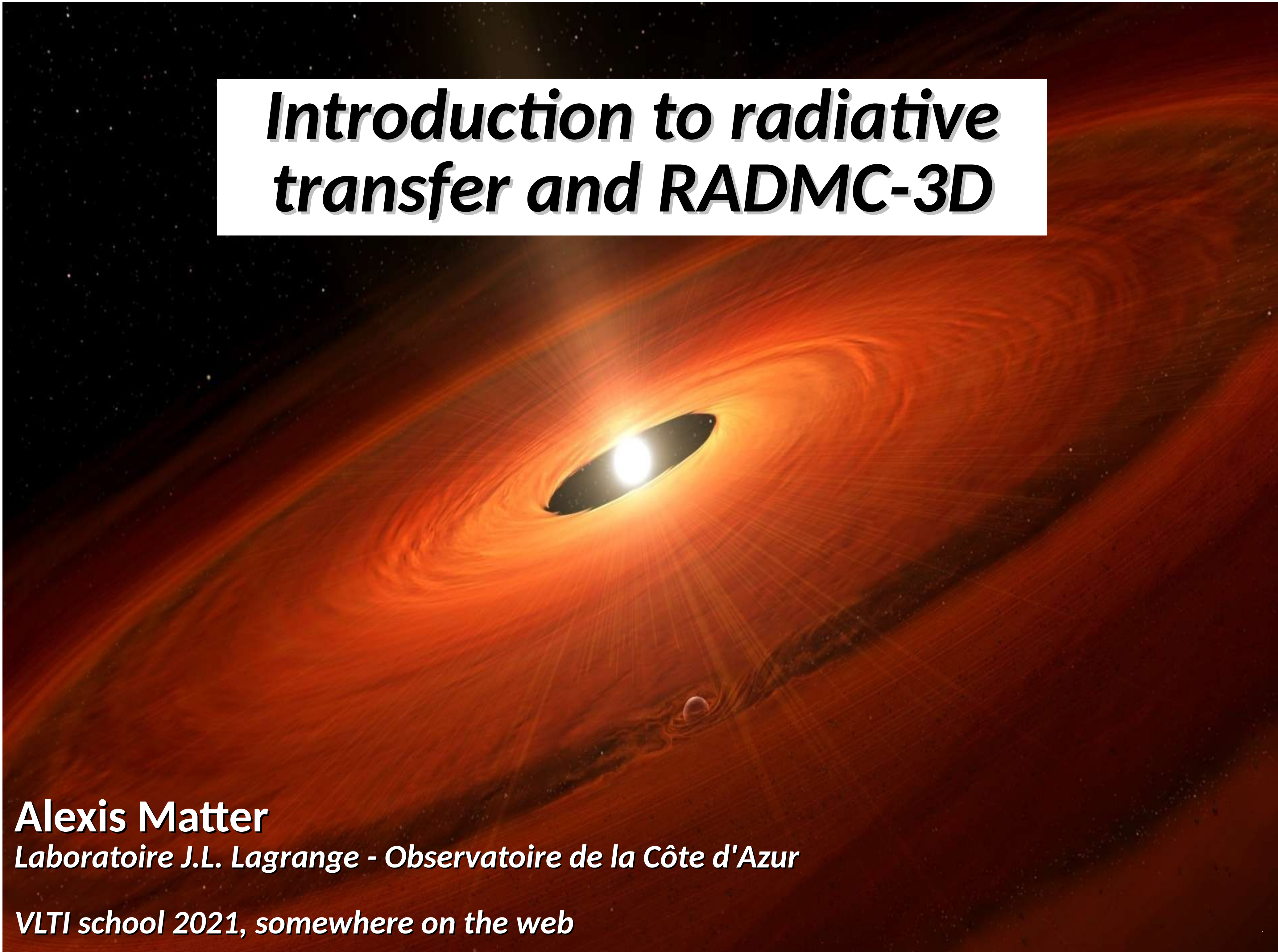


Introduction to radiative transfer and RADMC-3D

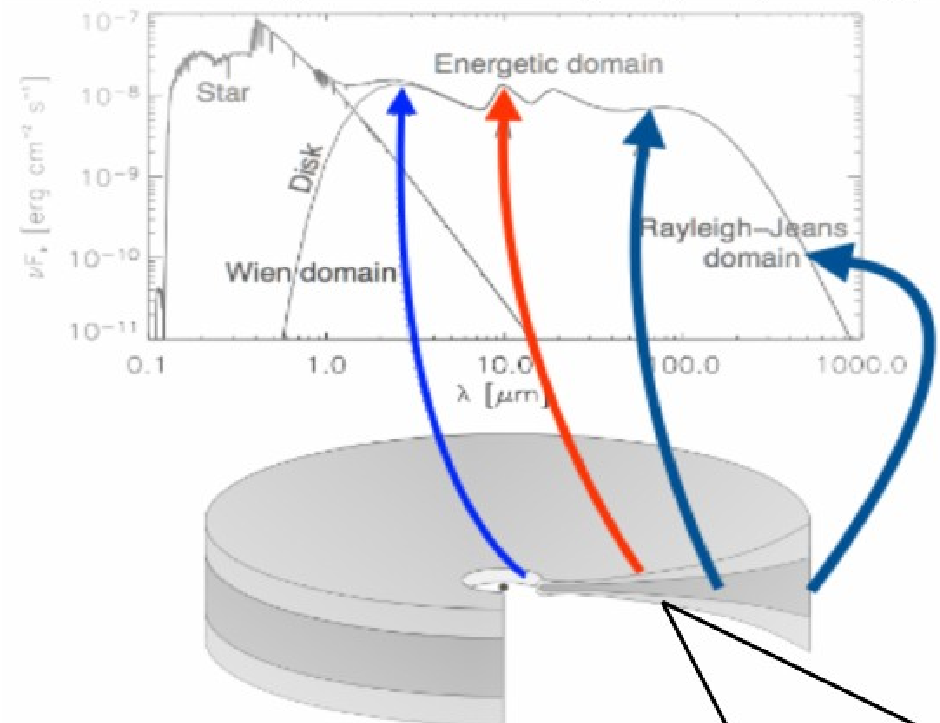
Alexis Matter

Laboratoire J.L. Lagrange - Observatoire de la Côte d'Azur

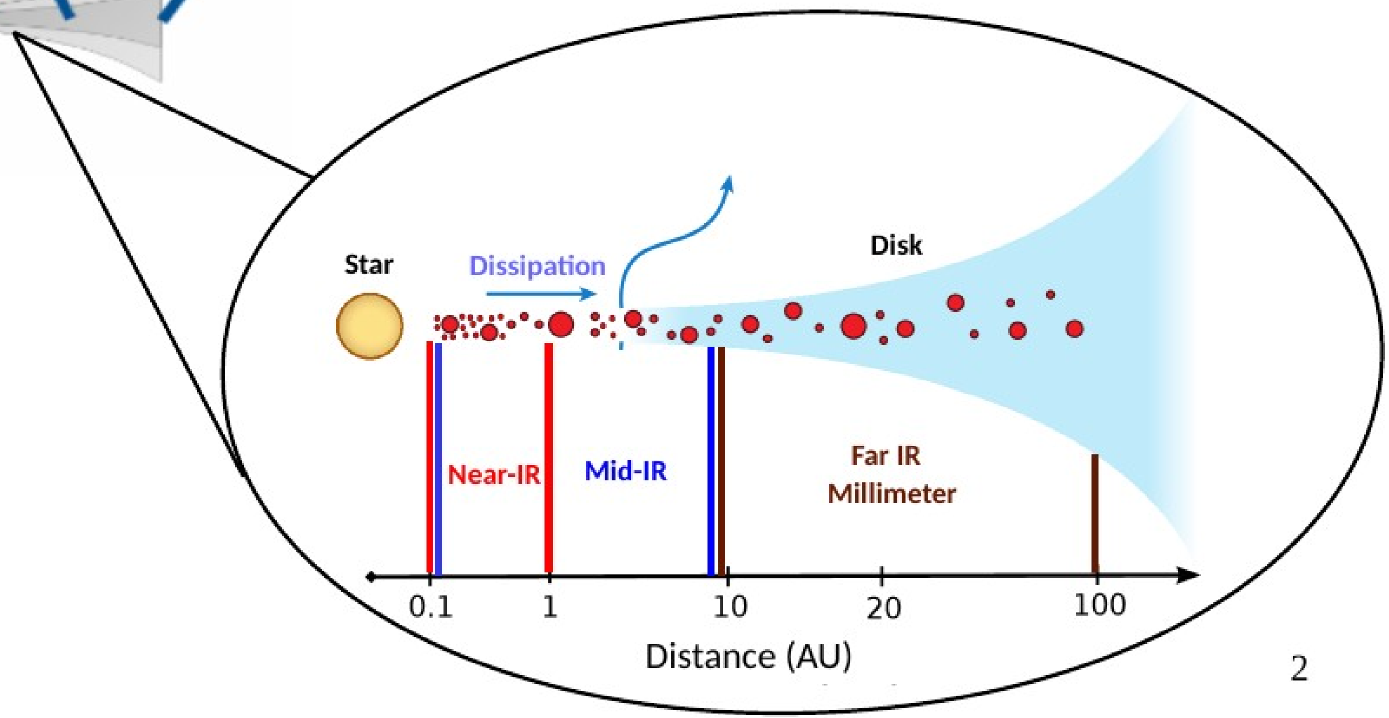
VLTI school 2021, somewhere on the web

Circumstellar disks – Spatial distribution and emission

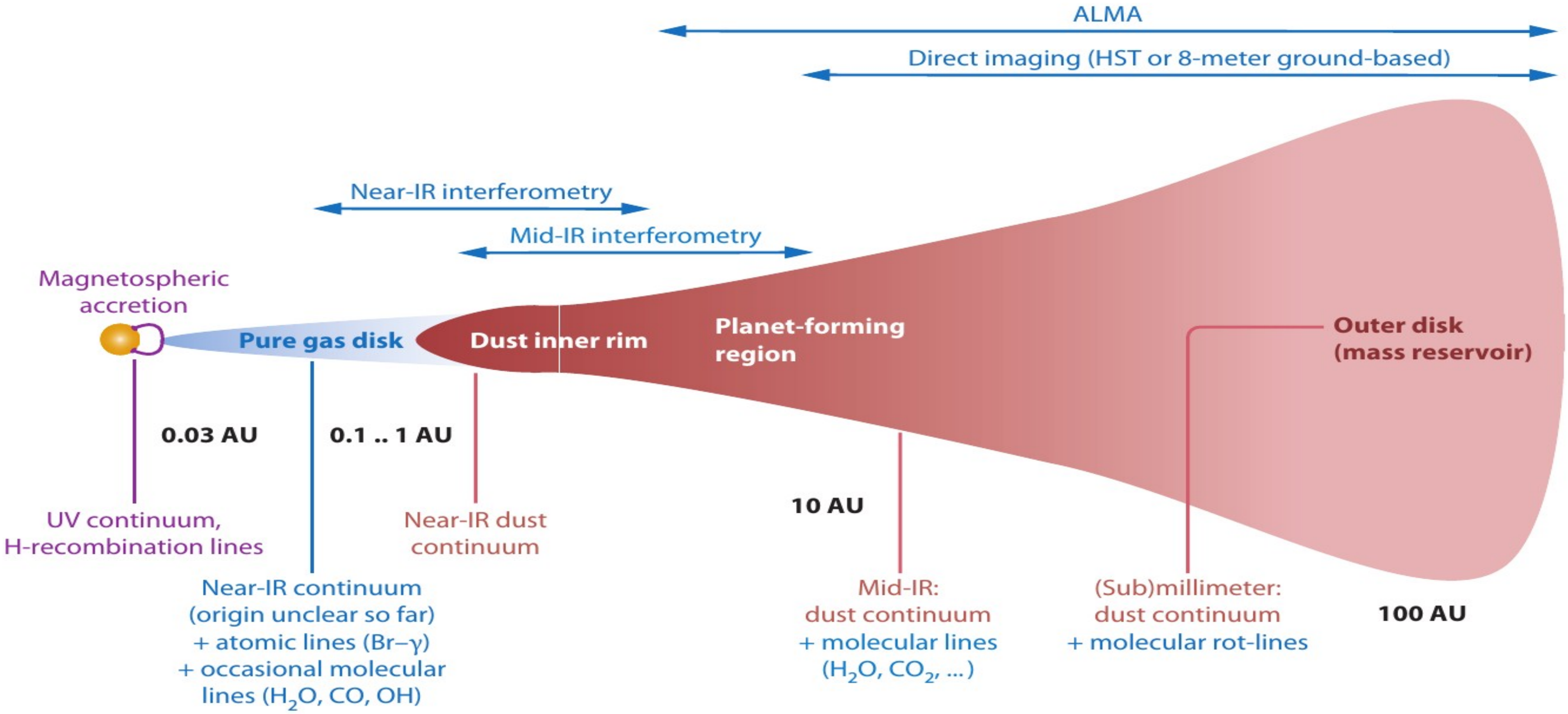
Spectral energy distribution of a young star (< 10 Myr)



Credit : Dullemond (2006)

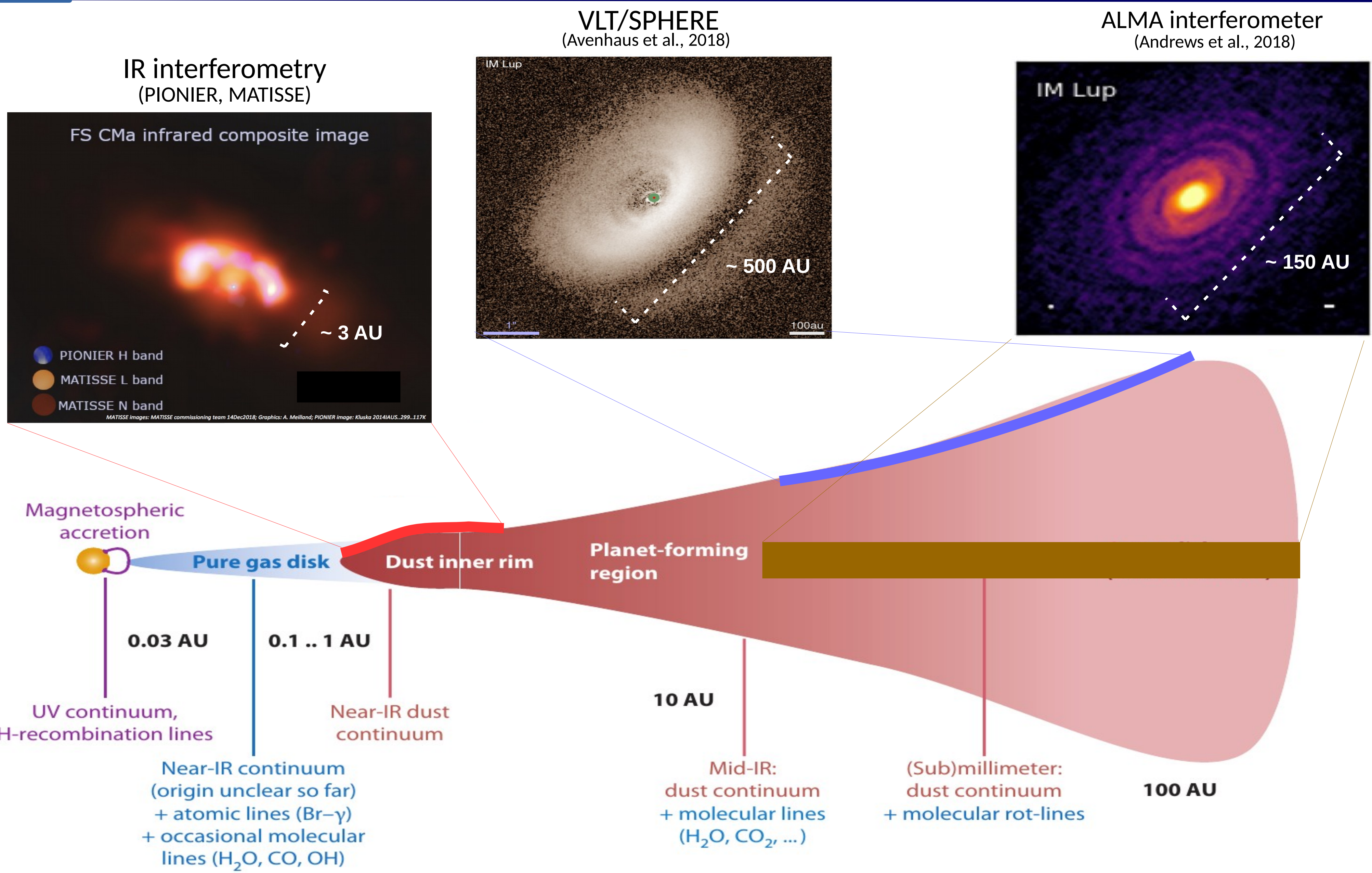


Circumstellar disks – Spatial distribution and emission



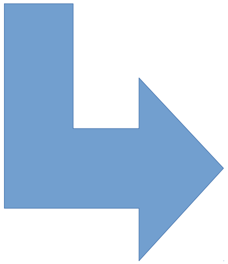
Taken from Dullemond & Monnier (2010)

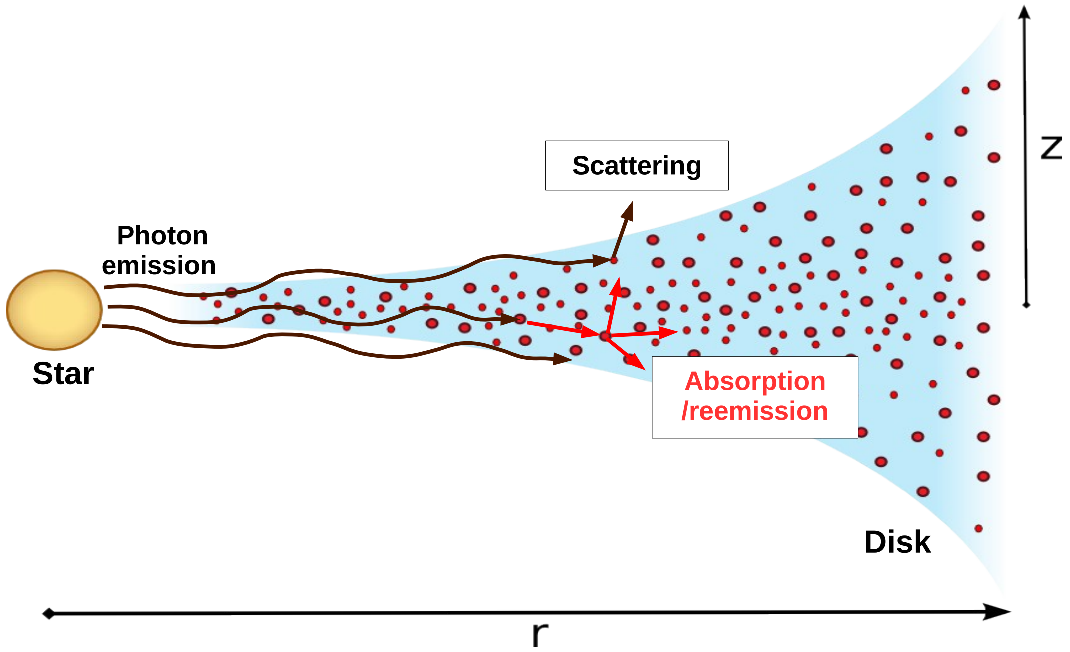
Circumstellar disks – Spatial distribution and emission



Taken from Dullemond & Monnier (2010)

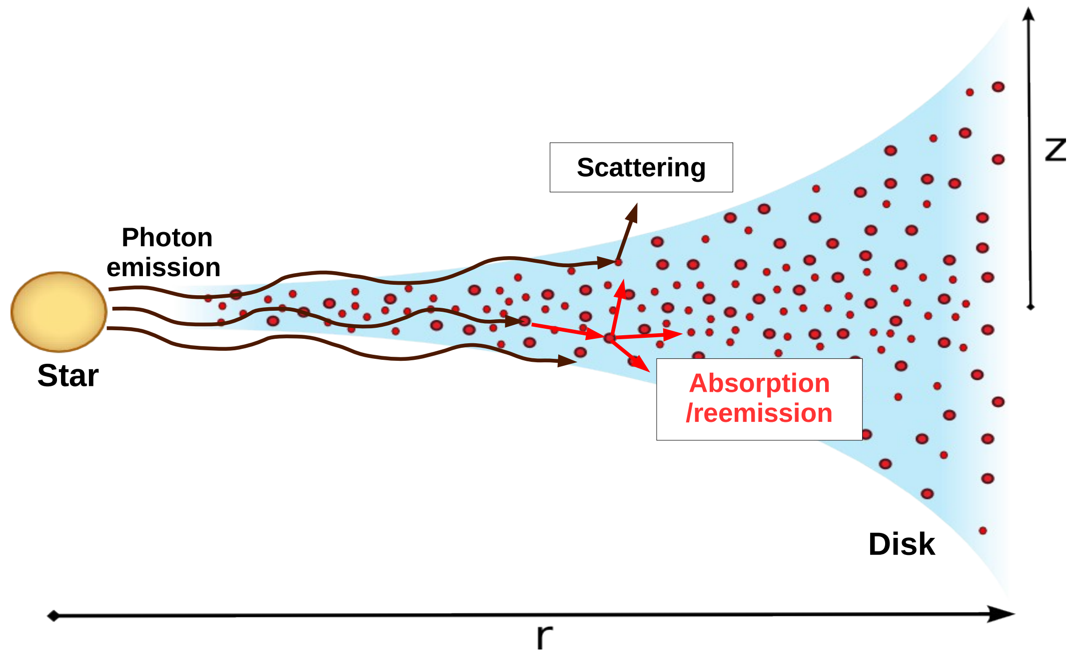
Circumstellar disks – Grain size regime and opacity





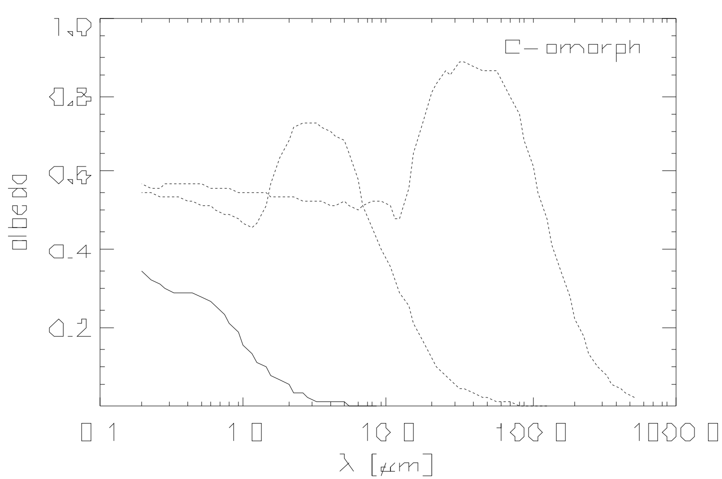
Circumstellar disks – Grain size regime and opacity

matter/radiation interaction

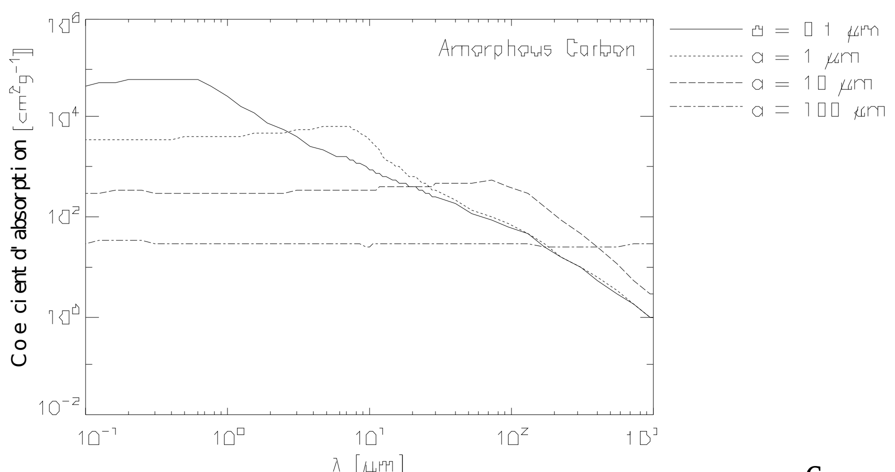


Dependence wavelength / dust grain size

Scattering



Absorption



Circumstellar disks – Grain size regime and opacity

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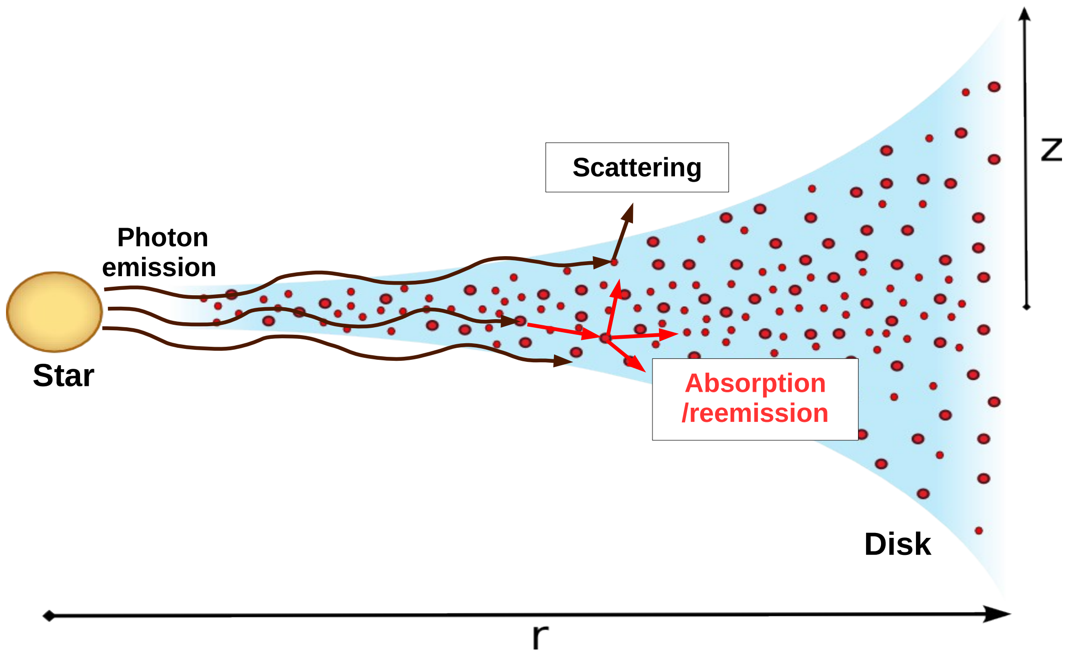

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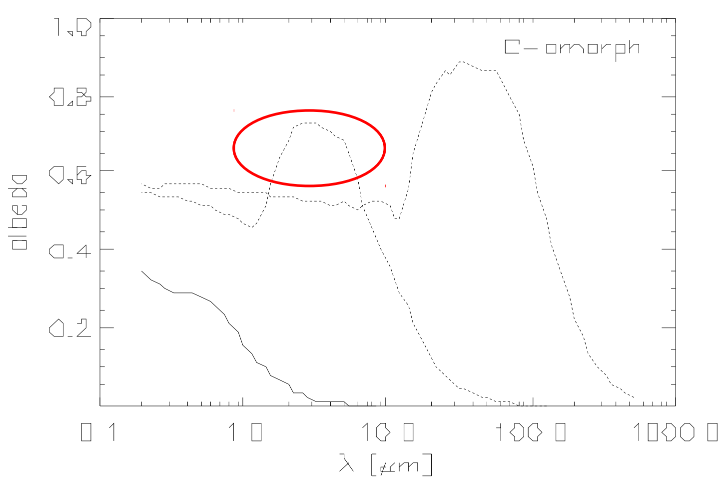
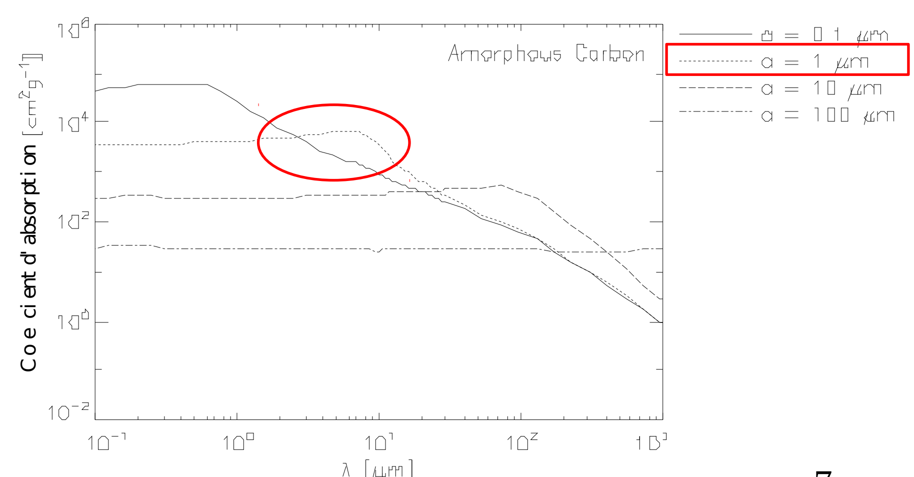
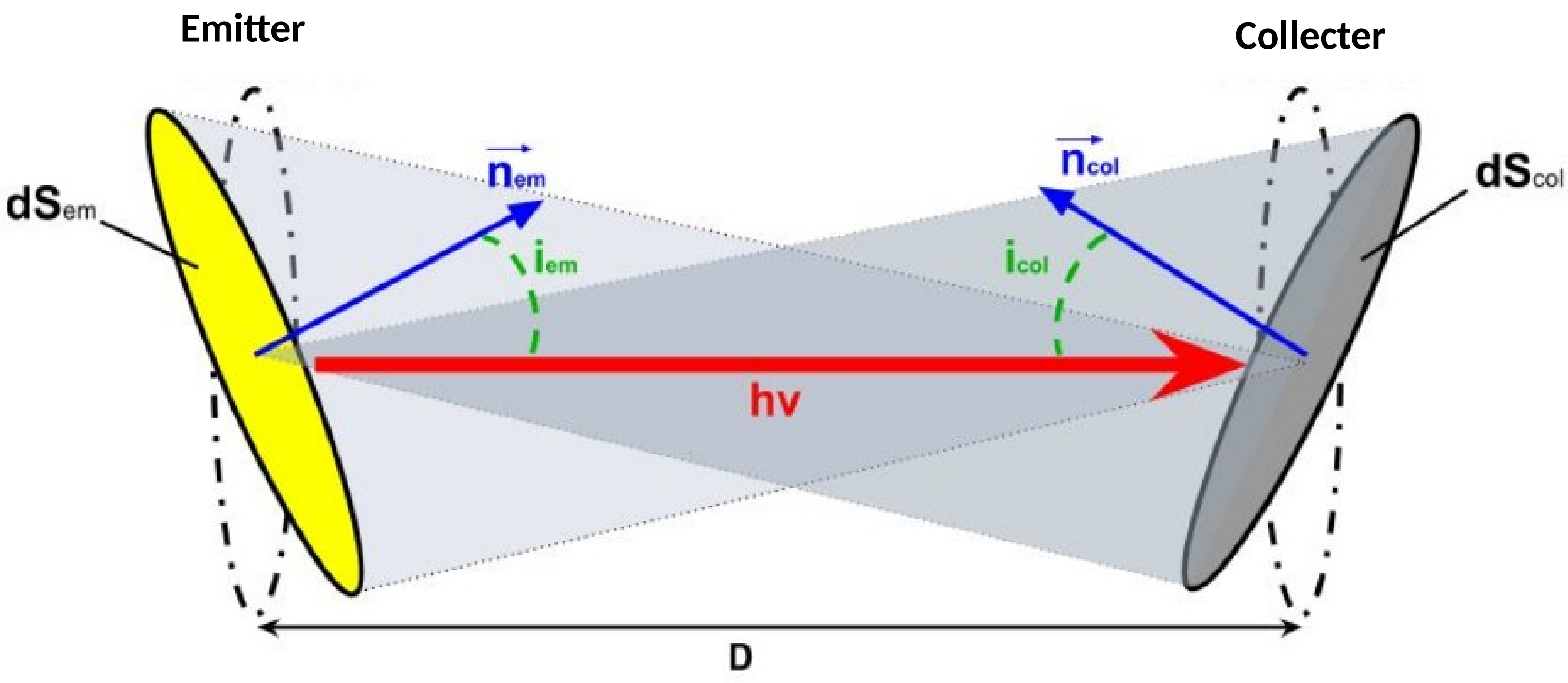


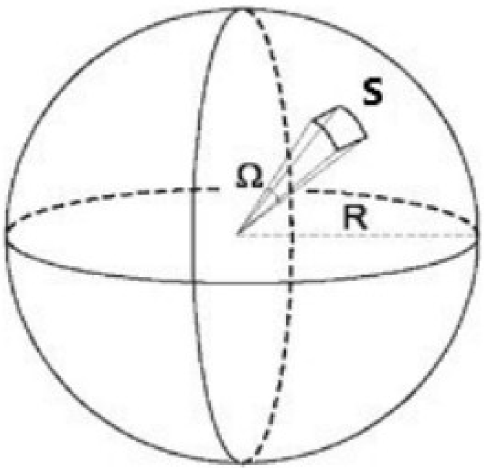
Absorption



Radiative quantities – the geometrical properties

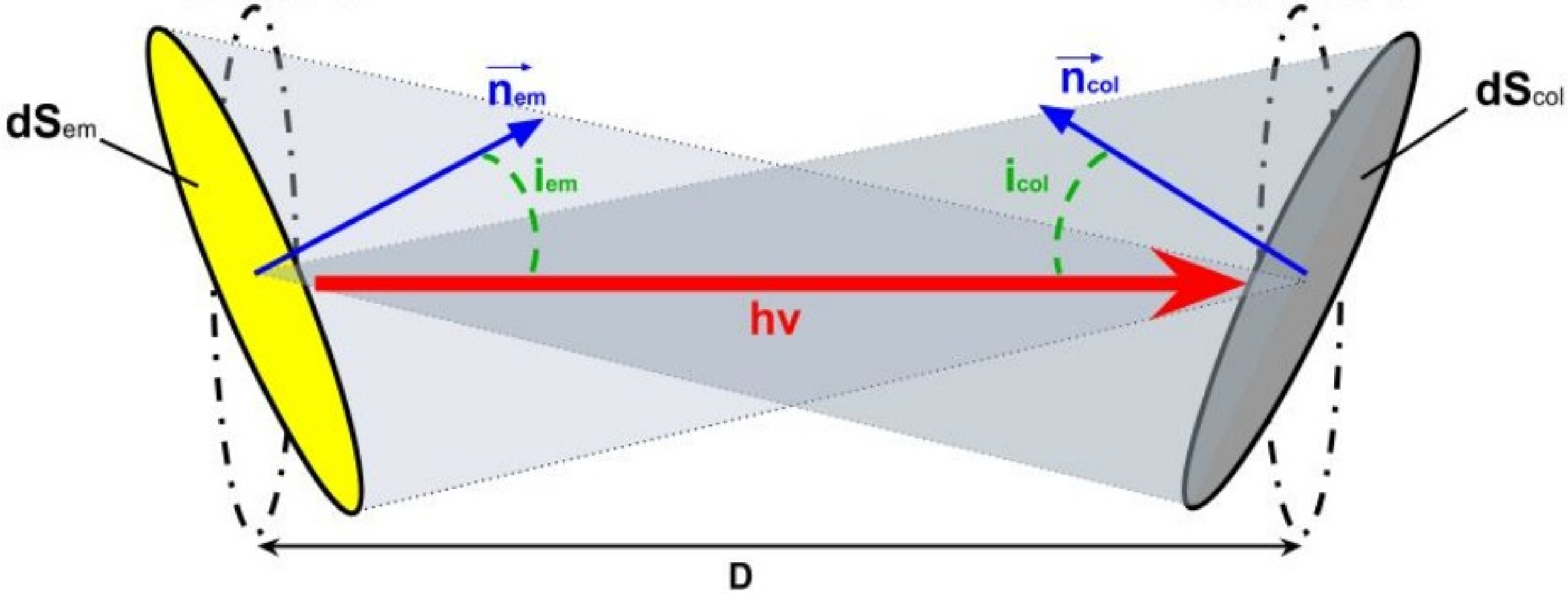


Radiative quantities – the geometrical properties

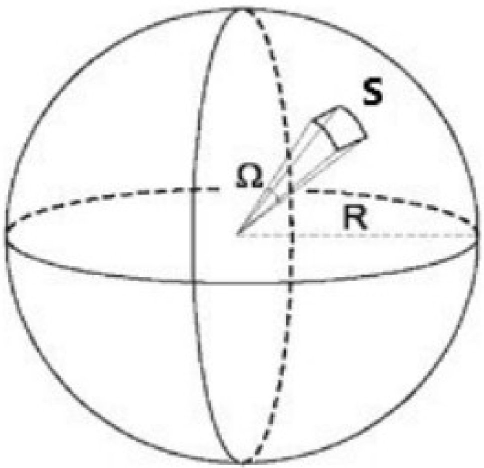


Solid angle

$$\boxed{\Omega = S / R^2}$$



Radiative quantities – the geometrical properties



Solid angle

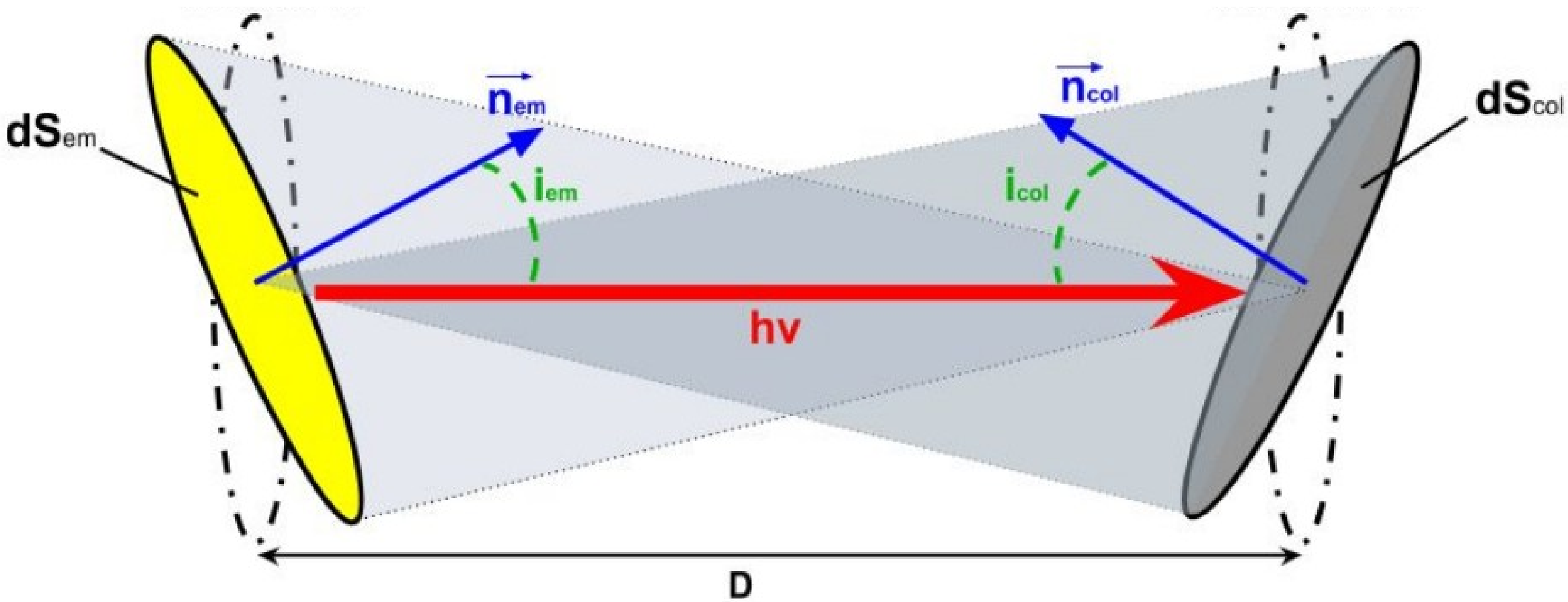
$$\boxed{\Omega = S / R^2}$$

Beam etendue

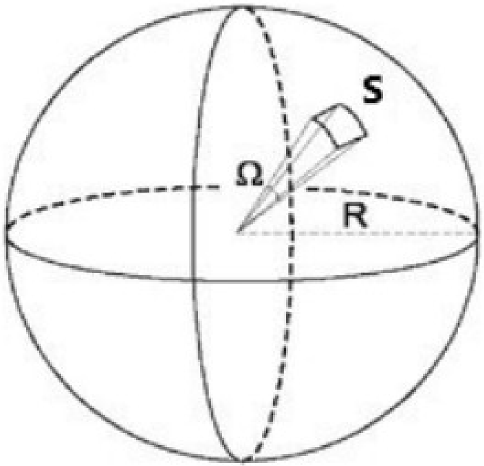
$$\boxed{\varepsilon = \frac{dS_{em}\cos(i_{em})\, dS_{col}\cos(i_{col})}{D^2}}$$

Emitter

Collecter



Radiative quantities – the geometrical properties



Solid angle

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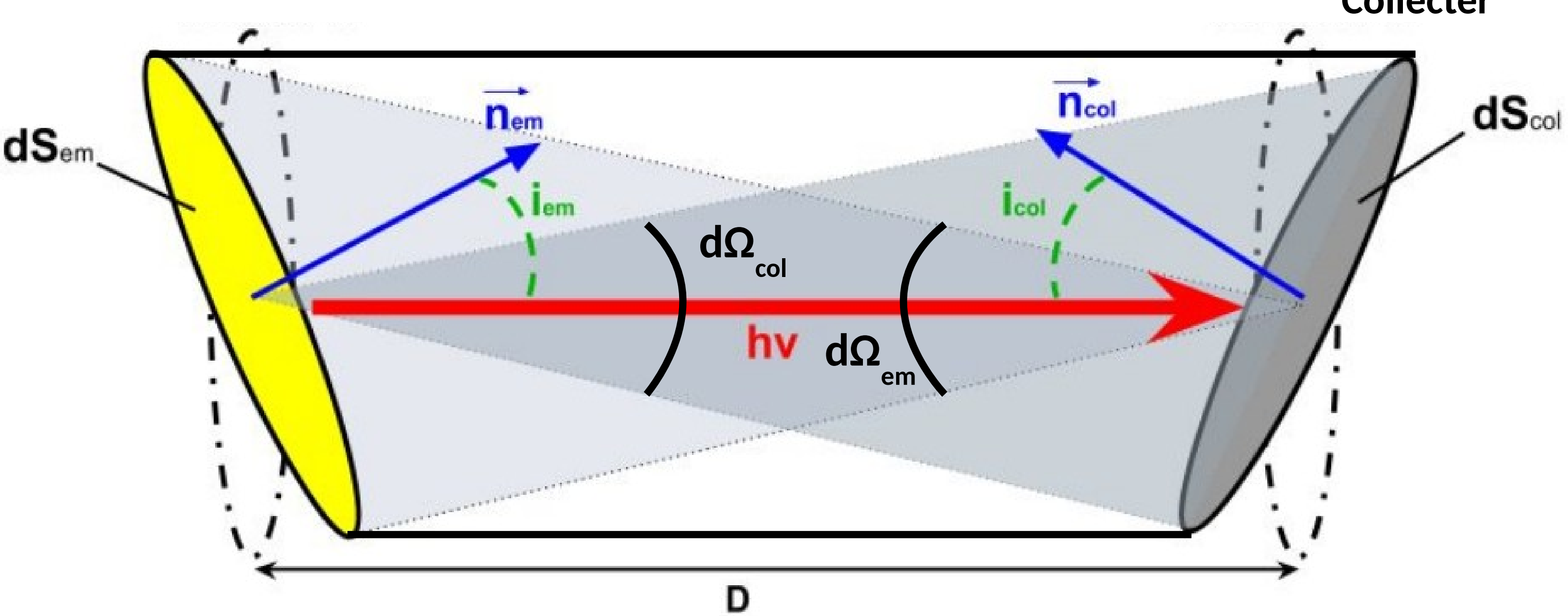
Beam etendue

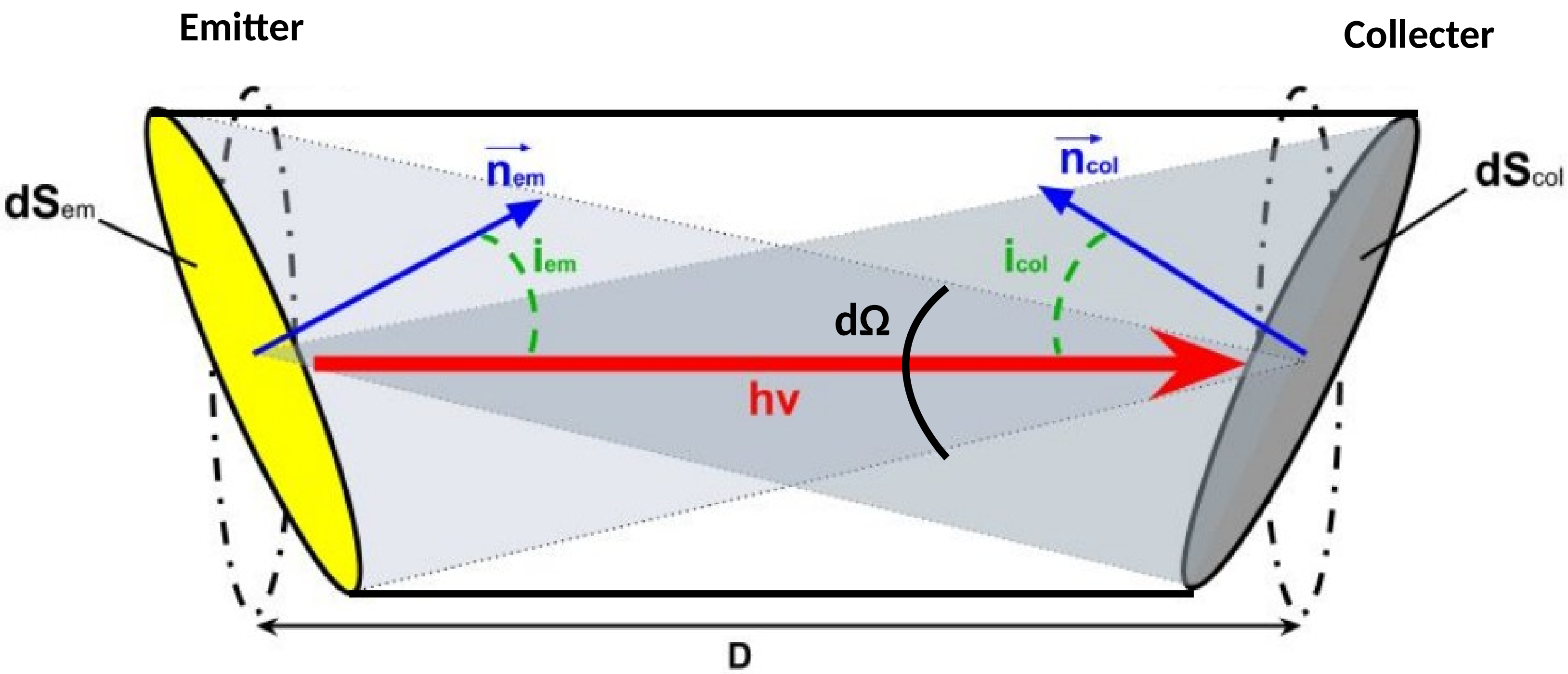
$$\boxed{\varepsilon = \frac{dS_{em} \cos(i_{em})\, dS_{col} \cos(i_{col})}{D^2}}$$

$$\boxed{\varepsilon = d\Omega_{em} * dS_{col}}$$

$$\boxed{\varepsilon = d\Omega_{col} * dS_{em}}$$

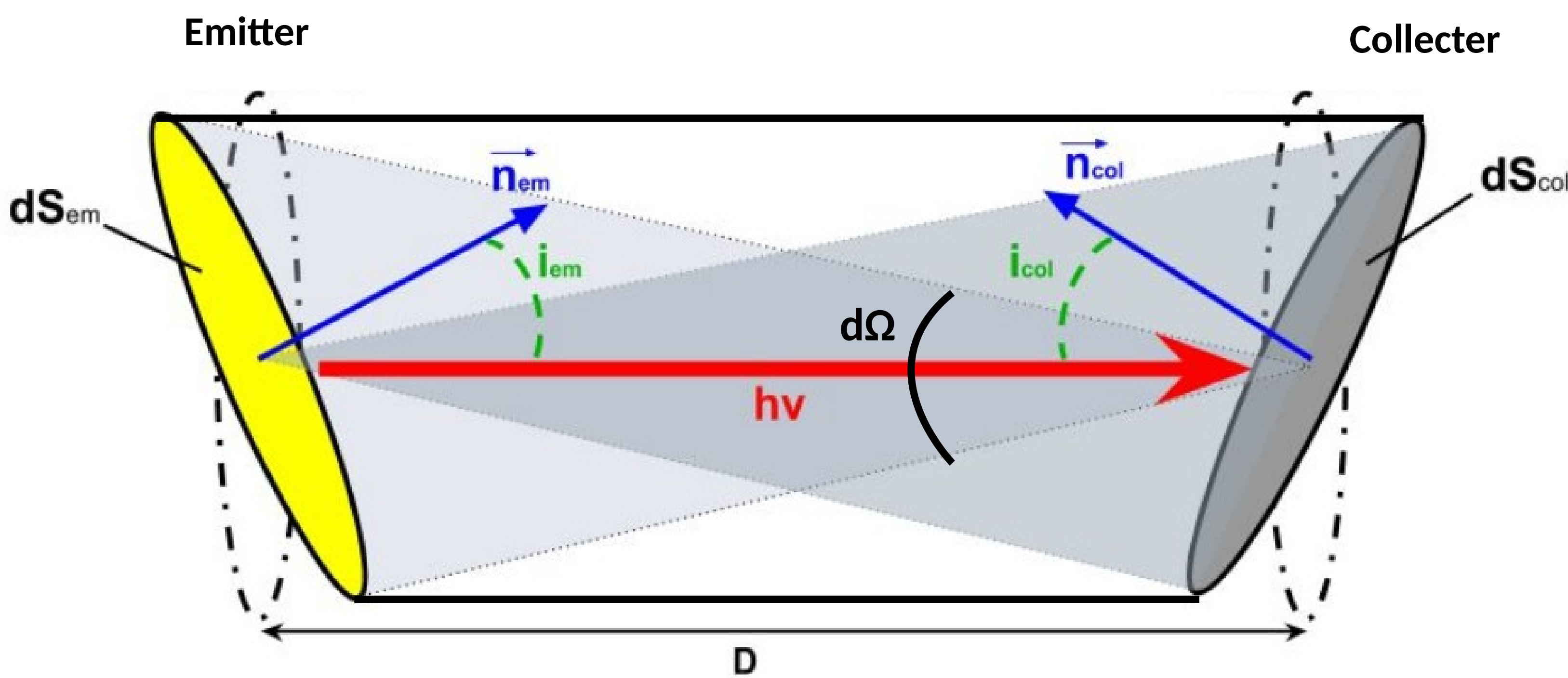


Radiative quantities – Definitions



Beam: « Tube » along which some energy is transported from an emitting surface to a collecting surface

Radiative quantities – Definitions

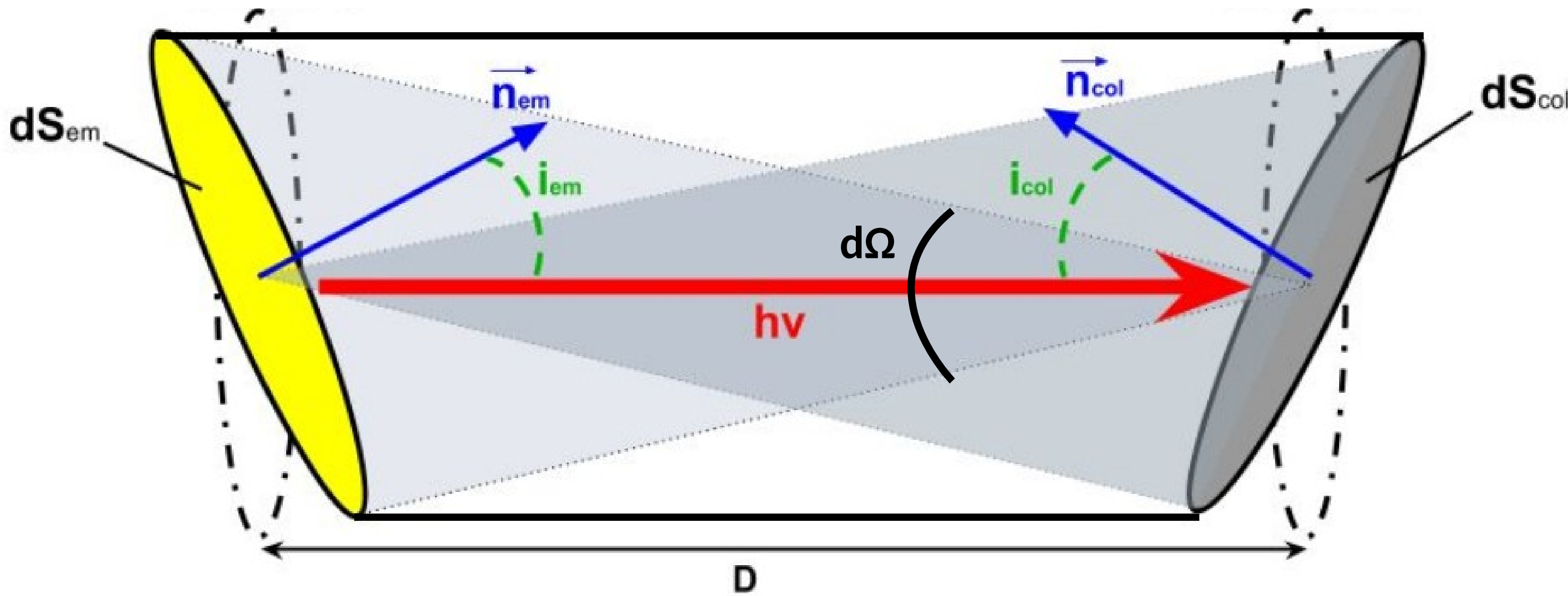


Beam: « Tube » along which some energy is transported from an emitting surface to a collecting surface

Energy $\Rightarrow$ $dE_\lambda = I_\lambda \, dt \, d\lambda \, dS_{col} \, d\Omega_{em}$ (unit : J)

$\Rightarrow$ I_λ : **Intensity** (unit : $J.s^{-1}.m^{-1}.m^{-2}.ste^{-1}$)

Radiative quantities – Definitions



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Energy → $dE_\lambda = I_\lambda \, dt \, d\lambda \, dS_{col} \, d\Omega_{em}$ (unit : J)

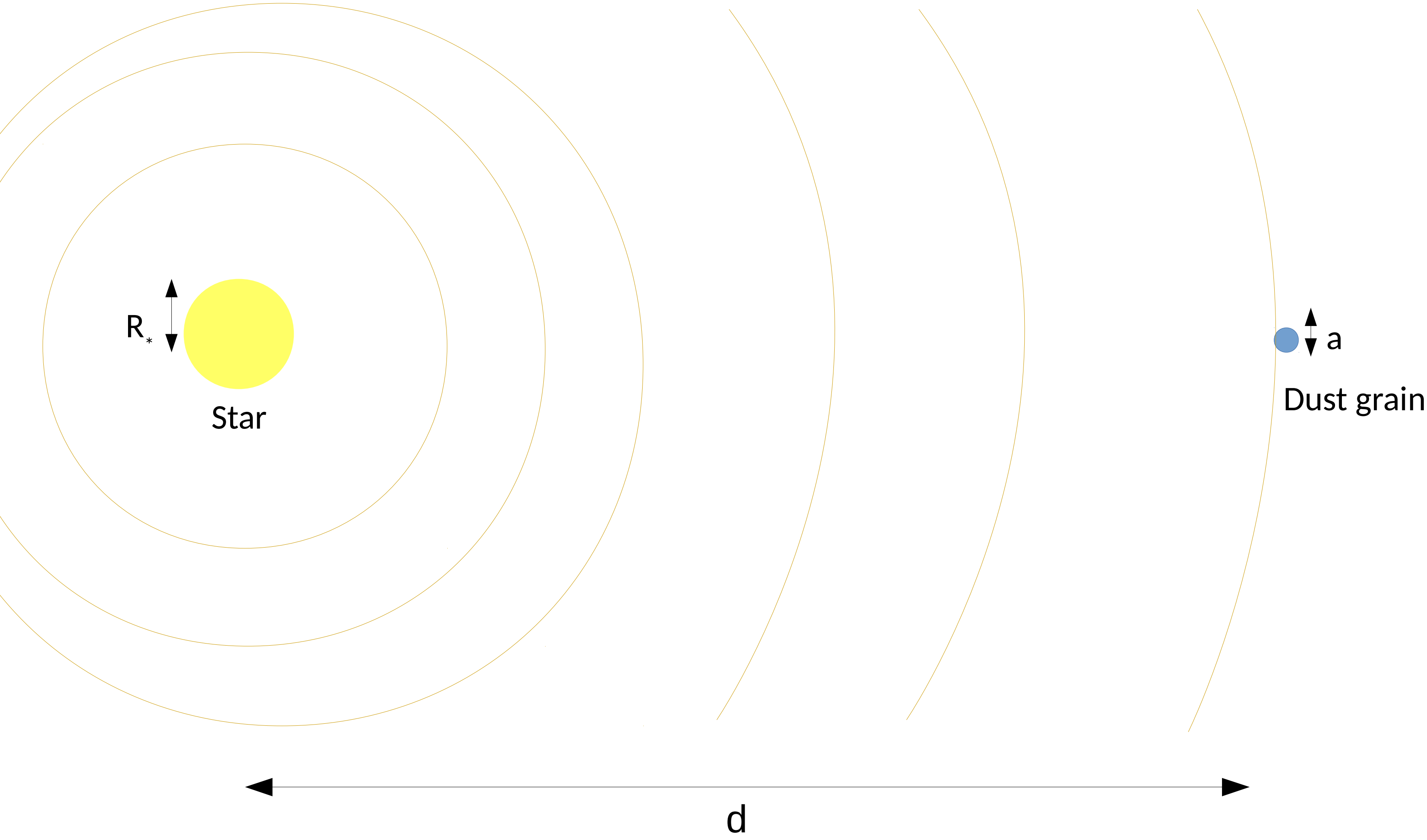
→ I_λ : **Intensity** (unit : $J.s^{-1}.m^{-1}.m^{-2}.ste^{-1}$)

Power → $dW_\lambda = I_\lambda \, d\lambda \, dS_{col} \, d\Omega_{em}$ (unit : $J.s^{-1}$)

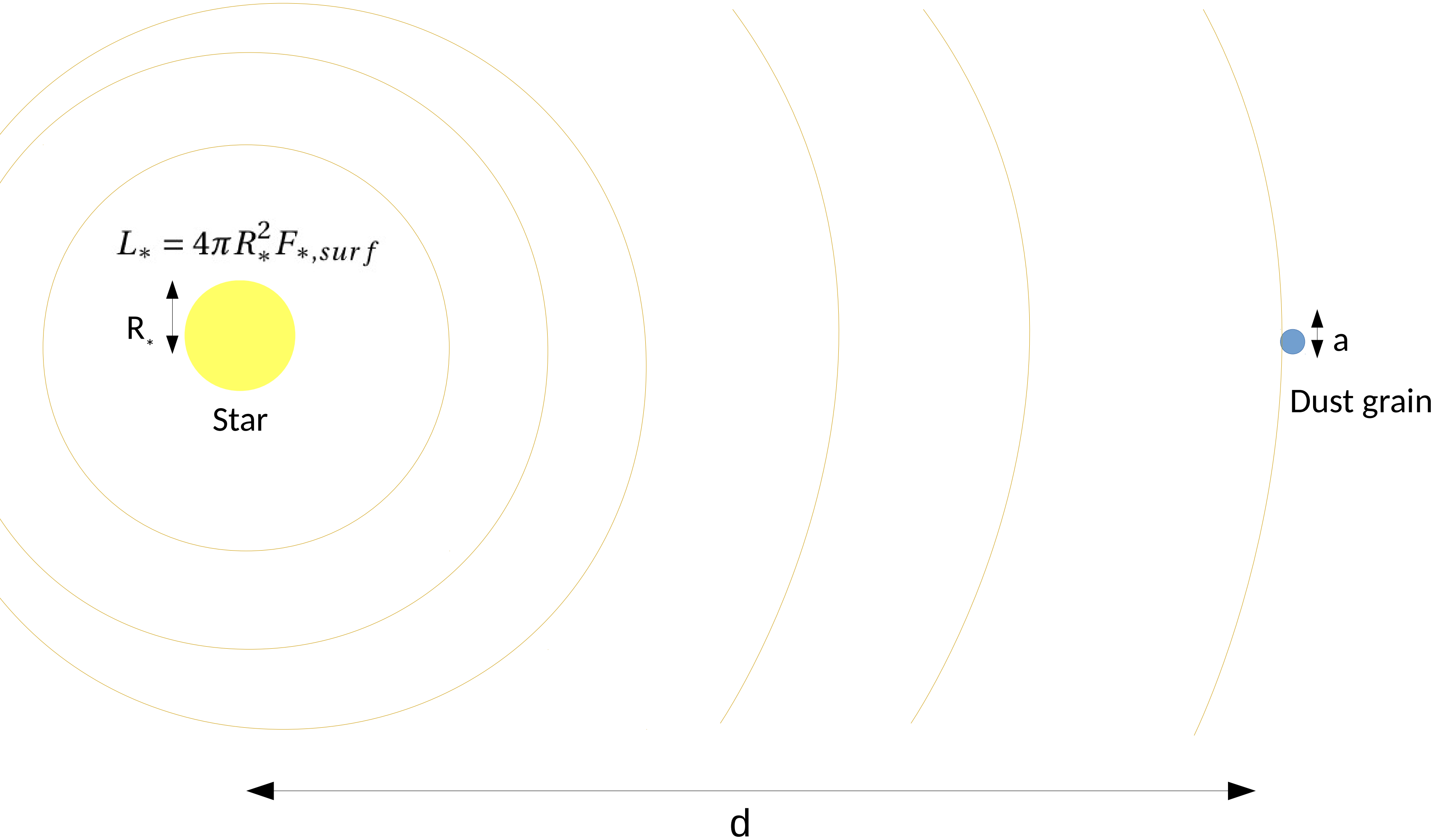
Bolometric flux → $dF_\lambda = I_\lambda \, d\lambda \, d\Omega_{em}$ (unit : $J.s^{-1}.m^{-2}$)

Monochromatic flux (or flux density) → $dF_\lambda = I_\lambda \, d\Omega_{em}$ (unit : $J.s^{-1}.m^{-1}.m^{-2}$)

Radiative quantities – Illustration

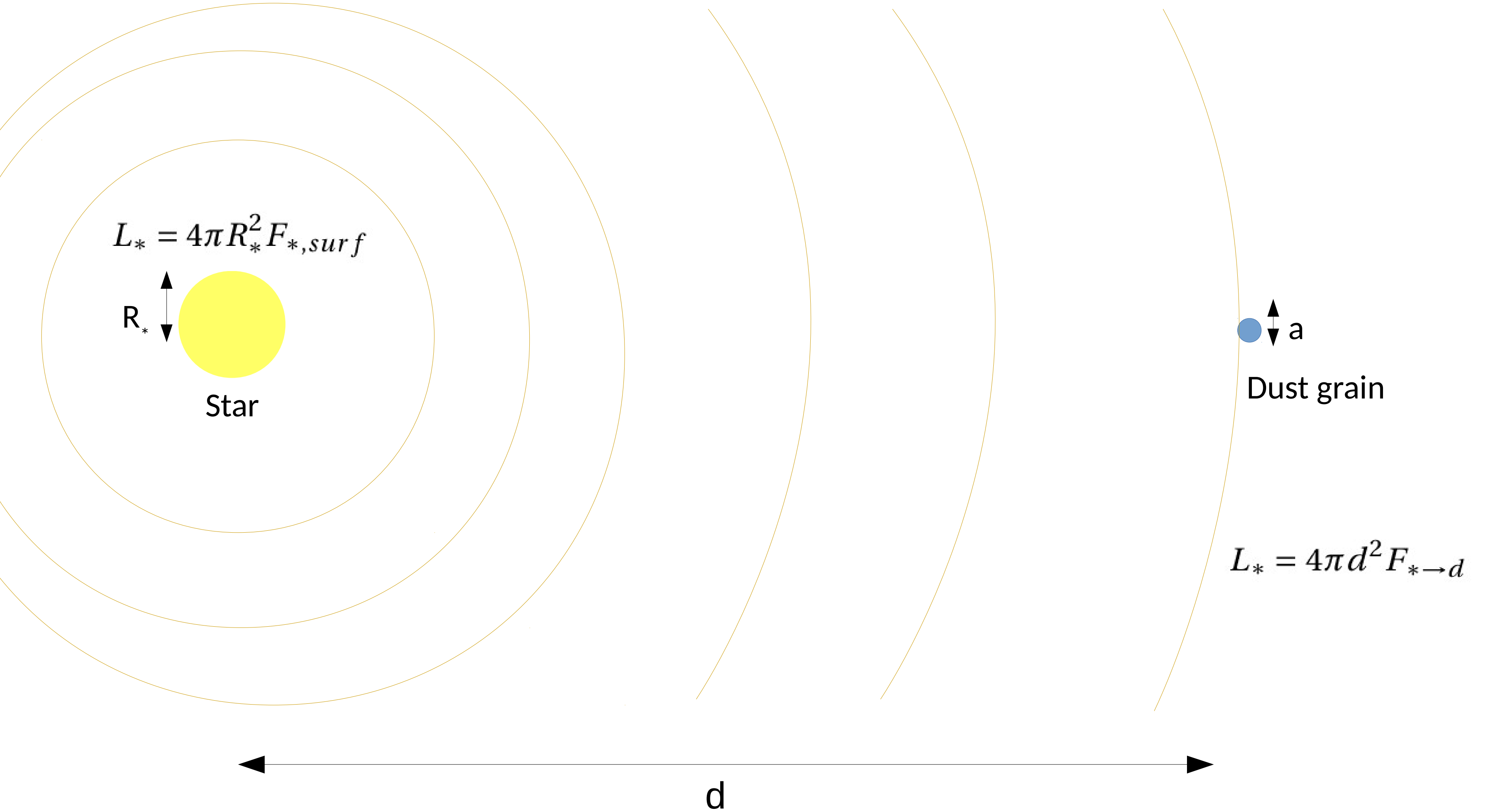


Radiative quantities – Illustration



$L_* = 4\pi R_*^2 F_{*,surf}$ Star luminosity = power emitted by the whole surface of the star

Radiative quantities – Illustration

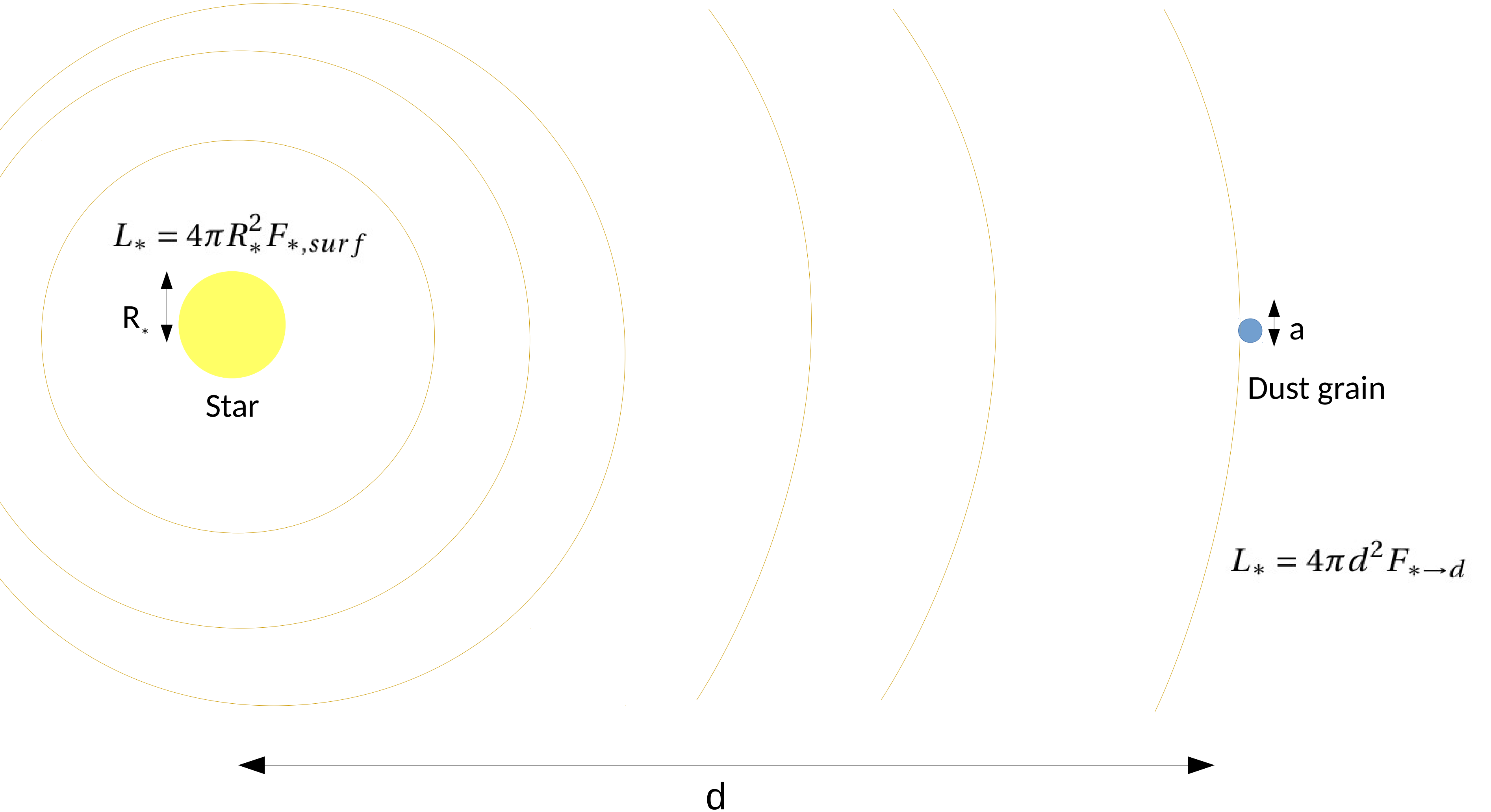


$L_* = 4\pi R_*^2 F_{*,surf}$ Star luminosity = power emitted by the whole surface of the star

$L_* = 4\pi d^2 F_{*\to d}$ Same power BUT diluted over a sphere of $4\pi d^2$

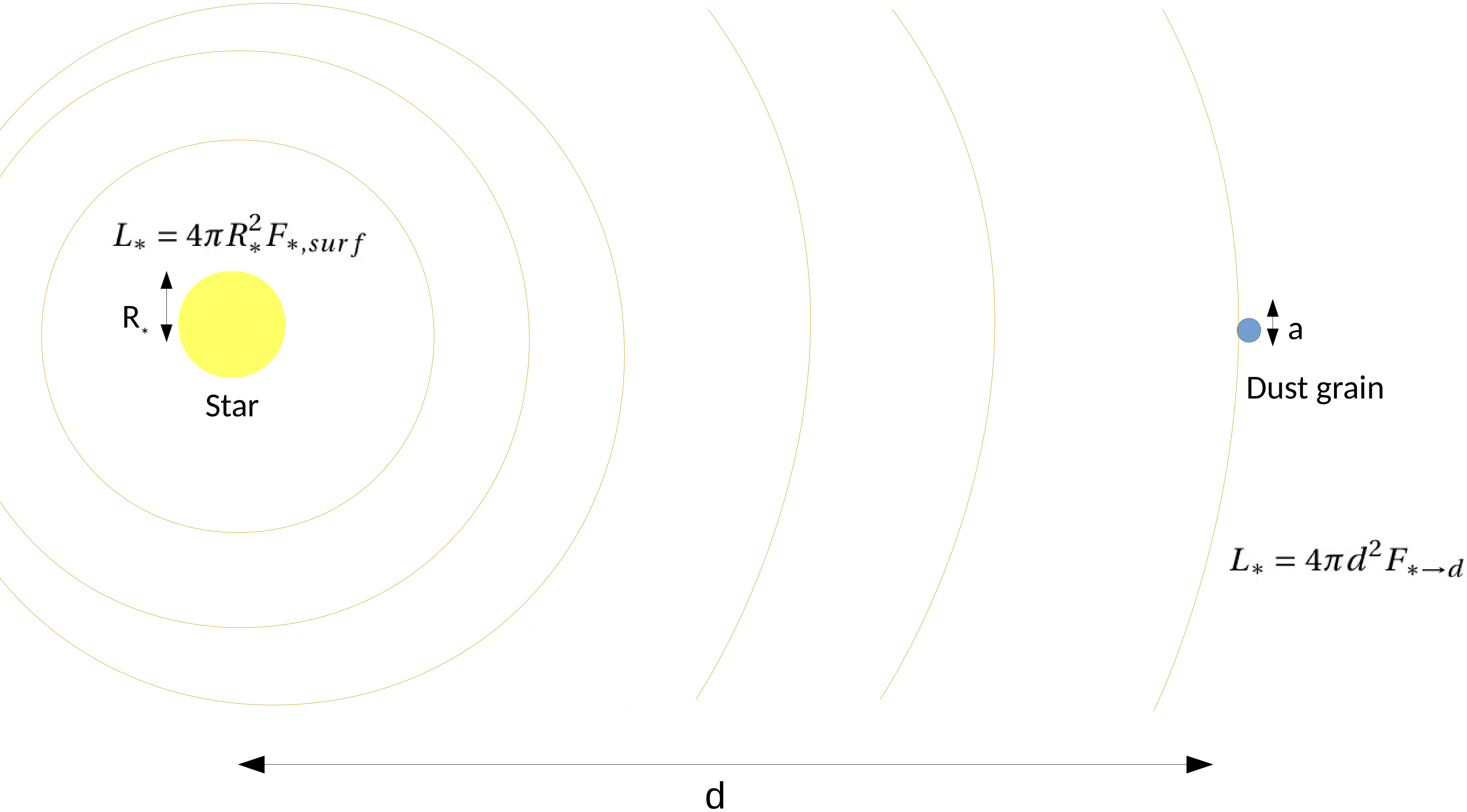
RADMC-3D

Radiative quantities – Illustration



L_*	$=$	L_*
$F_{*,surf}$	$\neq$	$F_{*\to d}$

Radiative quantities – Illustration



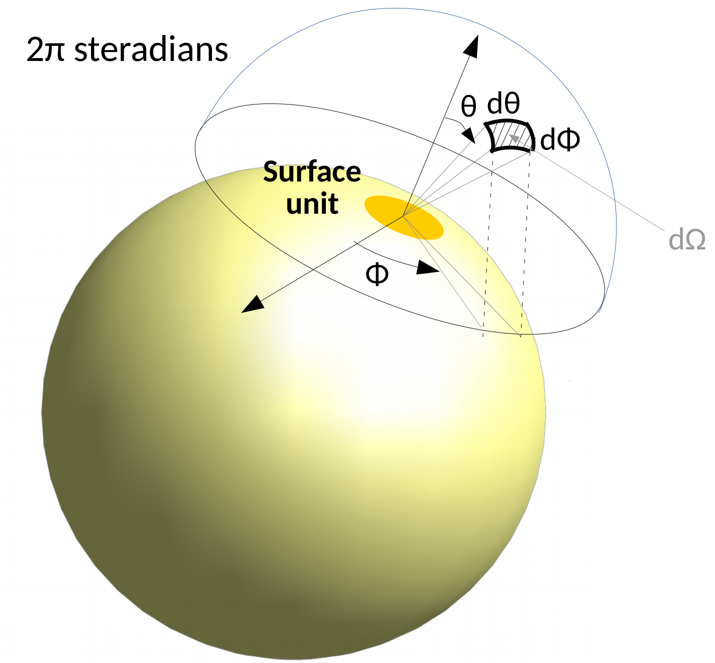
Flux irradiated by the star
AND received at the distance d

$$F_{*\to d} = \frac{4\pi R_*^2}{4\pi d^2} F_{*,surf} = \frac{R_*^2}{d^2} F_{*,surf}$$

Radiative quantities – Illustration

What is the expression of $F_{*,surf}$?

Flux = Energy radiated **OUTWARD / time unit / surf. unit**



Radiative quantities – Illustration

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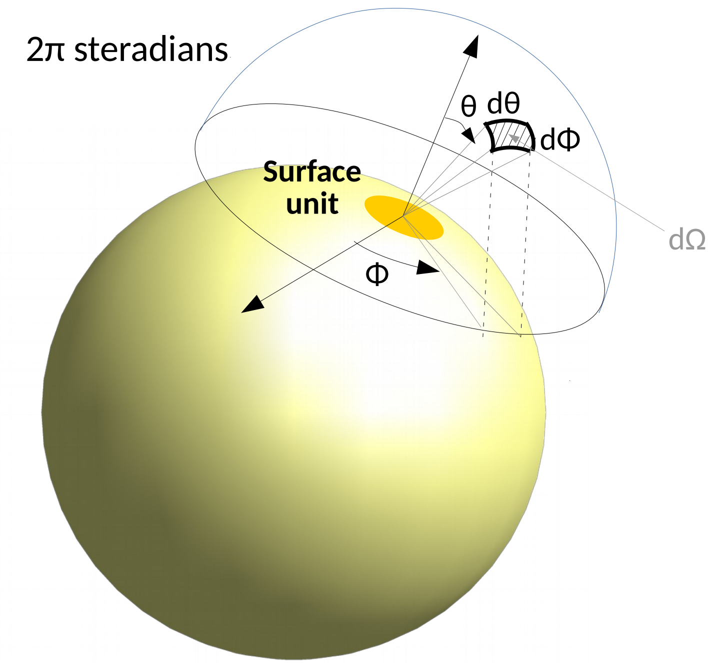

Flux = Energy radiated **OUTWARD / time unit / surf. unit**

$dF_\lambda = I_\lambda \, d\lambda \, d\Omega_{em}$

(unit : $J.s^{-1}.m^{-2}$)

Over a solid angle of 2π steradians above the surface unit

Radiative quantities – Illustration

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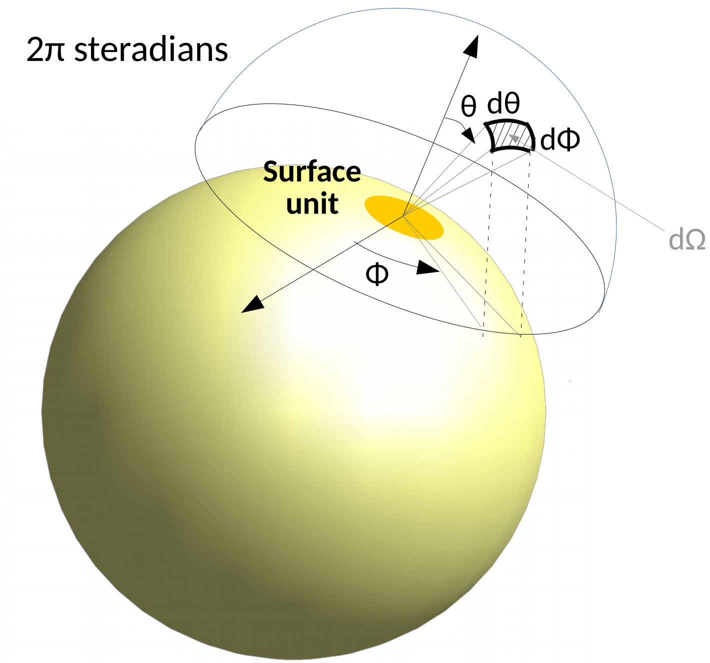

Star = Blackbody
→ $I(\lambda) = B_\lambda(T_*)$ (unit : $J.s^{-1}.m^{-1}.m^{-2}.ste$)

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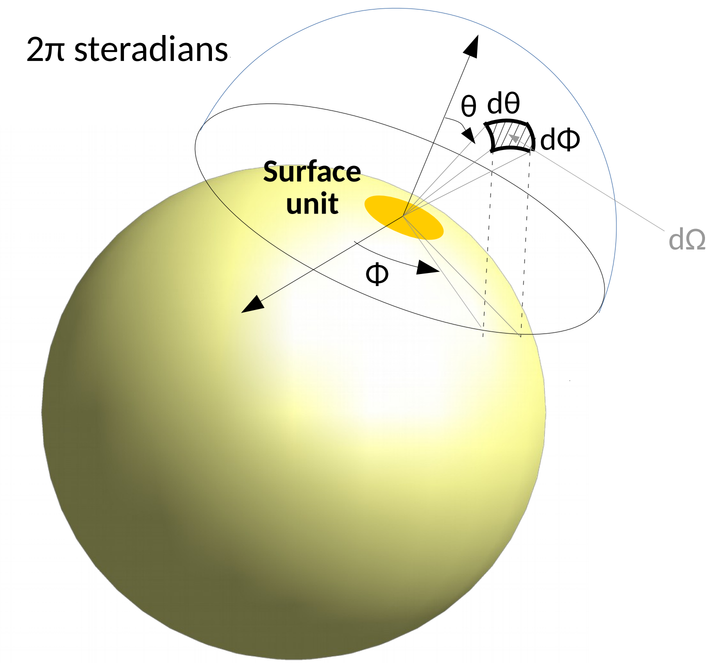

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Over a solid angle of 2π steradians above the surface unit

$$F_{*,surf} = \underbrace{\int_{\lambda=0}^{\lambda=\infty} B_\lambda(T_*)\, d\lambda}_{\text{Integration over } \lambda} \; \underbrace{\int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\frac{\pi}{2}} cos(\theta) sin(\theta)\, d\theta d\phi}_{\text{Integration over } 2\pi \text{ steradians}}$$

Radiative quantities – Illustration

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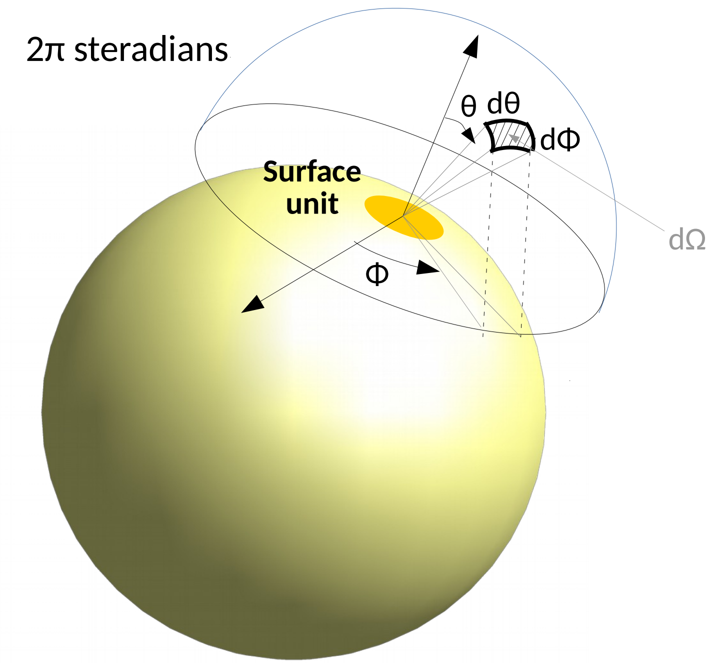

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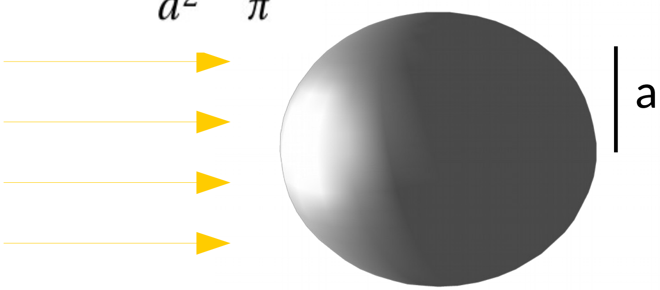
$$F_{*,surf} = \pi \int_0^\infty B_\lambda(T_*) d\lambda = \frac{\pi \sigma T_*^4}{\pi}$$

$$\boxed{F_{*,surf} = \sigma T_*^4}$$

(unit : $J.s^{-1}.m^{-2}$)

Radiative quantities – Illustration

$$F_{*\to d} = \frac{\pi R_*^2}{d^2}\frac{\sigma T_*^4}{\pi}$$

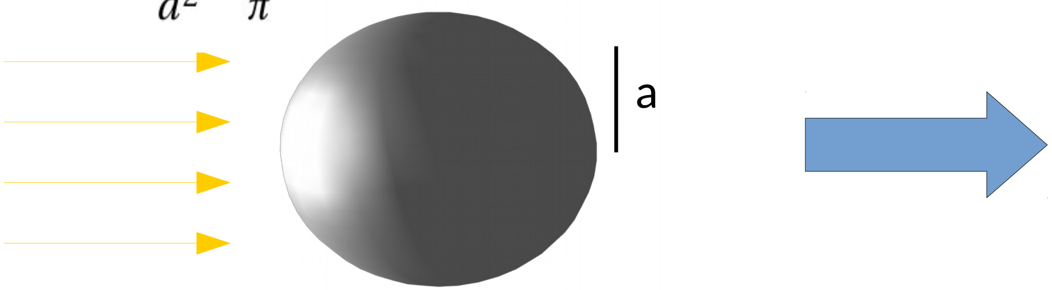


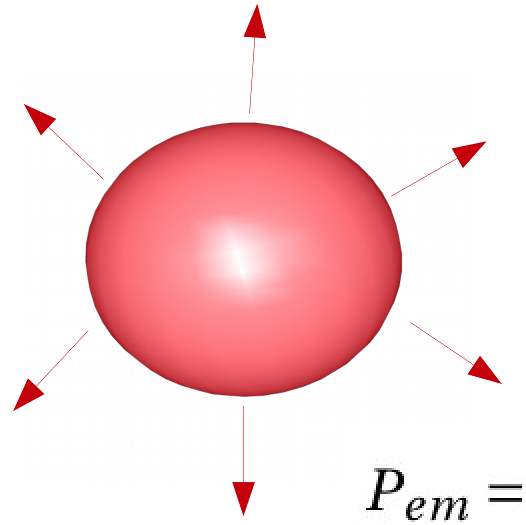
$$P_{col} = \frac{R_*^2 \sigma T_*^2}{d^2}\pi a^2$$

Collected power

Radiative quantities – Illustration

$$F_{*\to d} = \frac{\pi R_*^2}{d^2}\frac{\sigma T_*^4}{\pi}$$





$$P_{col} = \frac{R_*^2 \sigma T_*^2}{d^2}\pi a^2$$

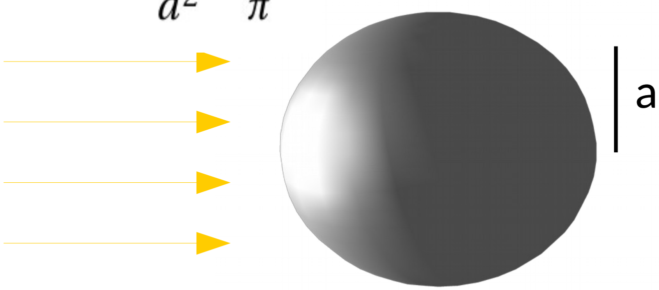
Collected power

$$P_{em} = 4\pi a^2 \sigma T_g^4$$

Reemitted power

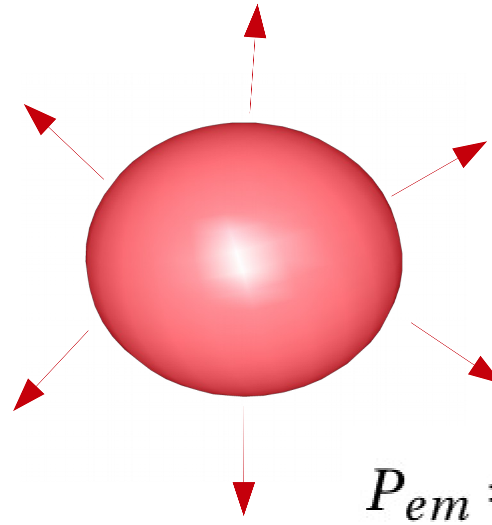
Radiative quantities – Illustration

$$F_{*\to d} = \frac{\pi R_*^2}{d^2}\frac{\sigma T_*^4}{\pi}$$



$$P_{col} = \frac{R_*^2 \sigma T_*^2}{d^2}\pi a^2$$

Collected power



$$P_{em} = 4\pi a^2 \sigma T_g^4$$

Reemitted power

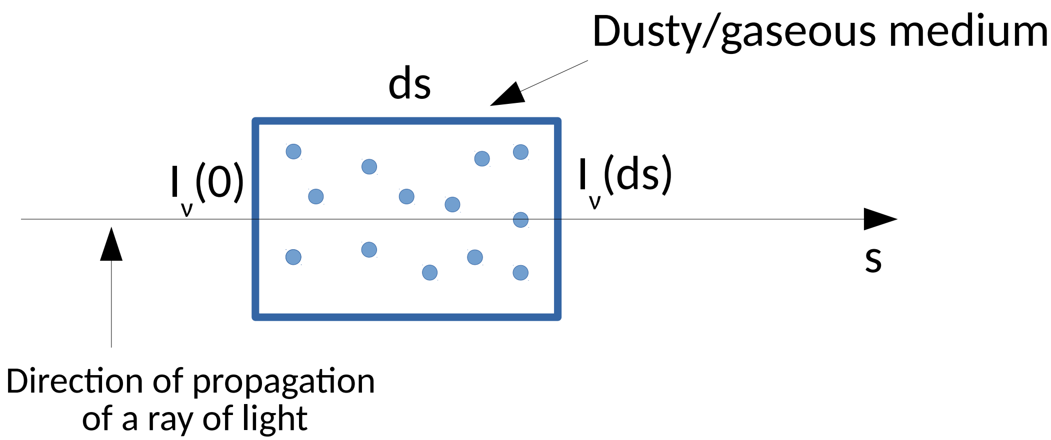
Radiative equilibrium

$$P_{col} = P_{em}$$

Equilibrium temperature of the dust grain

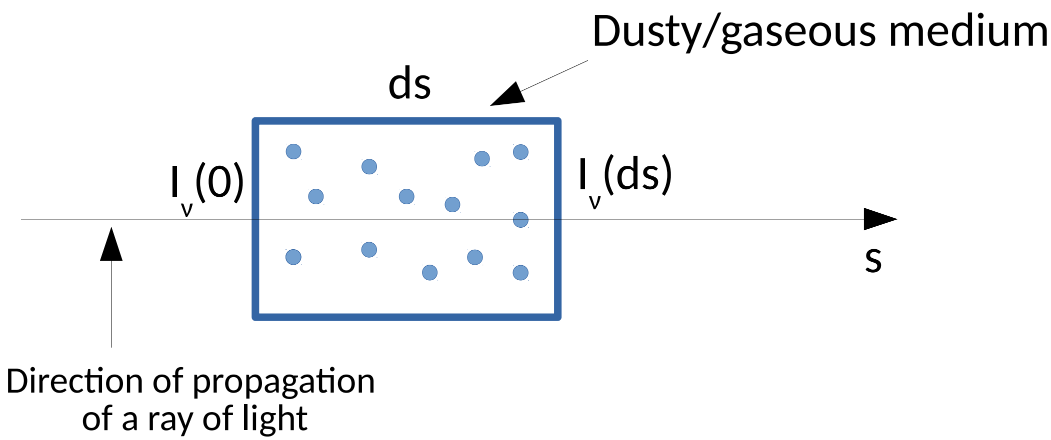
$$\boxed{T_g = \sqrt{\frac{R_*}{2d}}\,T_*}$$

Radiative Transfer – the problem



Interaction matter/radiation → $I_\nu(ds)$?

Radiative Transfer – the problem

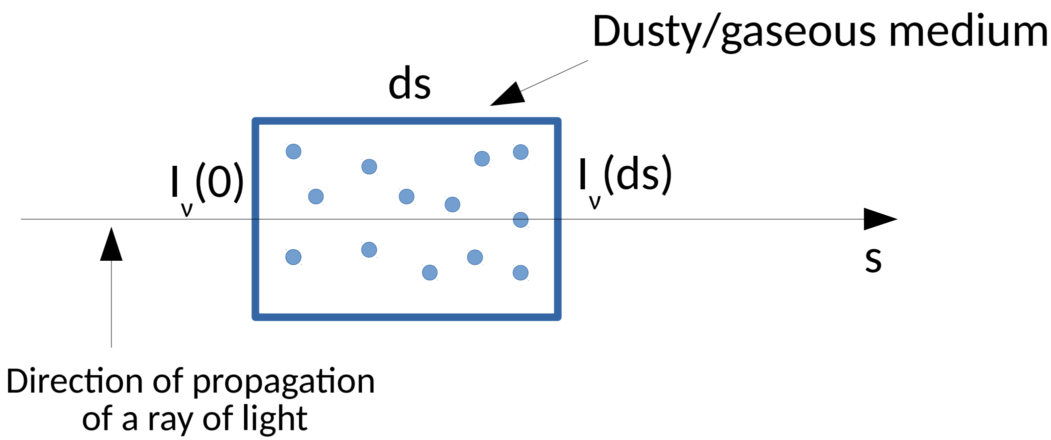


Interaction matter/radiation → $I_\nu(ds)$?

Oversimplified radiative transfer equation

$I_\nu(ds)$ - $I_\nu(0)$ = ΔI = what is added to the ray – what is removed from the ray

Radiative Transfer – the problem



Interaction matter/radiation → $I_\nu(ds)$?

$$\boxed{\text{Oversimplified radiative transfer equation}}$$

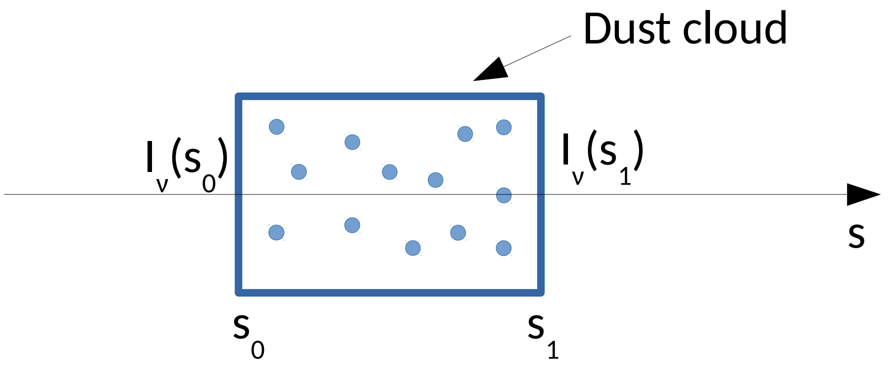
$I_\nu(ds) - I_\nu(0) = \Delta I$ = what is added to the ray – what is removed from the ray

$$\boxed{\text{Formal radiative transfer equation}}$$

$$\frac{dI_\nu(\vec{n},s)}{ds} = \underbrace{j_\nu(s)}_{\substack{\textbf{Source term} \\ \text{(add « energy » to that ray)}}} - \underbrace{\alpha_\nu(s)\, I_\nu(\vec{n},s)}_{\substack{\textbf{Extinction term} \\ \text{(remove « energy » from that ray)}}}$$

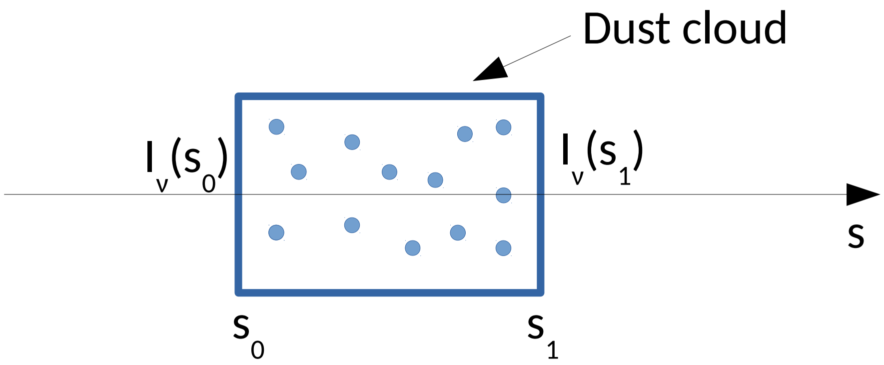
Radiative Transfer – the opacity

Geometric description



Radiative Transfer – the opacity

Geometric description

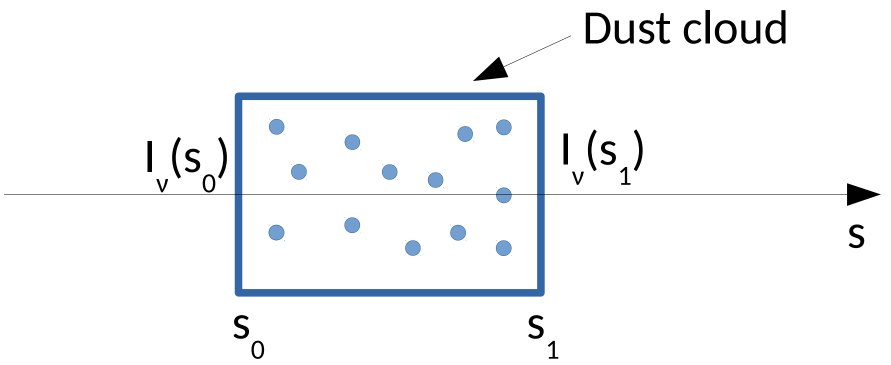


$\alpha_\nu(s)$ → **Opacity** (cgs unit : cm^{-1})

→ $\alpha_\nu = \dfrac{1}{l_{\nu,free}}$ with $l_{\nu,free}$: Mean free path of photons

Radiative Transfer – the opacity

Geometric description



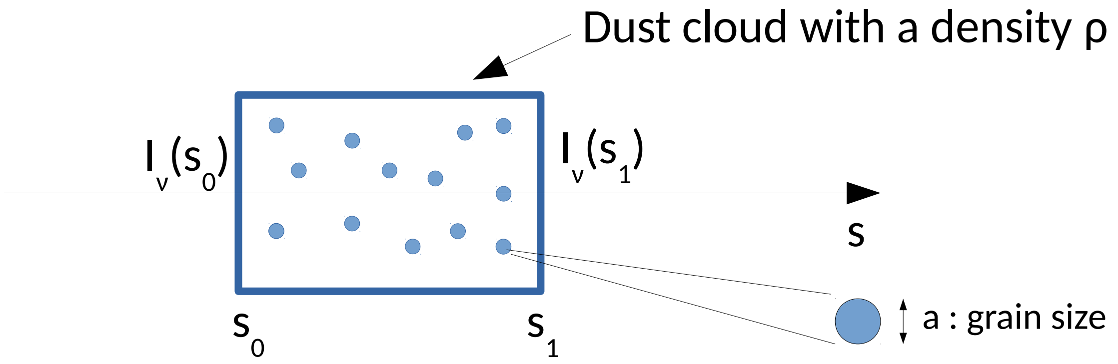
$\alpha_\nu(s)$ ➡ **Opacity** (cgs unit : cm^{-1})

→ $\alpha_\nu = \dfrac{1}{l_{\nu,free}}$ with $l_{\nu,free}$: Mean free path of photons

→ $\tau_\nu(s_0, s_1) = \displaystyle\int_{s_0}^{s_1} \alpha_\nu(s)\,ds$ Optical depth : number of mean free paths

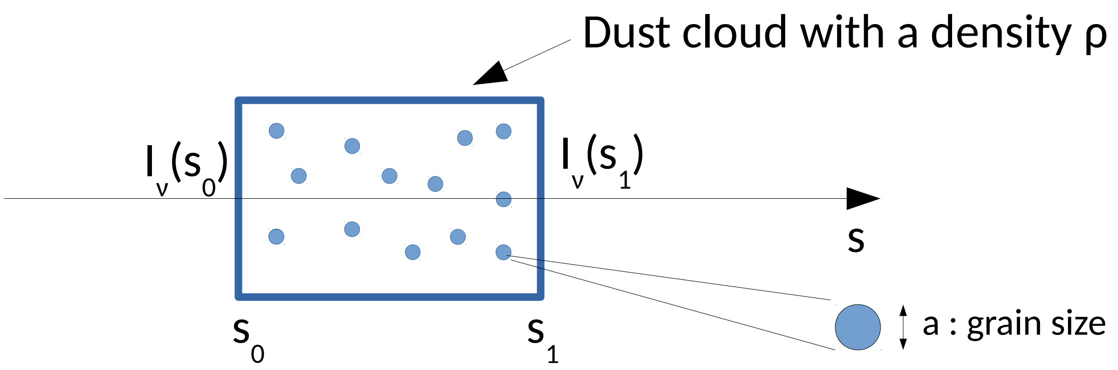
Radiative Transfer – the opacity

' Particle/density ' description



Radiative Transfer – the opacity

' Particle/density ' description

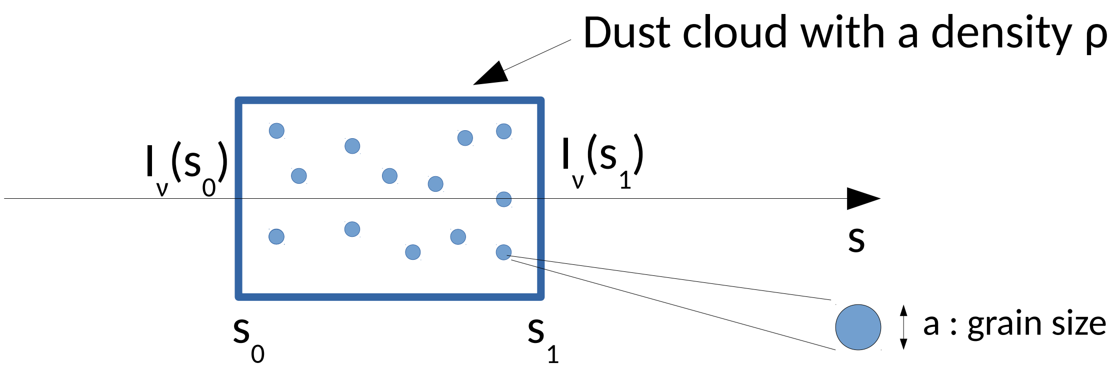


$\alpha_\nu(s)$ ⟹ **Opacity** (cgs unit : cm^{-1})

→ $\alpha_\nu = \rho\kappa_\nu$ With ρ the cloud density [g.cm^{-3}] and κ_ν the mass extinction cross section [cm^2.g^{-1}]

Radiative Transfer – the opacity

' Particle/density ' description



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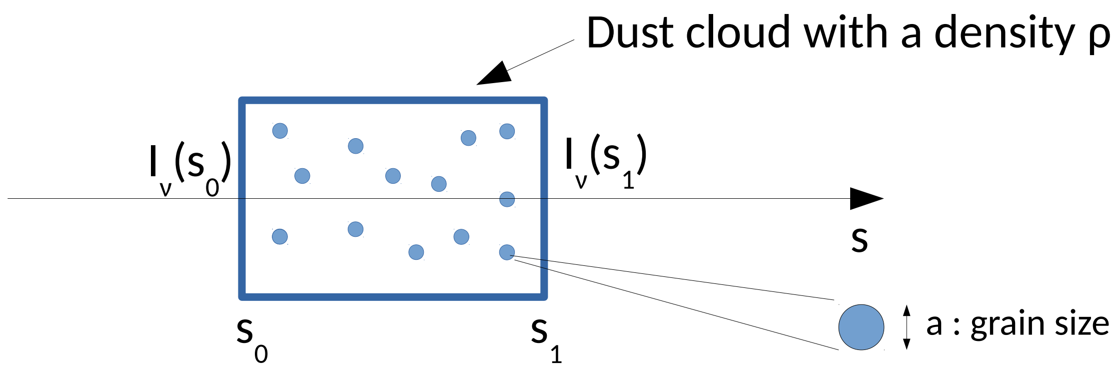
► $\alpha_\nu = \rho\kappa_\nu$ With ρ the cloud density [g.cm^{-3}] and κ_ν the mass extinction cross section [cm^2.g^{-1}]

► $$\tau_\nu(s_0, s_1) = \int_{s_0}^{s_1} \rho\kappa_\nu \; ds = \kappa_\nu \int_{s_0}^{s_1} \rho \, ds \; = \kappa_\nu \times \underbrace{\Sigma}$$

Surface density [g.cm^{-2}]
(volume density integrated along the line of sight)

Radiative Transfer – the opacity

' Particle/density ' description



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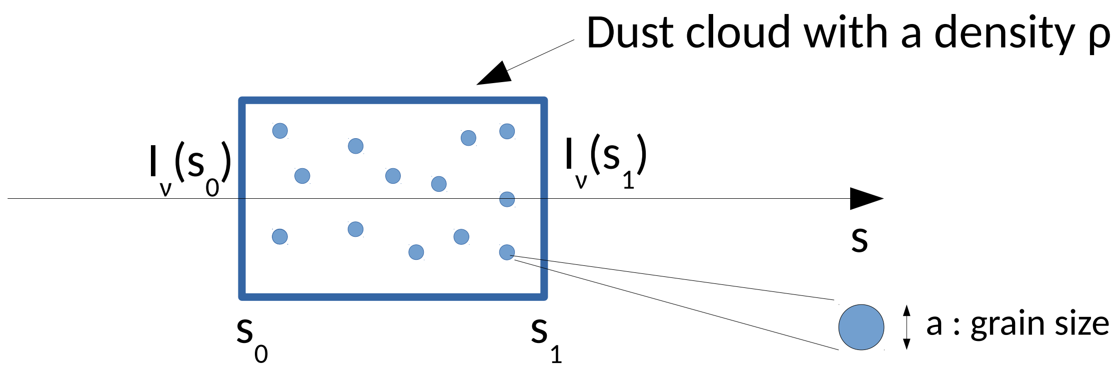
► $\alpha_\nu = \rho\kappa_\nu$ With ρ the cloud density [g.cm^{-3}] and κ_ν the mass extinction cross section [cm^2.g^{-1}]

► $\kappa_\nu = \dfrac{\sigma_\nu}{m}$ With m the grain mass [g] and σ_ν the grain extinction cross section [cm^2]

RADMC-3D

Radiative Transfer – the opacity

' Particle/density ' description



$\alpha_\nu(s)$ → **Opacity** (cgs unit : cm^{-1})

→ $\alpha_\nu = \rho\kappa_\nu$ With ρ the cloud density [g.cm^{-3}] and κ_ν the mass extinction cross section [cm^2.g^{-1}]

→ $\kappa_\nu = \dfrac{\sigma_\nu}{m}$ With m the grain mass [g] and σ_ν the grain extinction cross section [cm^2]

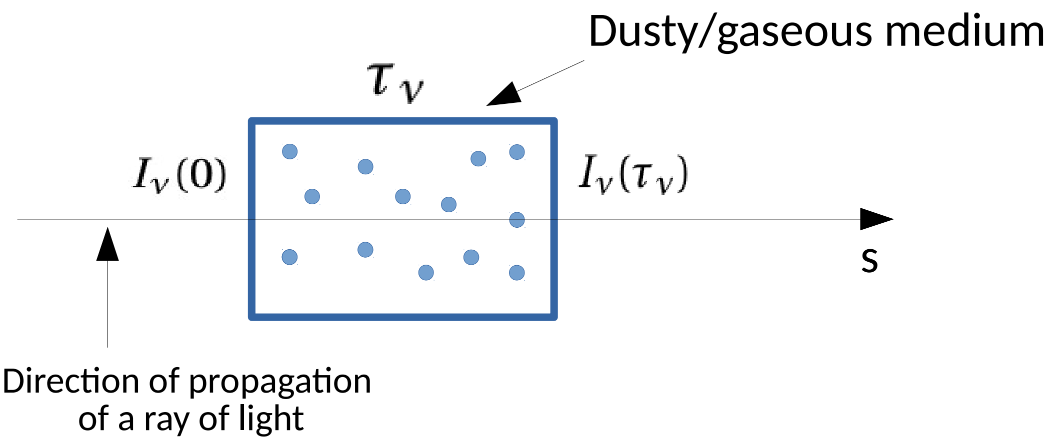
σ_ν

Interaction surface with radiation of frequency ν

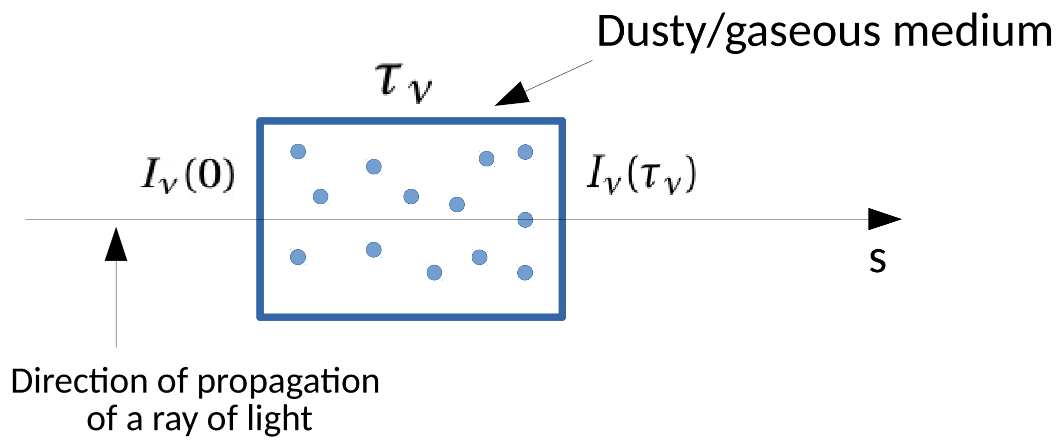
→ $a \gg \lambda$ → $\sigma_\nu = \sigma = \pi a^2$ 'grey' grain

→ $a \ll \lambda$ → $\sigma_\nu \ll \pi a^2$ 'chromatic' grain

Radiative Transfer – solving the equation

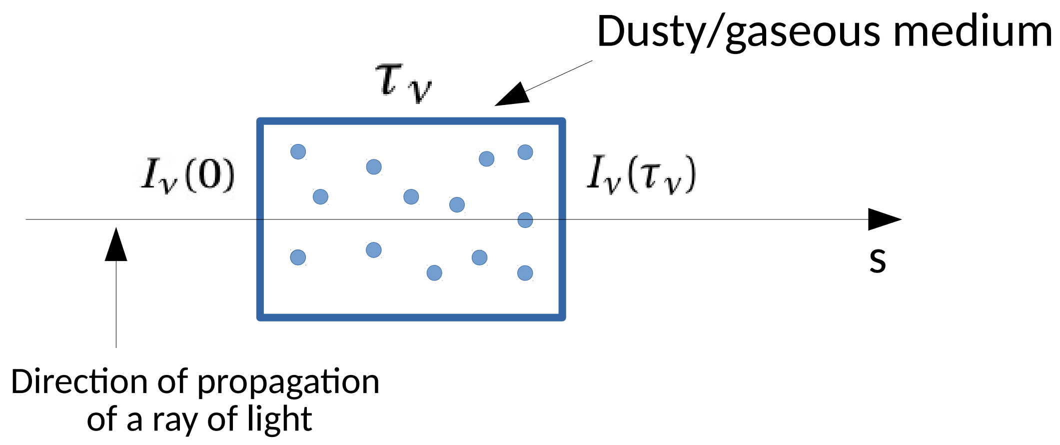


Radiative Transfer – solving the equation



$$\frac{dI_\nu(\vec{n},s)}{ds} = j_\nu(s) - \alpha_\nu(s) I_\nu(\vec{n},s)$$

Radiative Transfer – solving the equation



$$\frac{dI_\nu(\vec{n},s)}{ds} = j_\nu(s) - \alpha_\nu(s) I_\nu(\vec{n},s)$$

Division by α_ν

$$\frac{dI_\nu(s)}{d\tau_\nu} = \underbrace{S_\nu(s)} - I_\nu(s)$$

$S_\nu = \frac{j_\nu}{\alpha_\nu}$ ' Source function ' (has the dimension of an intensity)

Radiative Transfer – solving the equation

1) General solution without second member

$$\frac{dI_\nu}{I_\nu} = -d\,t_\nu$$

t_ν : Local optical depth in the cloud → integration variable

Radiative Transfer – solving the equation

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$$\frac{dI_\nu}{I_\nu} = -d t_\nu \quad \Rightarrow \quad \int_0^{\tau_\nu} \frac{dI_\nu}{I_\nu} = -\int_0^{\tau_\nu} d t_\nu \quad \Rightarrow \quad I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} = I_g$$

t_ν : Local optical depth in the cloud → integration variable

Radiative Transfer – solving the equation

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t_ν : Local optical depth in the cloud → integration variable

2) Particular solution with second member

Let us consider a solution of the form $I_p(\tau_\nu) = k(\tau_\nu)e^{-\tau_\nu}$

Radiative Transfer – solving the equation

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t_ν : Local optical depth in the cloud → integration variable

2) Particular solution with second member

Let us consider a solution of the form $I_p(\tau_\nu) = k(\tau_\nu)e^{-\tau_\nu}$

Inject it into RT equation $\longrightarrow$ $\dfrac{d(k(t_\nu)e^{-t_\nu})}{dt_\nu} + k(t_\nu)e^{-t_\nu} = S_\nu(t_\nu)$

Radiative Transfer – solving the equation

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Find particular solution $\longrightarrow$ $$I_p = \int_O^{\tau_\nu} S_\nu(t_\nu)e^{-(\tau_\nu - t_\nu)}dt_\nu$$

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$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t_\nu)e^{-(\tau_\nu - t_\nu)}dt_\nu$$

Radiative Transfer – Explicit solution (in a simple case)

Let us consider $S_\nu(t_\nu) = S_\nu$ (Source function constant throughout the cloud)

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

RADMC-3D

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$$S_\nu^{ETL} = \frac{j_\nu^{ETL}}{\alpha_\nu^{ETL}} = B_\nu(T)$$

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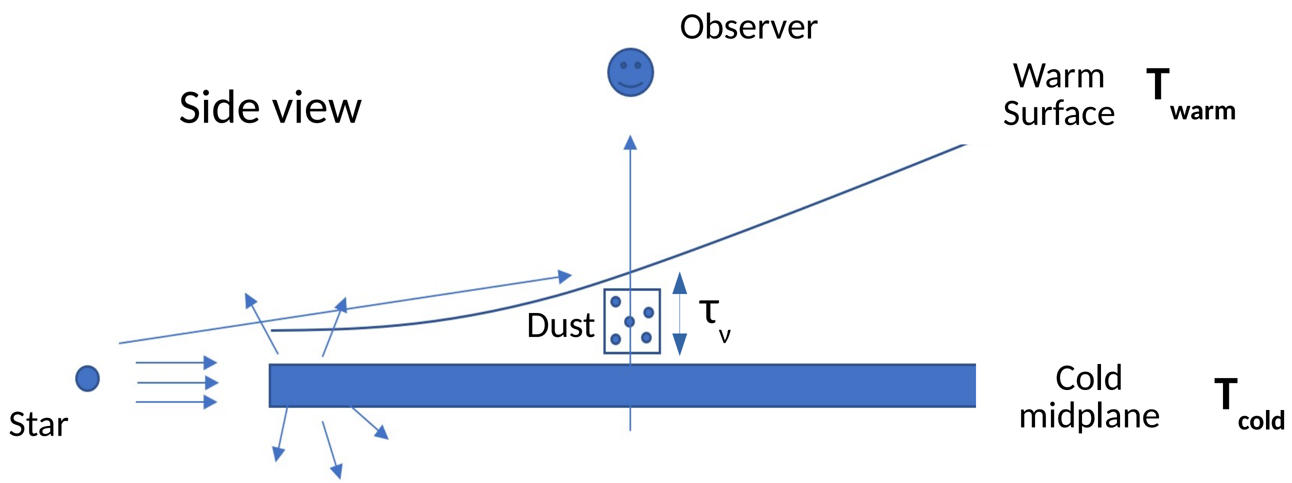
Explicit solution for a medium in thermal equilibrium (e.g., an isothermal dust cloud)

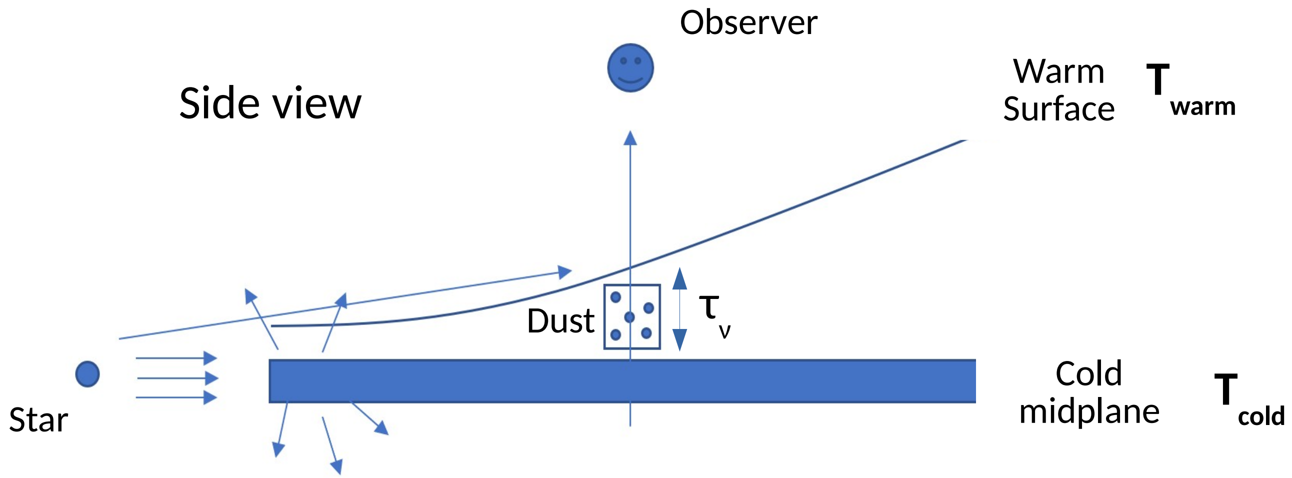
$$\boxed{I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + B_\nu(T)(1 - e^{-\tau_\nu})}$$

Simple model of disk emission



Simple two-layer disk model

Simple model of disk emission



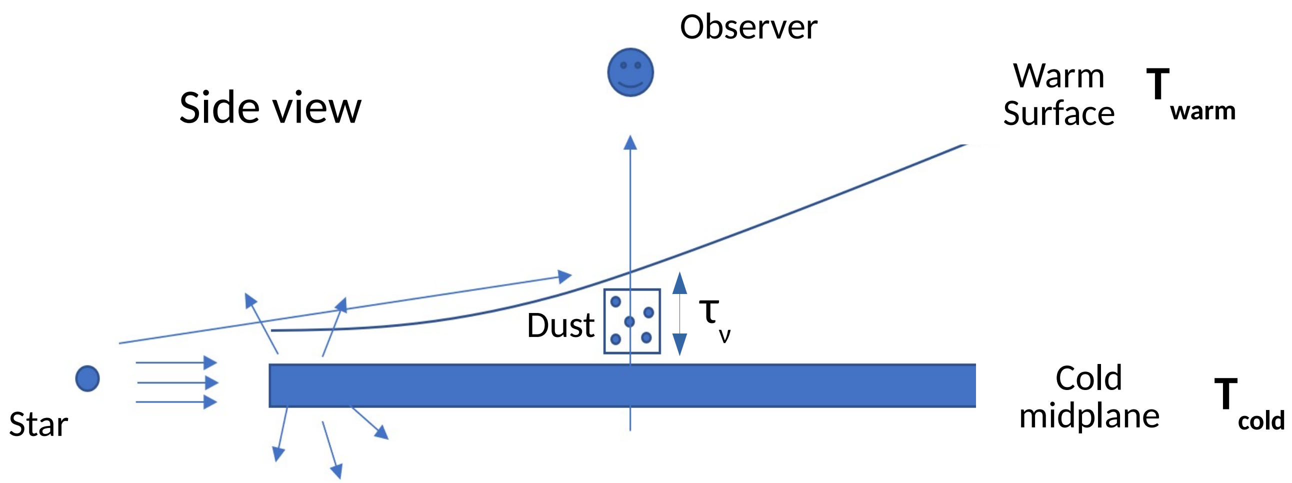
Simple two-layer disk model

$$\boxed{I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + B_\nu(T)(1 - e^{-\tau_\nu})}$$

Total intensity (towards observer) | Background intensity (cold midplane) | Foreground intensity (warm surface)

Simple model of disk emission

Simple two-layer disk model



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Total intensity
(towards observer)

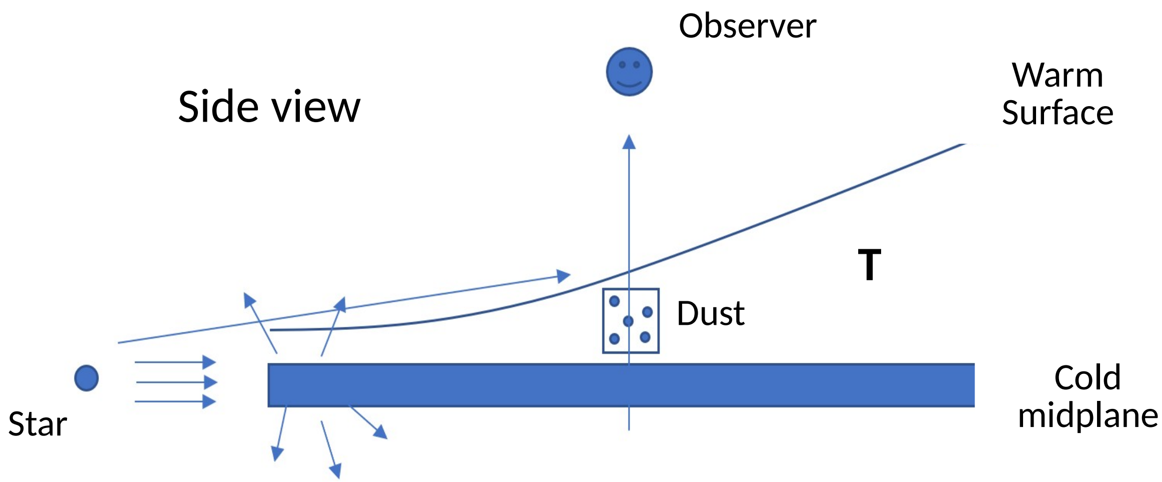
Background intensity
(cold midplane)

Foreground intensity
(warm surface)

$T_{warm} >> T_{cold}$

$$\boxed{I_\nu(\tau_\nu) \approx B_\nu(T)(1-e^{-\tau_\nu})}$$

Simple model of disk emission



$$\boxed{I_\nu(\tau_\nu) \approx B_\nu(T)(1 - e^{-\tau_\nu})}$$

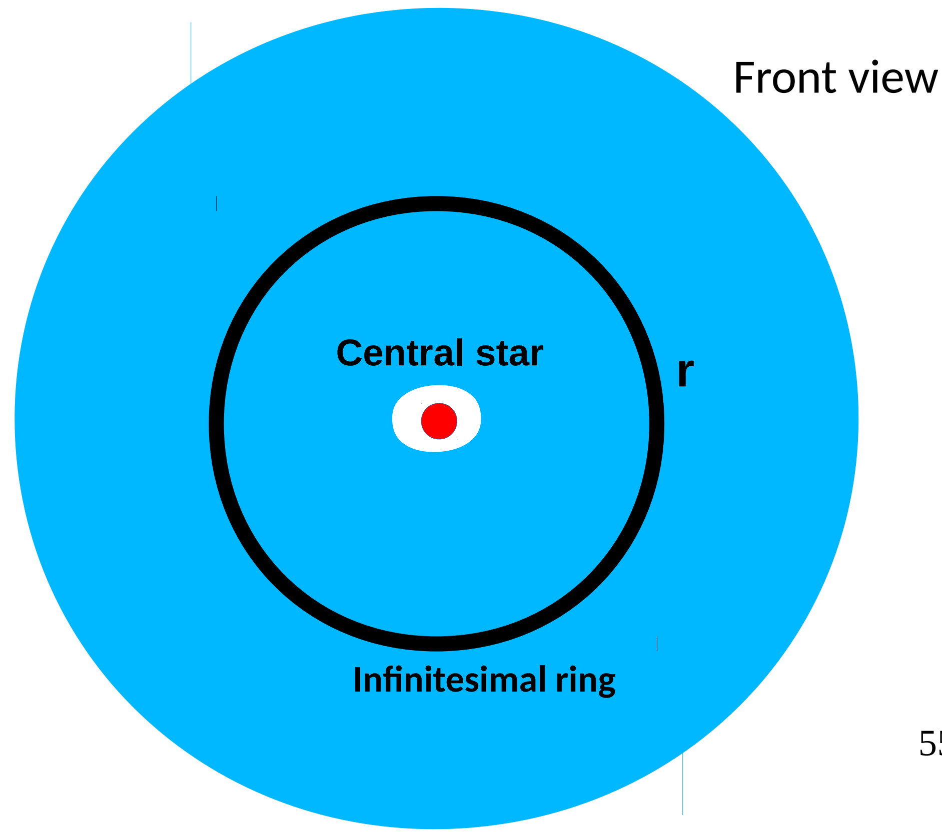
Temperature-gradient model

$$T_r = T_0 r^{-q}$$

Temperature radial profile

$$\Sigma_r = \Sigma_0 r^{-p}$$

Surface density profile



Radiative transfer modeling with RADMC-3D

Introduction

RADMC-3D is a software package for astrophysical radiative transfer calculations in arbitrary 1-D, 2-D or 3-D geometries. It is mainly written for continuum radiative transfer in dusty media, but also includes modules for gas line transfer and gas continuum transfer.

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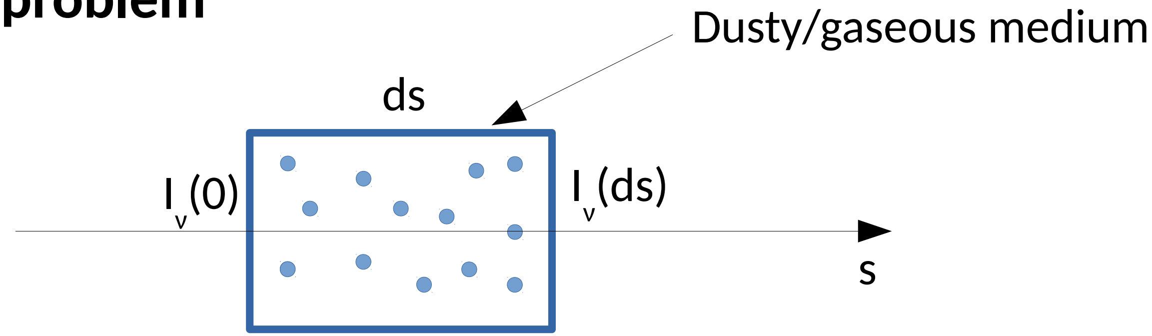
The radiative transfer problem



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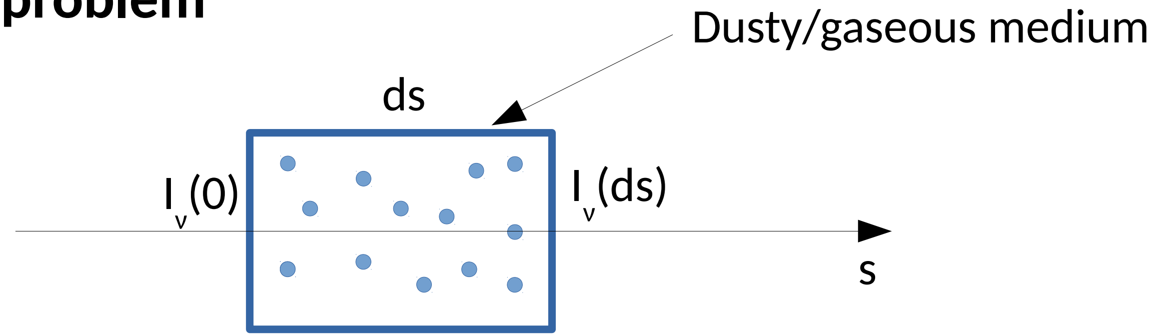

$$\frac{dI_\nu}{ds} = j_\nu^{\mathrm{therm}} + j_\nu^{\mathrm{scat}} - (\alpha_\nu^{\mathrm{abs}} + \alpha_\nu^{\mathrm{scat}}) I_\nu$$

Thermal 'source' function

scattering 'source' function

Absorption

Scattering

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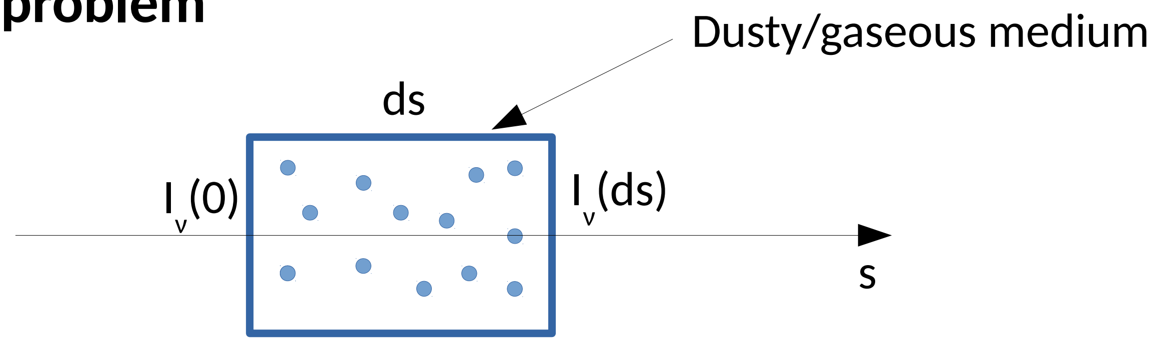

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Thermal 'source' function — scattering 'source' function — Absorption — Scattering

Source term
(add « energy » to that ray)

Extinction term
(remove « energy » from that ray)

RADMC-3D – model setup

Which spatial distribution of dust/gas do i want for my model ?

- Coordinate system = cartesian (x,y,z) or spherical (r,θ,φ)
- Spatial grid = 1D (spherical), 2D (spherical), or 3D (spherical or cartesian)

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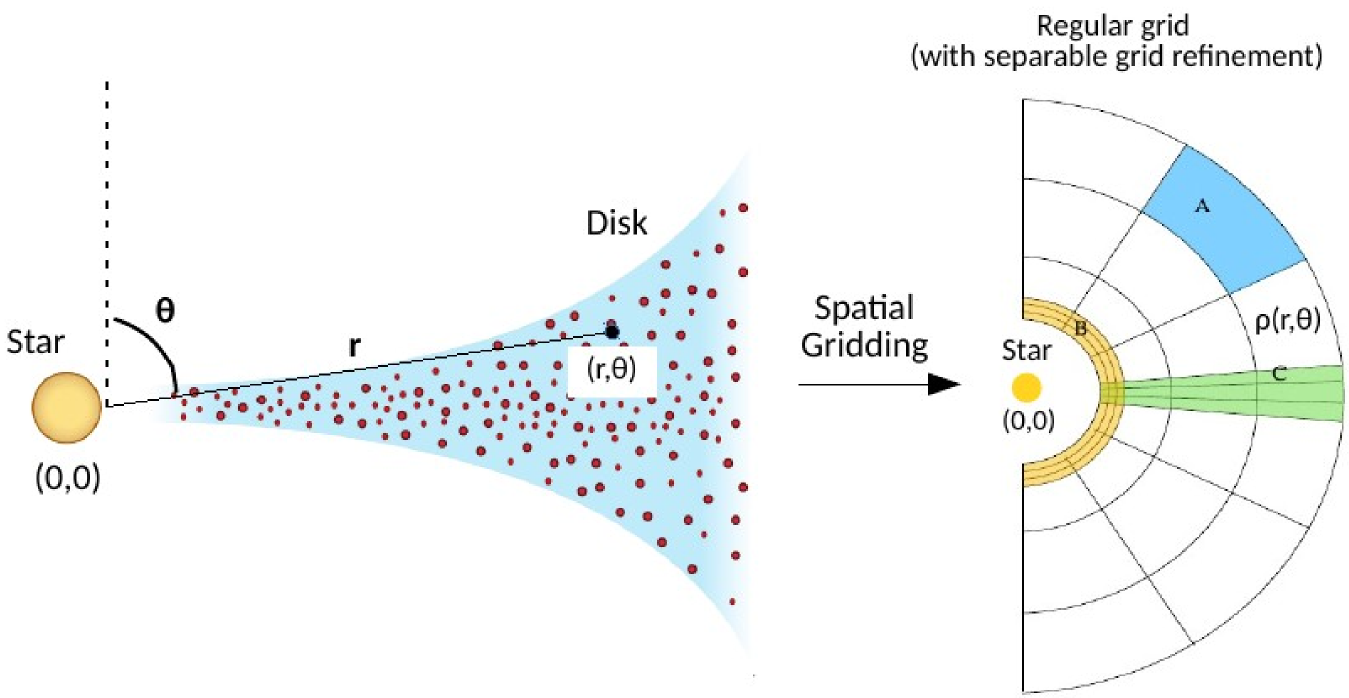
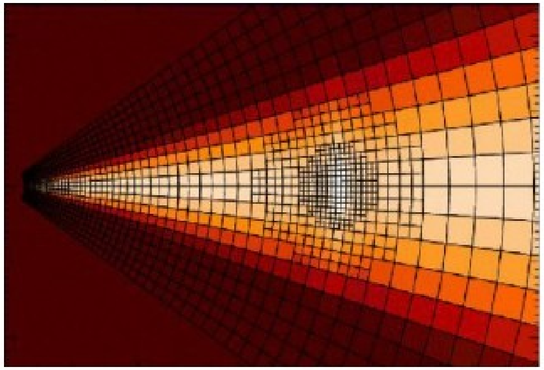
- Spatial grid = 1D (spherical), 2D (spherical), or 3D (spherical or cartesian)

Example : 2D model (axial symmetry)



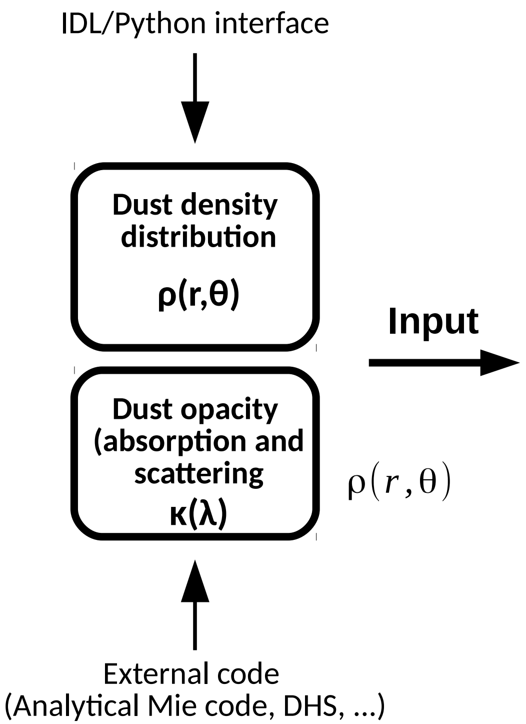
Possibility of adaptive mesh refinement



Credit : RADMC-3D manual

RADMC-3D – Monte Carlo run

Then, what is the dust EQUILIBRIUM temperature everywhere in the object ?



RADMC-3D – Monte Carlo run

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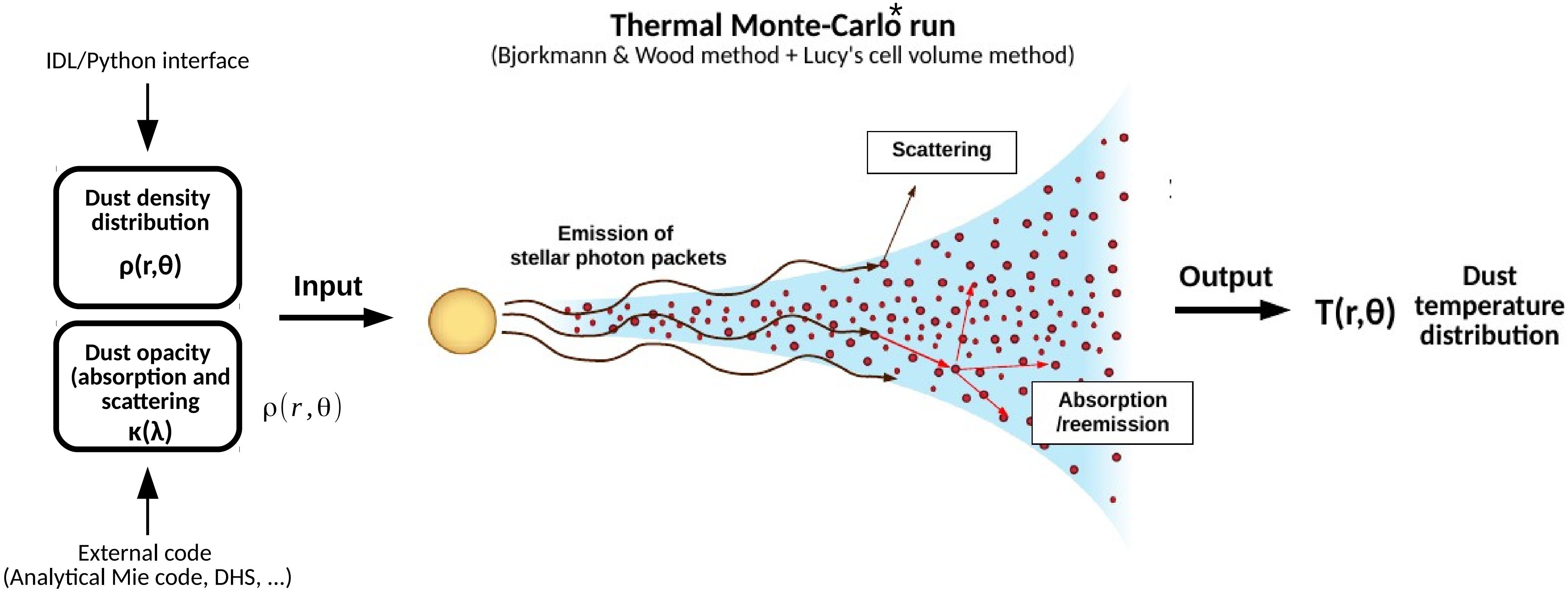
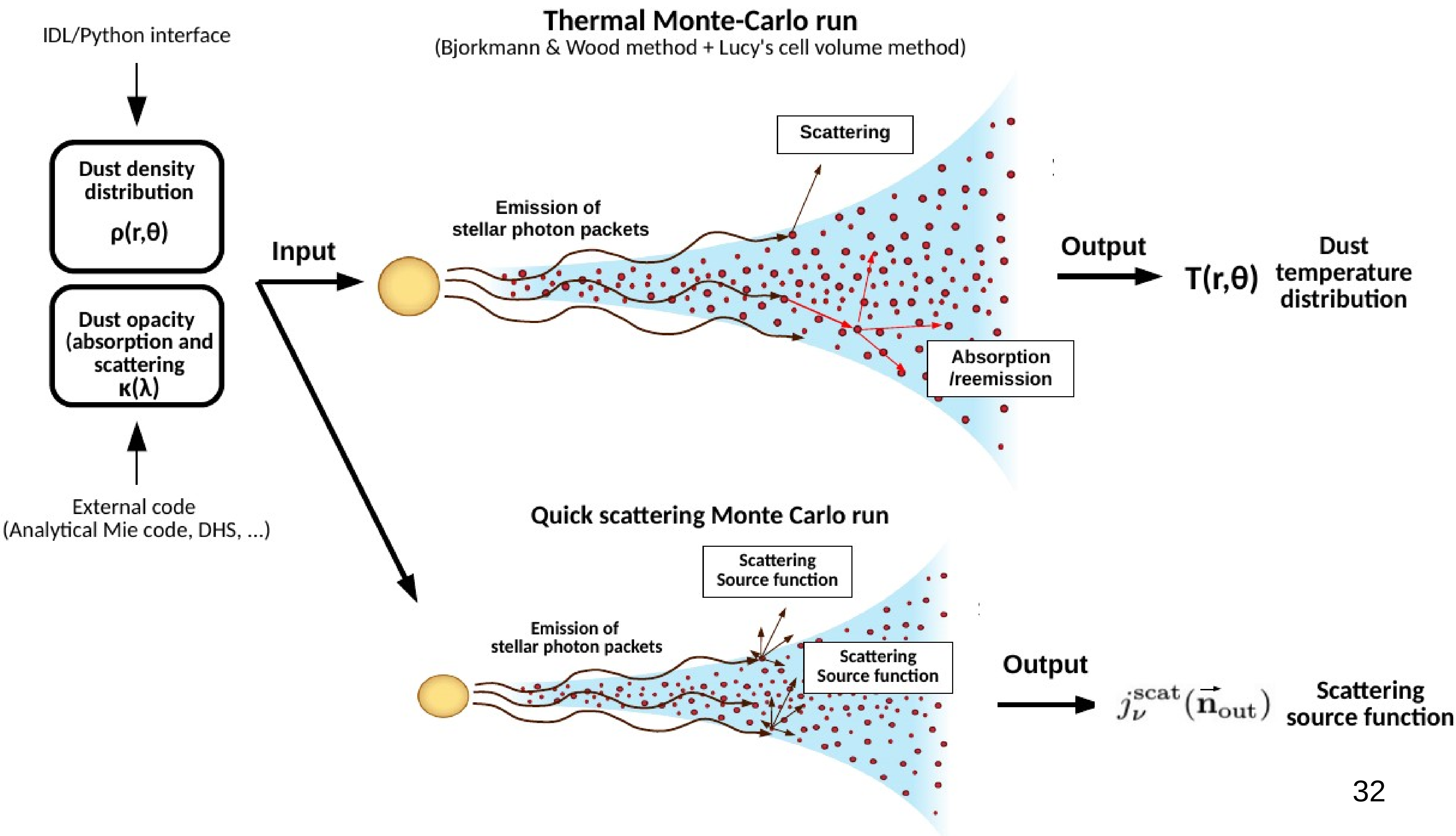


* **Monte Carlo methods :** broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomnness to solve problems that might be deterministic in principle.

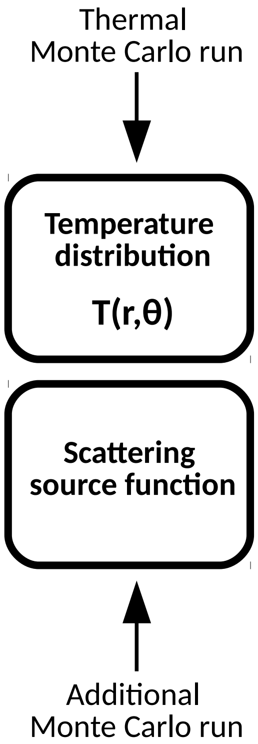
RADMC-3D – Monte Carlo run

Then, what is the scattering source function everywhere in the object ?



RADMC-3D – Making images and SED

Finally, how will my object look like ? What is the I(λ,x,y) ?



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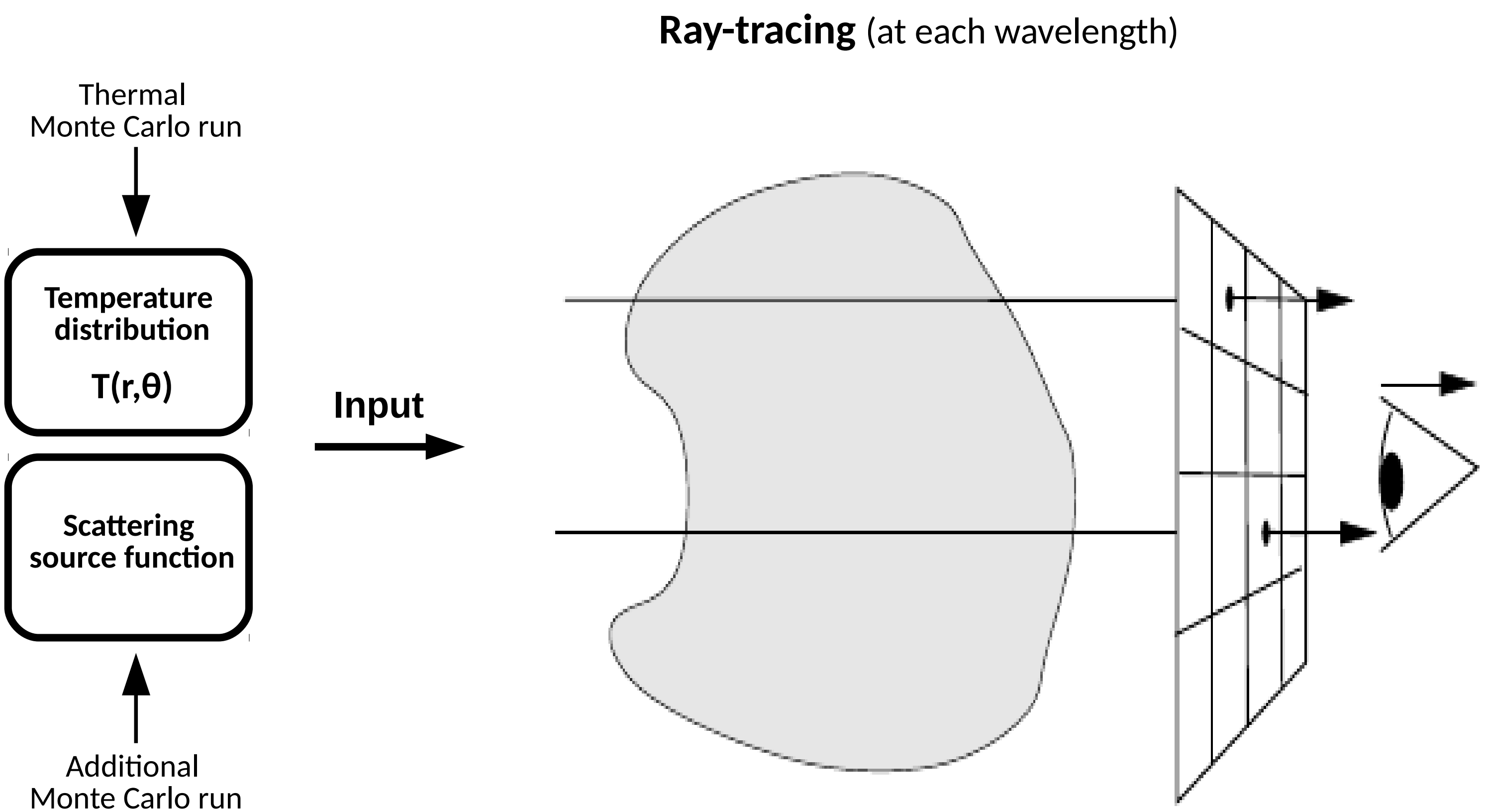


Credit : RADMC-3D manual

RADMC-3D – Making images and SED

Finally, how will my object look like ? What is the I(λ,x,y) ?

Thermal
Monte Carlo run

Temperature distribution

T(r,θ)

Scattering source function

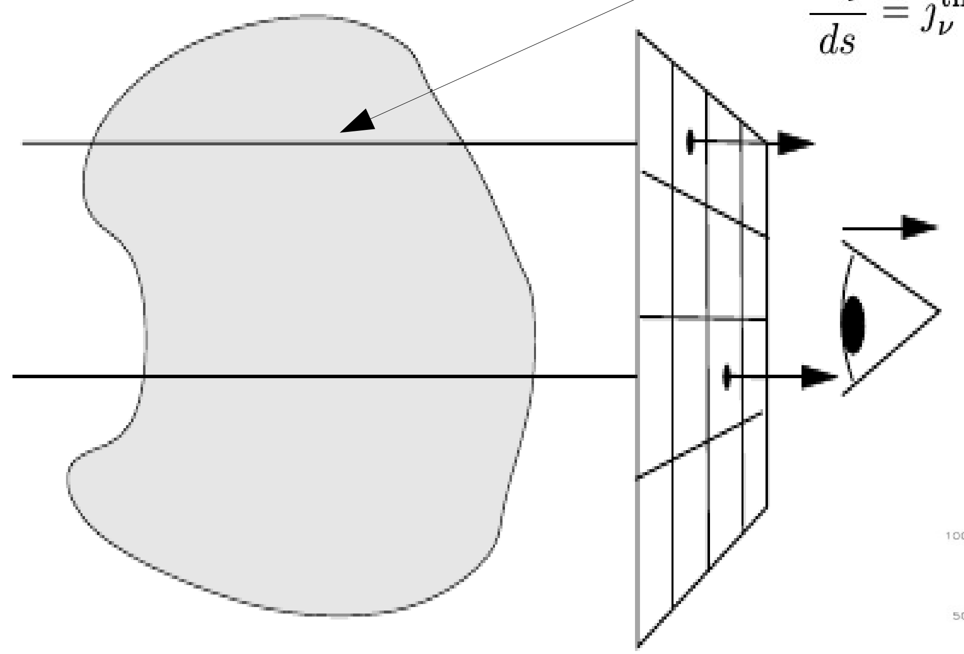
Additional
Monte Carlo run

Input

Ray-tracing (at each wavelength)

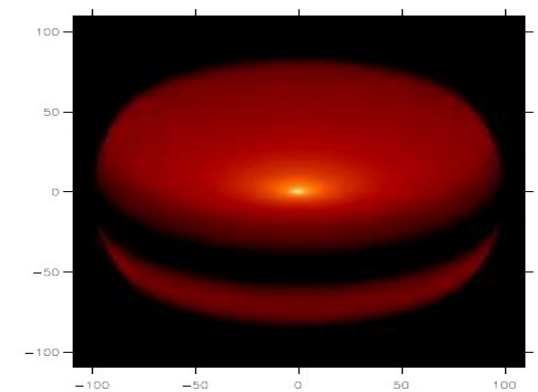
Integration along a ray

$$\frac{dI_\nu}{ds} = j_\nu^{\rm therm} + j_\nu^{\rm scat} - (\alpha_\nu^{\rm abs} + \alpha_\nu^{\rm scat}) I_\nu$$



Output

I(λ,x,y)



Credit : RADMC-3D manual

More detailed explanations of the ray-tracing approach

Credit : RADMC-3D manual

The easiest way is to perform a process known as *volume rendering*, also called *forward ray tracing*. It is very simple: we simply integrate the formal transfer equation

$$\frac{dI_\nu(s)}{ds} = \alpha_\nu(\mathbf{x}(s))[B_\nu(T(\mathbf{x}(\mathbf{s}))) - I_\nu(s)] \quad (5.15)$$

along a ray starting behind our cloud, going through the cloud and ending up at the observer. We only have to figure out how to choose the rays such that each ray belongs to a specific pixel in the image we wish to make.

The other way is to put the observer "at infinity". In this case the rays of the image will all be parallel. The image plane is again put perpendicular to the rays, but since no "depth perspective" is present in this case, it does not matter where we put it. This is illustrated in the other margin figure. The pixel size can no longer be expressed in terms of angle. Instead must specify *pixel size in centimeters*. We can thus compare pixel size with actual size scales of our object.

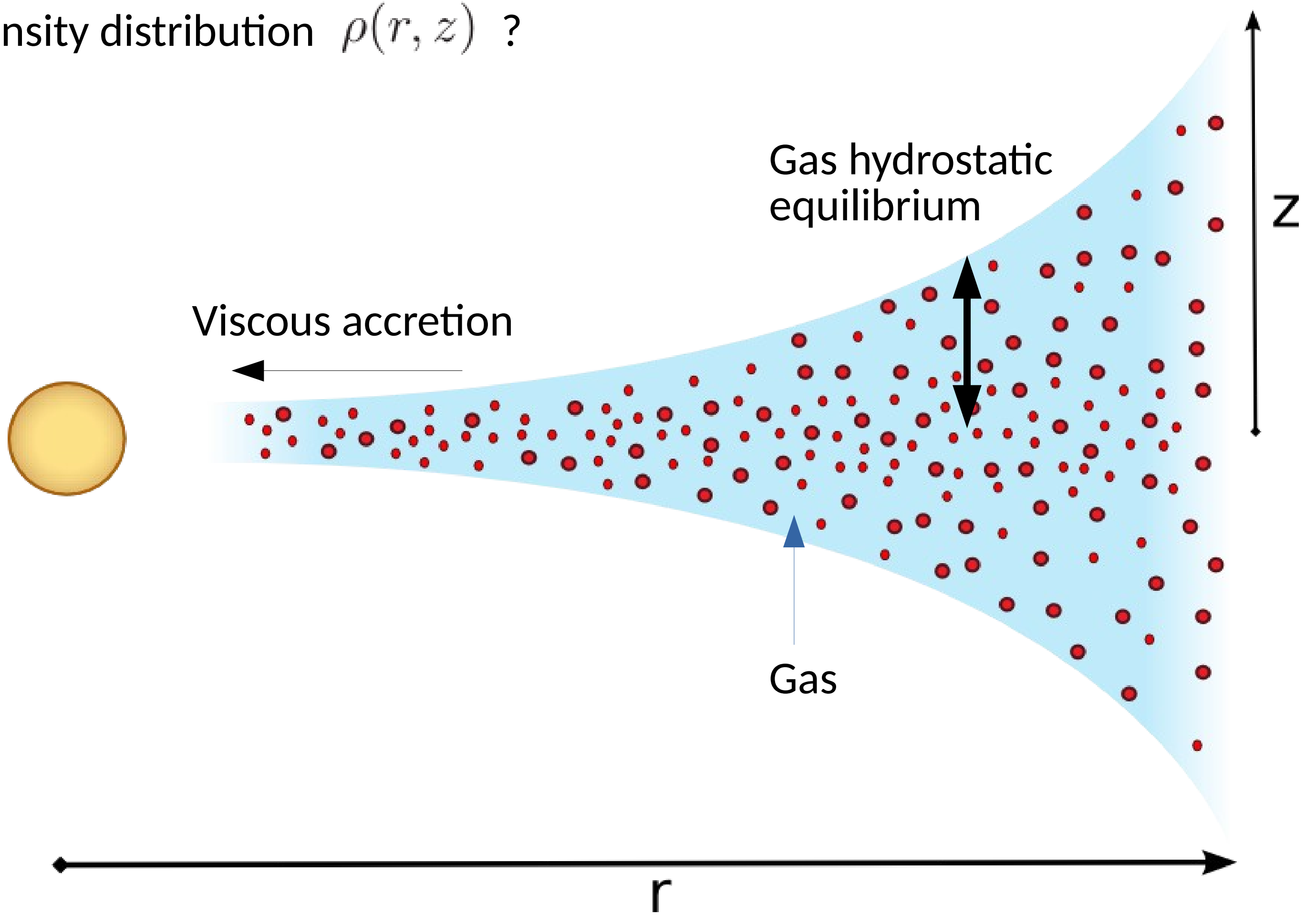
In reality the observer is, of course, never truly at infinity. But for most cases this is a good approximation. We can always convert the pixel size from cm to radian if we specify the distance d to the observer: $\Delta x^{\mathrm{rad}} = \Delta x^{\mathrm{cm}}/d$.

The advantage of using the "observer at infinity" approach, and specifying the image pixel size in centimeters instead of radian, is that we can compute the image first, and worry about the distance of the observer later, as long as the observer is in the far-field.

What the ray tracing method give you is the intensity at the center of each pixel of your image. In order to make a true image out of this we make the *assumption* that this intensity represents the *average intensity* of the pixel. If the cloud that we are observing is smooth enough, and if the pixels of our image are small enough, then this assumption is probabily fine. In that case we can safely use Eq. (5.16) to compute the flux from an image.

RADMC-3D – A typical disk model

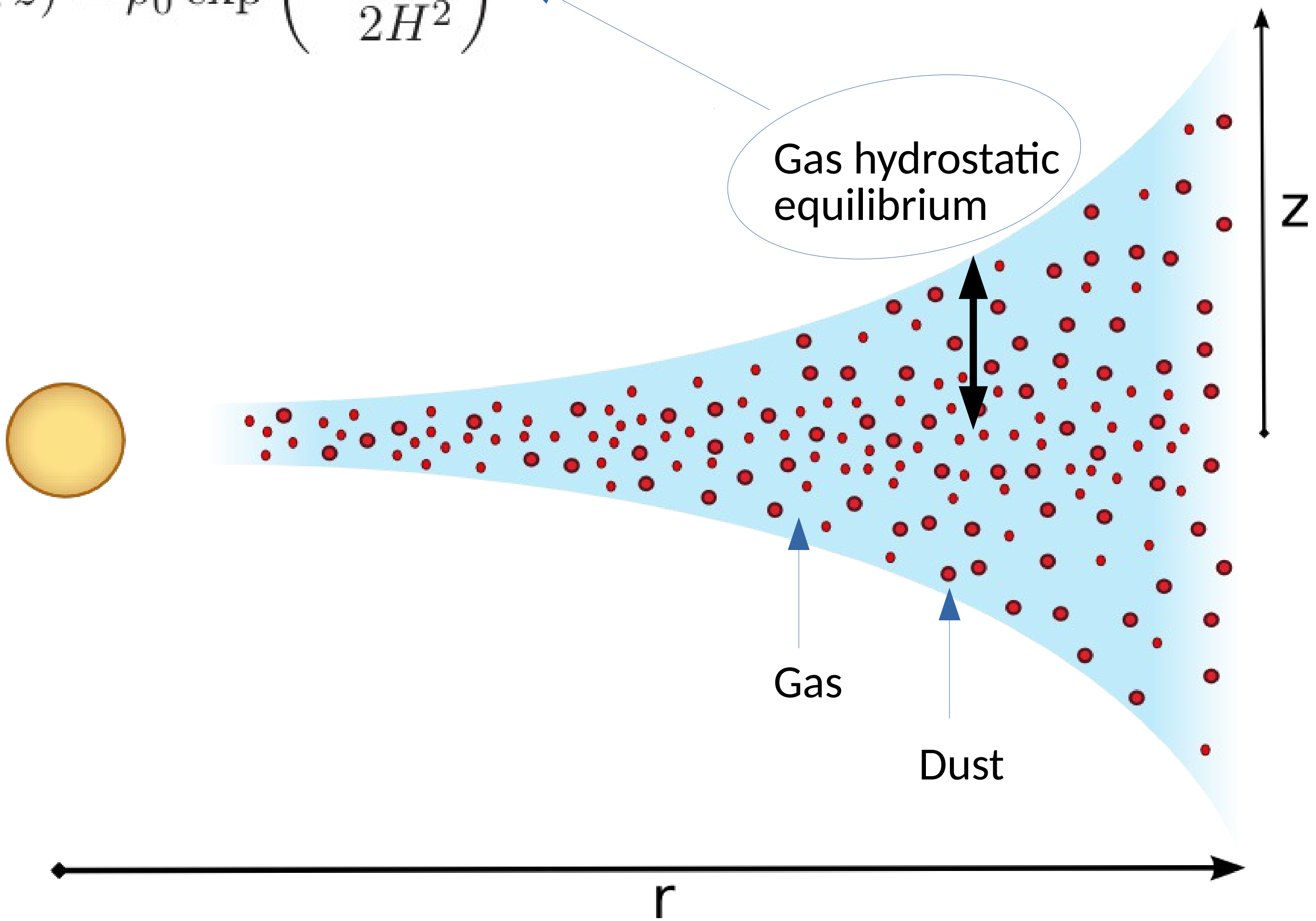
Gas density distribution $\rho(r, z)$?



RADMC-3D – A typical disk model

Vertical disk structure

$$\rho(r,z) = \rho_0 \exp\left(-\frac{z^2}{2H^2}\right)$$



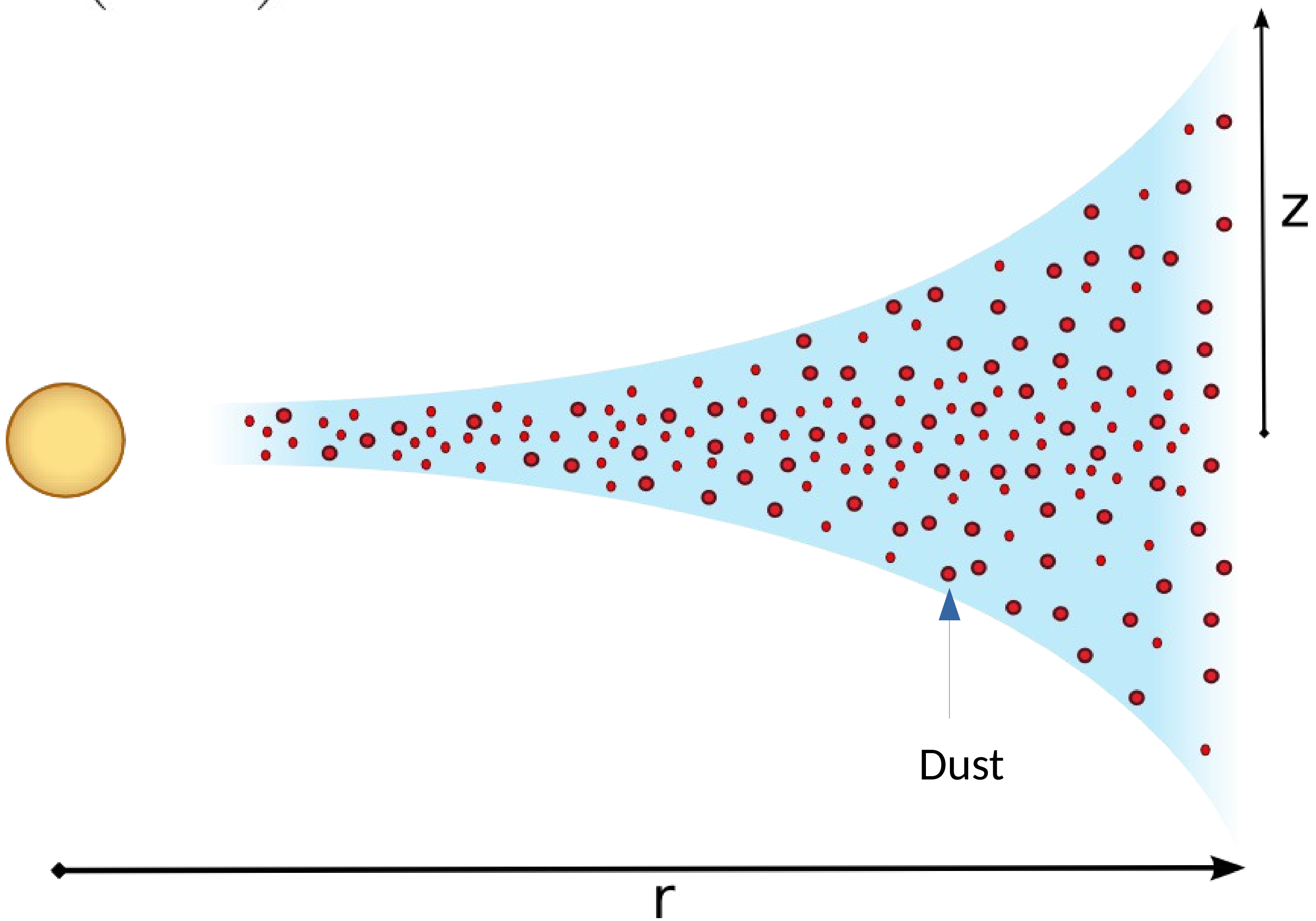
RADMC-3D – A typical disk model

Vertical disk structure

$$\rho(r,z) = \rho_0 \exp\left(-\frac{z^2}{2H^2}\right)$$

Gas Pressure scale height $H = \frac{c_s}{\Omega_K}$ Sound speed

Keplerian angular frequency



RADMC-3D – A typical disk model

Vertical disk structure

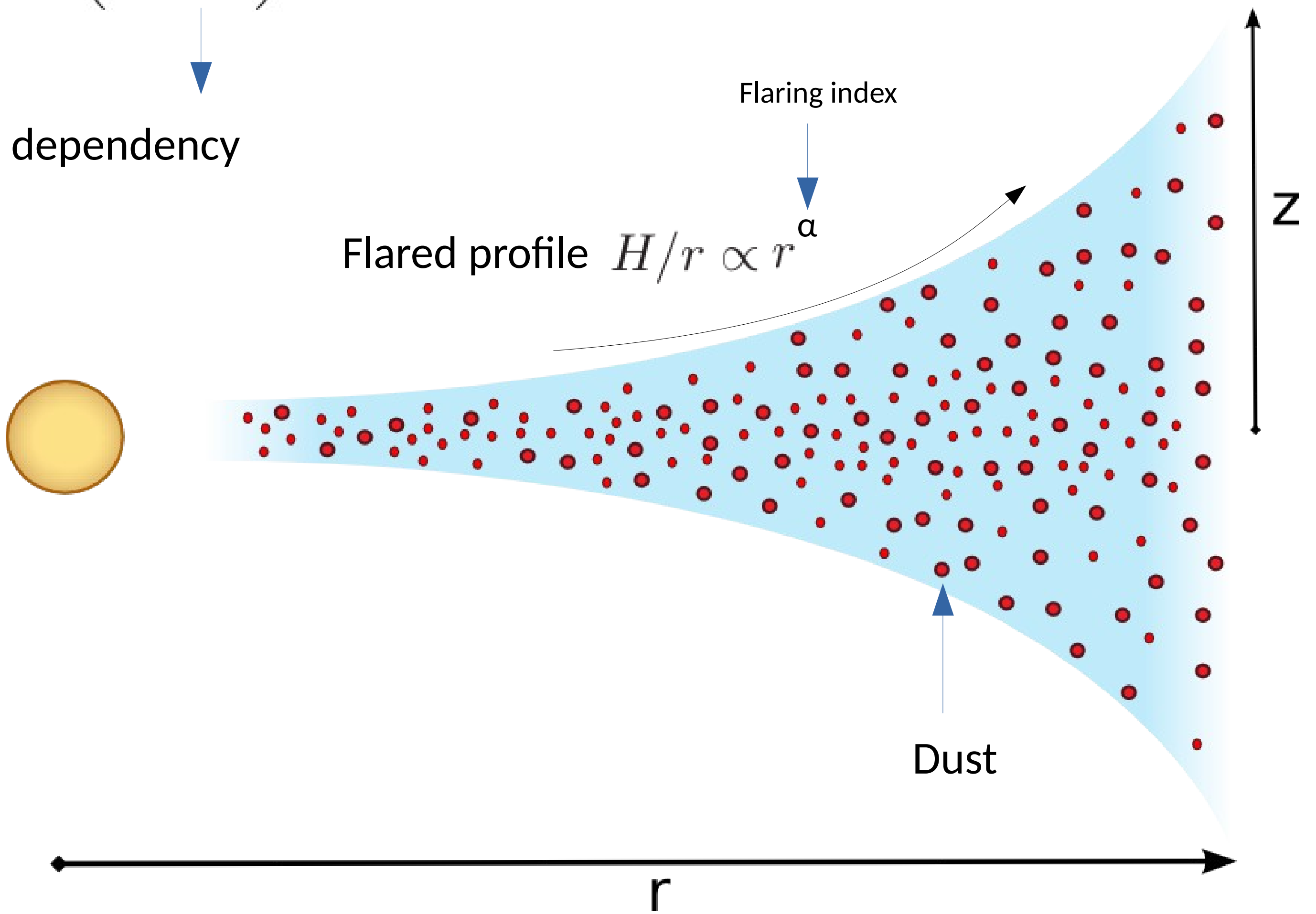
$$\rho(r,z) = \rho_0 \exp\left(-\frac{z^2}{2H^2}\right)$$

! Radial dependency

Pressure scale height $H = \frac{c_s}{\Omega_K}$ Sound speed; Keplerian angular frequency

Flaring index

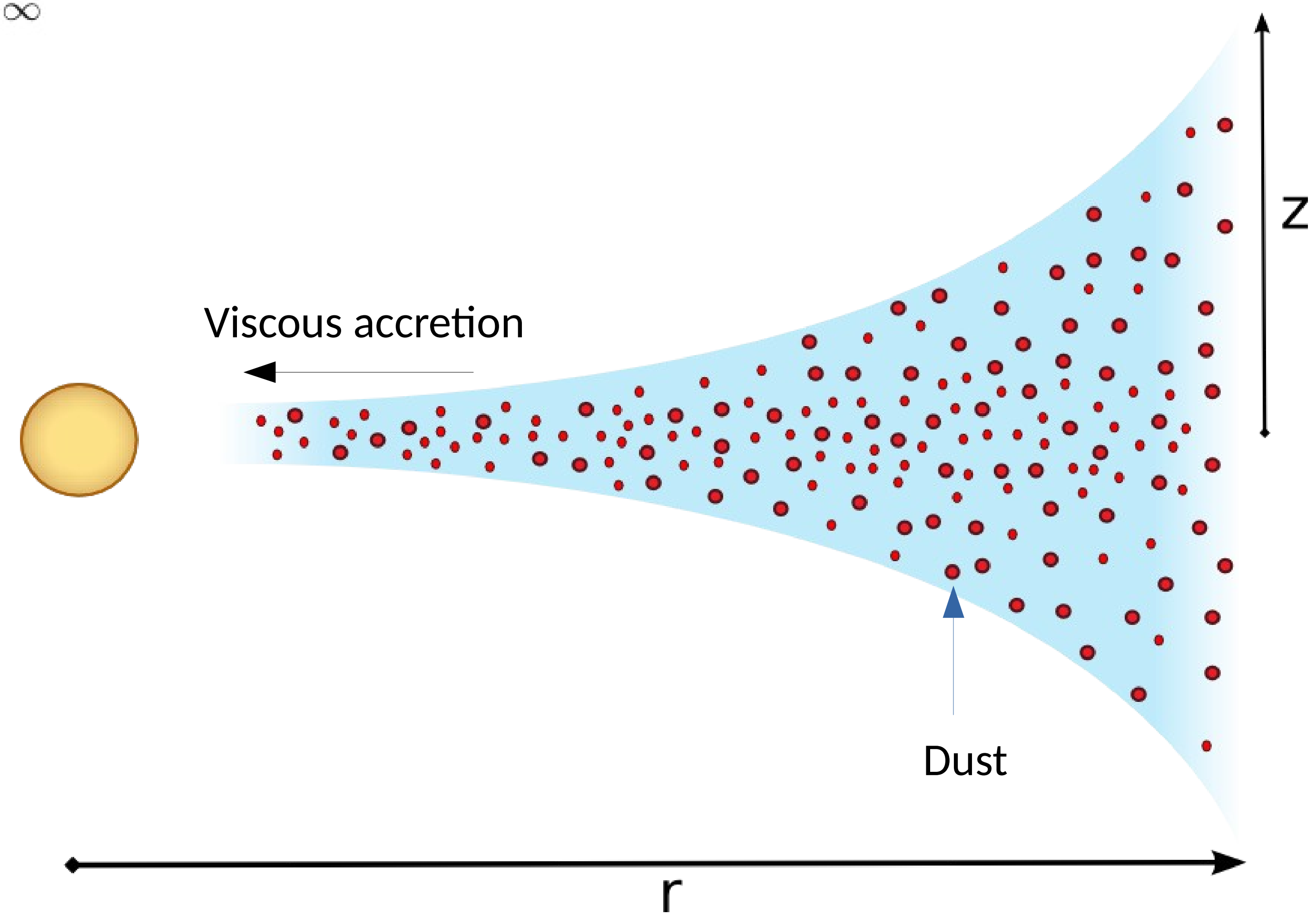
Flared profile $H/r \propto r^{\alpha}$



RADMC-3D – A typical disk model

Radial disk structure

$$\Sigma(r) \equiv \int_{-\infty}^{+\infty} \rho(r,z)dz = \rho_0 H \sqrt{2\pi}$$

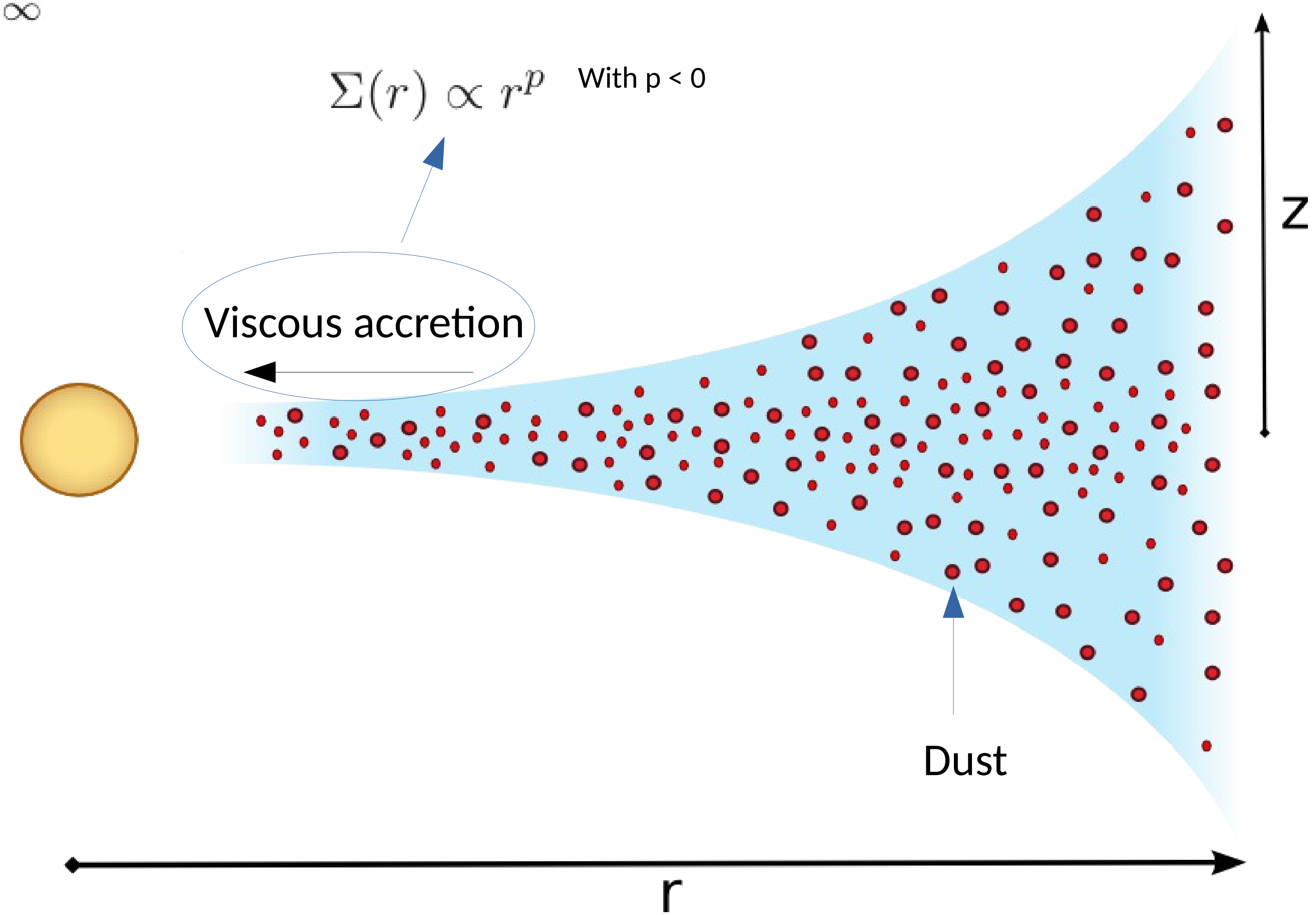


RADMC-3D – A typical disk model

Radial disk structure

$$\Sigma(r) \equiv \int_{-\infty}^{+\infty} \rho(r,z)dz = \rho_0 H \sqrt{2\pi}$$

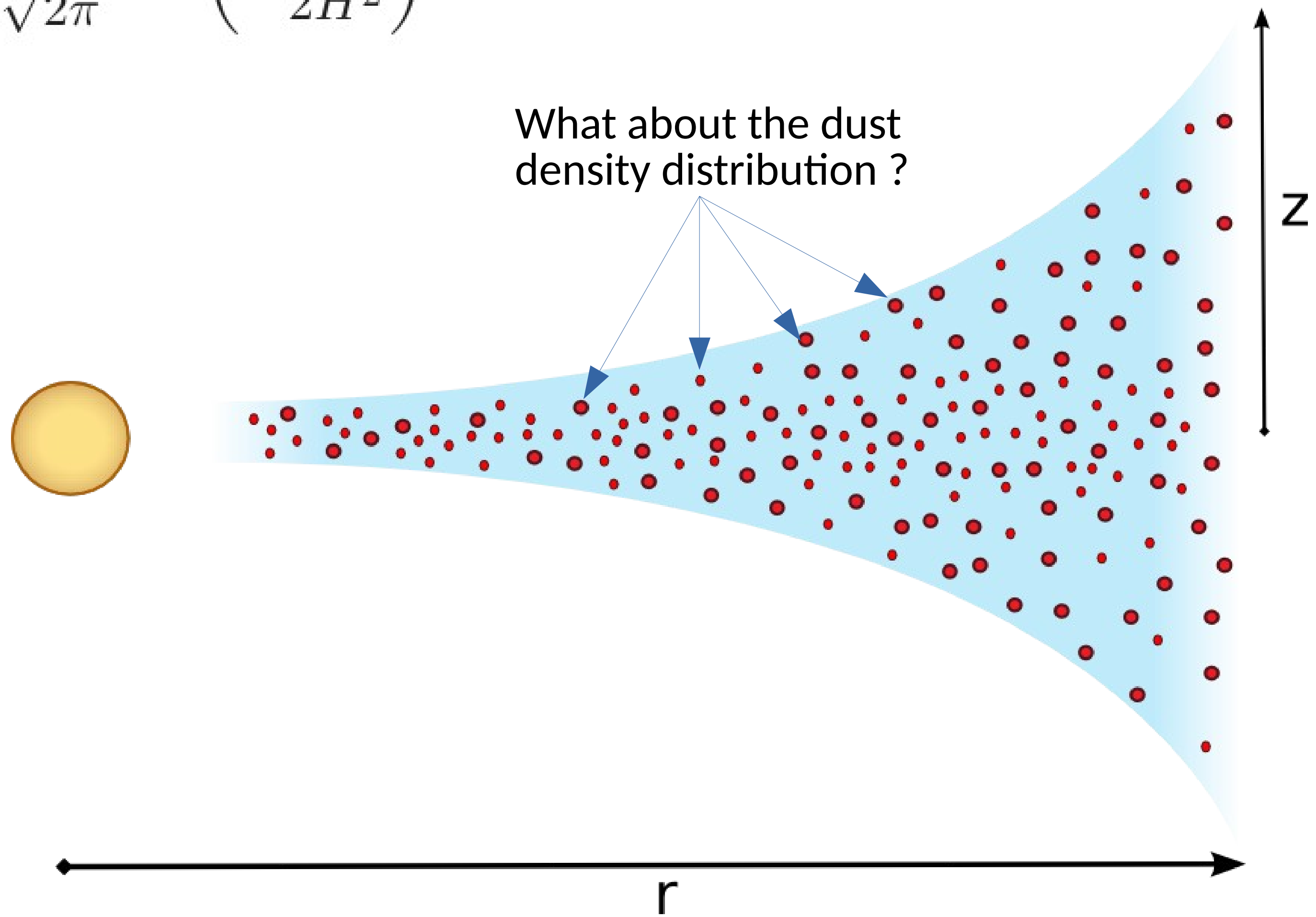
$$\Sigma(r) \propto r^p$$ With p < 0



RADMC-3D – A typical disk model

Gas density distribution

$$\rho(r,z) = \frac{\Sigma(r)}{H\sqrt{2\pi}}\exp\left(-\frac{z^2}{2H^2}\right)$$



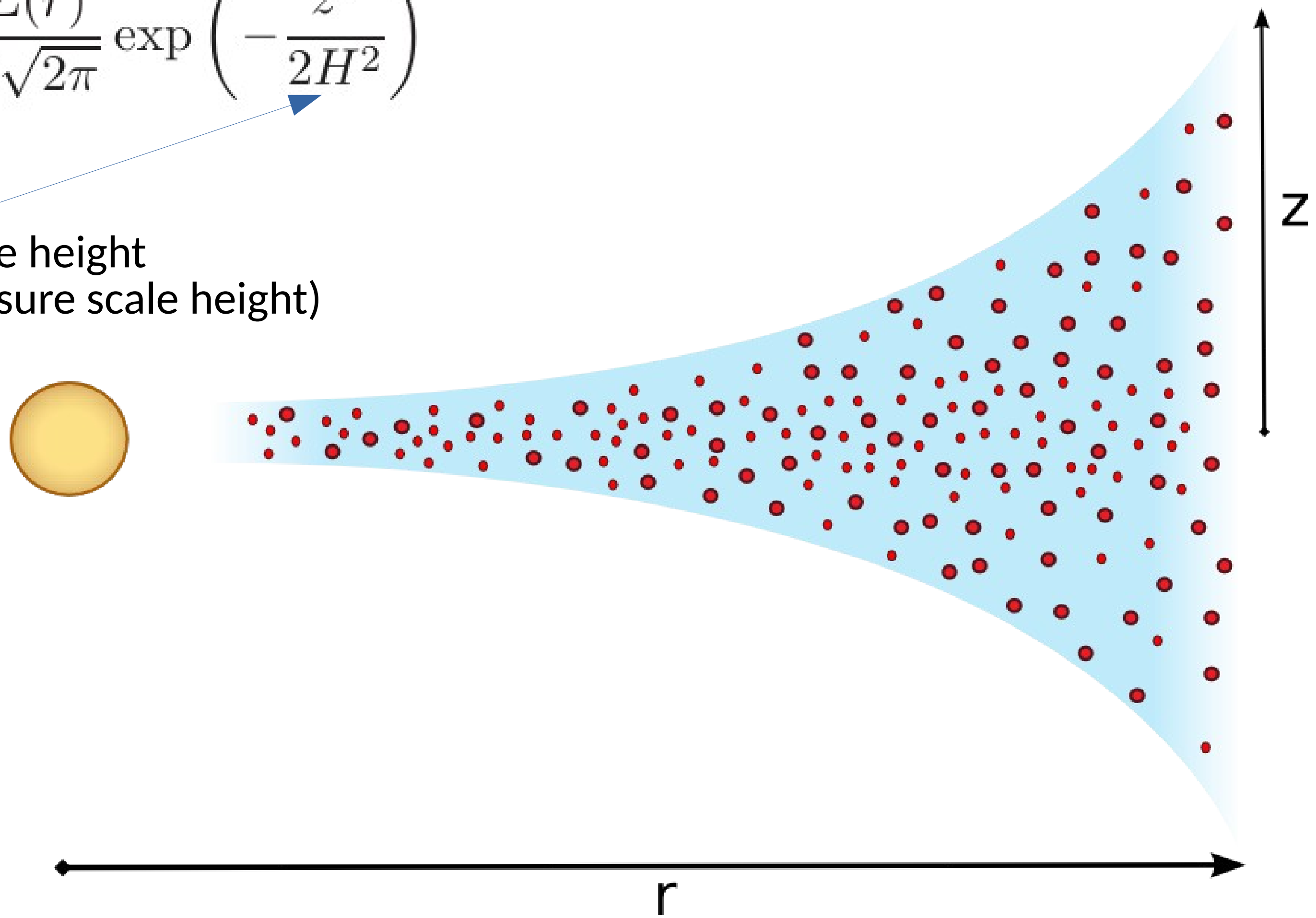
RADMC-3D – A typical disk model

Let us assume dust follows a similar type of vertical and radial structure

Dust surface density (≠ gas surface density)

$$\rho(r,z) = \frac{\Sigma(r)}{H\sqrt{2\pi}} \exp\left(-\frac{z^2}{2H^2}\right)$$

~ Dust scale height
(≠ gas pressure scale height)



RADMC-3D – last but not least

Parallelization
✔ Monte Carlo runs → OpenMP parallelized version

Availability of the code
✔ Publicly available and free of charge
✔ Package downloadable at: https://www.ita.uni-heidelberg.de/~dullemond/software/radmc-3d/

User support
✔ Detailed manual included in the package
✔ Mailing list for updates and bug reports

Not included yet
✔ Radiative transfer computation for quantum heated grains (PAHs, VSGs)
✔ No self-consistent computation of the disk vertical structure (hydrostatic equilibrium)
✔ No dust/gas dynamics considered ('static' models)