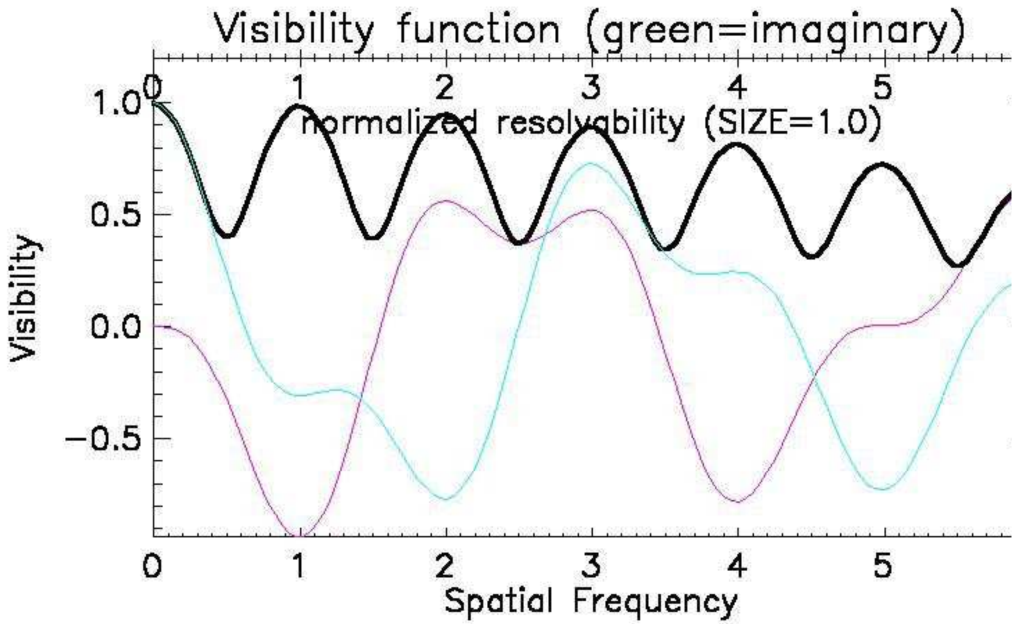
- 5 Light coherence
- **5.5 Aperture synthesis**

Exercises (…): point-like source?, double point-like source with a flux ratio = 1?, gaussian-like source?, uniform disk source?, …

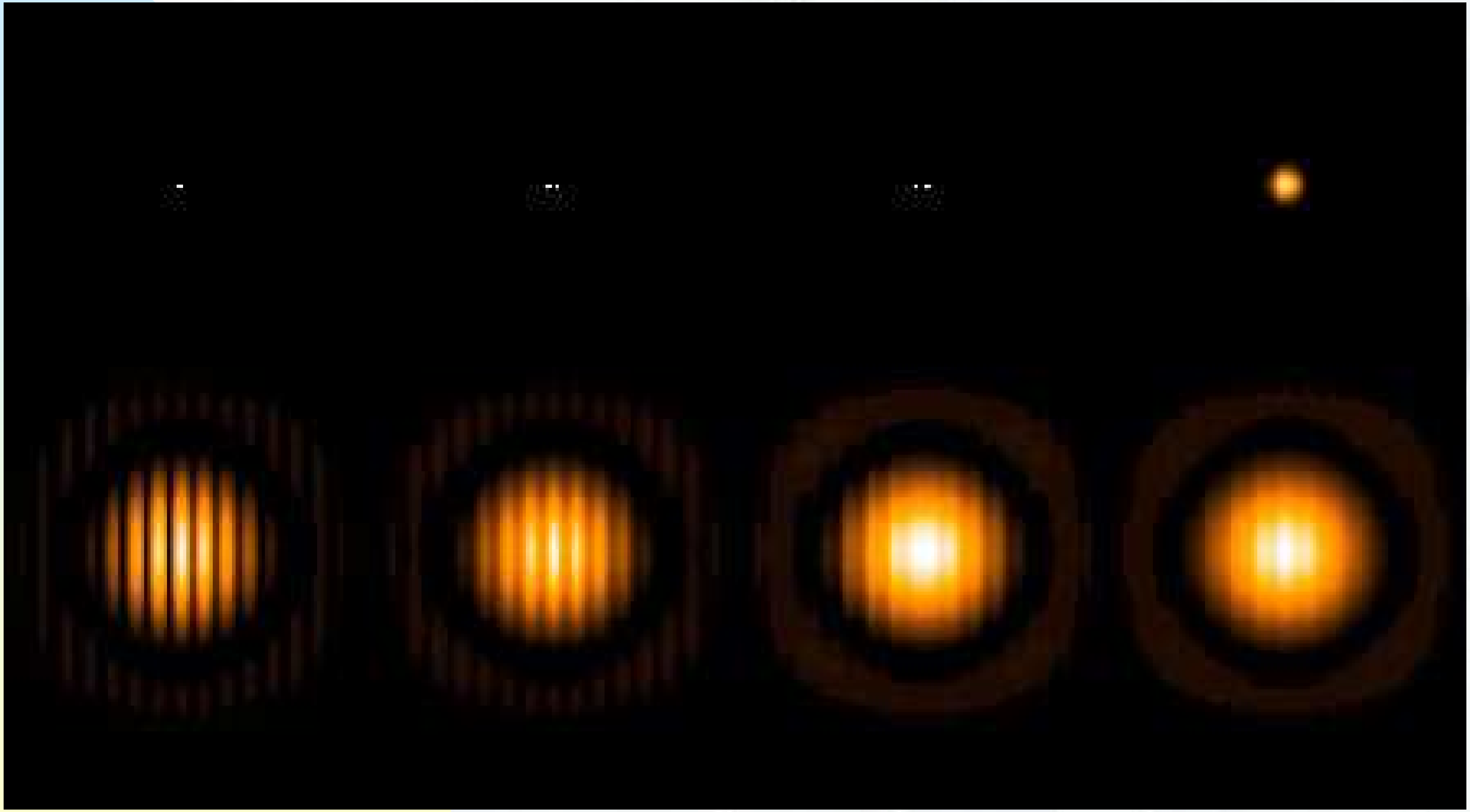
Case of a double point-like source with a flux ratio = 1



Case of a double point-like source with a flux ratio 0.7/0.3

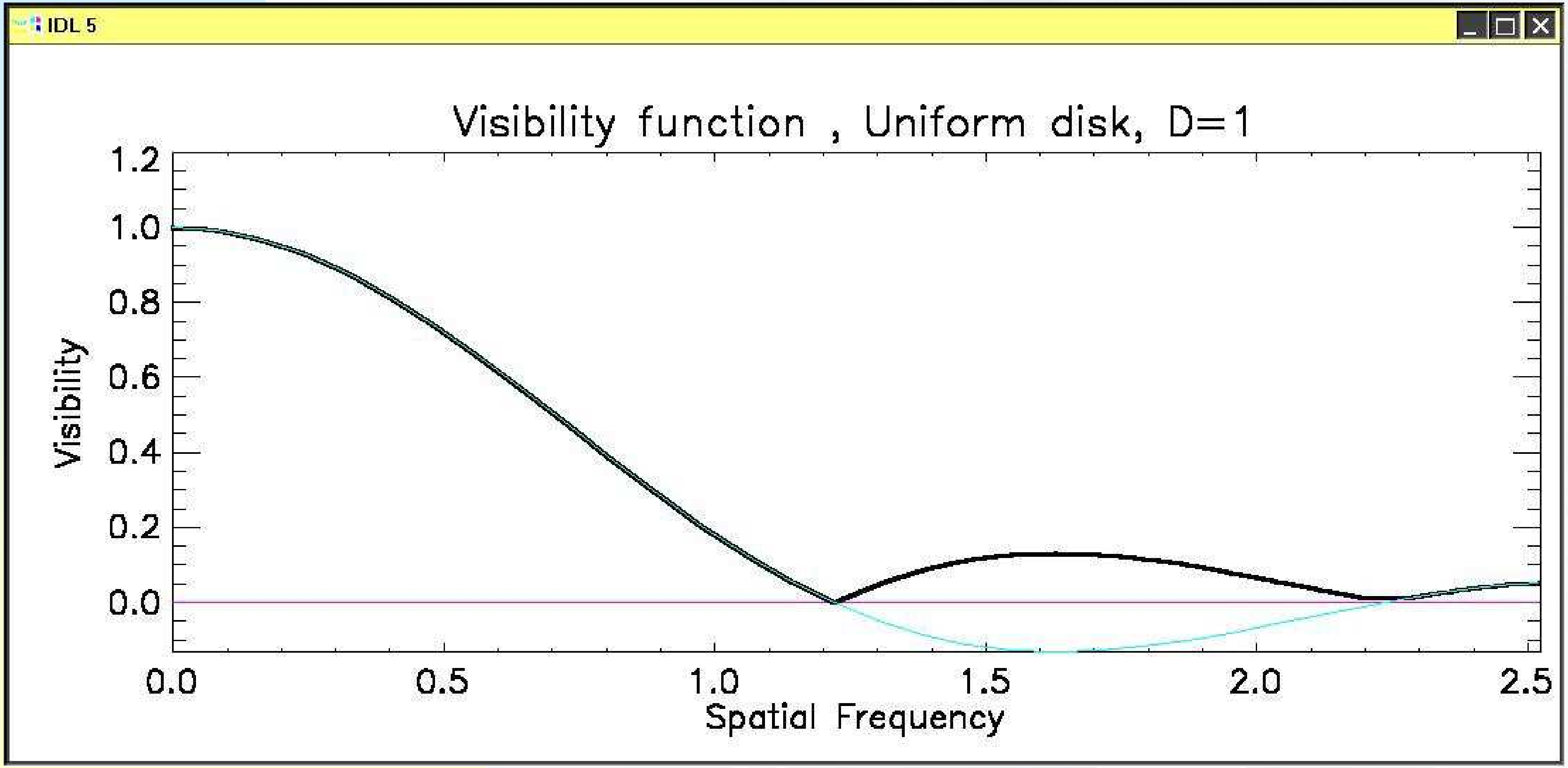


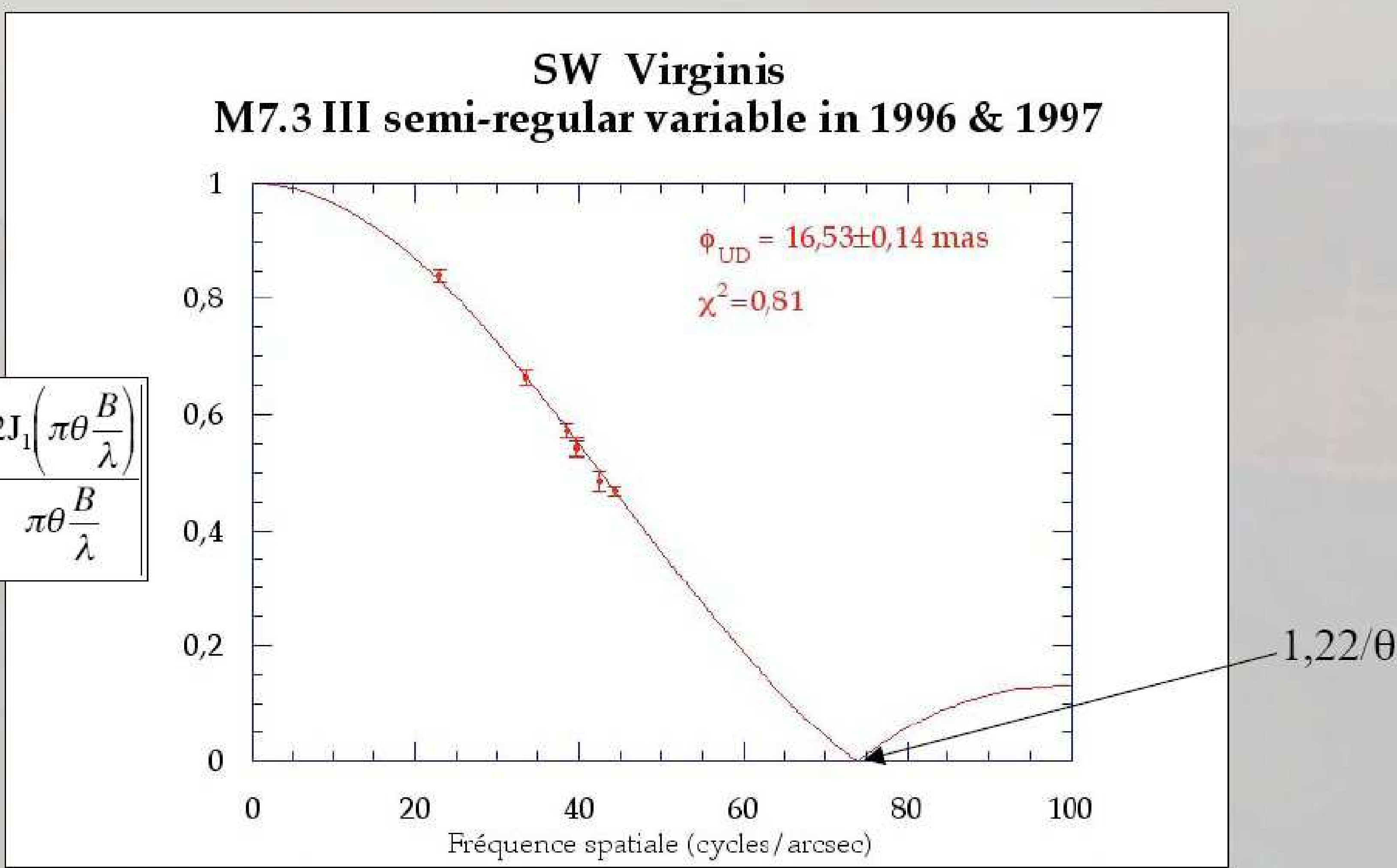
Variation of the fringe contrast as a function of the angular separation between the two stars:



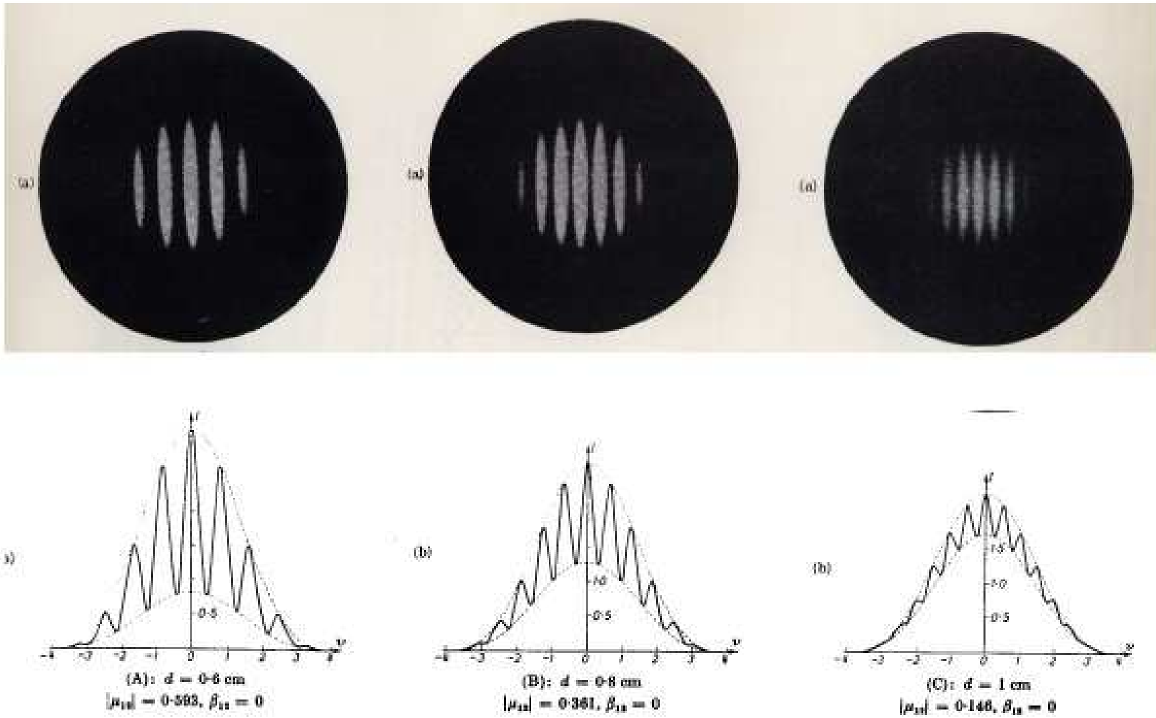
If the source is characterized by a uniform disk light distribution, the corresponding visibility function is given by

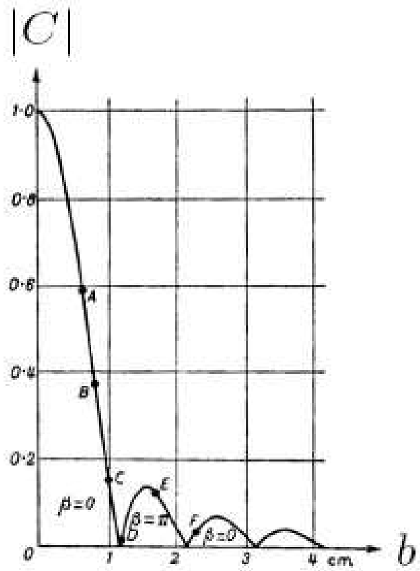
$$\upsilon = \left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \right) = \left| \gamma_{12}(0) \right| = TF(I) = \frac{2 J_1(\pi \theta_{UD} B / \lambda)}{\pi \theta_{UD} B / \lambda}$$



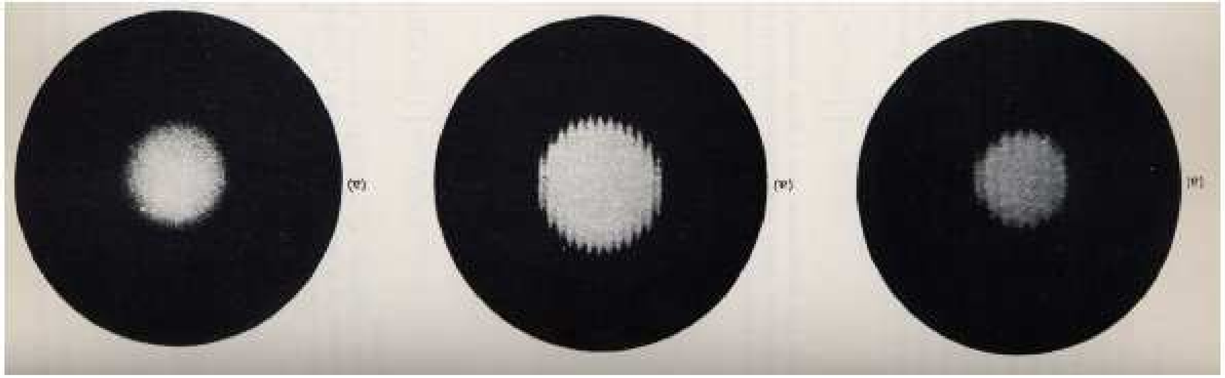


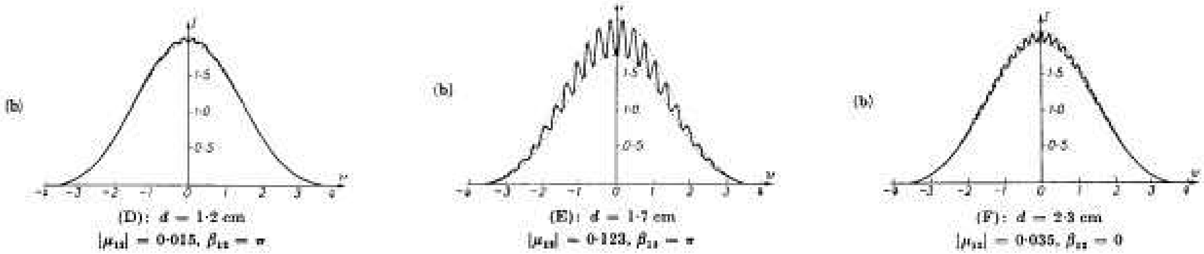
$$V_{DU}(B) = \left| \frac{2J_1\left(\pi\theta\frac{B}{\lambda}\right)}{\pi\theta\frac{B}{\lambda}} \right|$$

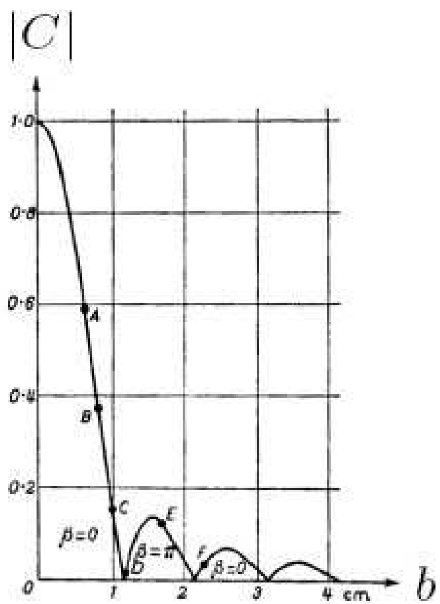




$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$







$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C|\cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

For the case of the Sun:

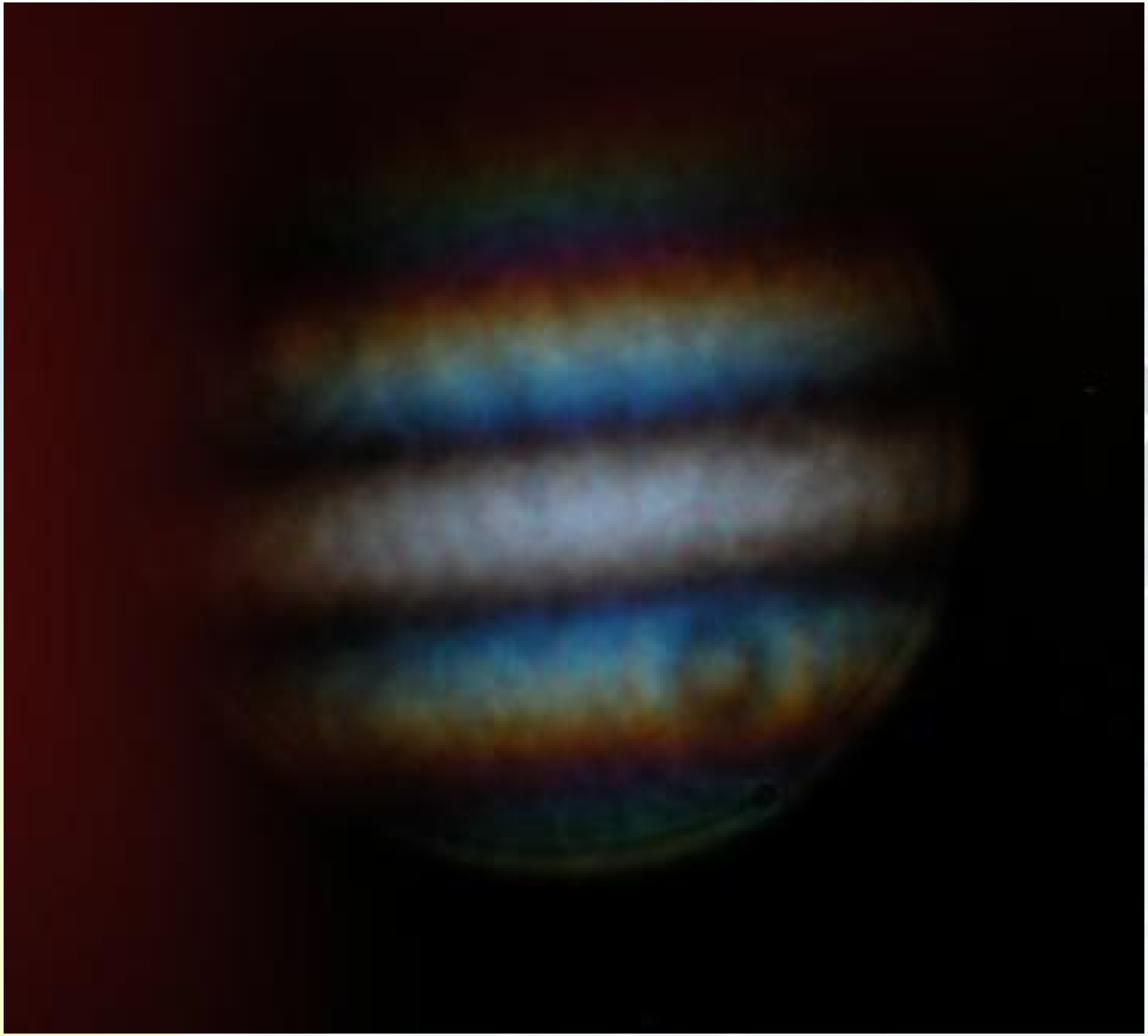
$\vartheta_{UD} = 1.22\ \lambda\ /\ B = 1.22\ 0.55\ /\ B(\mu) = 30'\ x\ 60''\ /\ 206265$

$B(\mu) = 206265\ x\ 1.22\ x\ 0.55\ /\ (30\ x\ 60) = 76.9\ \mu$

$d(\mu) = 7.2$ or $14.4\ \mu$ ➔ $\sigma = 2.44\ \lambda\ /\ d = 7.8^\circ$ or 3.9°

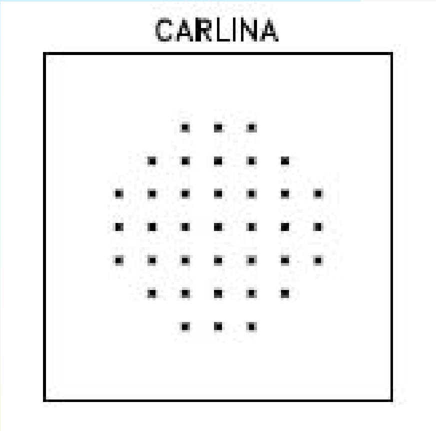
See the masks!

First fringes for the Sun: 9/4/2010



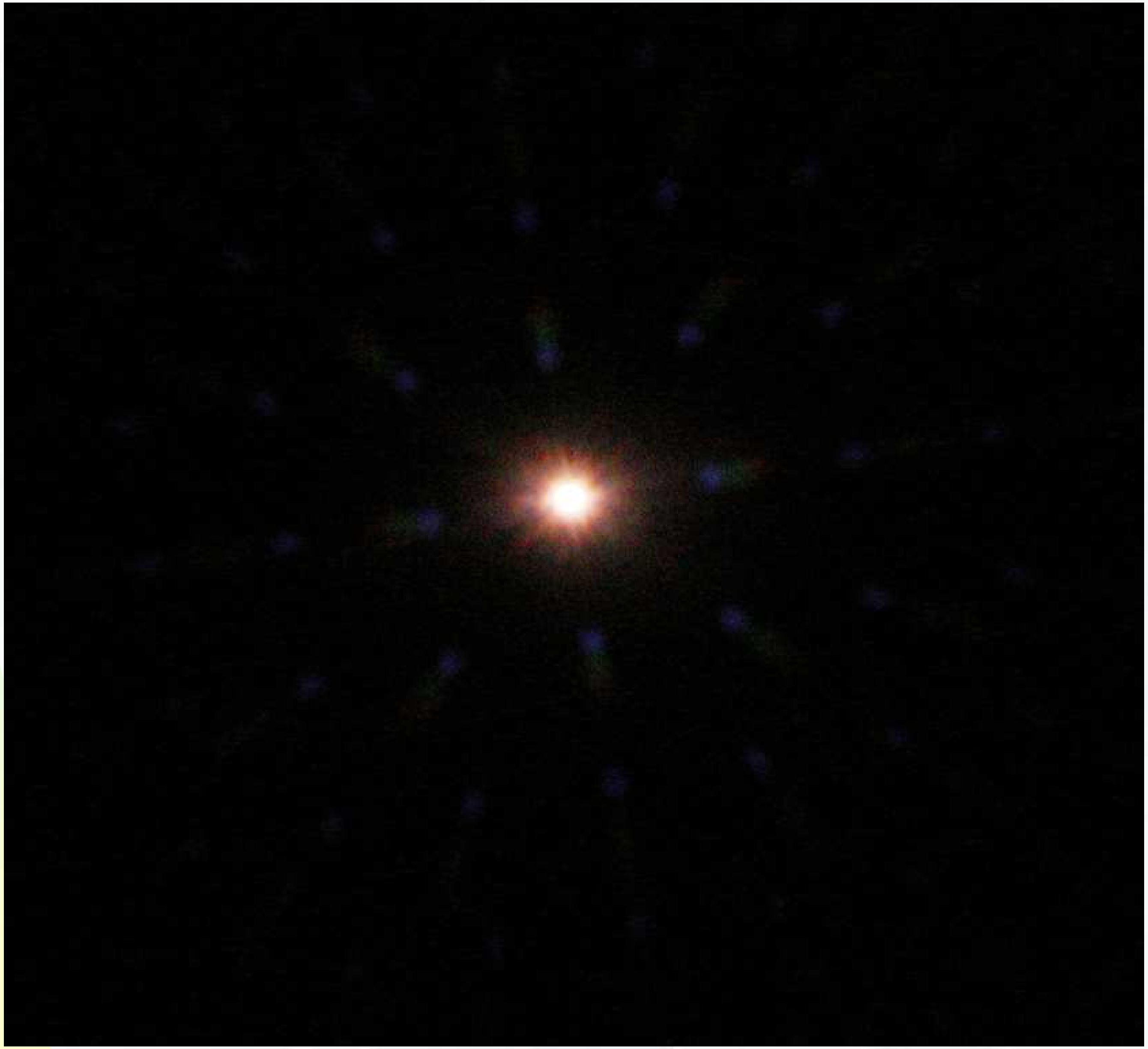
B = 29.4μ
d = 11.8μ

Carlina PSF

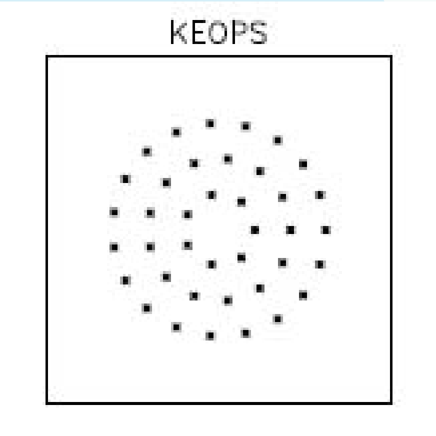


↔ 50µ

. 14µ



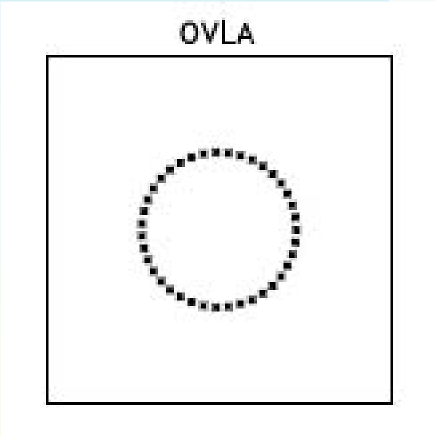
KEOPS PSF



↔ 50µ

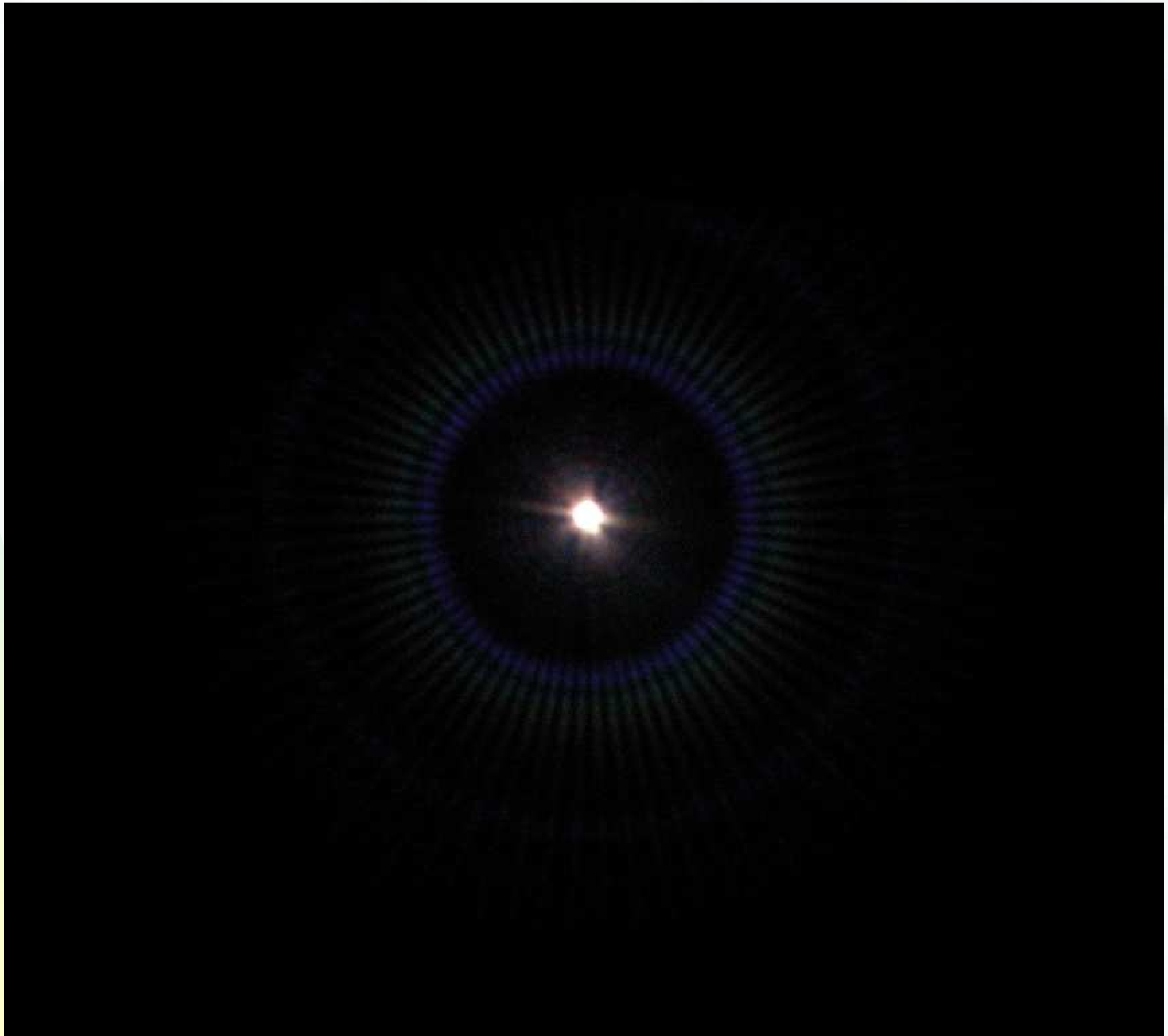
• 14µ

OVLA PSF

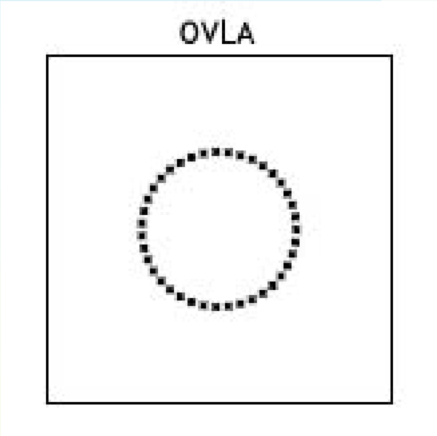


↔ 50µ

• 14µ



OVLA PSF

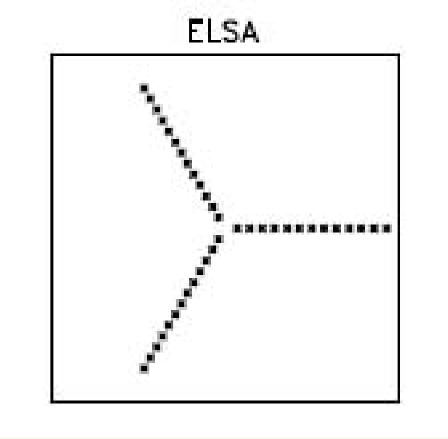


↔ 50μ

. 14μ

OVLA_Sun_2

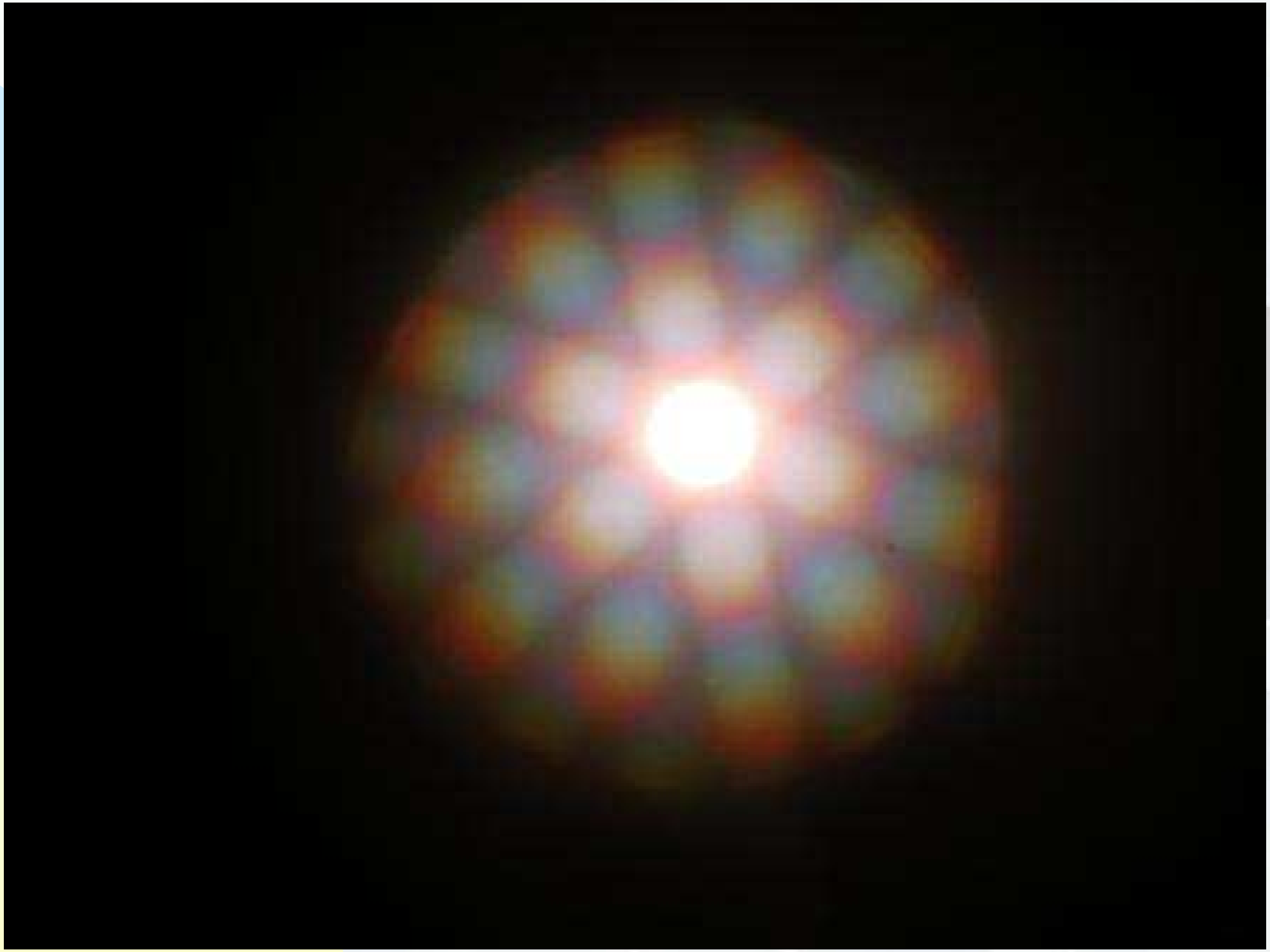
ELSA PSF



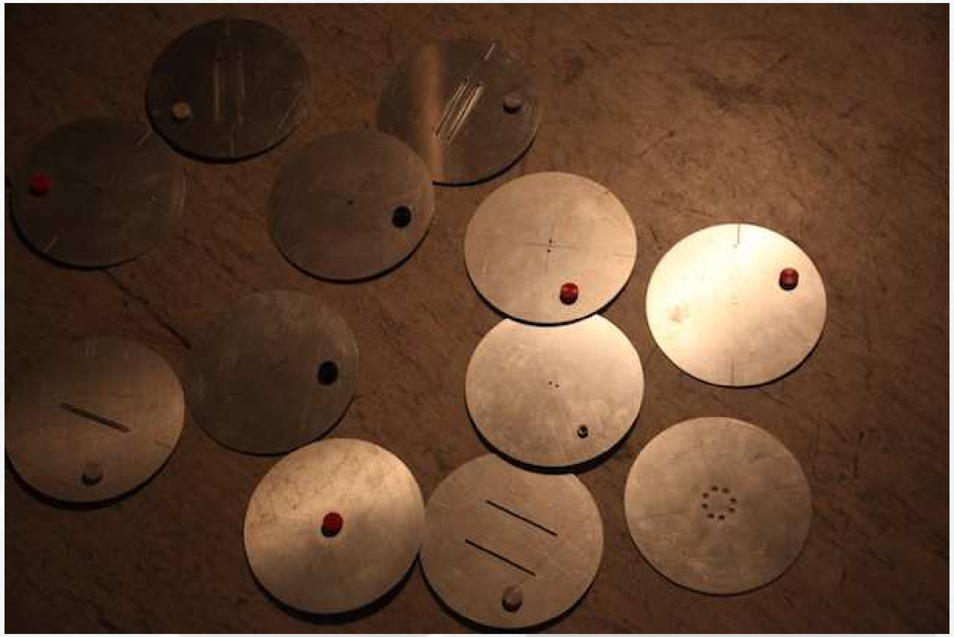
↔ 50μ

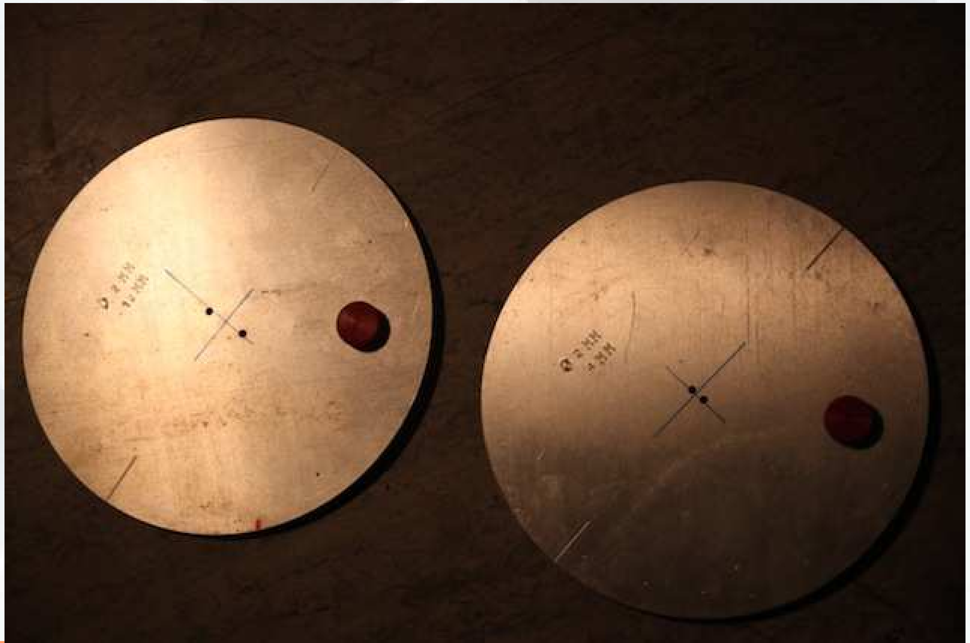
• 14μ

ELSA_Sun_24

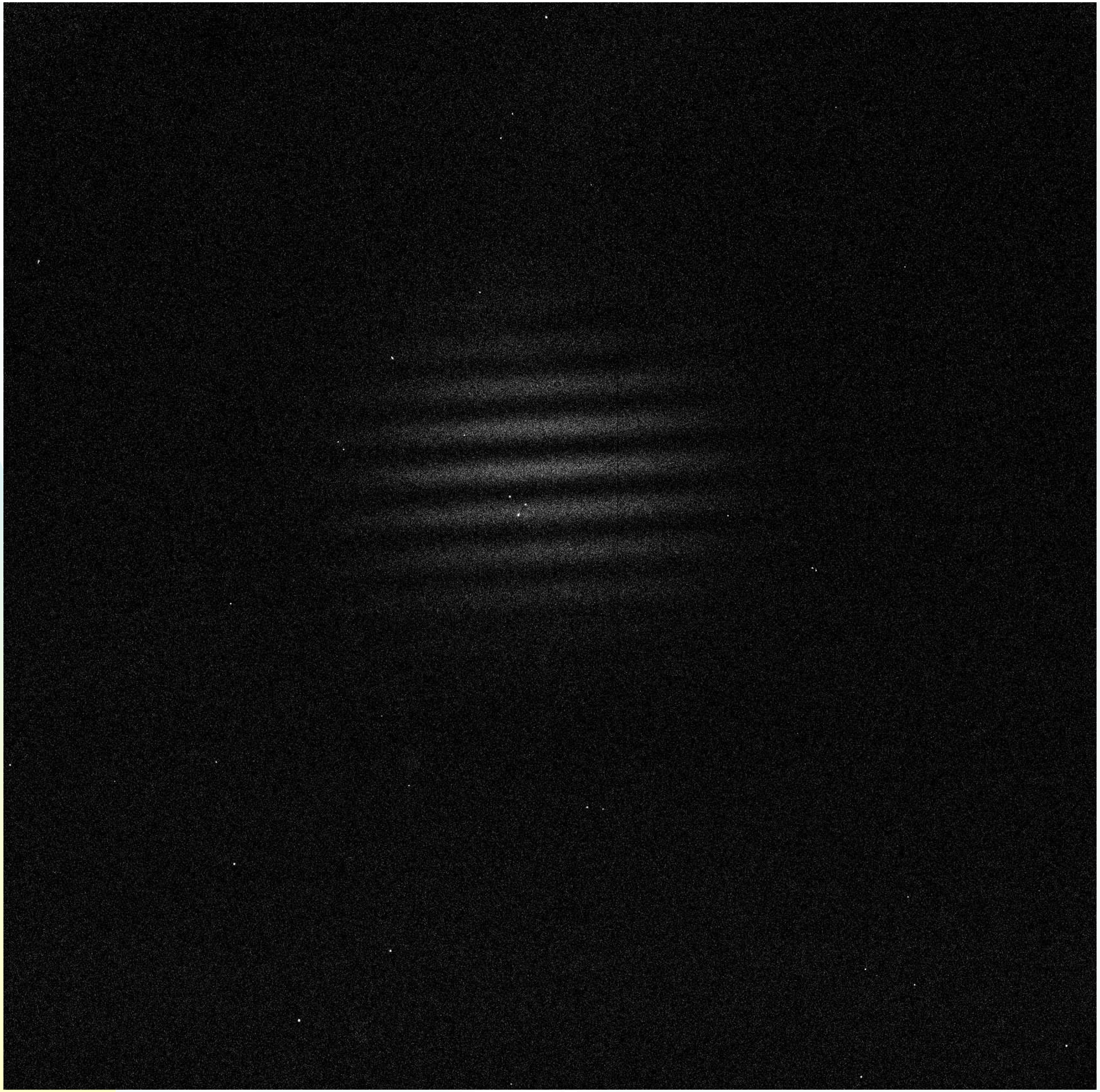


Interferometric observations on 10/4/2010 of Procyon, Mars and Saturn, using the 80cm telescope at Haute-Provence Observatory and adequate masks …



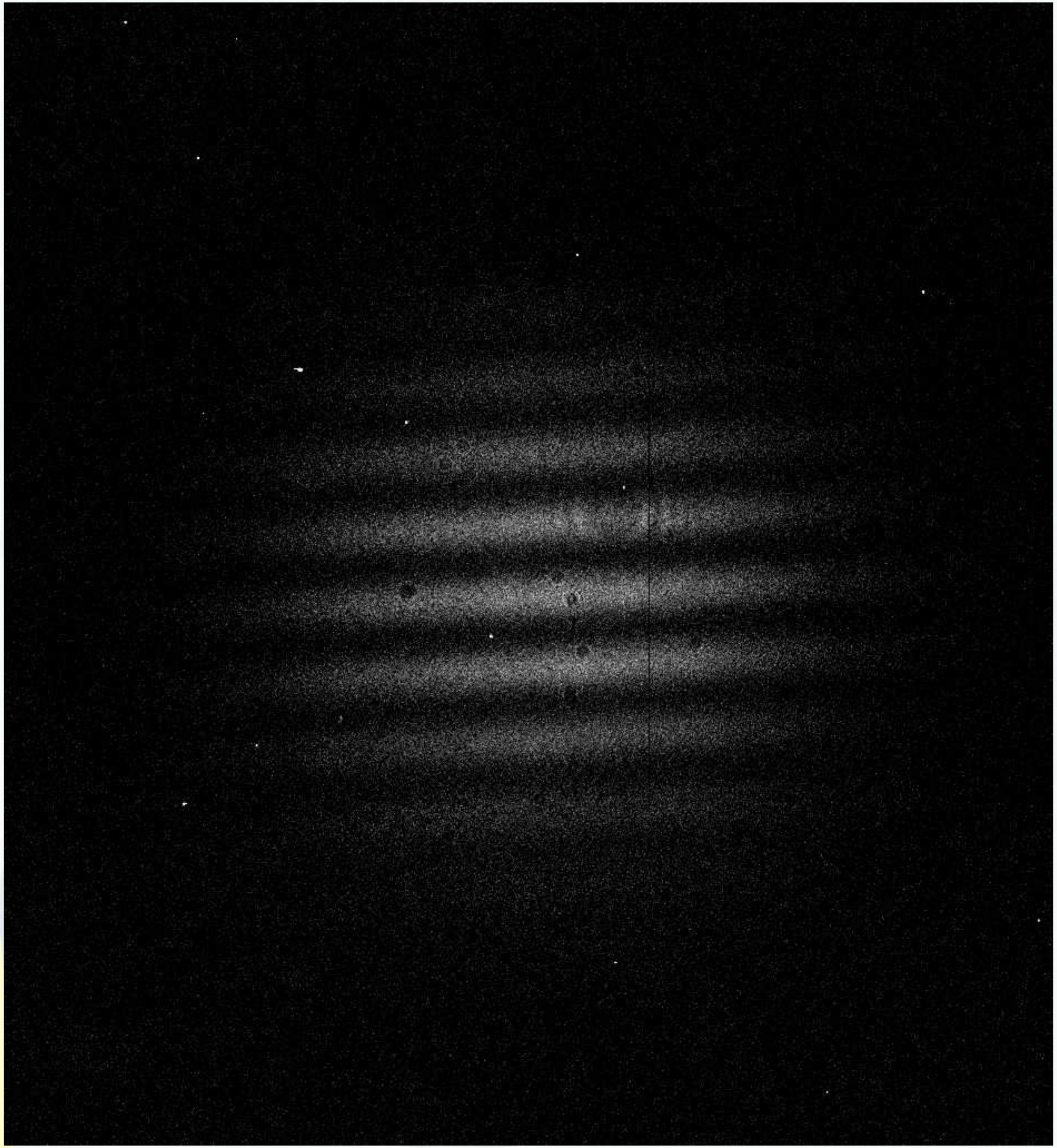


Procyon
B = 12 mm
d = 2 mm



17-28/4/2010

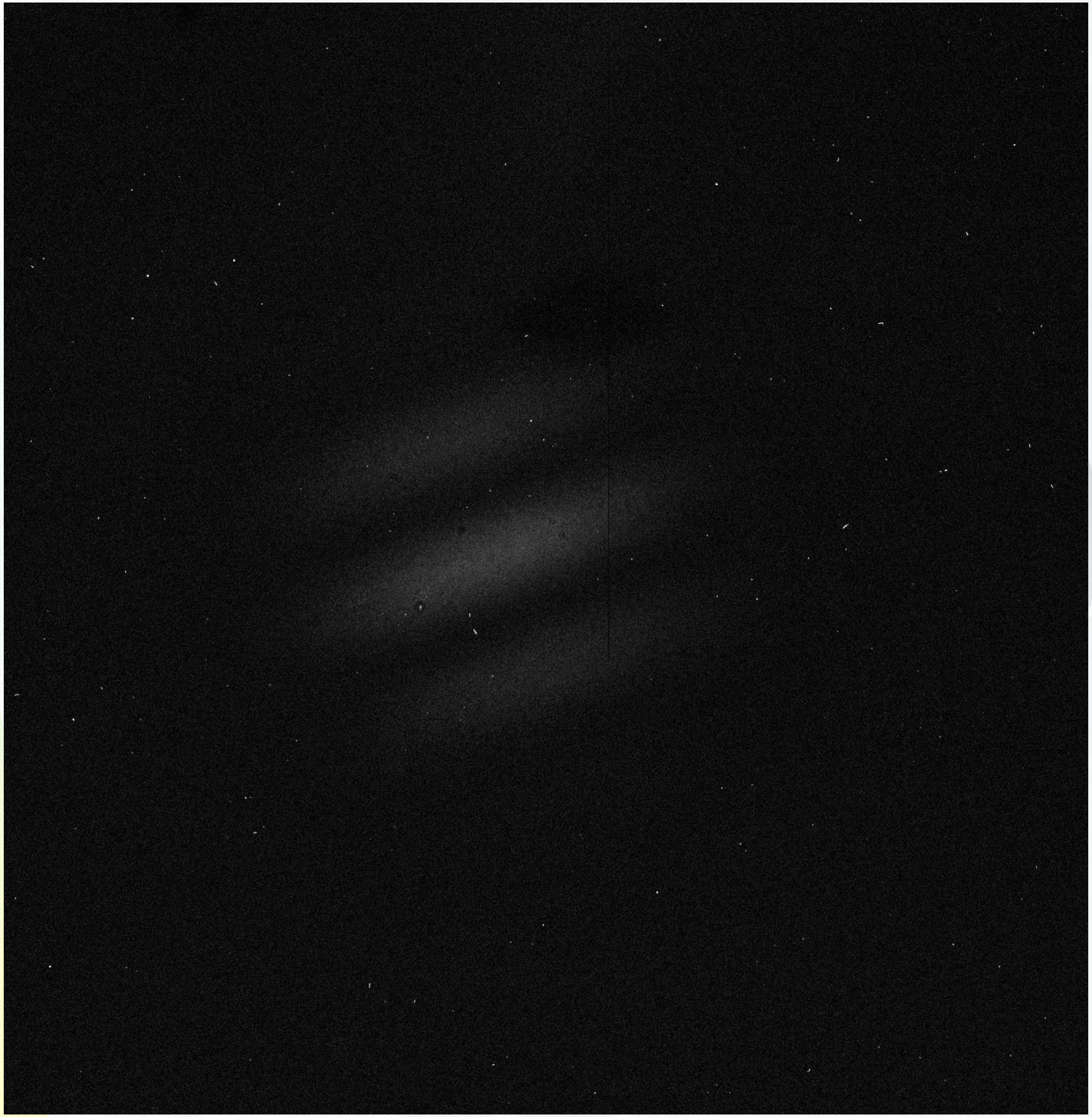
Mars
B = 12 mm
d = 2 mm



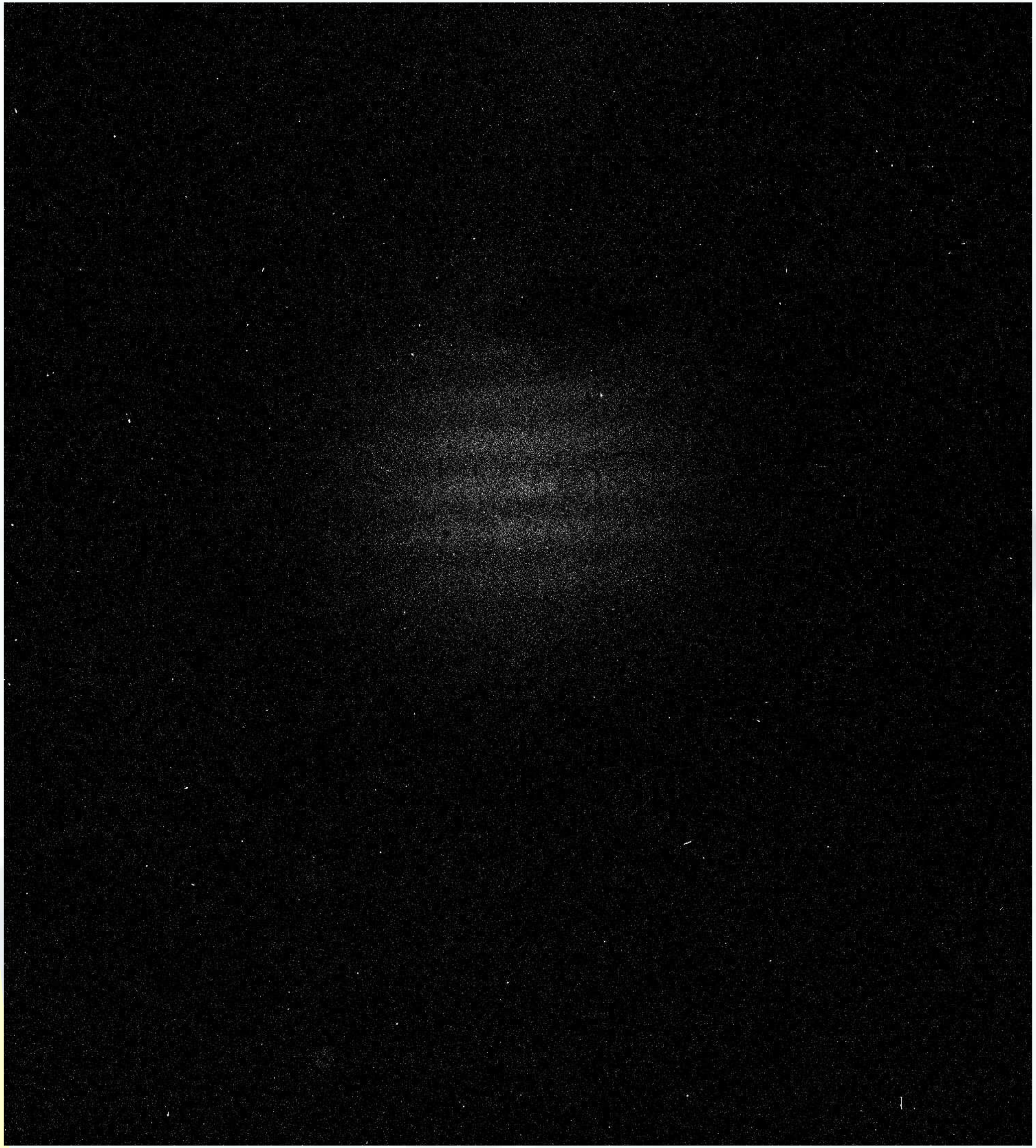
Saturn

B = 4 mm

d = 2 mm

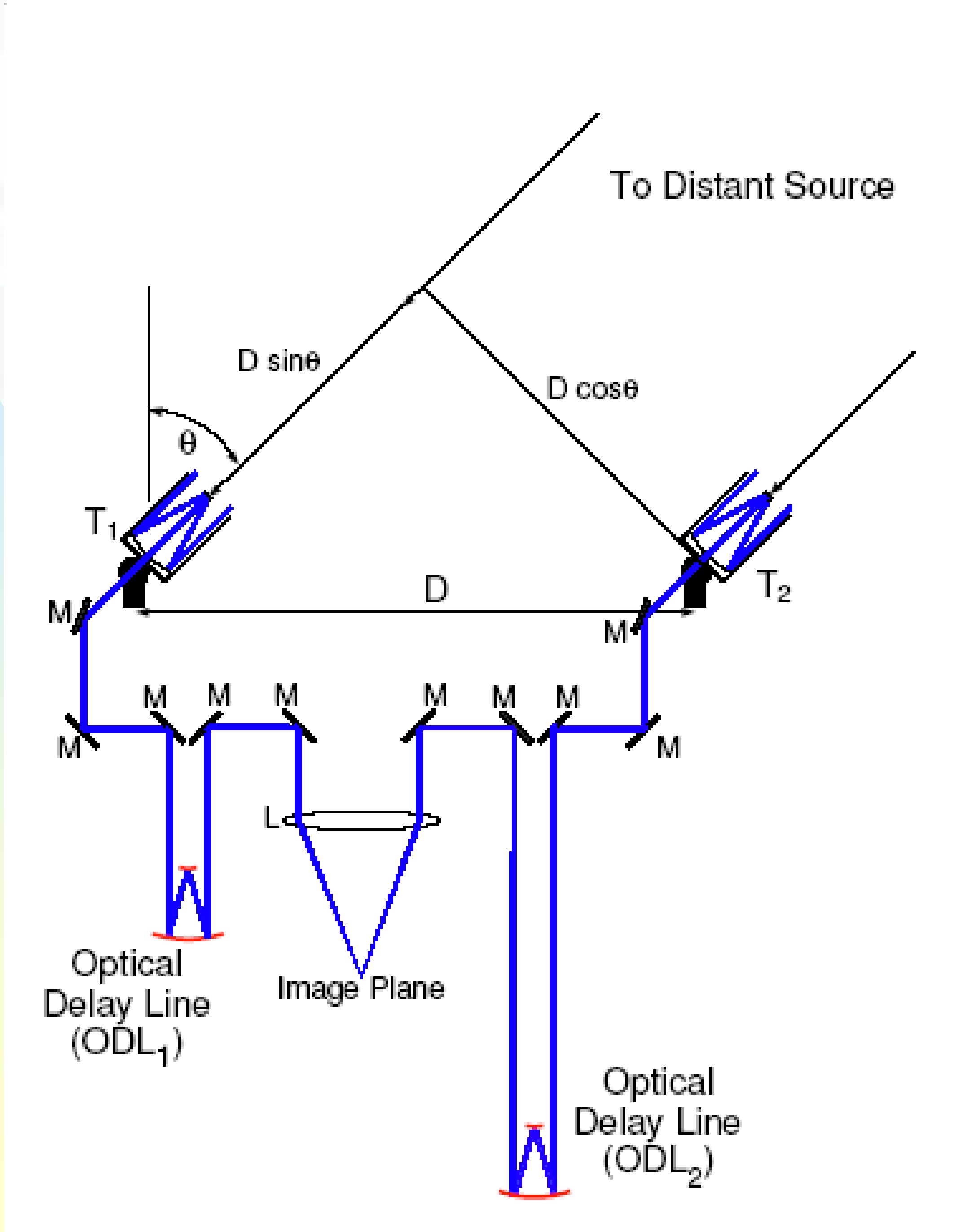


Saturn
B = 12 mm
d = 2 mm

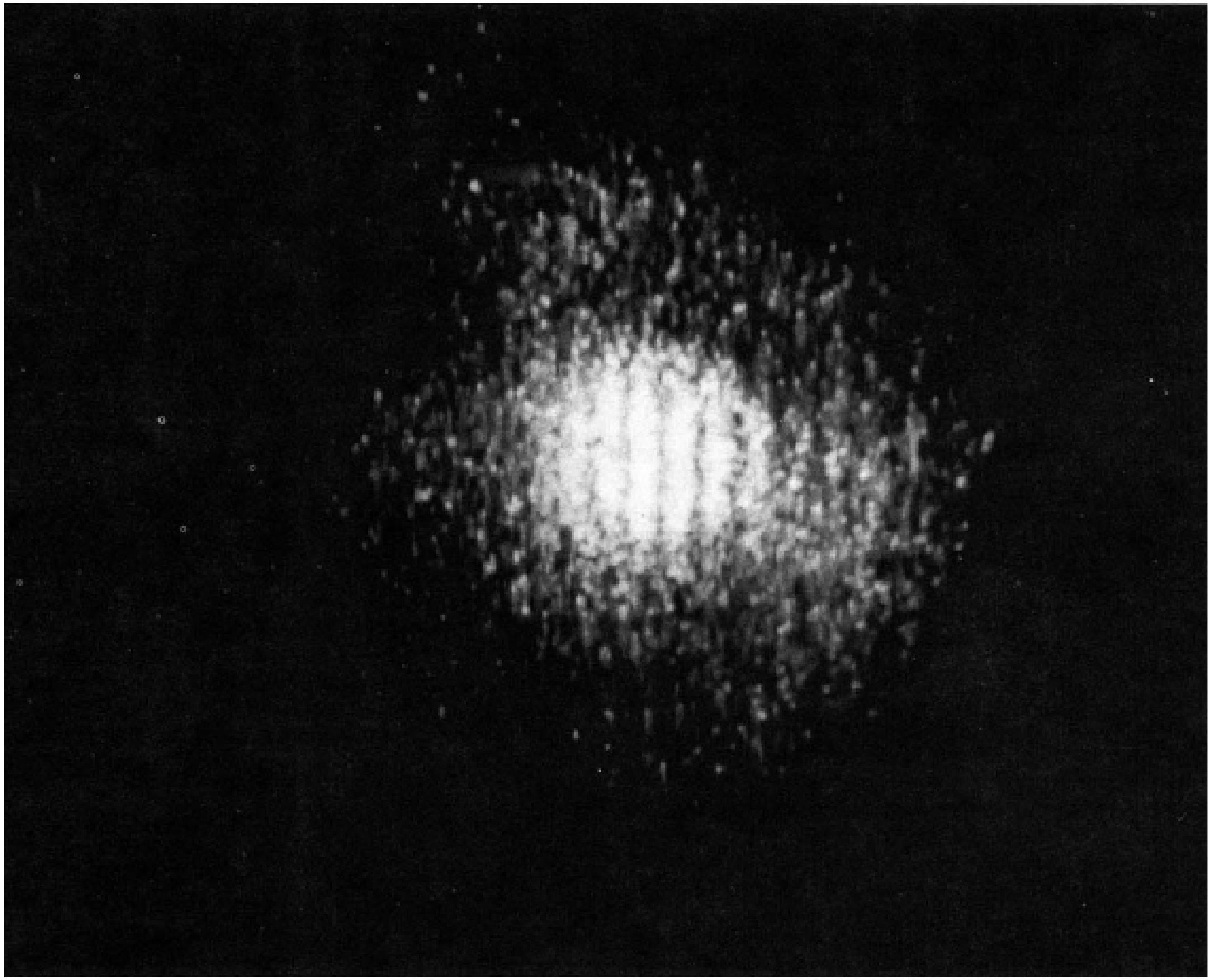


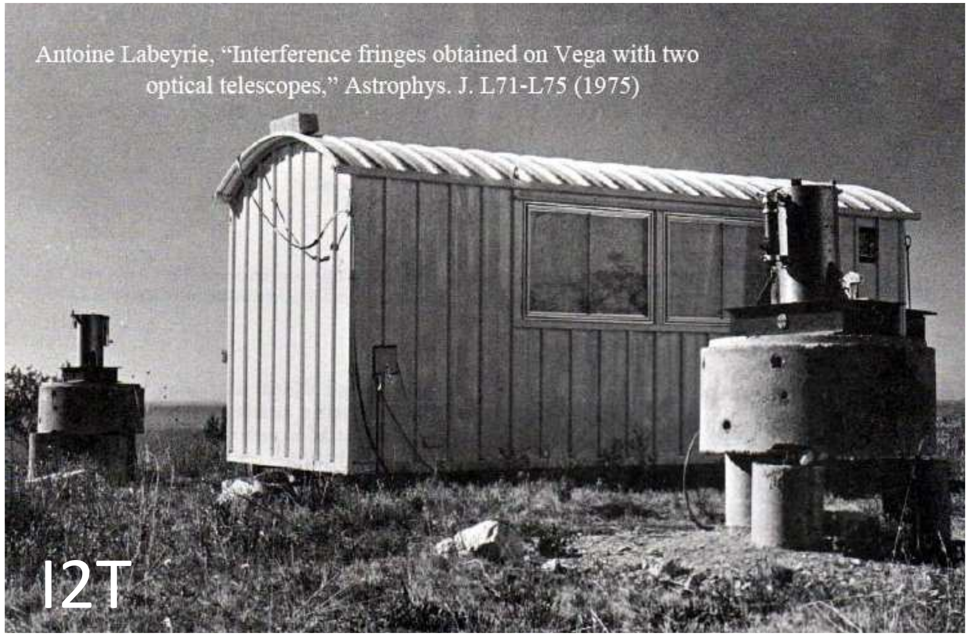
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers


To Distant Source
D sinθ
D cosθ
θ
T1
T2
D
M
L
Optical Delay Line (ODL1)
Image Plane
Optical Delay Line (ODL2)

First fringes with I2T

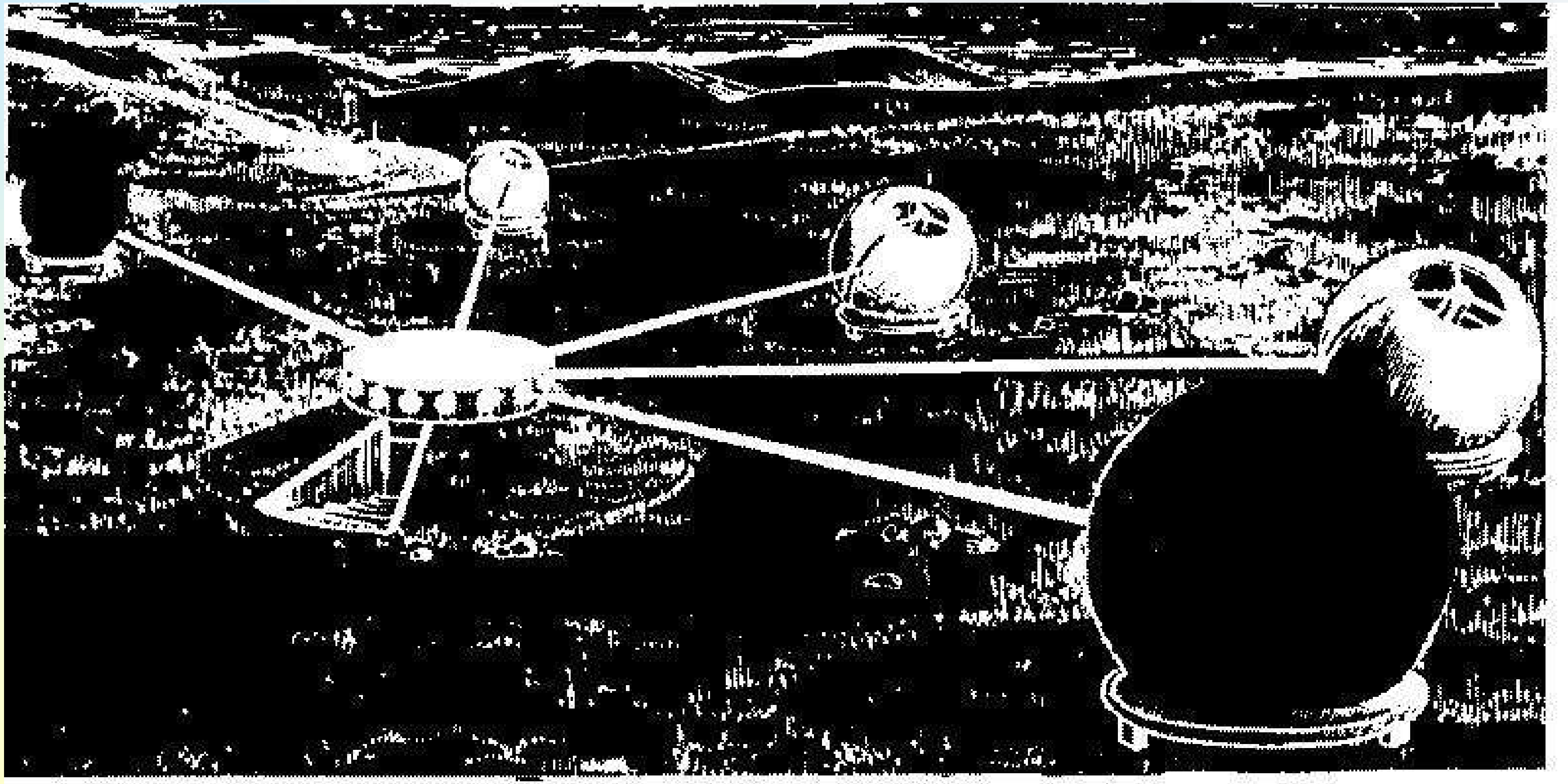



Antoine Labeyrie, "Interference fringes obtained on Vega with two optical telescopes," Astrophys. J. L71-L75 (1975)
I2T


GI2T

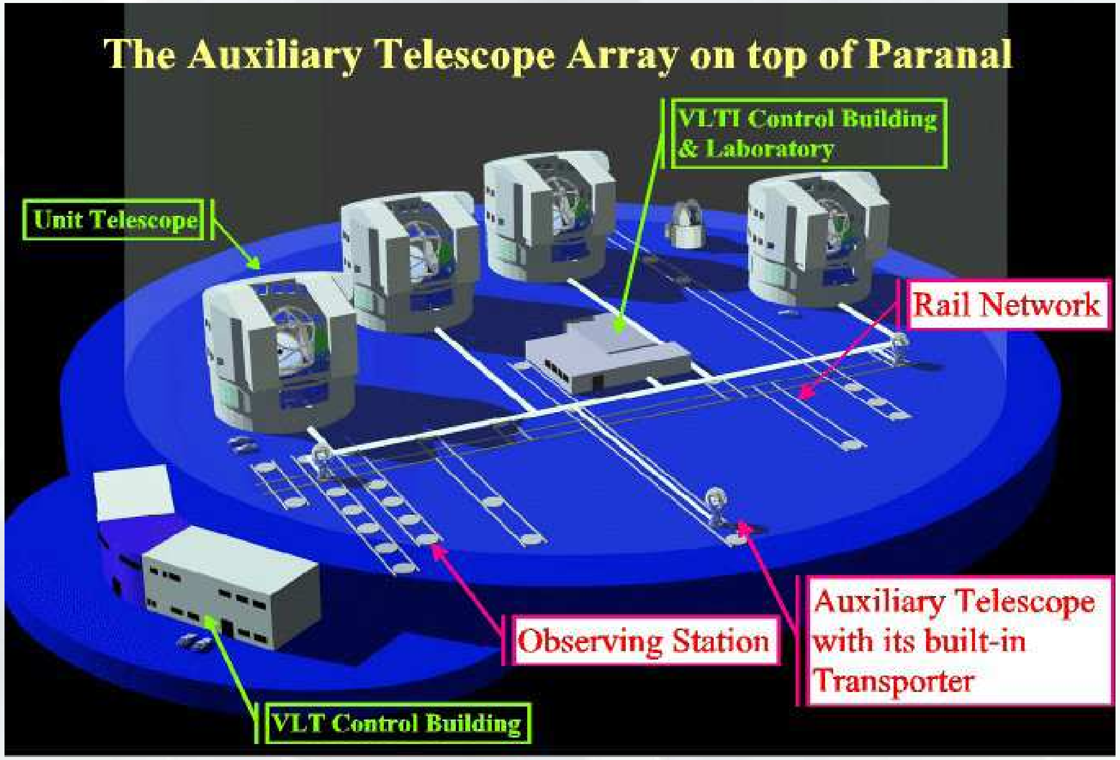
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers



An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers



http://www.aeos.ulg.ac.be/HARI/

An introduction to optical/IR interferometry

- 7 Some results

Star	Spectral type	Luminosity class	Angular diameter $\times 10^{-3}$ seconds of arc
α Boo	K2	Giant	20
α Tau	K5	Giant	20
α Sco	M1–M2	Super-giant	40
β Peg	M2	Giant	21
o Cet	M6e	Giant	47
α Ori	M1–M2	Super-giant variable	34→47

Table 2.1. Stars measured with Michelson's interferometer. From Pease (1931).

An introduction to optical/IR interferometry

- 7 Some results

Table 2. Diamètres stellaires mesurés à l'I2T

NOM	SPECTRE	DIAMÈTRE λ = 0,55 µm en ms. d'arc	MESURÉ λ = 2,2 µm en ms. d'arc	R/R ⊙	TEMPÉRATURE EFFECTIVE λ = 0,55 µm en degrés Kelvin	TEMPÉRATURE EFFECTIVE λ = 2,2 µm en degrés Kelvin	DISTANCE en parsecs (1 pc = 3,26 al)
α Cas	K0II	5.4 ± 0.6		26 ± 6	4700 ± 300		45 ± 9
β And	M0III	13.2 ± 1.7	14.4 ± 0.5	33 ± 9	3800 ± 250	3711 ± 64	23 ± 3
γ And	K3II	6.6 ± 0.6		60 ± 14	4650 ± 250		75 ± 15
α Per	F51b	2.9 ± 0.4		55 ± 9	7000 ± 600		176 ± 6
α Cyg	A2Ia	2.7 ± 0.3		145 ± 45	8200 ± 800		500 ± 100
α Ari	K2III	7.6 ± 1		15 ± 5	4300 ± 350		23 ± 4
β Gem	K0III	7.8 ± 0.6		8 ± 2	4900 ± 220		11 ± 1
β Umi	K4III	8.9 ± 1		30 ± 9	4220 ± 300		31 ± 11
γ Dra	K5III	8.7 ± 0.8	10.2 ± 1.4	45 ± 10	4300 ± 230	3960 ± 270	59 ± 21
δ Dra	G9III	3.8 ± 0.3		16 ± 5	4530 ± 220		36 ± 8
µ Gem	M3III		14.6 ± 0.8	94 ± 30		3980 ± 95	60 ± 15
α Tau	K5III		20.7 ± 0.4	47 ± 7		3904 ± 34	21 ± 3
α Boo	K2III		21.6 ± 1.2	26 ± 6		4240 ± 120	11 ± 2
α Aur$_{a}$	G5III	8.0 ± 1.2		11.7 ± 2	6400 ± 200		13.7 ± 0.6
α Aur$_{b}$	G0III	4.8 ± 1.5		7.1 ± 2	5950 ± 200		13.7 ± 0.6
α Lyr	A0V	3.0 ± 0.2		2.6 ± 0.2			8.1 ± 0.3

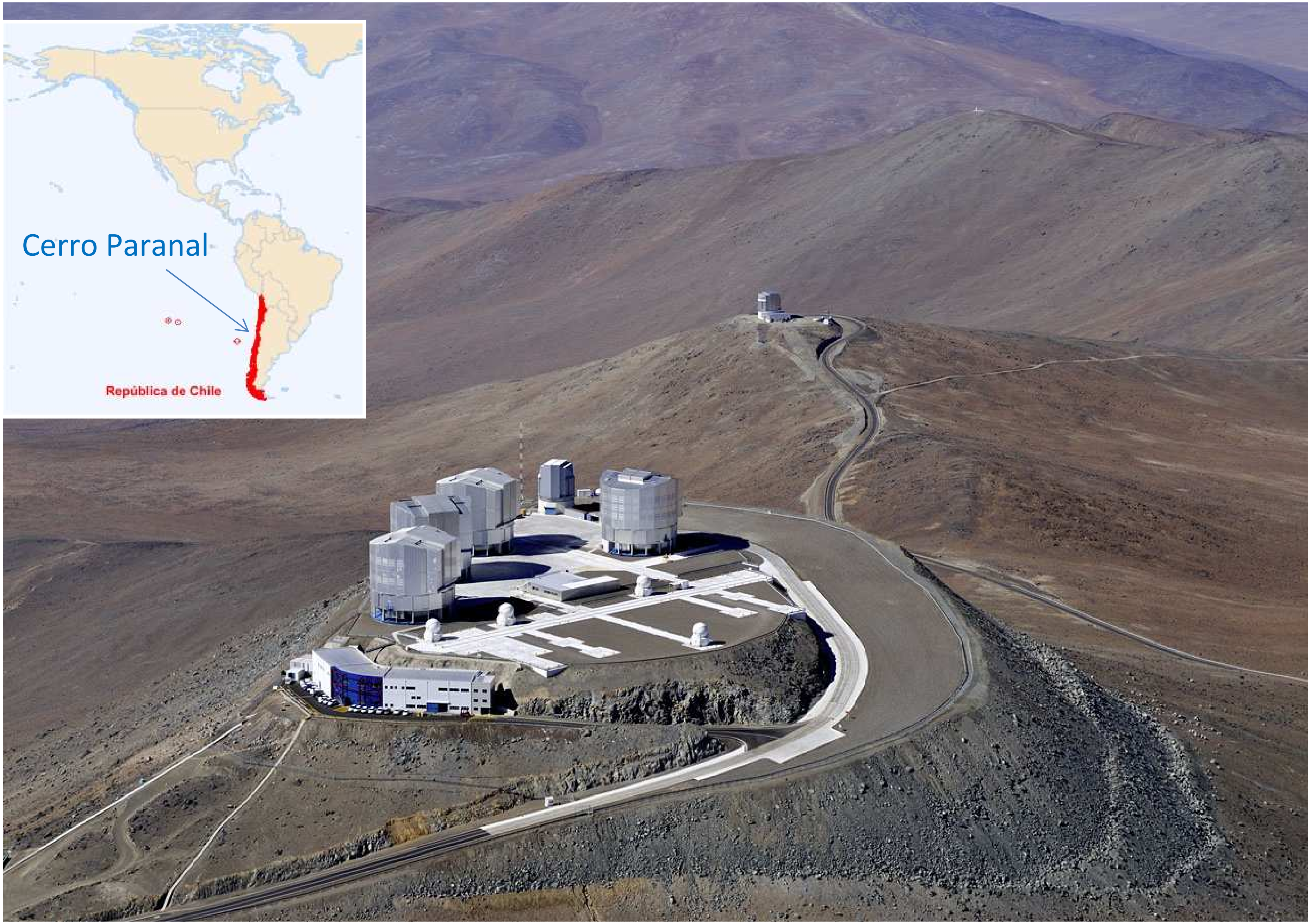
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day is:

Very Large Telescope Interferometer (VLTI)

- 4 x 8.2m UTs
- 4 x 1.8m ATs
- Max. Base: 200m


Cerro Paranal
República de Chile

An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

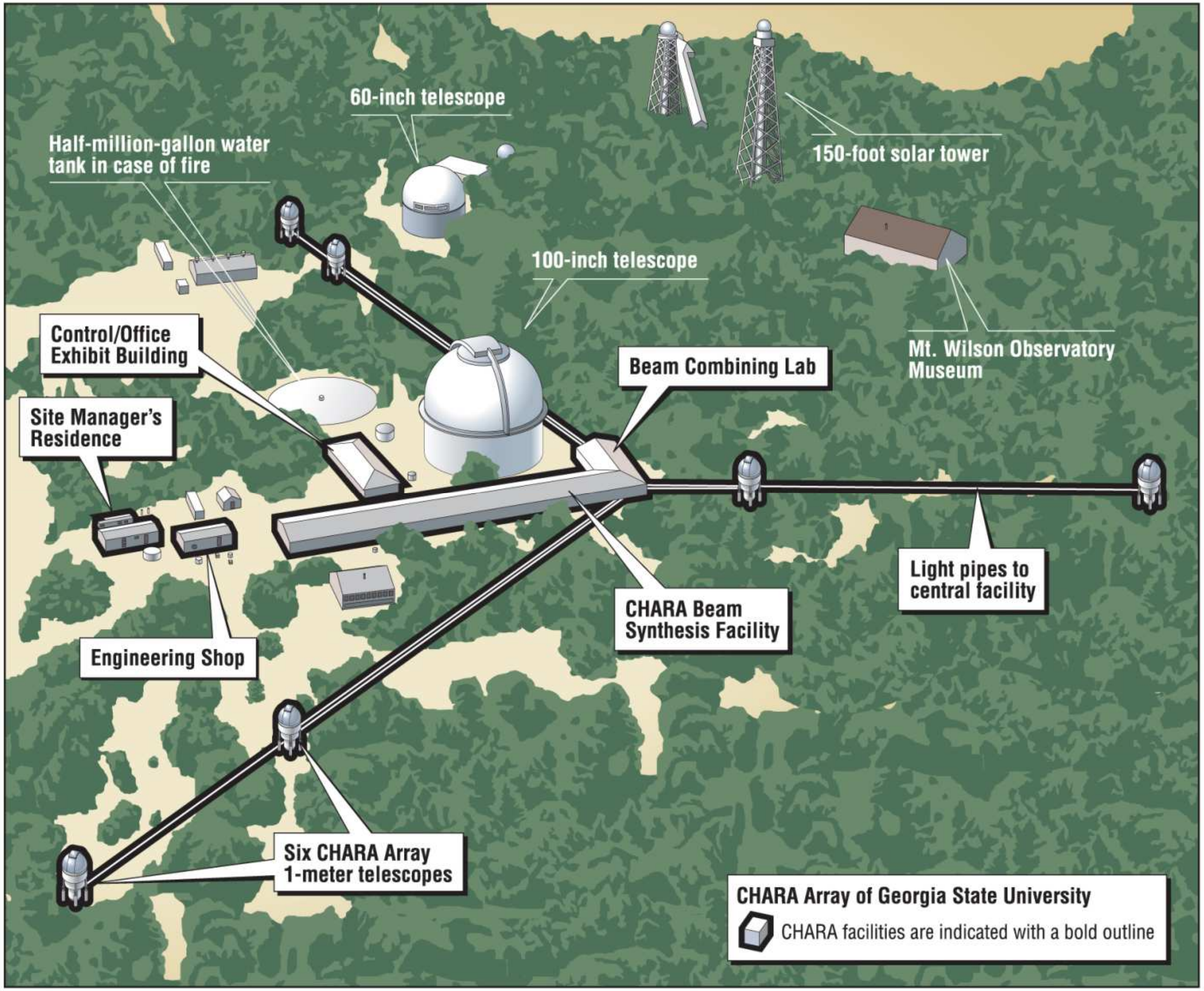
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day is also:

The CHARA interferometer

• 6 x 1m telescopes
• Max. Base: 330m


60-inch telescope
Half-million-gallon water tank in case of fire
150-foot solar tower
100-inch telescope
Mt. Wilson Observatory Museum
Control/Office Exhibit Building
Beam Combining Lab
Site Manager's Residence
Light pipes to central facility
CHARA Beam Synthesis Facility
Engineering Shop
Six CHARA Array 1-meter telescopes
CHARA Array of Georgia State University
CHARA facilities are indicated with a bold outline

An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day is also:

Palomar Testbed Interferometer (PTI)

- 3 x 40cm telescopes
- Max. Base: 110m

An introduction to optical/IR interferometry

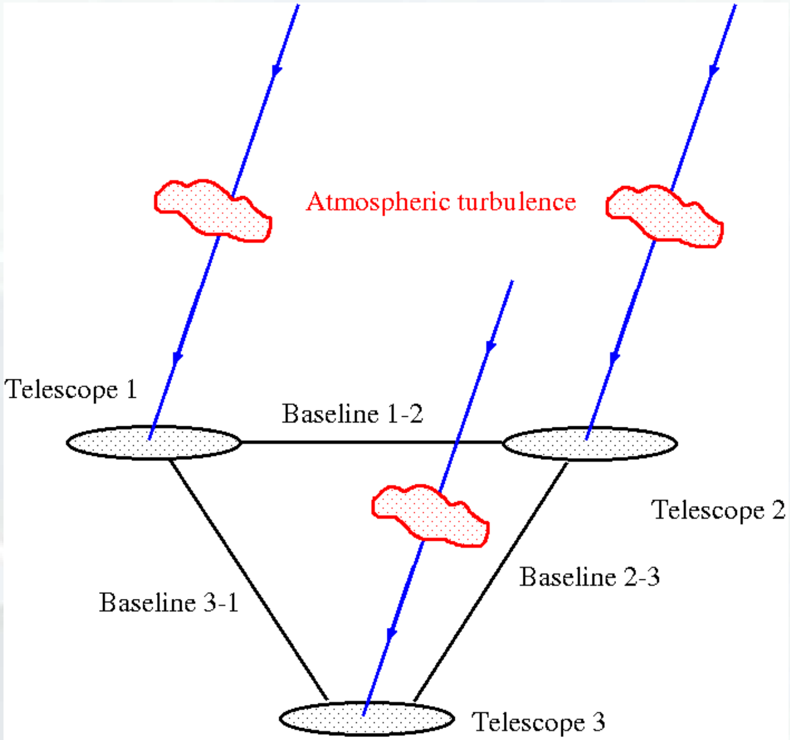
- 6 Some examples of optical interferometers

Interferometry to-day is also:

Keck interferometer

• 2 x 10m telescopes
• Base: 85m

Closure phases – what are these?



• Measure visibility phase (Φ) on three baselines simultaneously.

• Each is perturbed from the true phase (ϕ) in a particular manner:

$$\Phi_{12} = \phi_{12} + \varepsilon_1 - \varepsilon_2$$
$$\Phi_{23} = \phi_{23} + \varepsilon_2 - \varepsilon_3$$
$$\Phi_{31} = \phi_{31} + \varepsilon_3 - \varepsilon_1$$

• Construct the linear combination of these:

$$\Phi_{12} + \Phi_{23} + \Phi_{31} = \phi_{12} + \phi_{23} + \phi_{31}$$

The error terms are antenna dependent – they vanish in the sum.
The source information is baseline dependent – it remains.
We still have to figure out how to use it!

Closure phase is a peculiar linear combination of the true Fourier phases:

– In fact, it is the argument of the product of the visibilities on the baselines in question, hence the name triple product (or bispectrum):

$$V_{12}V_{23}V_{31} = |V_{12}|\,|V_{23}|\,|V_{31}| \exp(i2\pi\,[\Phi_{12} + \Phi_{23} + \Phi_{31}]) = T_{123}$$

– So we have to use the closure phases as additional constraints In some nonlinear iterative inversion scheme.

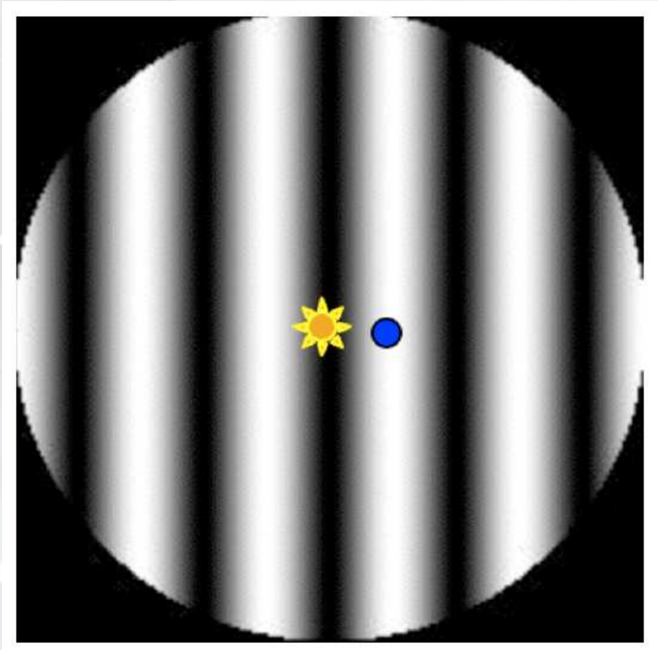
An introduction to optical/IR interferometry

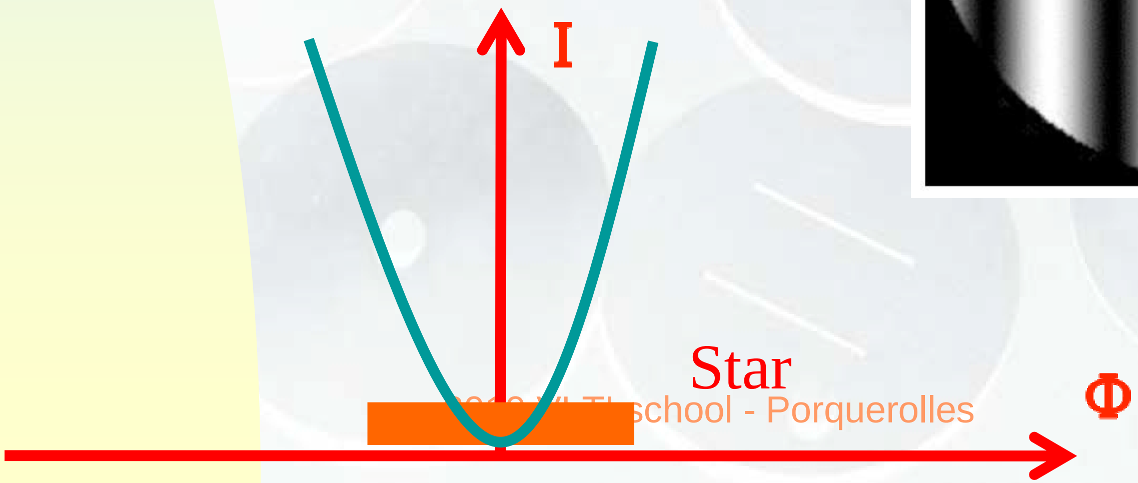
- 6 Some examples of optical interferometers

Interferometry to-day is also:

Nullin interferometry

• Measurement of « stellar leakage »
• Allow to resolve stars with a
a small size interferometer





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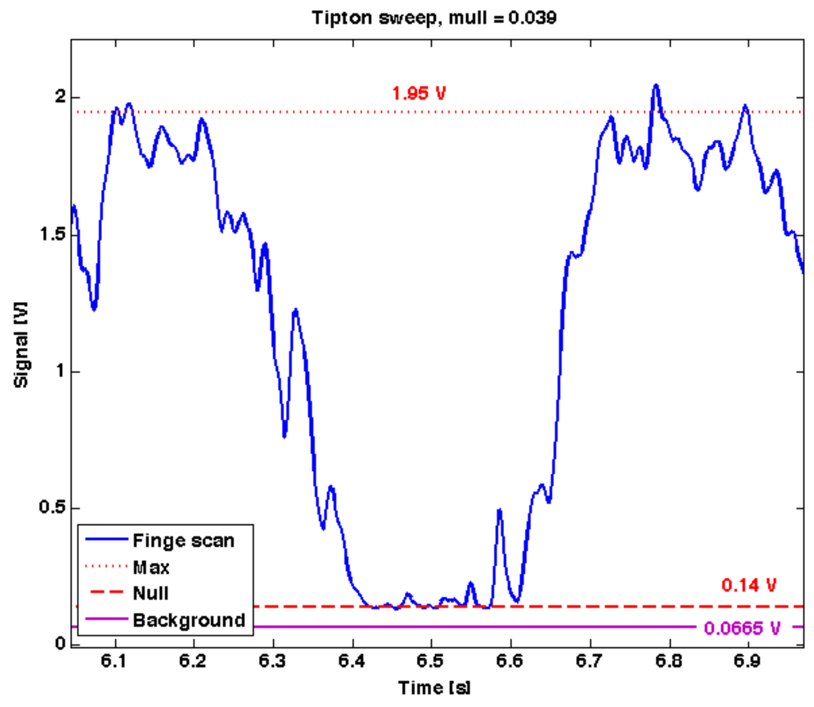
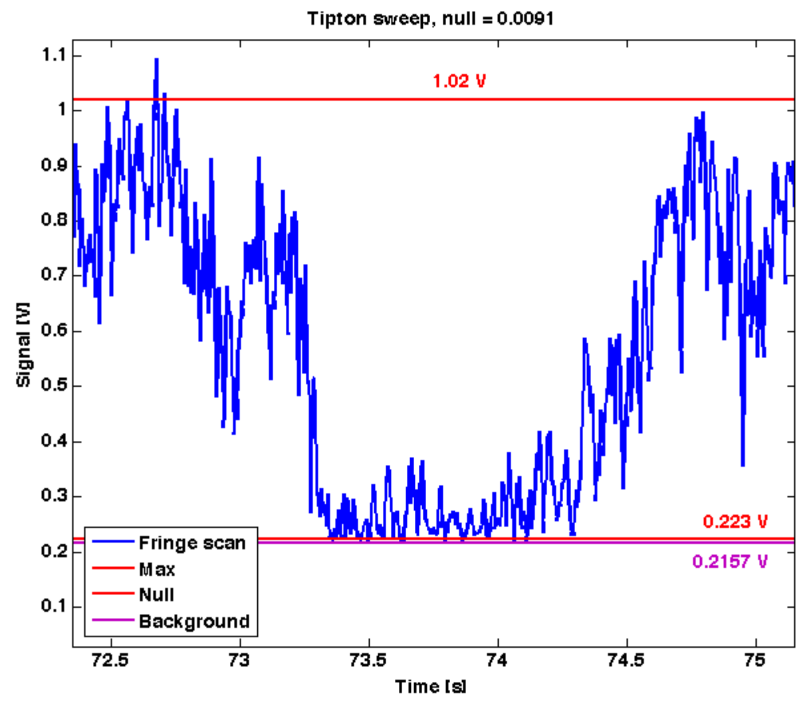
- 6 Some examples of optical interferometers

Interferometry to-day is also:





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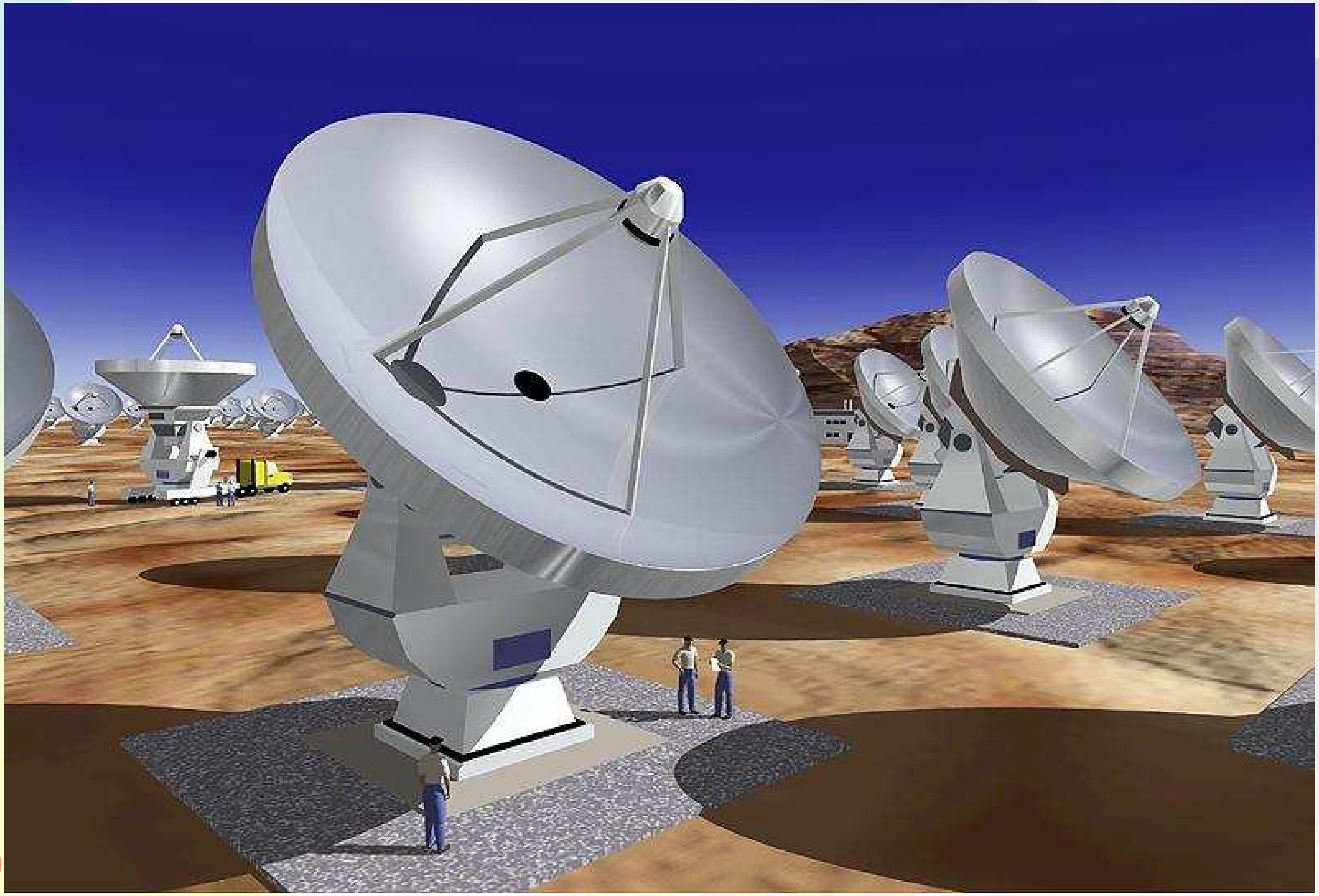
- 6 Other examples of interferometers: ALMA



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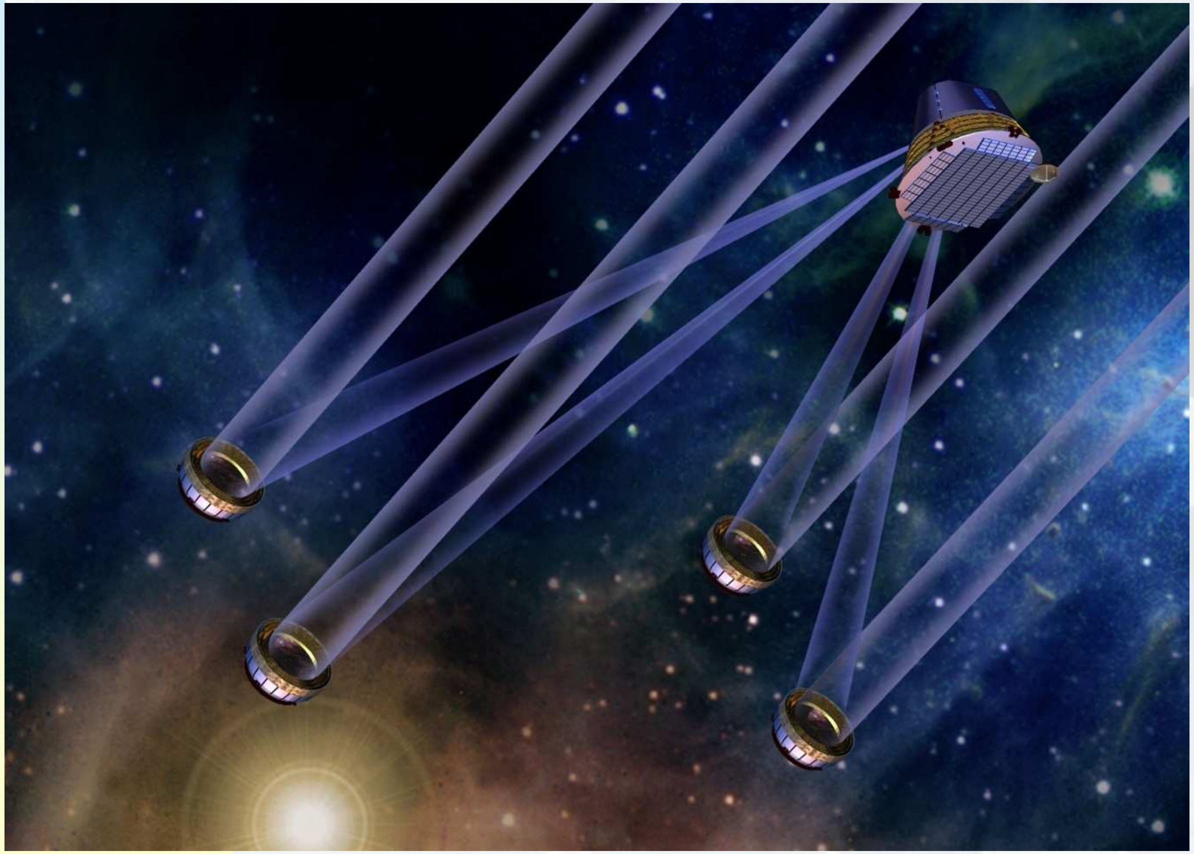
- 6 Other examples of interferometers: DARWIN



An introduction to optical/IR interferometry

8 Three important theorems … and some applications

8.1 The fundamental theorem

8.2 The convolution theorem

8.3 The Wiener-Khintchin theorem

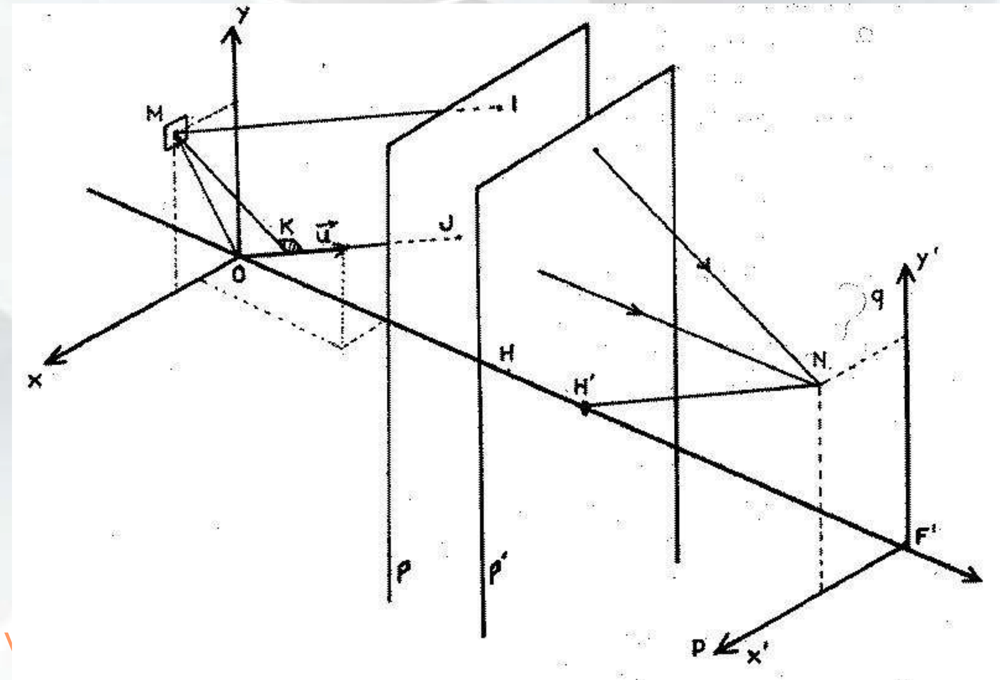
Réf.: P. Léna; Astrophysique: méthodes physiques de l'observation (Savoirs Actuels / CNRS Editions)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

a(p,q) = TF_(A(x,y))(p,q),

$$a(p,q) = \int_{R^2} A(x,y)\exp[-i2\pi(px+qy)]dxdy,$$



with

p = x'/(λ f)

q = y'/(λ f)

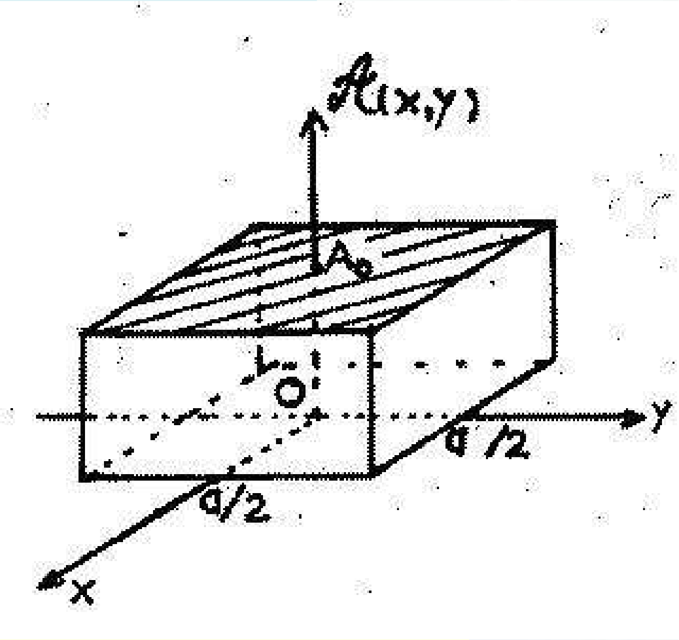
An introduction to optical/IR interferometry

8.1 The fundamental theorem

The distribution of the complex amplitude a(p,q) in the focal plane is given by the Fourier transform of the distribution of the complex amplitude A(x,y) in the entrance pupil plane.

8.1 The fundamental theorem

Application: Point Spread Function determination



$$A(x,y) = A_0\, P_0(x,y), \qquad (8.1.1)$$

$$P_0(x,y) = \Pi(x/a)\, \Pi(y/a). \qquad (8.1.2)$$

An introduction to optical/IR interferometry

8.1 The fundamental theorem

$$a(p,q) = TF\_[A(x,y)](p,q) = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2} A_0 \exp[-i2\pi(px+qy)]dxdy \qquad (8.1.3)$$

$$a(p,q) = A_0 \int_{-a/2}^{a/2} \exp[-i2\pi px]dx \int_{-a/2}^{a/2} \exp[-i2\pi qy]dy \qquad (8.1.4)$$

$a(p,q) = A_0\ a^2\ [\sin(\pi pa) / (\pi pa)]\ [\sin(\pi qa) / (\pi qa)].$ (8.1.5)

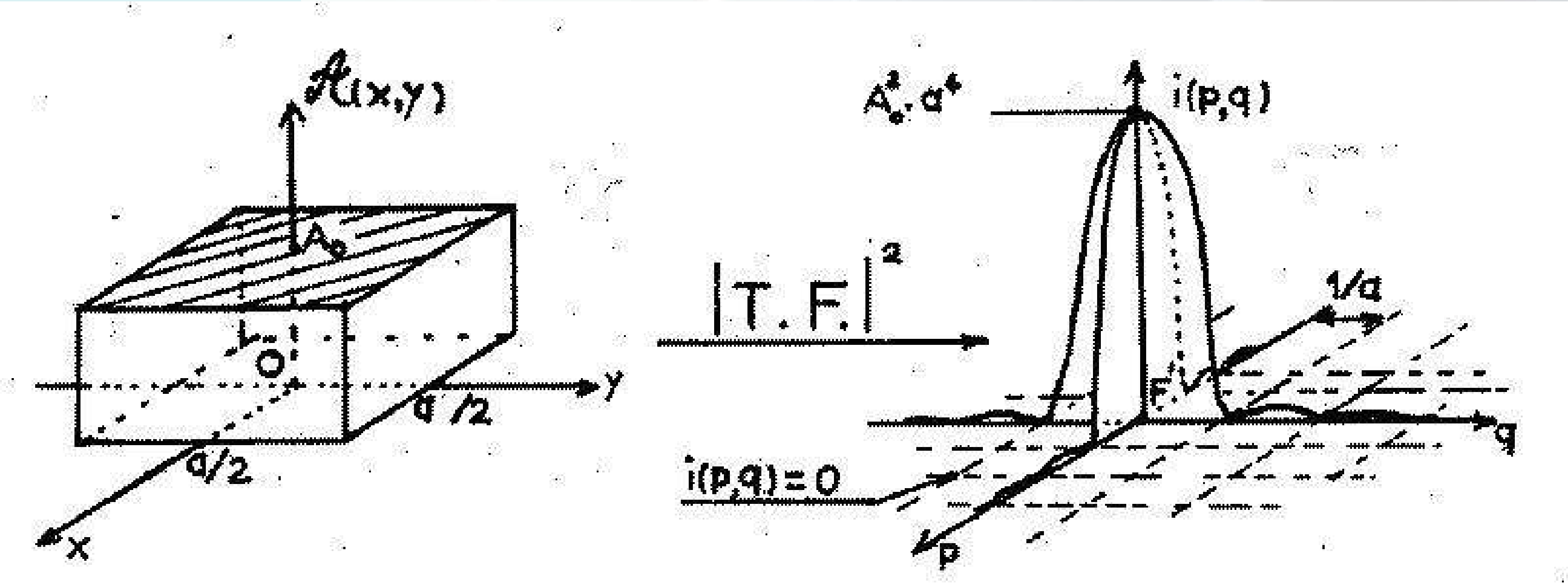
$i(p,q) = a(p,q)\ a^*(p,q) = |a(p,q)|^2 = |h(p,q)|^2 =$

$= i_0\ a^4\ [\sin(\pi pa) / (\pi pa)]^2\ [\sin(\pi qa) / (\pi qa)]^2.$ (8.1.6)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination

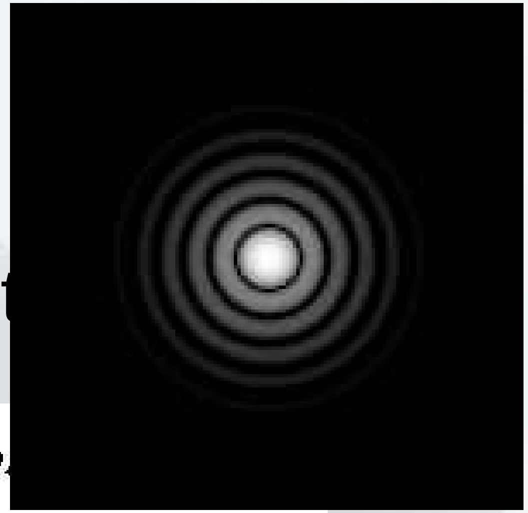


$$\Delta p = \Delta q = \Delta x'/(\lambda f) = \Delta y'/(\lambda f) = 2/a \Rightarrow \Delta\phi_{x'} = \Delta\phi_{y'} = 2\lambda/a \quad (8.1.7)$$

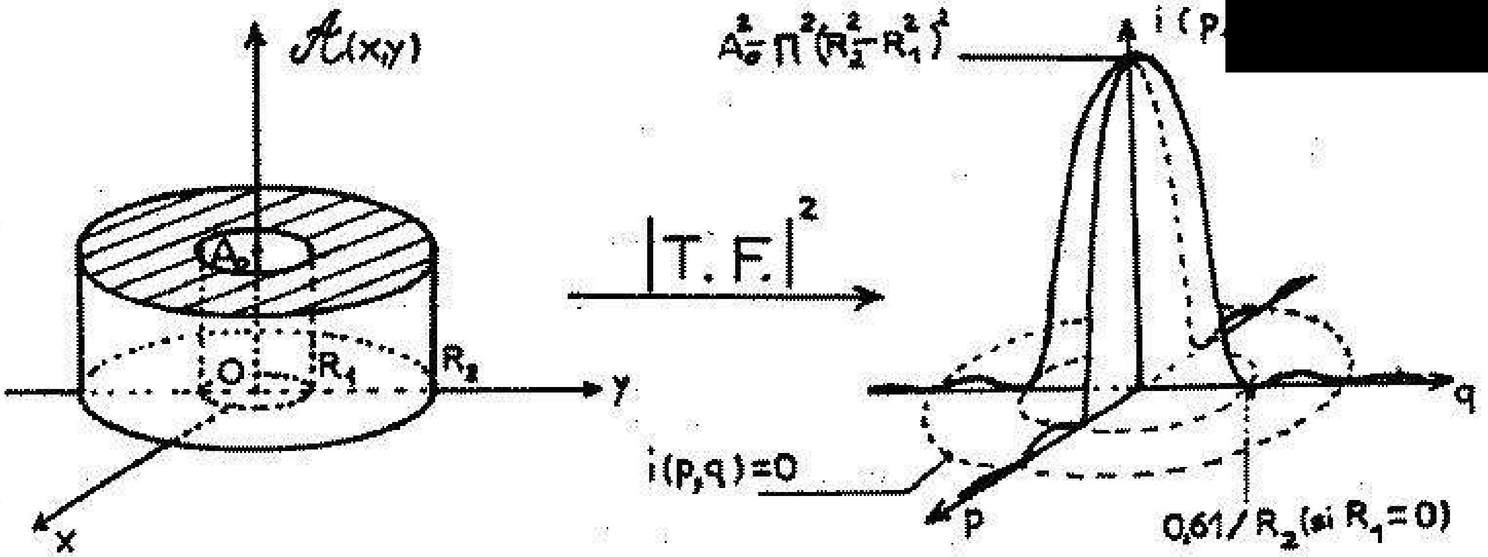
An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function det



h(p,q) = TF_(P(x,y))(p,q)



$$i(\rho') = |a(\rho')|^2 = (A_0\,\pi)^2\,[R_2^2\,2\,J_1(Z_2)\,/\,Z_2\; - R_1^2\,2\,J_1(Z_1)\,/\,Z_1]^2, \quad (8.1.8)$$

with $Z_2 = 2\pi\, R_2\, \rho' / (\lambda f)$ and $Z_1 = 2\pi\, R_1\, \rho' / (\lambda f)$. (8.1.9)

An introduction to optical/IR interferometry

BESSEL FUNCTIONS (REMINDER)

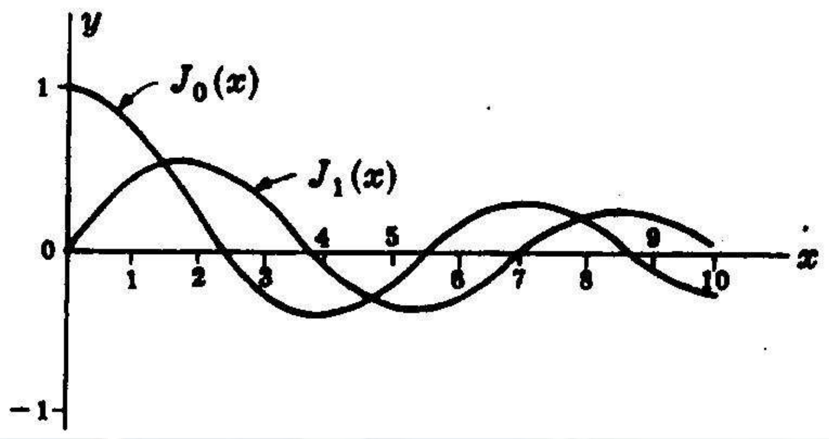
Integral representation of the Bessel functions

$$J_0(x) = \frac{1}{\pi}\int_0^\pi \cos[x\sin(\vartheta)]d\vartheta \qquad J_n(x) = \frac{1}{\pi}\int_0^\pi \cos[n\vartheta - x\sin(\vartheta)]d\vartheta$$

Undefined integral

$$\int x' J_0(x')dx' = x J_1(x)$$

Graphs of the $J_0(x)$ and $J_1(x)$ functions



Series development (x ~ 0):

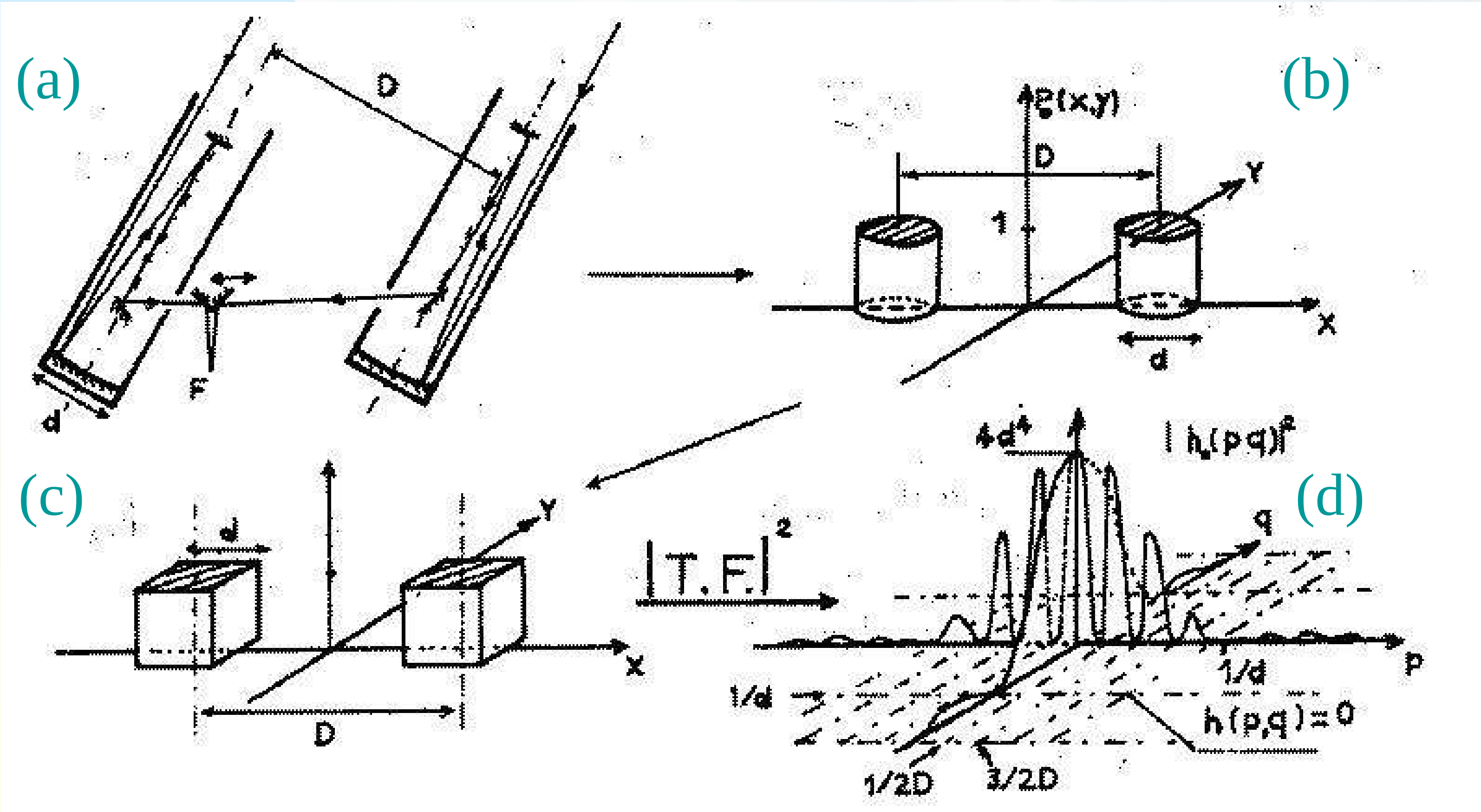
$J_0(x) = 1 - x^2/2^2 + x^4/(2^2 4^2) - x^6/(2^2 4^2 6^2) + \ldots$

$J_1(x) = x/2 - x^3/(2^2 4) + x^5/(2^2 4^2 6) - x^7/(2^2 4^2 6^2 8) + \ldots$

$J_n(x) = (2 / (\pi x))^{1/2} \cos(x - n\pi/2 - \pi/4)$ … and when x is large!

An introduction to optical/IR interferometry

8.1 The fundamental theorem: 2 telescope interferometer



Two coupled optical telescopes: simplified optical scheme (a). Distribution of the complex amplitude for the case of two circular (b) or square (c) apertures and corresponding impulse response (d).

An introduction to optical/IR interferometry

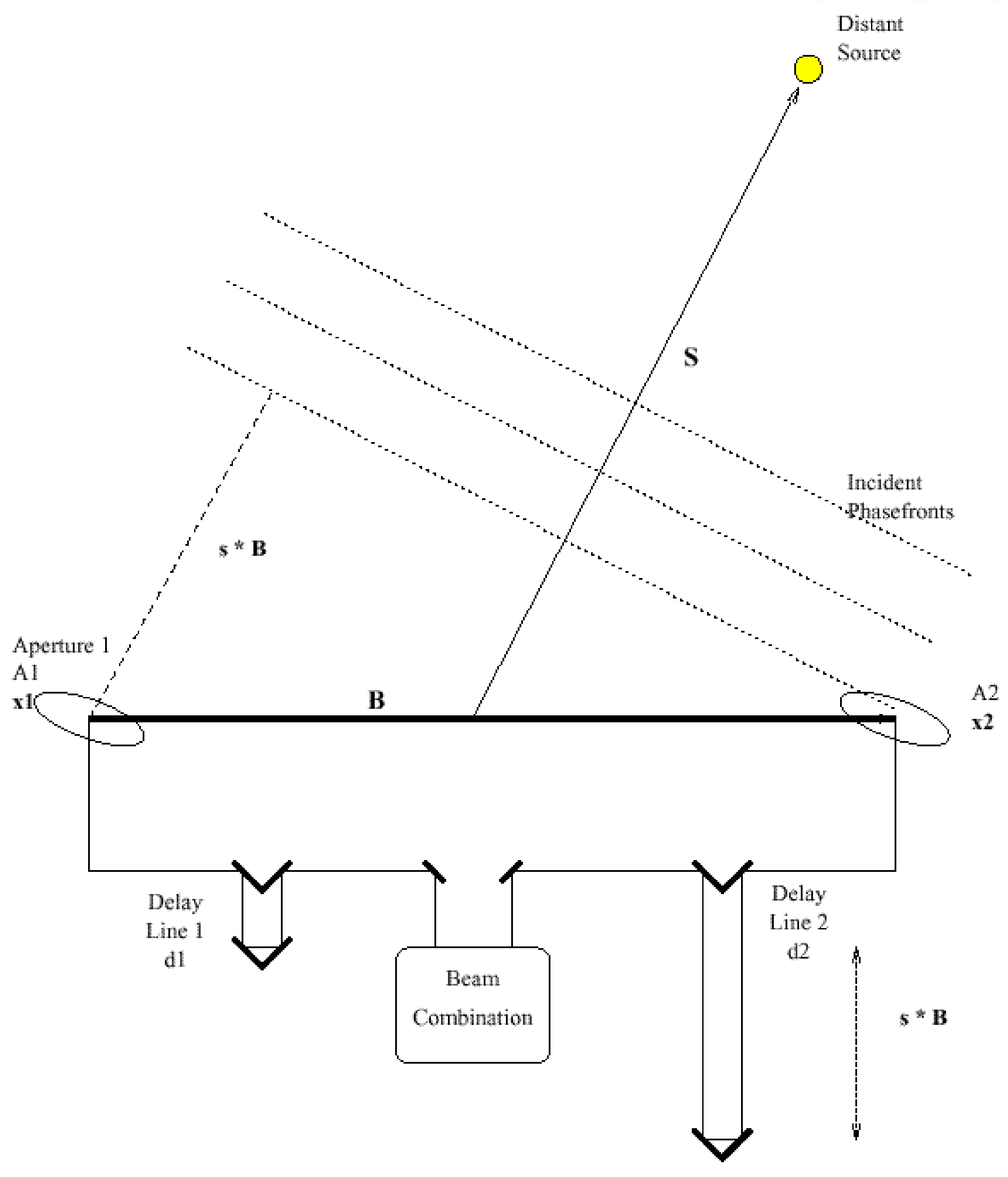
8.1 The fundamental theorem: 2 telescope interferometer

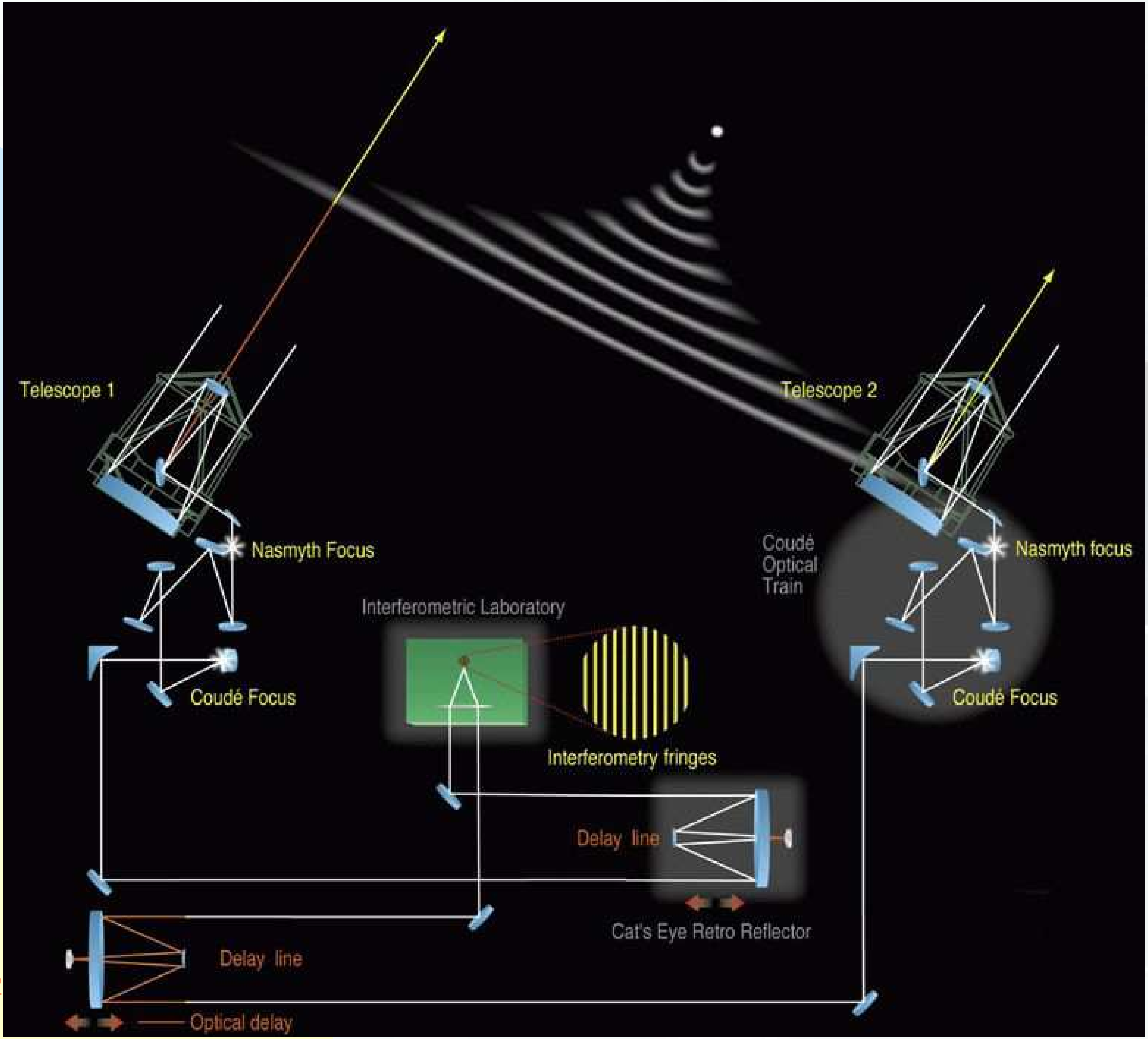
$$h\,(p,q) = TF(P(x,y)(p,q) = \int_{R^2} P\,(x,y)\exp[-i2\pi(px+qy)]dxdy \qquad (8.1.10)$$

$$\begin{aligned}
&h\,(p,q) = TF(P_0(x+D/2)+P_0(x-D/2))(p,q) = \\
&TF(P_0(x+D/2))(p,q)+TF(P_0(x-D/2))(p,q) = \\
&\exp(i\pi D)\,TF(P_0(x))(p,q)+\exp(-i\pi D)\,TF(P_0(x))(p,q) = \\
&(\exp(i\pi D)+\exp(-i\pi D))\,TF(P_0(x))(p,q) = \\
&2\cos(\pi D)\,TF(P_0(x))(p,q)
\end{aligned} \qquad (8.1.11)$$

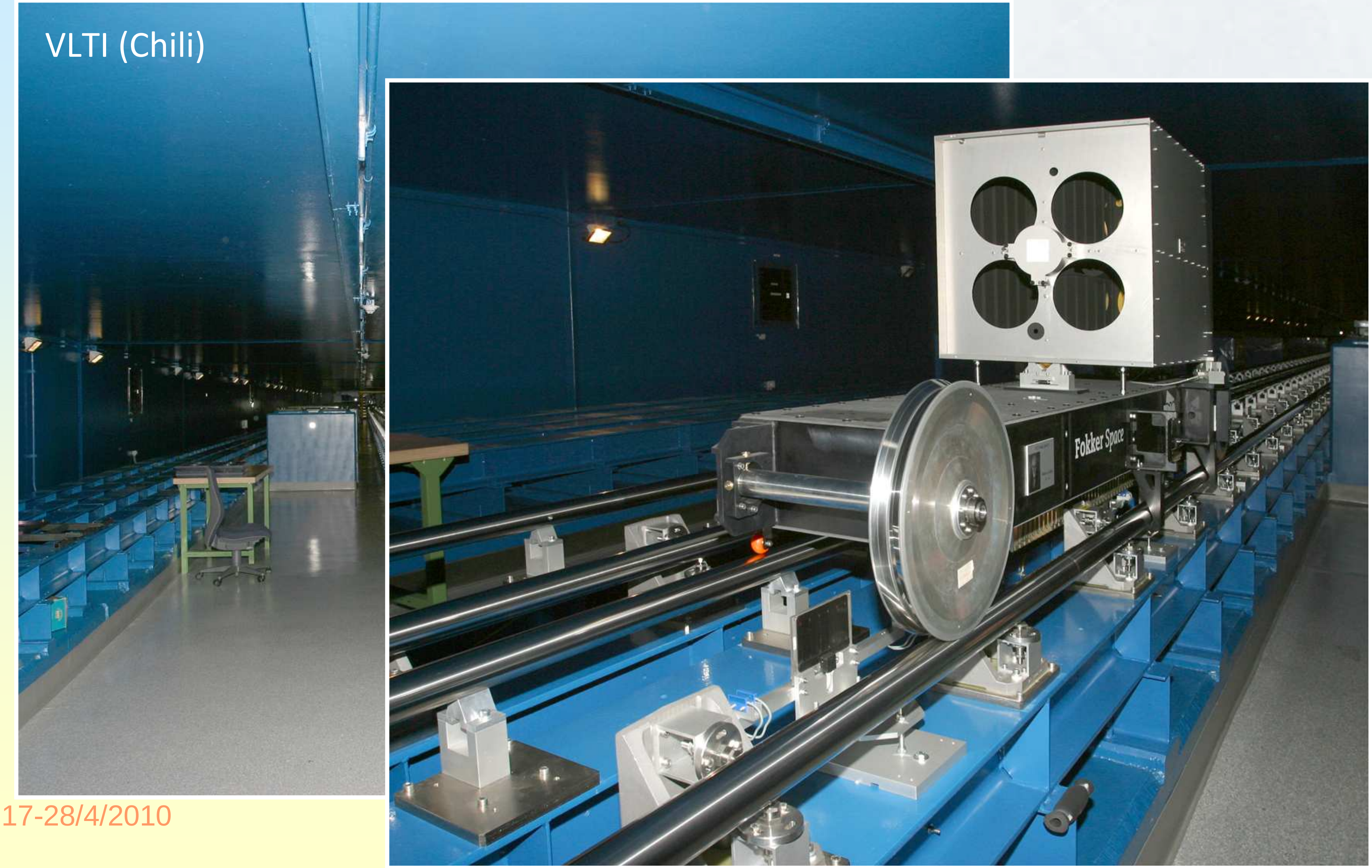
For the particular case of two square apertures:

$$i(p,q) = \left|h\,(p,q)\right|^2 = 4\cos^2(\pi pD)\,d^4\left(\frac{\sin(\pi qd)}{\pi qd}\right)^2\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2 \qquad (8.1.12)$$

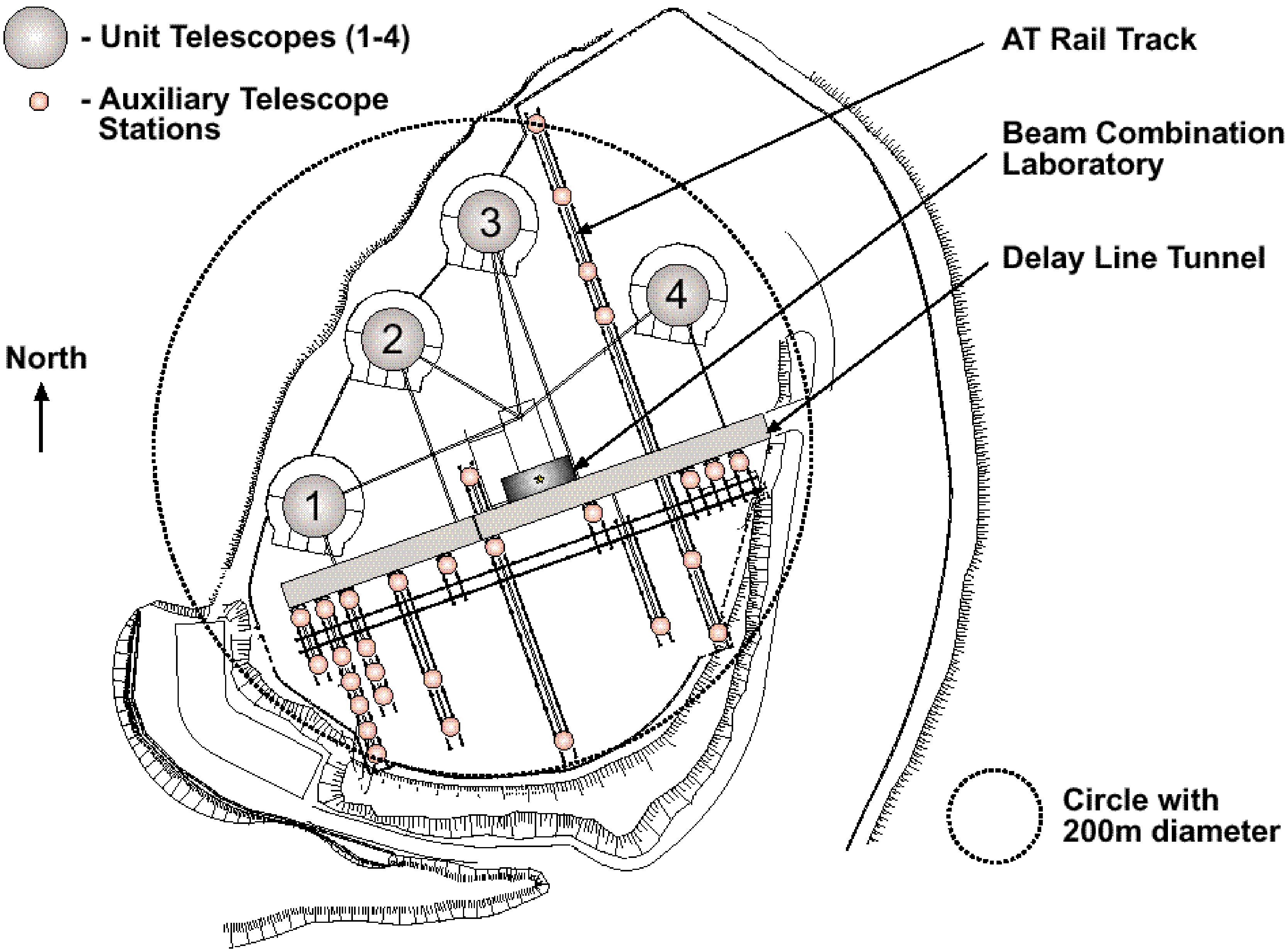

Distant
Source
S
Incident
Phasefronts
s * B
Aperture 1
A1
x1
B
A2
x2
Delay
Line 1
d1
Beam
Combination
Delay
Line 2
d2
s * B

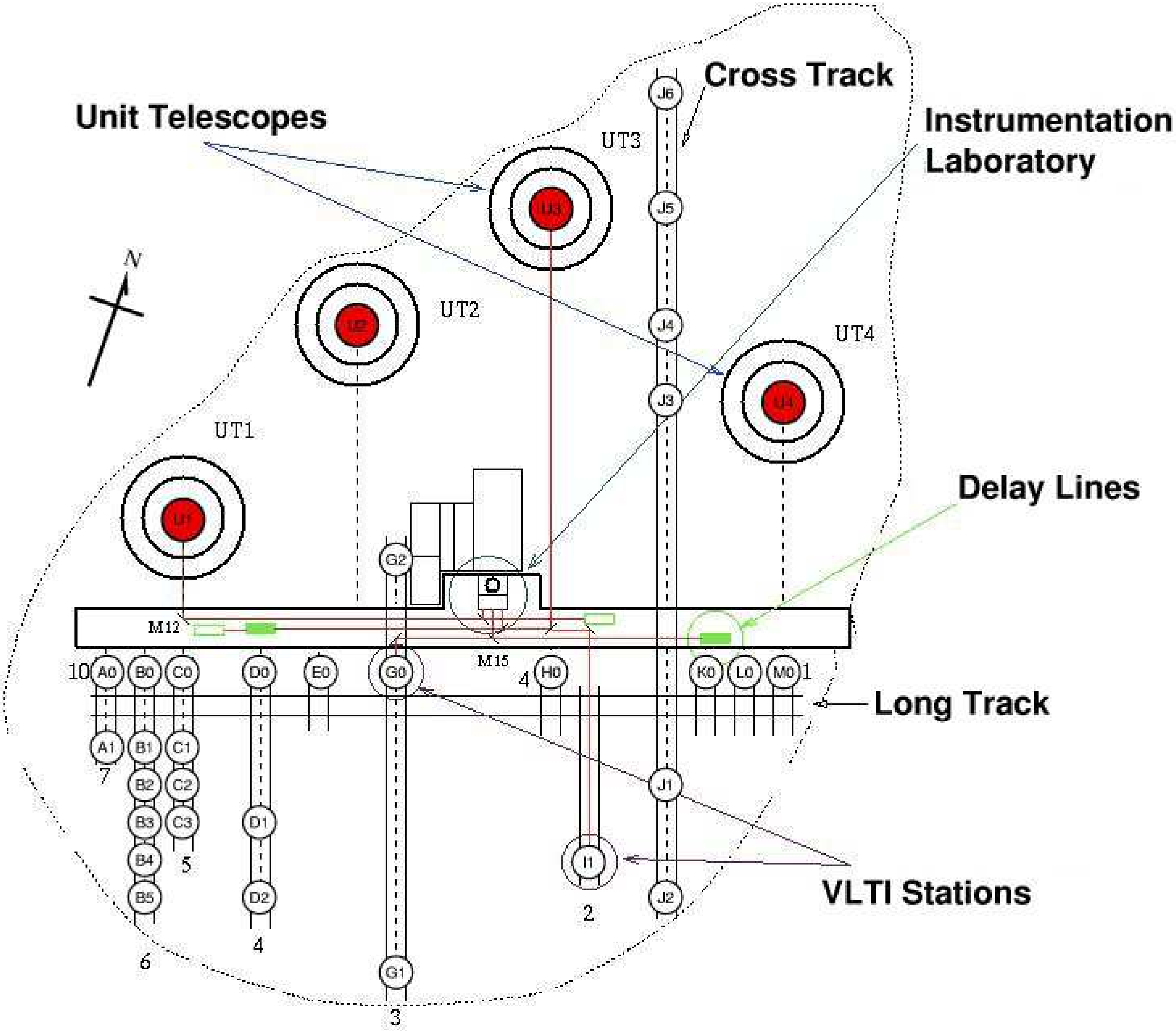

Telescope 1
Telescope 2
Nasmyth Focus
Nasmyth focus
Coudé Optical Train
Interferometric Laboratory
Coudé Focus
Coudé Focus
Interferometry fringes
Delay line
Cat's Eye Retro Reflector
Delay line
Optical delay

Delay lines at the VLTI

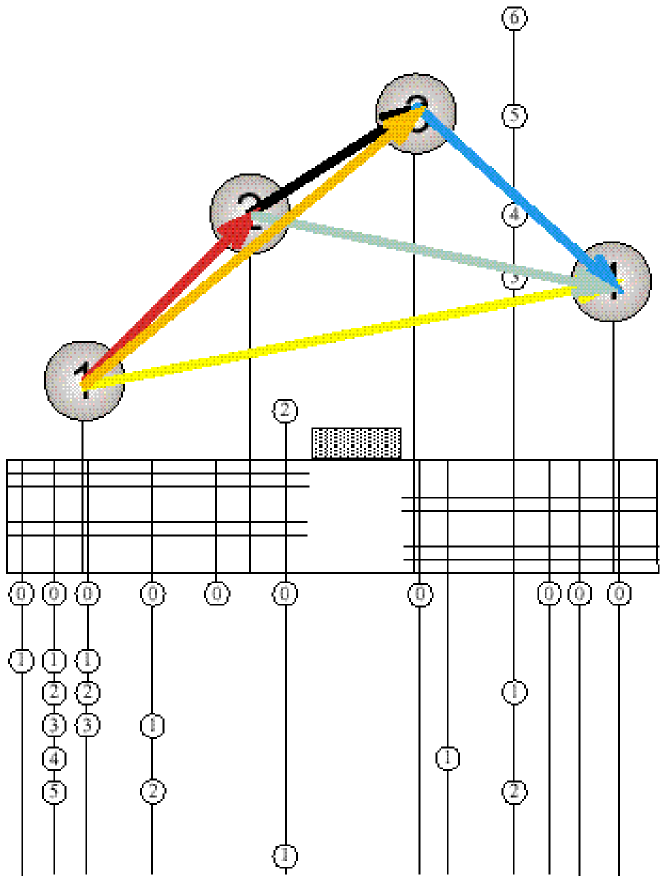


What do the VLTI telescope locations look like?

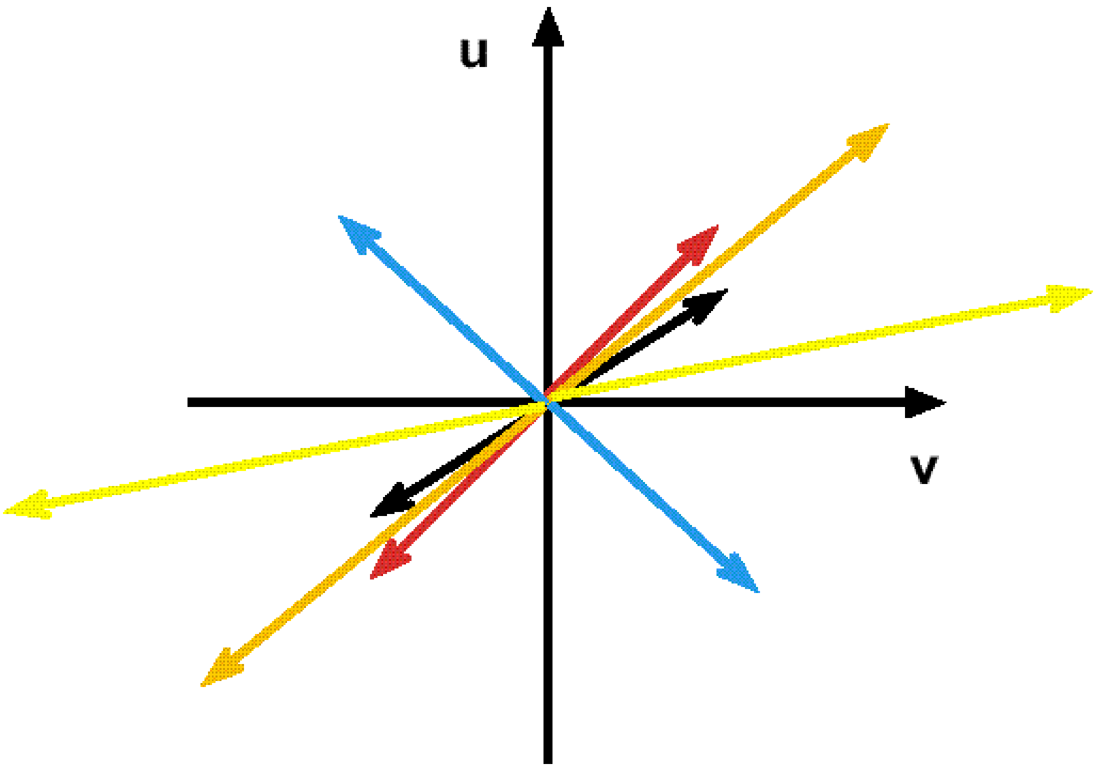



Unit Telescopes
Cross Track
Instrumentation Laboratory
UT1
UT2
UT3
UT4
Delay Lines
M12
M15
Long Track
VLTI Stations
A0
B0
C0
D0
E0
G0
H0
K0
L0
M0
A1
B1
C1
B2
C2
B3
C3
B4
B5
D1
D2
G1
G2
I1
J1
J2
J3
J4
J5
J6

How are those locations related to the *uv* coverage?



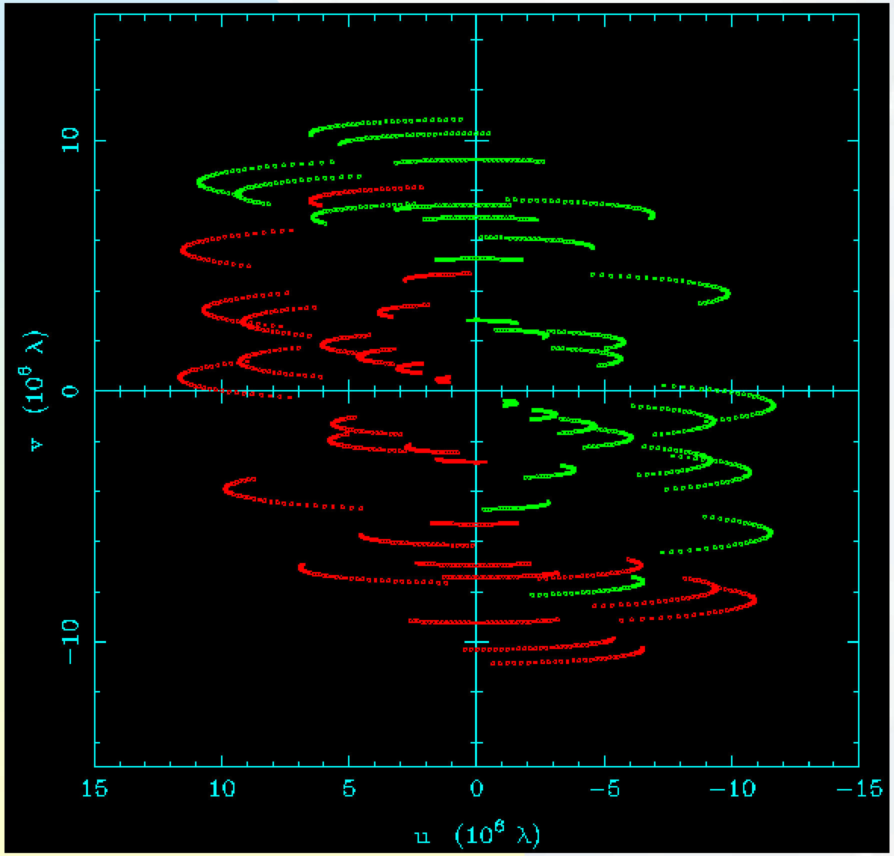
This is the uv-plane:



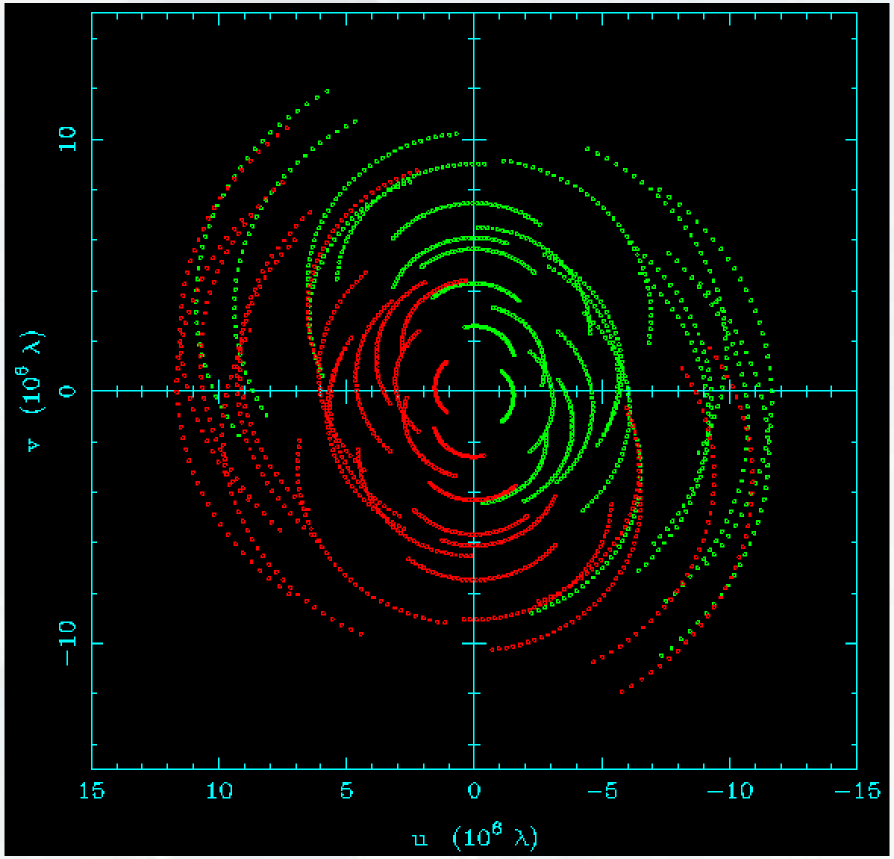
Note: This is the uv-plane for an object at zenith.
In general, the projected baselines have to be used.

Examples of Fourier plane coverage

Dec -15

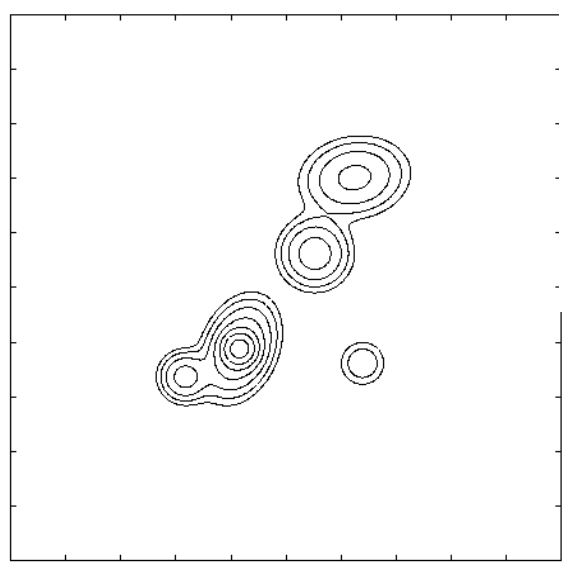


Dec -65

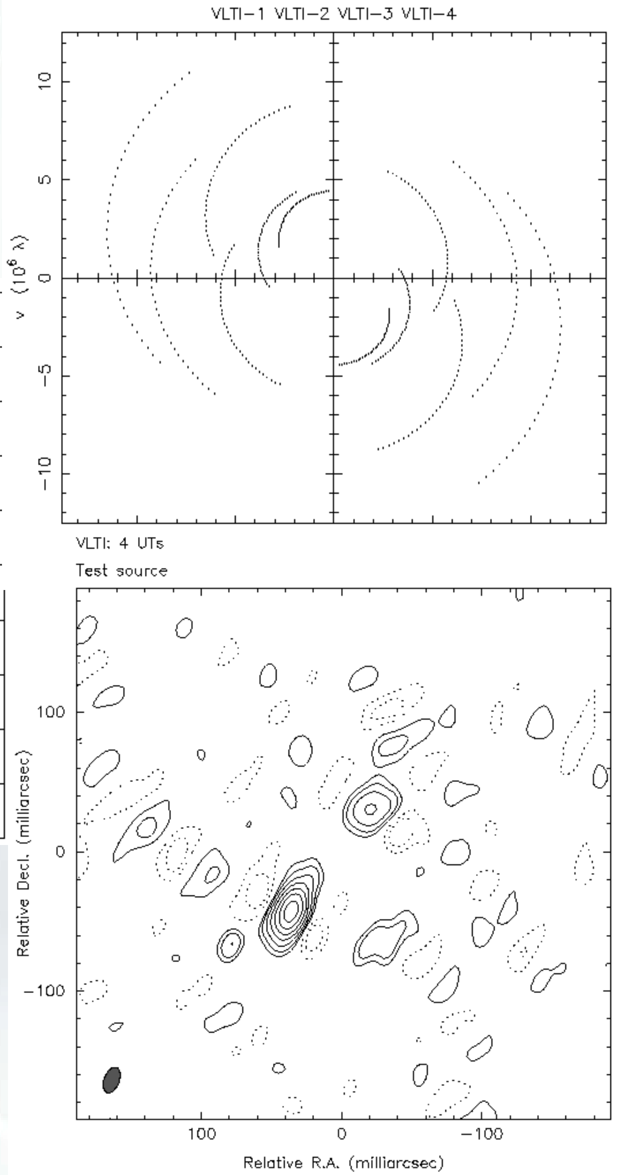


How does the uv plane coverage impact imaging?

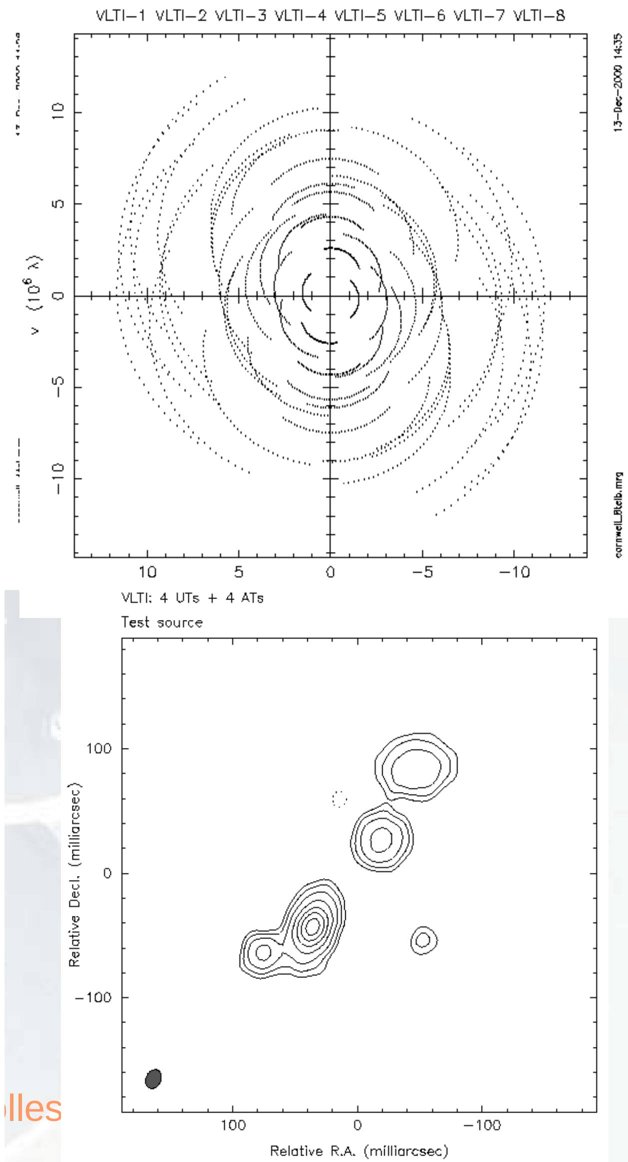
Model



4 telescopes, 6 hours



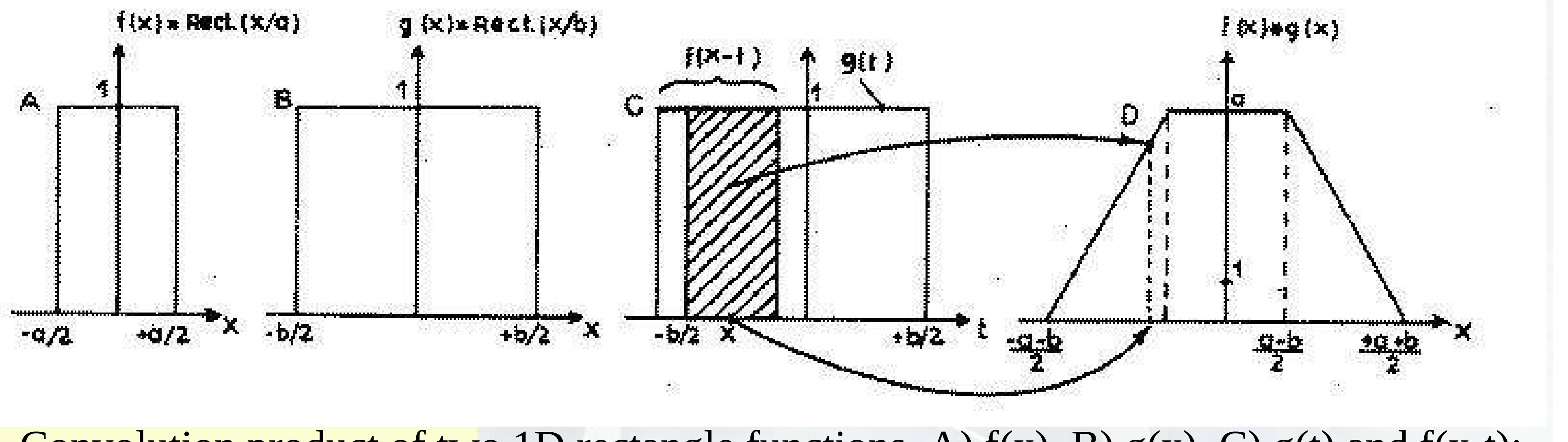
8 telescopes, 6 hours



An introduction to optical/IR interferometry

8.2 The convolution theorem

$$f(x) * g(x) = (f * g)(x) = \int_{R^n} f(x-t)g(t)dt$$

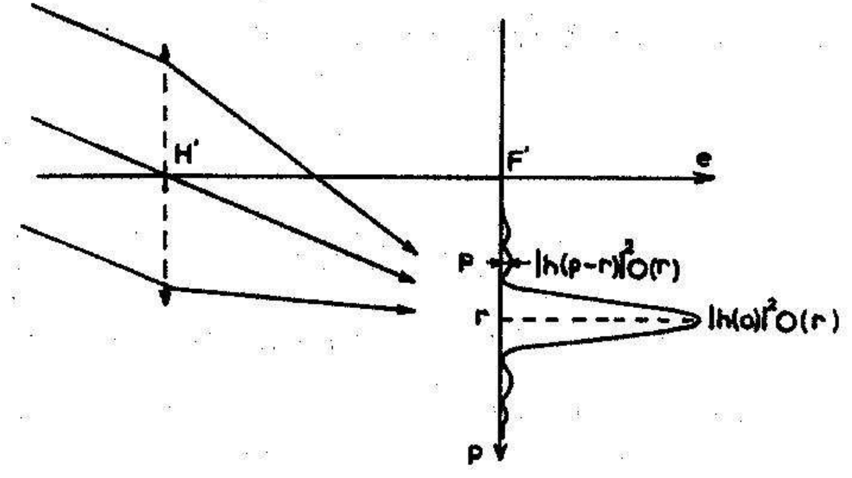


Convolution product of two 1D rectangle functions. A) f(x), B) g(x), C) g(t) and f(x-t); the dashed area represents the integral of the product of f(x-t) and g(t) for the given x offset, D) f(x)∗g(x) = (f∗g)(x) represents the previous integral as a function of x.

An introduction to optical/IR interferometry

8.2 The convolution theorem

$e(p,q) = O(p,q) * |h(p,q)|^2,$



$$e(p,q) = \int_{R^2} O(r,s) \left| h(p-r, q-s) \right|^2 drds$$

An introduction to optical/IR interferometry

8.2 The convolution theorem

For the case of a point-like source:

$O(p,q) = E\,\delta(p,q),$ (8.2.1)

[$\delta(x) = 0$ if $x \neq 0$, $\delta(x) = \infty$ if $x = 0$] and (8.2.2)

$e(p,q) = O(p,q) * |h(p,q)|^2 = E\,\delta(p,q) * |h(p,q)|^2 = E\,|h(p,q)|^2$ (8.2.3)

An introduction to optical/IR interferometry

8.2 The convolution theorem

More generally, since
$TF\_(f * g) = TF\_(f)\ TF\_(g)$. (8.2.4)

We find, because
$e(p,q) = O(p,q) * |h(p,q)|^2$ (8.2.5)

that:
$TF\_(e(p,q)) = TF\_(O(p,q))\ TF\_(|h(p,q)|^2)$, (8.2.6)

and, finally,
$O(p,q) = TF^{-1}\,[TF\_(e(p,q)) / TF\_(|h(p,q)|^2)]$. (8.2.7)

An introduction to optical/IR interferometry

8.2 The convolution theorem

$O(p,q) = (\lambda^2 E / \phi^2)\ \Pi(p\lambda/\phi)\ \Pi(q\lambda/\phi).$ (8.2.8)

$e(p,q) = O(p,q) * |h_0(p,q)|^2.$

$e(p) = O(p) * |h_0(p)|^2,$ (8.2.9)

$$e(p) = 2d^2(\lambda/\phi)\sqrt{E}\int_{p-\phi/2\lambda}^{p+\phi/2\lambda}\left(\frac{\sin(\pi rd)}{\pi rd}\right)^2 \cos^2(\pi rD)dr \quad (8.2.10)$$

$\left(\frac{\sin(\pi rd)}{\pi rd}\right)^2 \approx$ Cte sur $[p-\phi/2\lambda, p+\phi/2\lambda]$, et (8.2.11)

$$e(p) = 2d^2(\lambda/\phi)\sqrt{E}\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2 \int_{p-\phi/2\lambda}^{p+\phi/2\lambda} \cos^2(\pi rD)dr. \quad (8.2.12)$$

An introduction to optical/IR interferometry

8.2 The convolution theorem

$$e(p) = 2d^2\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2 \left[O(p) * \cos^2(\pi pD)\right], \qquad (8.2.13)$$

$$e(p) = 2d^2\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2 \left[\frac{1}{2}\int_R O(p)dp + \frac{1}{2}O(p) * \cos(2\pi pD)\right] \qquad (8.2.14)$$

$$e(p) = A\left[B + \frac{1}{2}\mathrm{Re}\big(O(p) * \exp(i2\pi pD)\big)\right], \qquad (8.2.15)$$

$$A = 2d^2\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2 \quad et \quad B = \frac{1}{2}\int_R O(p)dp, \qquad (8.2.16)$$

An introduction to optical/IR interferometry

8.2 The convolution theorem

$$e(p) = A\left[B + \frac{1}{2}\mathrm{Re}\left(\int_R O(r)\exp\left(i2\pi(p-r)D\right)dr\right)\right], \qquad (8.2.17)$$

$$e(p) = A\left[B + \frac{1}{2}\cos(2\pi pD)\,TF\_(O(r))(D)\right], \qquad (8.2.18)$$

$$\gamma(D) = (e_{max} - e_{min}) / (e_{max} + e_{min}), \qquad (8.2.19)$$

$$\gamma(D) = TF\_(O(r))(D) / (2B) = TF\_(O(r))(D) / \int O(p)\, dp. \qquad (8.2.20)$$

An introduction to optical/IR interferometry

8.3 The Wiener-Khintchin theorem

The **Wiener–Khinchin theorem** states that the power spectral density of a wide-sense-stationary random process is the Fourier transform of the corresponding autocorrelation function. **In our case, this theorem merely states that the Fourier tranform of the PSF (see Eq. (8.2.7)) is the auto-correlation function of the distribution of the complex amplitude in the pupil plane**:

$$TF\left(\left|h(p,q)\right|^2\right) = \iint A^*(x,y)\,A(x+p,y+q)\,dx\,dy$$