Practical Introduction to Model Fitting

examples of model fitting, all on real data : to be made **successively**

1. fitting of a **simple** model on **one** file
2. fitting with **parameters sharing** on several files
3. fitting with **degeneracies**
4. fitting **a star + environment**
5. Additional exercises, if time and for fun

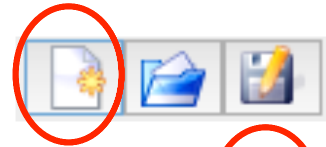
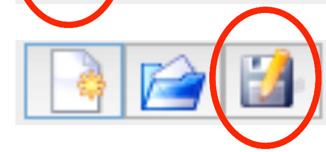
In red, questions or suggestions

You may ask a teacher at every moment.

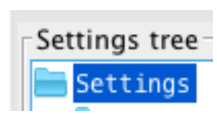
"wrap-up pauses" during the session

slides in blue: **comment for the teachers**, in green: **results & complements**

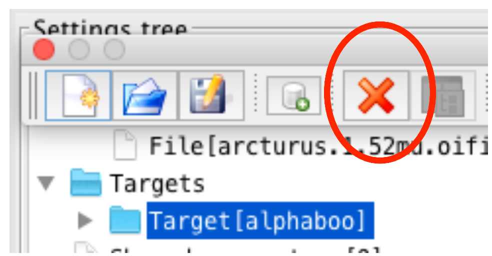
Preliminaries

- Directives and data at https://app.nuclino.com/VLTIschool/School-workspace/ → Model-fitting
- Launch LITpro, OIFITsExplorer, SAOImageDS9
- One setting by exercise
  - open settings *File > New settings* or shortcut 
  - click *Load oifiles* for loading data
  - save settings *File > Save settings* or shortcut 

    (you may name the setting with the name of exercise)

- *Personal notebook* visible at any time from 

  & almost any Panel
  - initially empty
  - saved with the setting
- Possibility to remove elements of the tree (except Files) if selected



- Multiple **?** for help

Exercise 1- simple fit 1.1

- Load the file **arcturus.1.79mu.oifits**

  What kind of data did you load ?

- Explore the data
  - Settings Tree →*Files*

    in *Files Panel*, *Plot VIS2DATA…, UV coverage,…*

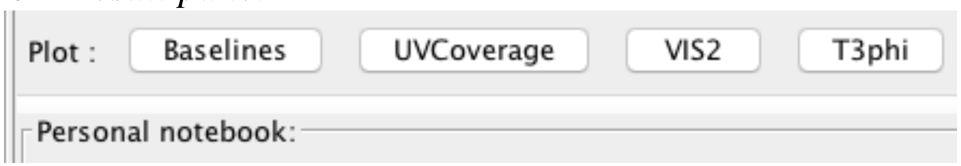
  What clue do you get from the OI_T3 data ?

- Build a model
  - Settings Tree →*Targets*
  - *Add new target*
  - In Target Panel :
    - */Fitter setup*: *Normalize total flux* ON, select the data to fit (for ex. VIS2)
    - */Model List*: add a model function (for ex. disk)
  - Initialize the parameters (for ex. diameter = 0, 10, 20 mas ..)

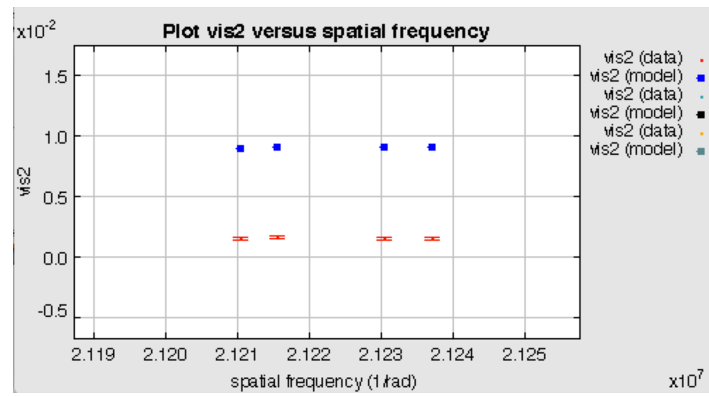
- *Run fit* (bottom of the Settings tree)

Exercise 1- simple fit 1.1

- Visualize the result of the fit:
  - Settings Tree →*Results* (and personal notebook)
    What do you can deduce from Chi2 value?
  - with plots from *Plot model Panel* : Plot Radial (ex: VIS2, try "Residuals", "overplot model")
  - To Notice: plotting the fitted data or the array is also possible from *Result panel*



- if VIS2 plotted, residuals are also automatically plotted (other plot to open from Settings tree/Results/Fit Results
- possibility to zoom (draw with the mouse the zone to be zoomed. (Reset zoom button on right top corner)



- Try a fit after removing the setting "Normalize total flux":
  Explain the value of the flux_weight parameter and of Chi2

*** wrap-up pause***

Exercise 1- simple fit 1.1

- What kind of data did you load ?

  VIS2 and T3phi

- What clue do you get from the OI_T3 data ?

  T3 null then centro-symmetrical model suitable for fitting

- What do you can deduce from Chi2 value?

  final reduced Chi2 ~ 0.5 with σ ~0.3 → $Chi2_r$ <1 means a low probability to have this model, even if the fit result seems good. That can indicate that the errors on the data are too large.

- Explain the value of the flux_weight parameter and of Chi2

  Flux_weight has no effect in the Chi2 calculation

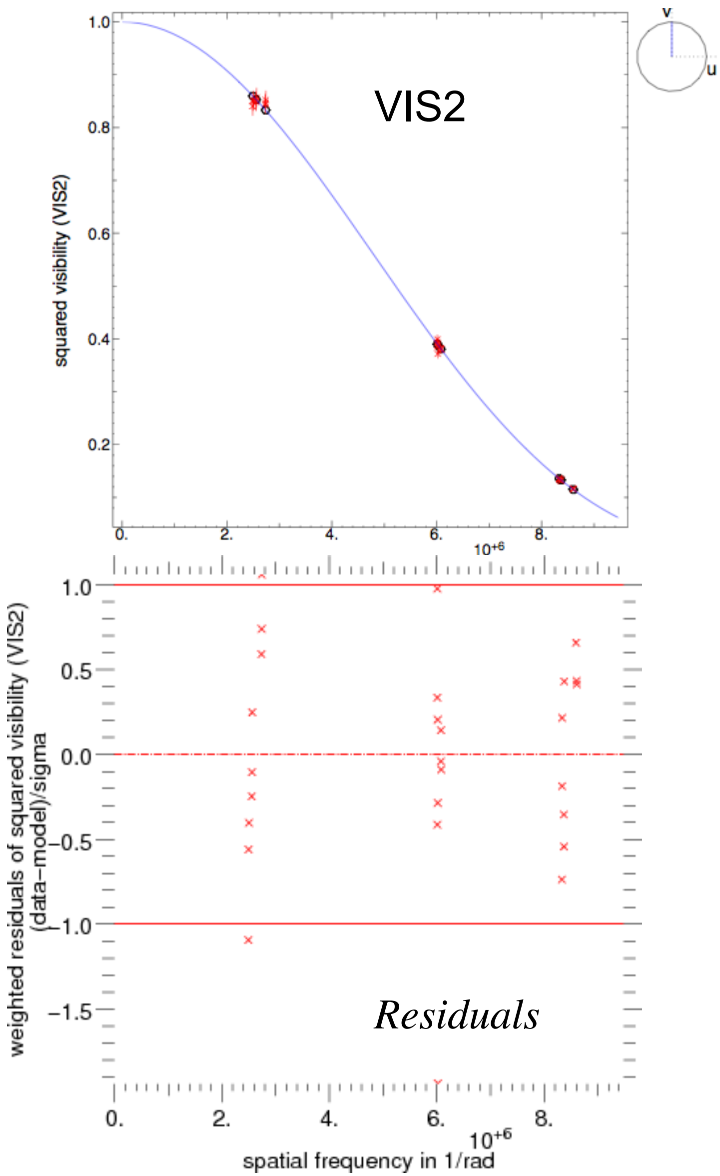
  There is no constraint on the flux in the interferometric data.

  If "Normalize total flux" is removed:

  o no constraint on the flux, thus *flux_weight* is undetermined

  o flux_weight has no effect on data, thus chi2 is the same.

Exercise 1 - simple fit 1.1



arcturus.1.79mu.oifits
+ disk

fit result:

reduced Chi2: final= 0.4392 - sigma= 0.2828
Number of degrees of freedom = 25

Final values and standard deviation for fitted parameters:
diameter1 = 20.348 +/- 0.0267 mas
diameter1 ≃ 20.35 +/_ 0.03 mas

Concerning the normalization:
- Flux is not constrained by interferometric data
- Either:
  - o fix the flux
  - o use *Normalize total flux* (add a constraint to chi2)

Exercise 1- simple fit 1.2

- Open a *New setting* and load the file **arcturus.1.52mu.oifits**

- Process as Exercise 1.1
  ex: same model function *disk*

- *Run fit* from various initial values of the diameter (*value* = 0 mas, 20 mas, 25 mas)
  How are the results of these fits ?

- Explore the "Chi2 space" for analyzis:

Plot Chi2 1D with Parameter[diameter1) (*log* & *reduced* selected) (min= 0, max= 30, #samples=100)

What do you observe ?

Why the final Chi2 is not so good when fitting from the global minimum ?

- Try another model: a center to limb-darkening model, for ex. *limb_power*

*** wrap-up pause***

Exercise 1- simple fit 1.2

- How are the results of these fits ?

There are different.

==> highlight the importance of the initialization, the question "how to start the fit" should come, and the necessity to explore the chi2 space

- What do you observe ?

one global minimum exists @20mas but a minimum exists also @13.5, another @29.5mas

- Why the final Chi2 is not so good when fitting from the global minimum ?

Starting the fit on the global minimum, the result is nevertheless not good, despite an apparently acceptable plot of VIS2. Necessity and usefulness to plot the residuals. On them, we see that the model is mostly below or above the data.

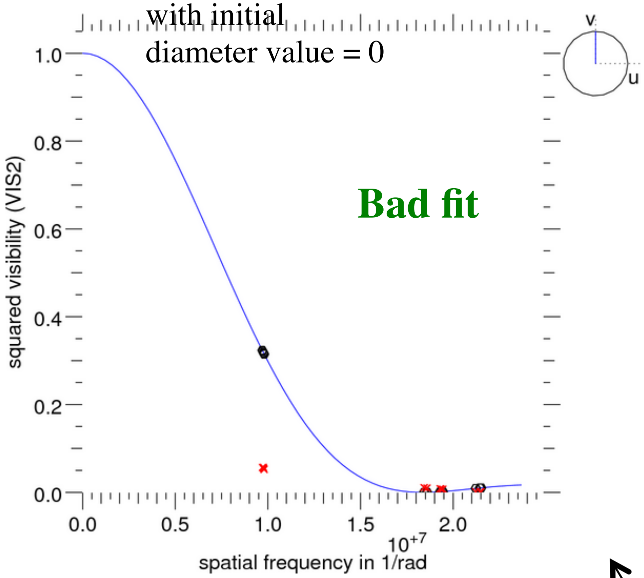
➔ the model of a disk is not good ➔try another model

Suggest to the student, if not found, a center to limb-darkening model : these function will attenuate the amplitude of the second lobe and then will allow a better fit. (there are different limb-functions, take for ex. the first one, with a power law)

*** wrap-up pause***

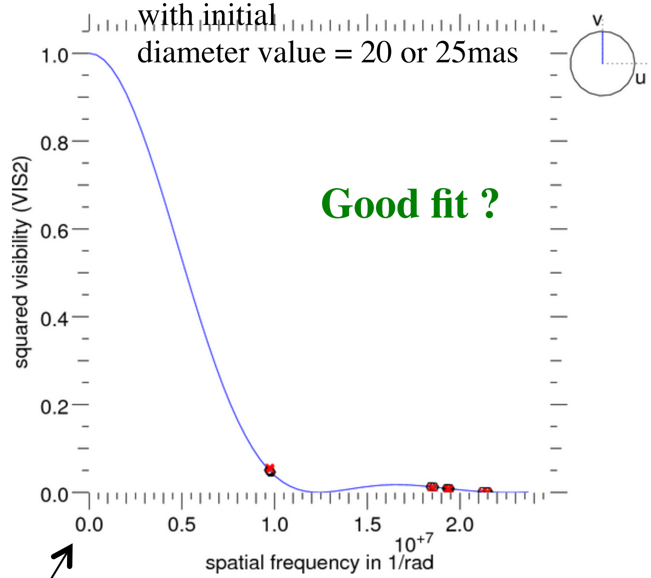
Exercise 1- simple fit 1.2

arcturus.1.52mu.oifits + disk

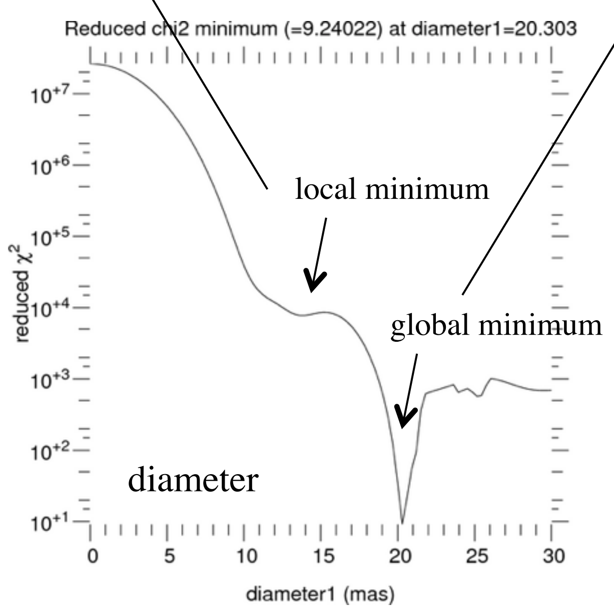


reduced Chi2: final= 1.043e+04

diameter1 = 13.694 +/- 0.445 mas



Plot Chi2 1D



No !

reduced Chi2: final= **11.87**

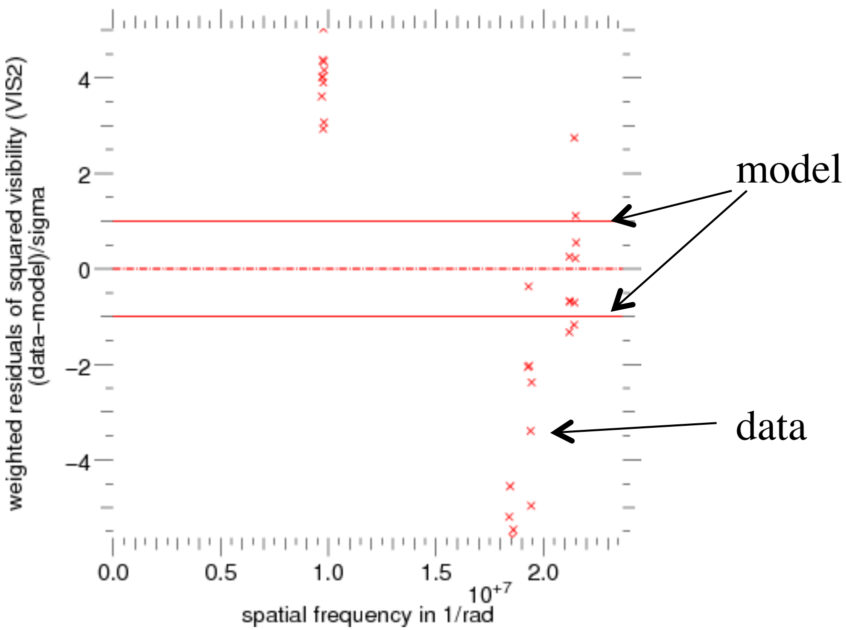
diameter1 = 20.333 +/- 0.0398 mas

Have a look on the residuals of the fit…

Exercise 1- simple fit 1.2

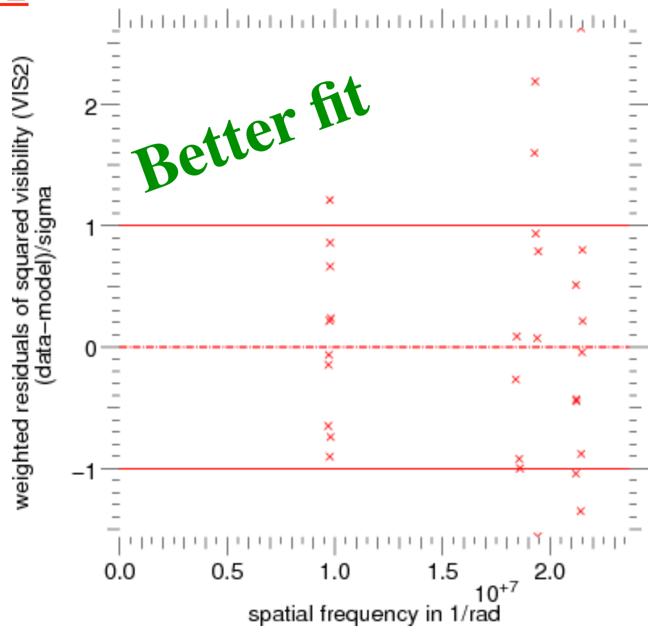
Weighted residuals of VIS2

arcturus.1.52mu.oifits + disk



reduced Chi2: final= 11.87 –
diameter1 = 20.333 +/- 0.0398 mas

arcturus.1.52mu.oifits + limb_power



reduced Chi2: final= 1.099
diameter1 = **20.68** +/- 0.0266 mas
power1 = 0.22869 +/- 0.0145
≃ **0.23 +/- 0.014**

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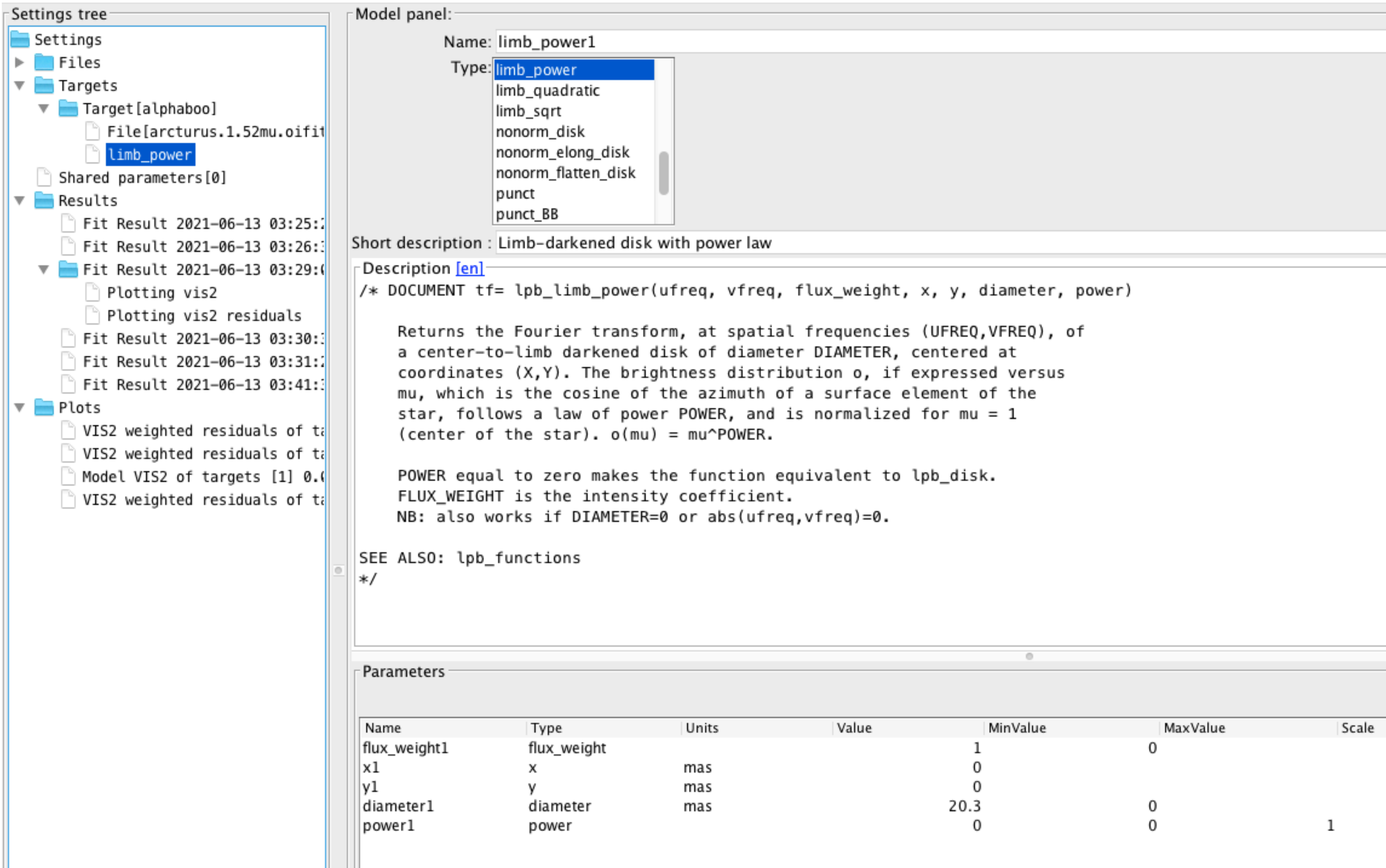
GUI functionality

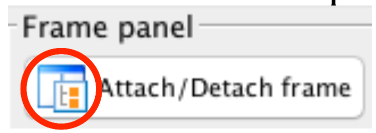
Exercise 1- simple fit 1.2

To know the description of the selected model, simple click on the name on Settings tree:

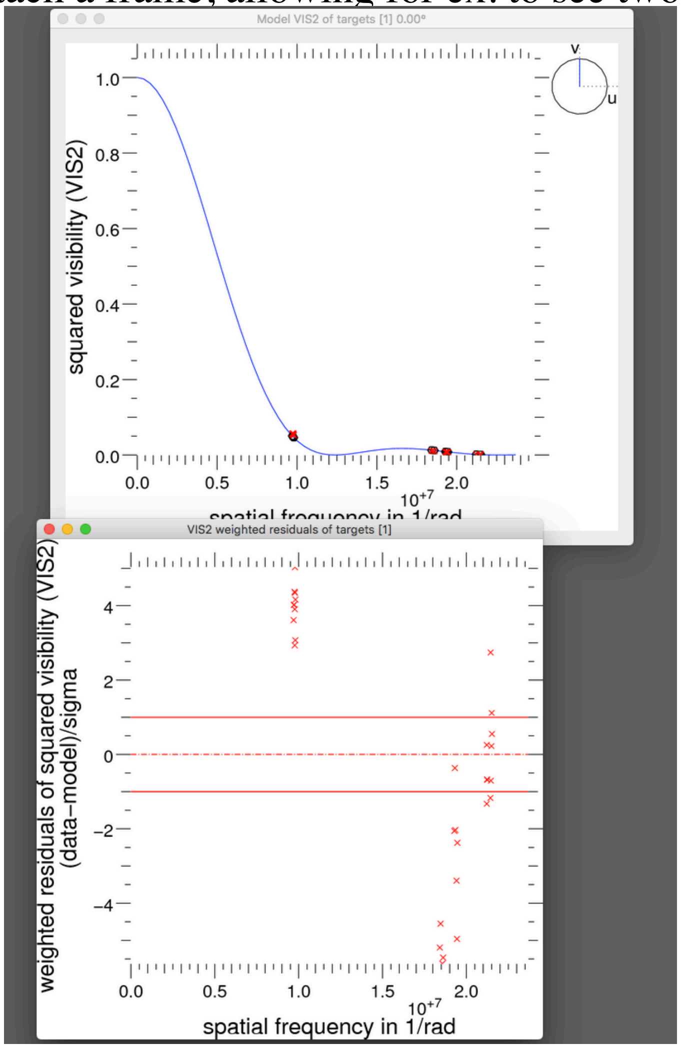


Exercise 1- simple fit 1.2

From the shortcut menu (detachable) or from each frame panel



possibility to detach/attach a frame, allowing for ex. to see two frames simultaneously



Exercise 2 - Fit with sharing of parameter

- **Aim** : on **2** data sets, one by wavelength, fit a model of center-to-limb darkening (e.g. power law) considering that:
  - the diameter of the photosphere (therefore common to both groups) is achromatic
  - the center-to-limb darkening coefficient is chromatic

Exercise 2 - Fit with sharing of parameter

- Open a New setting and load the files **arcturus.1.52mu.oifits** **and** **arcturus.1.79mu.oifits**
  - *Add new target* for file arcturus.1.52mu.oifits and select *limb_power* ---> group1
  - *Add new target* for file arcturus.1.79mu.oifits and select *limb_power* ---> group2

- Share the diameter between both groups, using contextual menu (mouse right click) in the Parameters table:
  - for *diameter1*: *share this parameter*
  - for *diameter2: link* it with *diameter1*
  - you may verify with *Shared parameters* of the Settings tree

- *Run fit*

- Plot all the data and fitted models on the **same** graph:
  - use the *Common plots panel* accessible when clicking on Settings Tree →*Plots*
  - select both targets

- Plot an image of the final model

*** wrap-up pause***

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Exercise 2 - Fit with sharing of parameter

- No real difficulty for this exercise. Only necessary to know the contextual menu active through the parameters panel.
- Always suggest to the students to plot the residuals
- To notice: the model for arcturus.1.79mu is not over-plotted to the highest spatial frequency since the data are at lower frequencies.
- Plot image ➔ to know the "Send FITS" to SAOImage DS9

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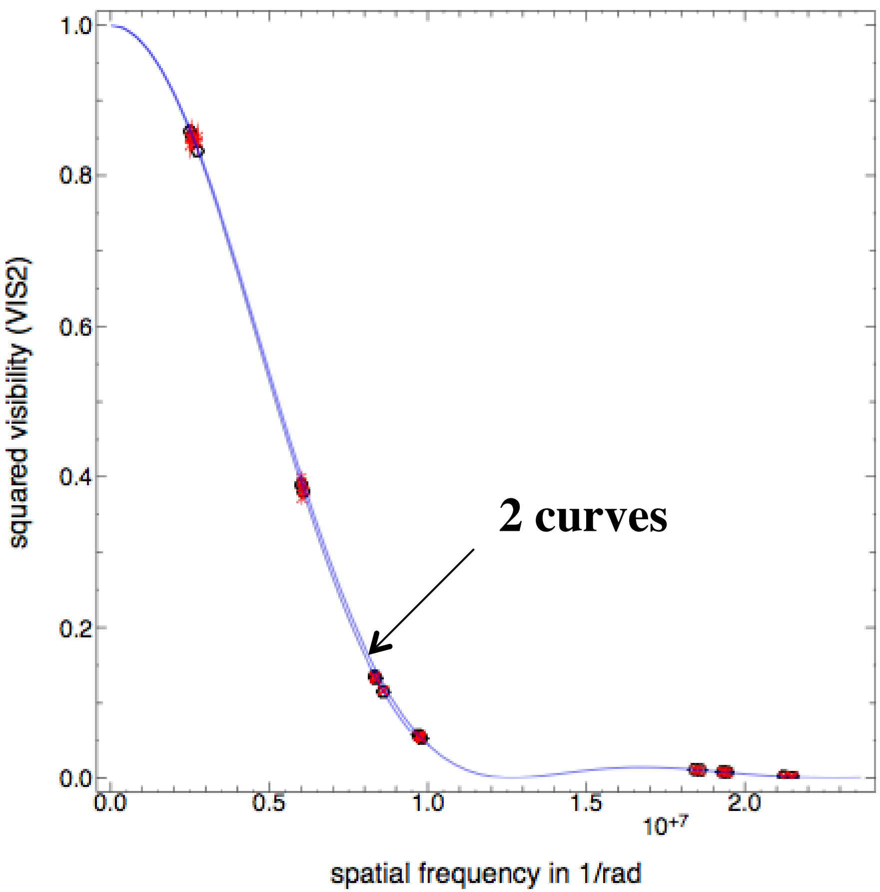
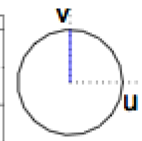
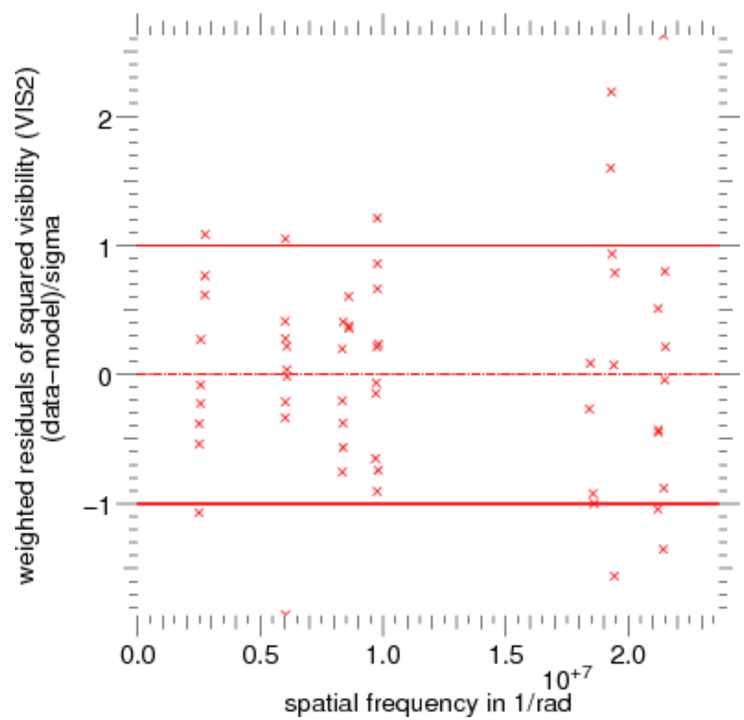
Exercise 2 - Fit with sharing of parameter





arcturus.1.52mu.oifits + *limb_powe r*
arcturus.1.79mu.oifits + *limb_power*
diameter shared

diameter1 = 20.68 +/- 0.025 mas
power1 = 0.229 +/- 0.014 @1.52mm
power2 = 0.118 +/- 0.013 @1.79 mm
reduced Chi2 = 0.84 sigma = 0.17



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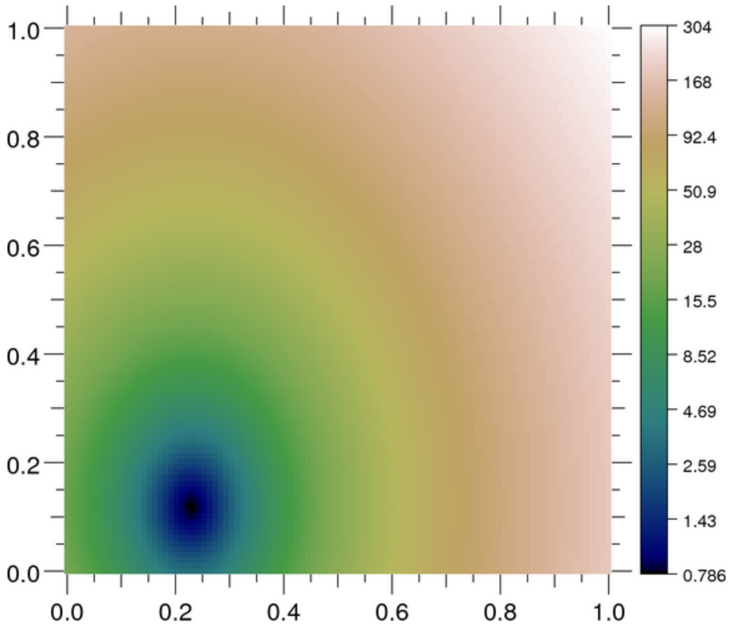
Exercise 2 - Fit with sharing of parameter

Complement

- if a student has explored the Chi2 space:

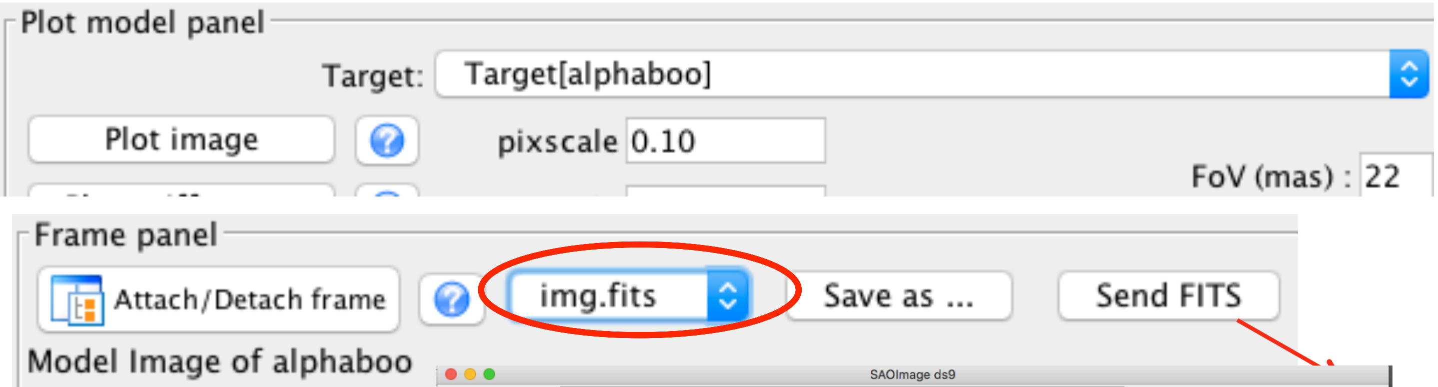
Chi2 map for the LD parameters, power2 versus power1

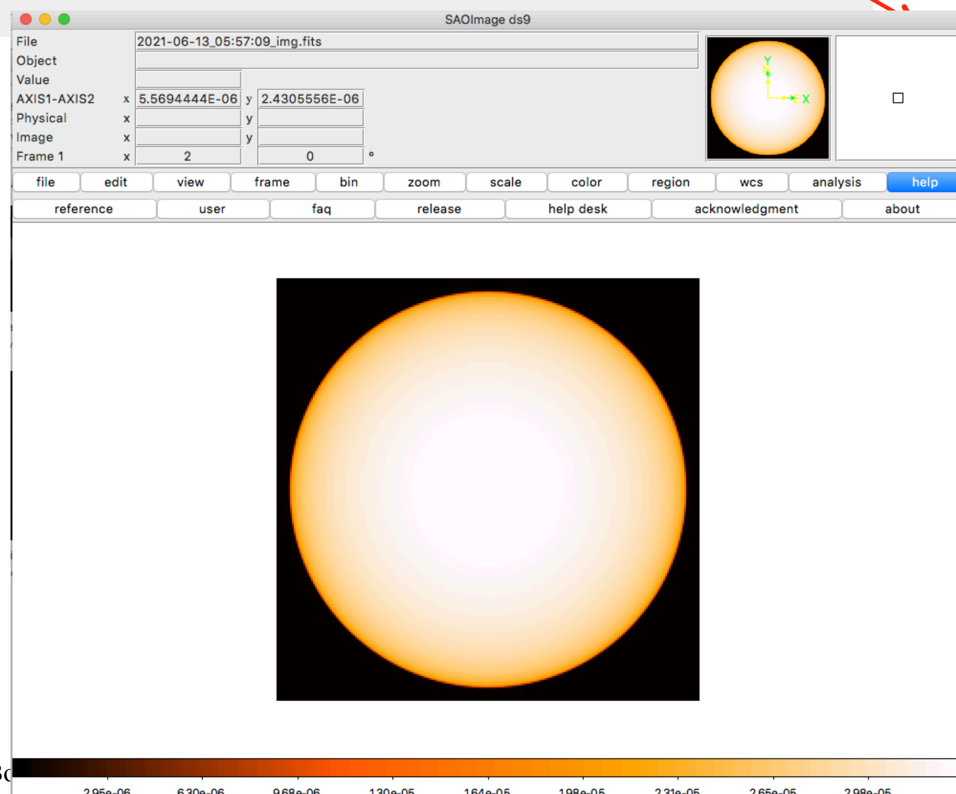
- Reduced chi2 first minimum at power1=0.232323, power2=0.121212



Exercise 2 - Fit with sharing of parameter

- Plot Image of the final limb-darkening model of Arcturus:





SAOImage DS9

go further in
SAOds9
to draw e.g. profiles

but later
not during the
session

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Exercise 3 - Fit with degeneracies 3.1

- **Aim**: estimate the separation of a binary
- Open a New setting and load the file **2018-07-17T085626_94Aqr_B2D0C1-mj2.fits**
- See the data (files → plot VIS2) Look at VIS2
  - Which indication do you deduce for the model of the object?
  - Coud you estimate roughly some parameters of the model?
  - Load the file in OIFITsExplorer for a better view of the data and easier measures.
  - You may take a pen for the formula and

click on different points on the *VIS2DATA vs SPATIAL_FREQ* view for the measurements.

*** wrap-up pause***

to see formula and some estimation

Exercise 3 - Fit with degeneracies 3.1

- Which indication do you deduce for the model of the object?

sinusoïdal variation ➔ binarity (remember D. Buscher's talk or Michel's talk or Practice session on *Interferometry basics with ASPRO* or simply that the Fourier transform of 2 Dirac delta functions is a sinusoïdal function.

Coud you estimate roughly some parameters of the model?

i.e. the flux ratio of the components $r = phi_2/phi_1 < 1$

and the order of magnitude of the angular separation ρ

after ~5min, wrap-up pause to see the method and results

*** wrap-up pause***

Exercise 3 - Fit with degeneracies 3.1

How to estimate roughly *r* and ρ of a binary

$O(\alpha) = \phi_1\delta(\alpha) + \phi_2\delta(\alpha - \rho)$ image plane

$\rightarrow TF(O(\alpha)) = \widehat{O(u)} = \phi_1 + \phi_2 e^{-2i\pi.u\rho}$ Fourier plane

$\rightarrow V^2(u) = \left|\widehat{O(u)}\right|^2$

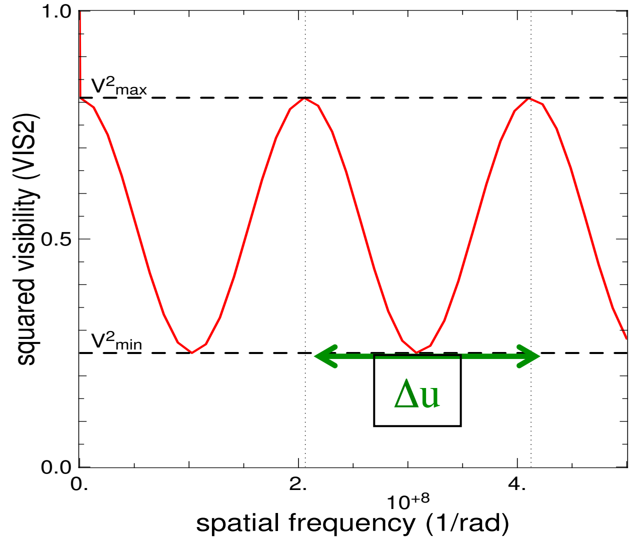
$V^2(u) = \phi_1^2 + \phi_2^2 + 2\phi_1\phi_2 cos(2\pi u\rho)$ ==> $\rho = \frac{1}{\Delta u}$

$\rightarrow V^2_{\max} = (\Phi_1 + \Phi_2)^2$

& $V^2_{\min} = (\Phi_1 - \Phi_2)^2$

$\rightarrow \Phi_1 = \frac{1}{2}\left(\sqrt{V^2_{\max}} + \sqrt{V^2_{\min}}\right)$

$\Phi_2 = \frac{1}{2}\left(\sqrt{V^2_{\max}} - \sqrt{V^2_{\min}}\right)$



Flux ratio of the components: $r = \frac{\phi_1}{\phi_2} < 1$ and normalization: $\phi_1 + \phi_2 = 1$

==> $r = \frac{\sqrt{V^2_{max}} - \sqrt{V^2_{min}}}{\sqrt{V^2_{max}} + \sqrt{V^2_{min}}}$ & $\phi_1 = \frac{1}{1+r}$

if r<<1 $\phi_1 \simeq 1 - r$ & $\phi_2 \simeq r$

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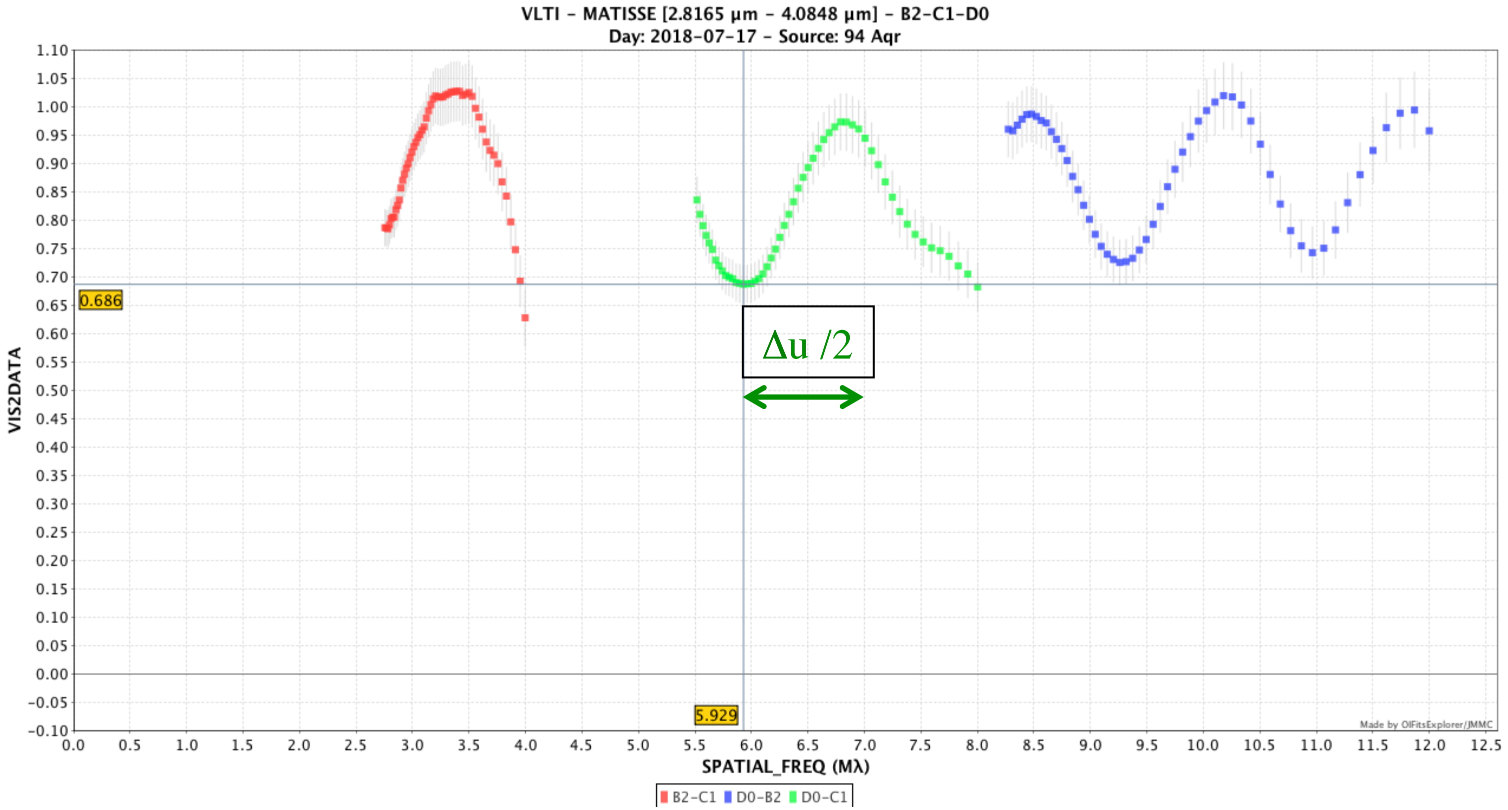
Exercise 3 - Fit with degeneracies 3.1



[(spatial_freq,$V2_{min)}$);(spatial_freq,$V2_{max}$)]

[(5.95e6 , 0.686);(6.82e6, 0.974)] → r = 0.0874 & ρ = 5.74713e-07 rad ~119 mas

[(9.27e6, 0.724);(1.02e7, 1.02)] → r = 0.0855 & ρ = 5.37634e-07 rad ~111 mas

$\phi_1 \simeq 0.92$

$\phi_2 \simeq 0.08$

take ρ > 120 mas

Exercise 3 - Fit with degeneracies 3.1

- Build the model:
  - combine 2 *puncts*
  - select **VIS2** and **T3phi**
  - leave the parameters x1,y1 fixed at 0: the main component is centered (it is the default case)
  - bound the flux_weight to [0,1] and take as initial value the ones you have estimated from the VIS2 plot
  - bound the parameters x2, y2 regarding the order of magnitude of ρ

- Use ***Plot Chi2 2D*** with parameters (x2, y2) - increase the sampling (eg #samples100)
- Observe the Chi2 map
  - explain the valleys and their orientation
- Take the values of (x2,y2) of the first minimum as initial values for the fit, and fit
  - what happens?

- So, what to do for resolving the binary 94 Aqr ?
  - you may ask a teacher

*** wrap-up pause***

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Exercise 3 - Fit with degeneracies 3.1

- explain the valleys and their orientation
  - any binary with the same y2/x2 ratio will give the same data since we have only one cut in the (u,v) plane. There is no single solution : there is **degeneracy**
  - invite to plot the uv-coverage. For ex: *Settings > Plots* ---> Common plots panel – Select Target[94 Aqr] ---> *Plot UV-coverage*
  - *Detach frame* .and approach it to the Chi2 map ==> perpendicularity valleys/ baseline

- What happens?
  - Flux_weight are fitted, but not [x2,y2].
  - Have a look to the correlation matrix: [x2,y2] are anti-correlated ($coef_{x2y2}$ of the correlation matrix = -1), related to the linear valleys of the Chi2 map and the colinear baselines.

- So, what to do for resolving the binary 94 Aqr?
  - add a concurrent baseline

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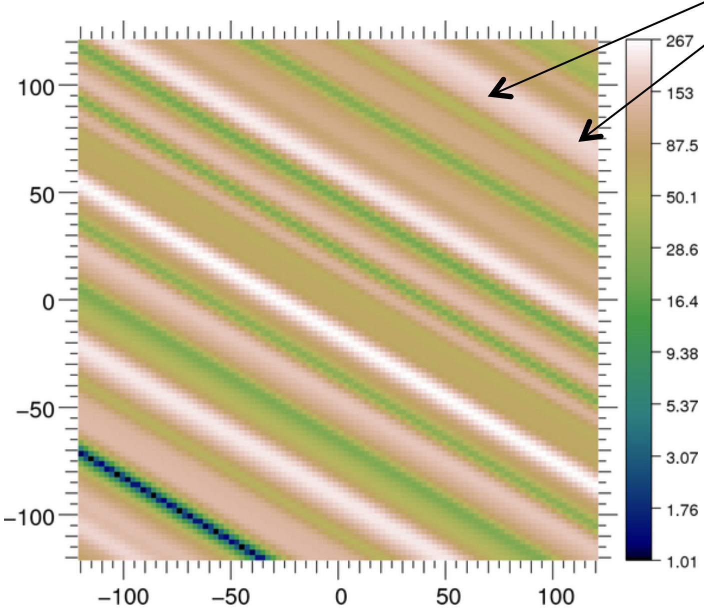
Exercise 3 - Fit with degeneracies 3.1

2018-07-17T085626_94Aqr_B2D0C1-mj2.fits

Plot Chi2 2D, x_2, y_2



infinity of [x2, y2] giving the same Chi2 minimum
➔no single solution = **degeneracy**

[x2, y2] are correlated

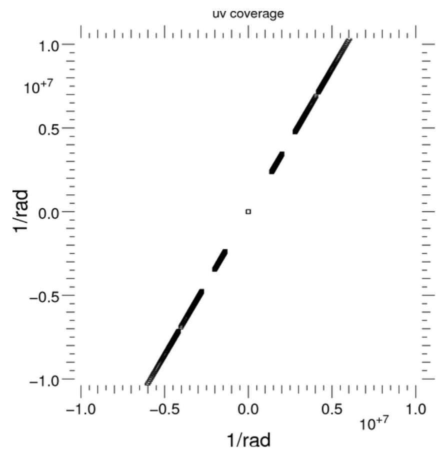
Example of a result of a fit:

flux_weight1 = 0.9167 +/- 0.0624
flux_weight2 = 0.083304 +/- 0.00571
x2 = -36.46 +/- 201 mas
y2 = -120 +/- 117 mas (*)
reduced Chi2 final= 0. 8872

--- Correlation matrix ---

	i1	i2	x2	y2
i1	1	0.99	-0.0035	0.0035
i2	0.99	1	0.038	-0.038
x2	-0.0035	0.038	**1**	**-1**
y2	0.0035	-0.038	**-1**	**1**

uv coverage



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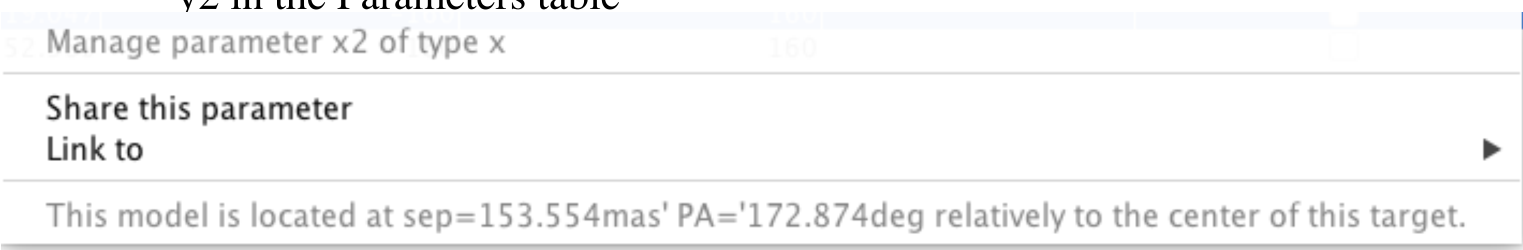
Exercise 3 - Fit with degeneracies 3.2

- In the current setting, add the file **2018-07-17T085626_94Aqr_A0-mj2.fits** and select it
- Use again ***plot chi2 2D*** with (x2, y2) (increase the bounds to for ex. **160mas**)
  - set the (x2,y2) corresponding to the best minimum of chi2, as well as both flux_weights and fit.
  - See the possibility to convert (x2,y2) in polar coordinates: use **contextual menu** on x2 or y2 in the Parameters table



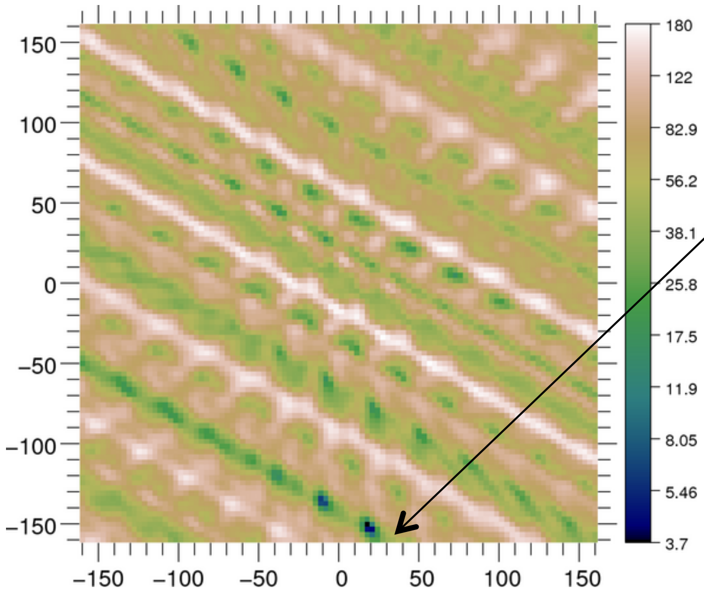
you may check your answer with a teacher

*** wrap-up pause***

Exercise 3 - Fit with degeneracies 3.2

Plot Chi2 2D, x_2, y_2

2018-07-17T085626_94Aqr_B2D0C1-mj2.fits + 2018-07-17T085626_94Aqr_A0-mj2.fits



global minimum →

Reduced chi2 first minima:

1. 3.69897 at x2=17.7778, y2=-150.303
2. 4.31356 at x2=21.0101, y2=-153.535
3. 4.68377 at x2=17.7778, y2=-153.535
4. 6.20896 at x2=-11.3131, y2=-134.141
5. 6.77393 at x2=-8.08081, y2=-137.374
6. 7.07319 at x2=-11.3131, y2=-137.374

flux_weight1 = 0.91358 +/- 0.0416
flux_weight2 = 0.086419 +/- 0.00397
x2 = 19.047 +/- 0.0485 mas
y2 = -152.37 +/- 0.0635 i.e. **[ρ, PA] =** [153.55 mas , 172.87°]
reduced Chi2 final ~1.12

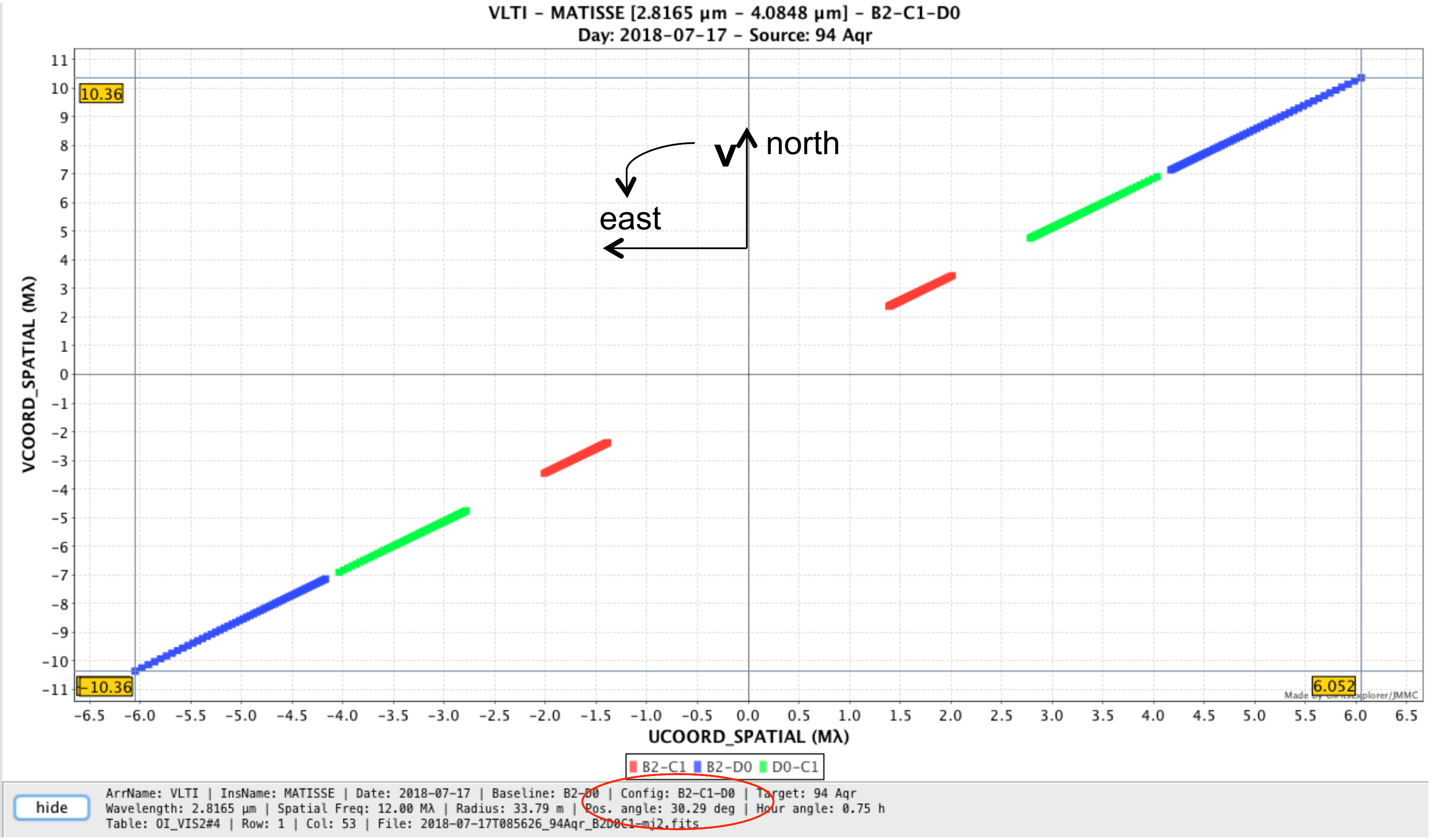
Docobo et al, The Astronomical Journal, The Astronomical Journal, 156:85, 2018
orbital parameters of 94 AqR A-B
==> 2018.5 et ρ = 0.153" : PA = 355.2 deg (to see, how is East axis oriented)

if Plot Image on SAOds9, be careful on the orientation (turned from180 degrees for the East direction).

Exercise 3 - Fit with degeneracies 3.2

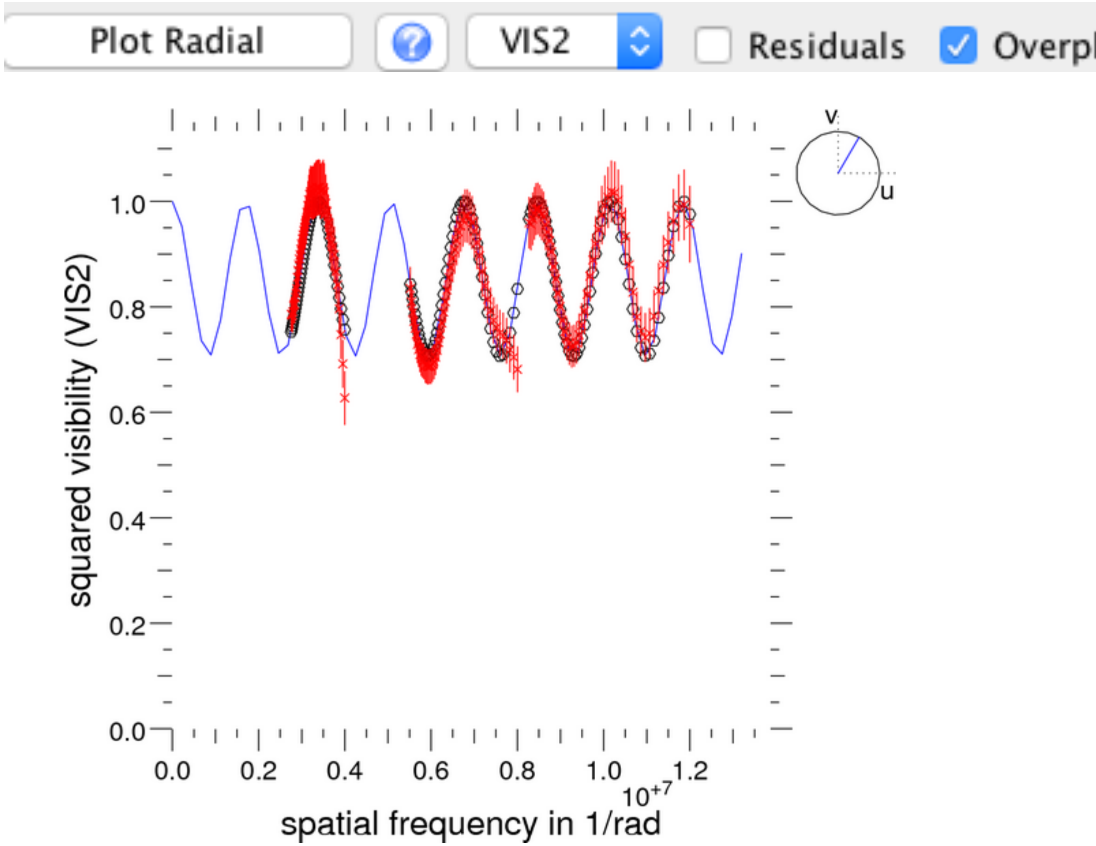
Some complements

If there is a question about the angle of the *Overplot model with cut angle*
To find the angle of a baseline: use OIFITsExplorer

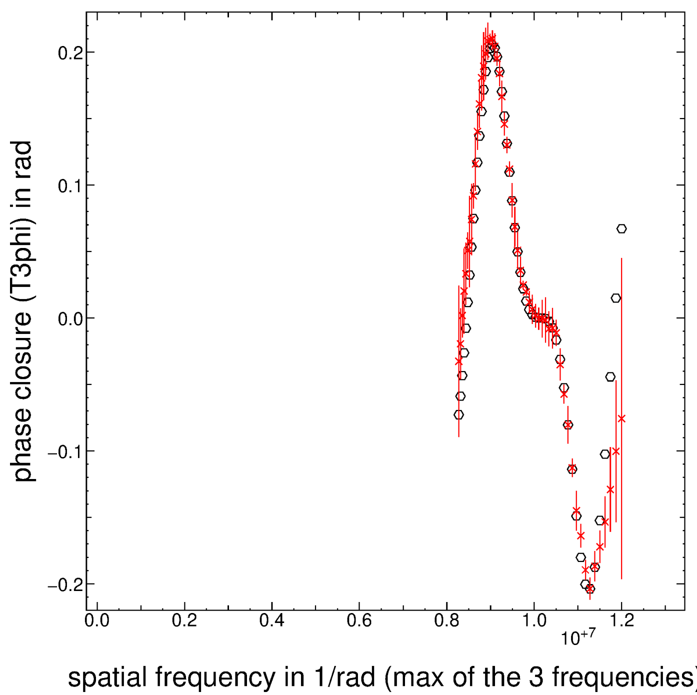


Exercise 3 - Fit with degeneracies 3.2

Some complements



T3phi



Exercise 4: star with circumstellar environment

- **Aim**: recognize artifacts from chromatic object in the data. Use of a chromatic model

- Open a new setting and load some V854Cen data : V854Cen-Hband-Kband.fits

  Real data from AMBER/VLTI

  V854Cn is a R Coronae Borealis (RCB) star (small group of carbon-rich supergiant)

Exercise 4: star with circumstellar environment 4.1

- See T3phi data
  - What could you deduce?
- Build a first model with an unresolved star and a simple environment (a shell)
  - punct + for ex. gaussian centered and both fixed at (0,0) or a disk
  - bound the parameters (flux_weights [0,1], fwhm of the shell [0,30mas]
  - select VIS2 and T3phi data for fittin
- ***Plot Chi2 1D*** (fwhm) → initial guess
- *Run fit* with this initial guess
- Observe the results and the radial plots of VIS2 and T3phi
  - How explain the commas in VIS2 data?

  i.e. the alignments of data points whose values increase with spatial frequency.
  - You may open files with OIFits Explorer
  - You may check with a teacher
- Conclusion?

*** wrap-up pause***

JMMC

Exercise 4: star with circumstellar environment 4.1

- What could you deduce from T3phi data
  - T3phi close to zero ===> excluding a close companion, circumstellar structure may be supposed centro-symmetrical.

- How explain the commas in VIS2 data?
  - open OIFits Explorer and add the file
  - on the Plot interface, choose *Show VIS2DATA, T3PHI vs SPATIAL_FREQ*

  and *Color by "Effective wavelength"*

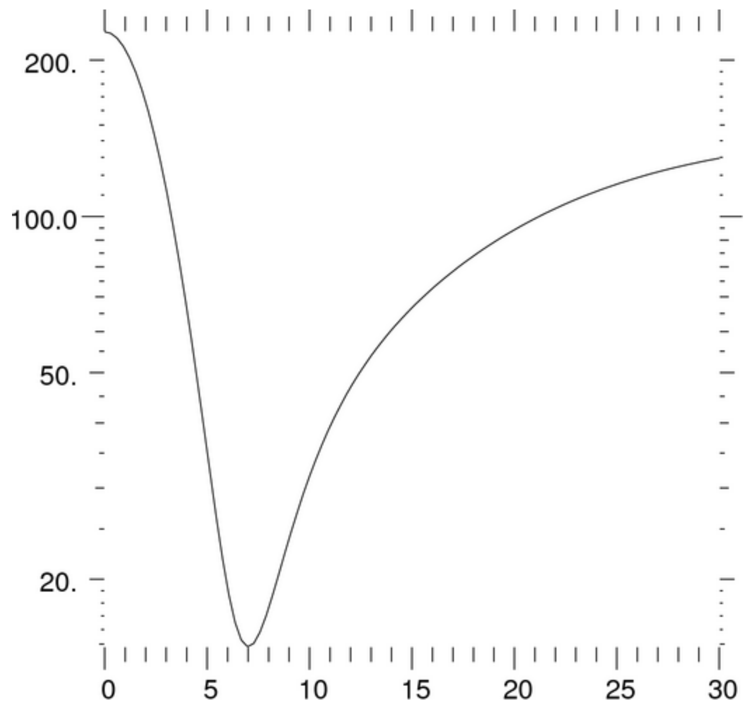
  → the form of the comma appears clearly as an effect of wavelengths

- Conclusion?
  - an achromatic model cannot fit well such chromatic data.

Exercise 4: star with circumstellar environment 4.1

Fit VIS2 & T3phi with central punct & or gaussian

Chi2 1D fwhm



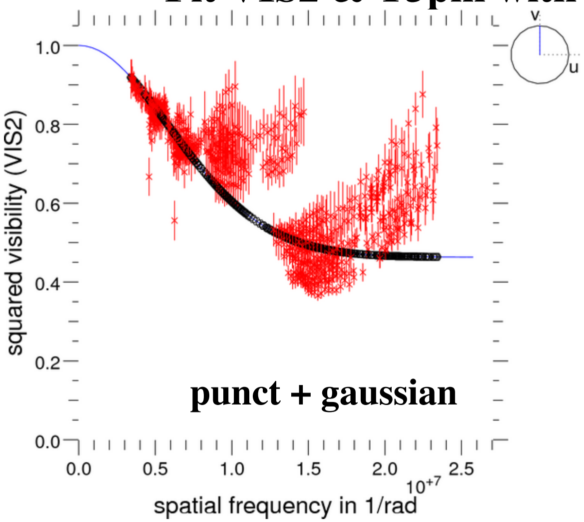
Reduced chi2 first minima:

14.8287 at fwhm2=6.9697

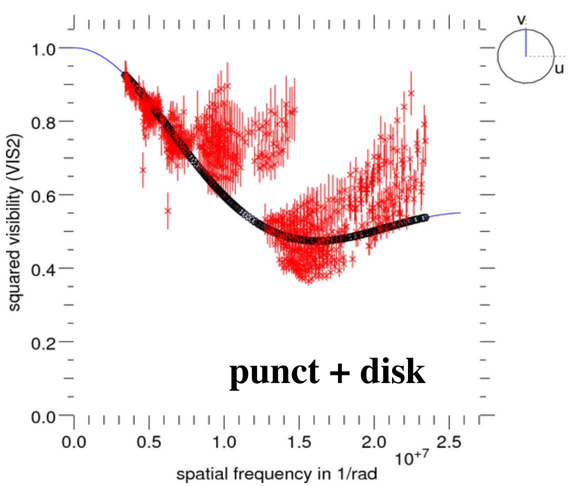
→ initial guess

Exercise 4: star with circumstellar environment 4.1

Fit VIS2 & T3phi with central punct & centered gaussian (or disk)



flux_weight1 = 0.68048 +/- 0.0636 central punct
flux_weight2 = 0.31951 +/- 0.0301 shell around
fwhm2 = 11.944 +/- 0.238 mas gaussian
reduced Chi2 final= 7.7 sigma= 0.05

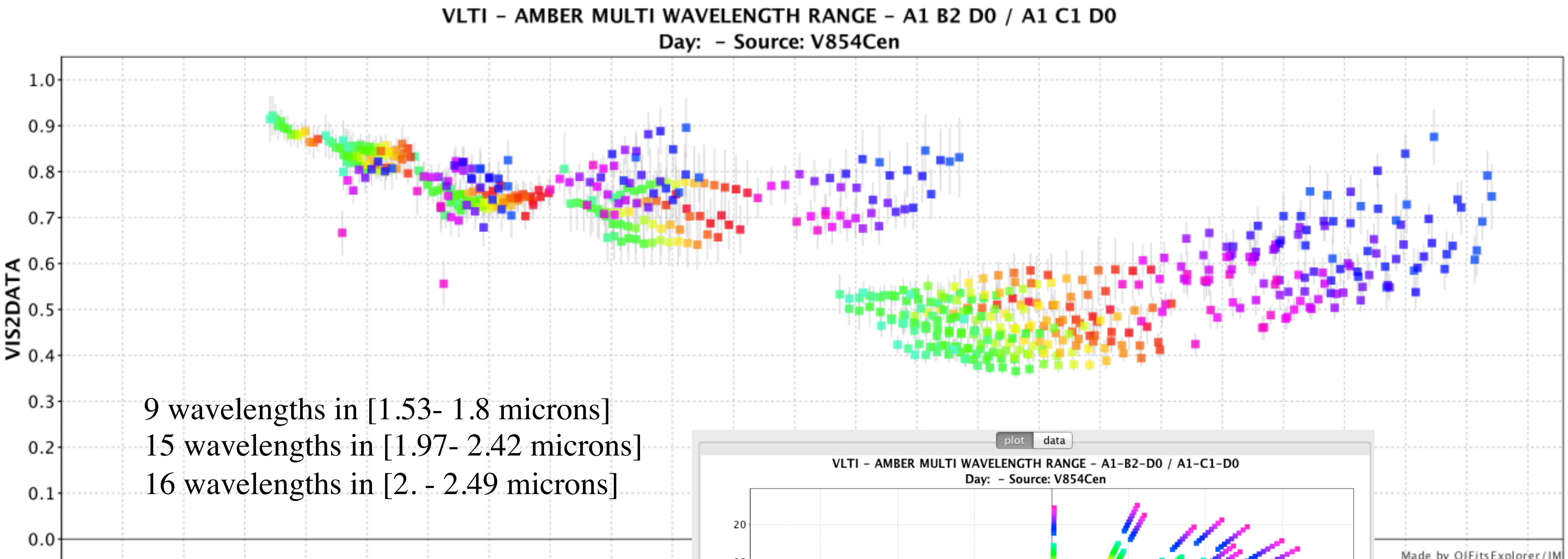


slightly better with a disk shell
flux_weight1 = 0.72504 +/- 0.0614
flux_weight2 = 0.27496 +/- 0.0234
diameter2 = 20.783 +/- 0.254 mas
reduced Chi2 final= 6.3 sigma= 0.05

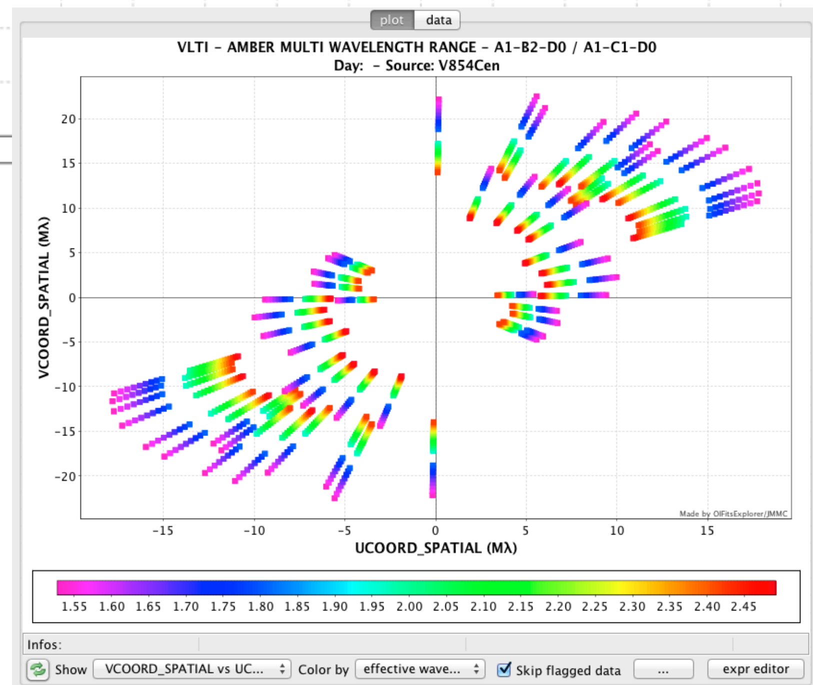
Such achromatic models seem to follow a mean behaviour of the data versus the spatial frequencies, but they cannot fit the "commas".
Those are due to the chromatism of the data.

Exercise 4: star with circumstellar environment

Use of OIFits Explorer 4.1



9 wavelengths in [1.53- 1.8 microns]
15 wavelengths in [1.97- 2.42 microns]
16 wavelengths in [2. - 2.49 microns]



"Commas" in VIS2 are a chromatic artifact

CRAL

Exercise 4: star with circumstellar environment 4.2

- Build a chromatic model with **blackbody functions** (with the "_BB" suffix)
  - **see the help** of these modeling functions to know how the blackbody is introduced
  - you **may start the fit from the achromatic solution in 4.1** with two radial components, but affected now by a Planck function (→ black body components)

    ex. central punct + gaussian or disk → *punct_BB* + *gaussian_BB* or *disk_BB*

Hypothesis

from *Bright, S. N., Chesneau, O., Clayton, G. C., et al. 2011, MNRAS, 414, 1195* : the temperature of the star is **6750K** →the temperature of punct_BB may be fixed for the fit.

- Leave "Normalize Total Flux" selected on VIS2 & T3phi
- But take care to not bound the flux_weights, which are no more the flux_ratio
  - you may see a teacher for explanation.

*** wrap-up pause***

Exercise 4: star with circumstellar environment 4.2

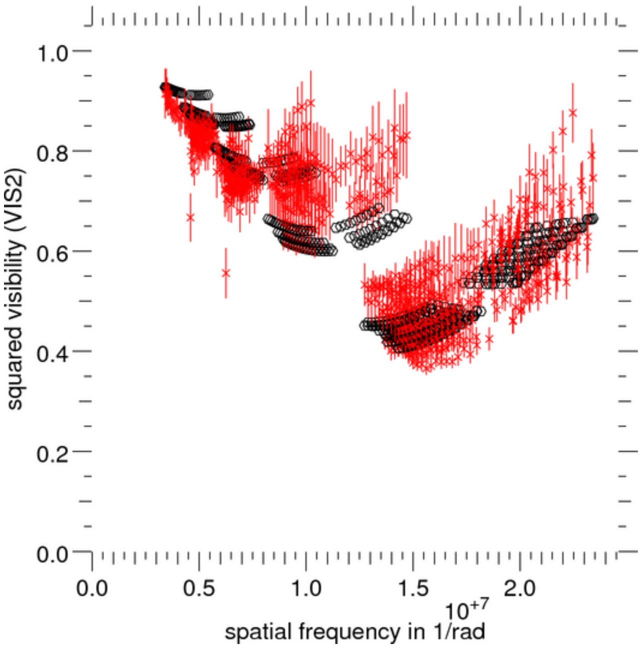
- *Chi2 slice 1D* on temperature_2 or *Chi2 slice 2D* on temperature_2 and fwhm_2 allows to have initial guess and to fit
- *Plot VIS2* shows that the model now varies with ~comma shape
- But need to improve the model because of a significative gap of VIS2 between the model and the data at some spatial frequencies.

Exercise 4: star with circumstellar environment 4.2

Use of models weighted by a black body

for example : **Fit VIS2 & T3phi with central punct_BB & centered gaussian_BB**

temperature1 = 6750 K – fixed -
flux_weight1 = 3.0552e+35 +/- 5.17
flux_weight2 = 2.1737e+34 +/- 3.13e+32
fwhm2 = 10.3 +/- 0.151 mas
temperature2 = 1917.5 +/- 34.7 Kelvin
reduced Chi2 final= 2.7 - sigma= 0.05



Exercise 4: star with circumstellar environment 4.3

- Could you improve the model ?
  - look again the VIS2 data with OIFITsExplorer *with Color by "Baseline or triplet"*
  - see for ex. the D0-C1 data : compare them with the model (with frame "Model VIS2 of target" of your previous fit.
  - conclusion ? which model could you build?

*** wrap-up pause***

Exercise 4: star with circumstellar environment 4.3

- Could you improve the model ?
  - the model gives shorter VIS2: at the frequencies of this baseline the object is smaller (real VIS2 higher) , it's not the case for D0-A1 or D0-B2
  - we can think about a asymetric shape, like a stretched_gaussian or _disk or equivalent (elong_, flatten_)
  - then the extension of the shell may be expected to be larger along the D0C1axis.

- Complements given if time:

How to know the ratio of flux in the central star versus the wavelength?

  - see Results – a common discussion on the slide would be better -

(function for the calculation not yet implemented).

*** wrap-up pause***

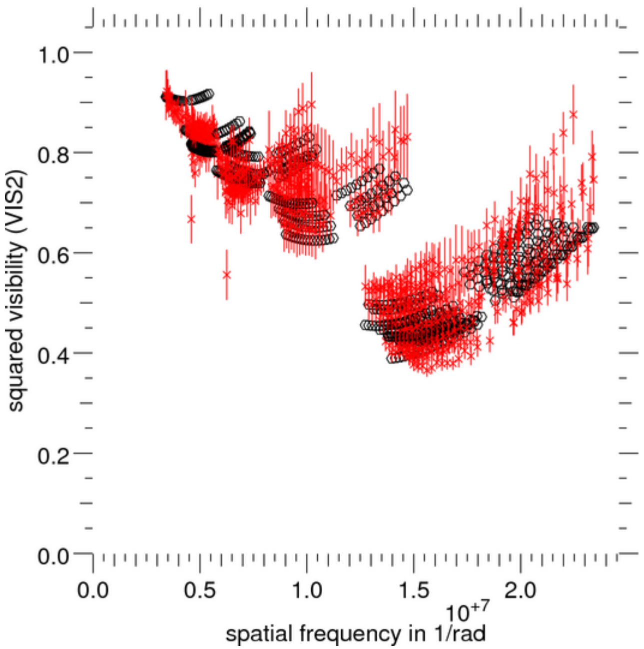
JMMC

Exercise 4: star with circumstellar environment 4.3

Fit VIS2 & T3phi with central punct_BB (T_1=6750 fixed) & stretched_gaussian_BB

reduced Chi2 final= 2.128
fwhm2 = 10.814 +/- 0.192 mas
flux_weight1 = 325.89 +/- 16.4 punct
flux_weight2 = 31.273 +/- 1.86 shell
stretch_pos_angle2 = **35.785** +/- 1.16 degree
stretch_ratio2 = **0.72**009 +/- 0.0119
temperature2 = 1606.2 +/- 35.6 Kelvin

see the help of the function streched_gaussian_BB for a better knowledge of the parameters



Comparison with published results *Chesneau O., Millour F. et al., A&A 569, L4 (2014)*

Extracted from the paper : "Our best-match model for the compact array is a two-component model, consisting of an unresolved uniform disk (Θ ≤ 2.5 mas, star component), and a flattened Gaussian (shell component) with a FWHM of the minor axis of 8 ± 1 mas, and a major axis of 11 ± 3 mas.

The orientation of the major axis is 126 ± 29°. The quality of the fit is relatively good with a reduced χ2 of 1.5."

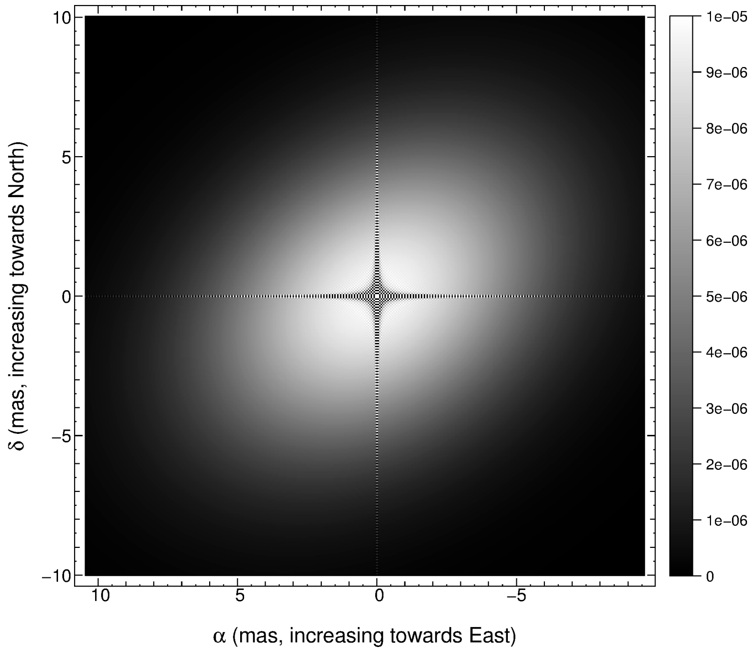
➔ stretch_ratio = **0.72** & orientation of the major axis =126-90 = **36 deg.** **Compatibility**

CRAL

Exercise 4: star with circumstellar environment 4.3

Some complements

- View of the image (need of playing with the levels scale)



final image of the model
(using SAOds9 implies a rotation for the orientation
-> symmetry around the North axis)

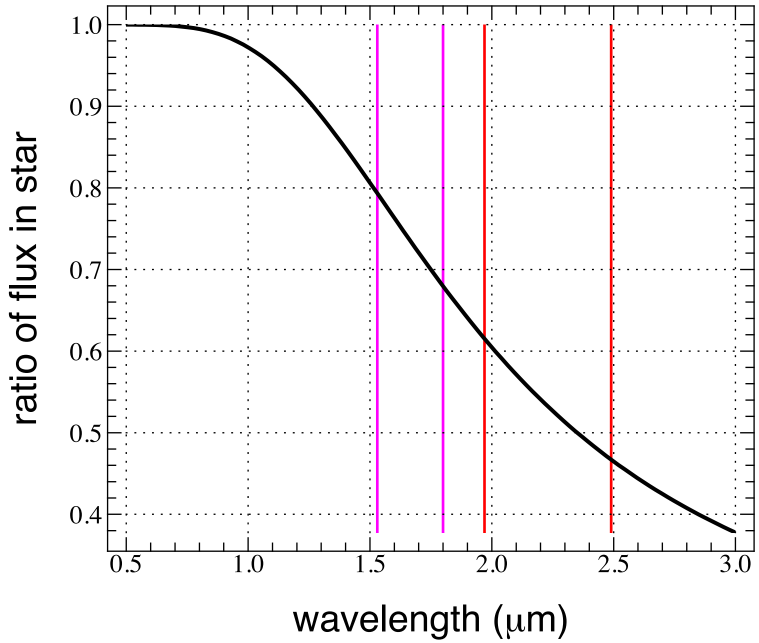
Exercise 4: star with circumstellar environment 4.3

Some complements

To calculate the flux of the star relative to the total flux, at a given wavelength :

$$\frac{\phi_1\ P_N(\lambda, T_1)}{\phi_1\ P_N(\lambda, T_1) + \phi_2\ P_N(\lambda, T_2)}$$

with φ1= flux_weight1, φ2= flux_weight2, and P_N the normalized Planck function.



Extracted from the paper :
"The star flux relative to the total flux steadily decreases from 85% at 1.53 μm to 42% at 2.49 μm , while the shell relative flux increases from 15% to 58%, respectively, with photometric errors at the level of 25%. "

➔ **compatibility with the paper results.**

5- Additional Exercises

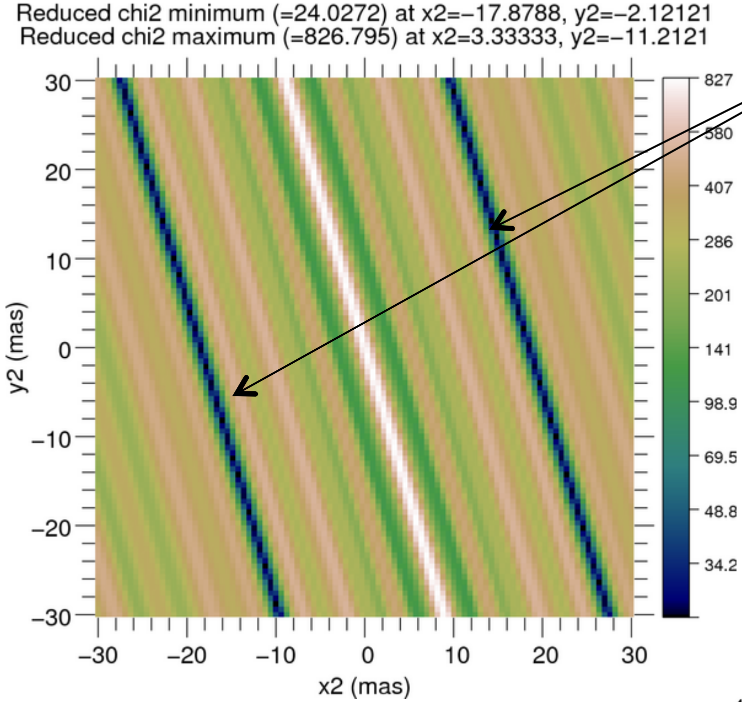
5- Additional Exercises
5.1 Theta Ori C

- Open a New setting and load the file **Theta1Ori2007Dec03_2.fits**
- See tha data and build a modl of a binary woth two unresolved components:
  - select **VIS2 only**
  - proceed as for Exercise 3 (***Plot Chi2 2D*** with parameters (x2, y2))
  - observe the symmetry of the Chi2 map
- To the binary, add the file **Theta1Ori2007Dec05_2.fits**
- Use again ***plot chi2 2D*** with (x2, y2)
  - set the (x2,y2) corresponding to the best minimum of chi2, as well as both flux_weights and fit the VIS2 only
- Find the two best solutions and compare them.
- On the same setting, add now the T3phi for the fit
  - run fit from one of the best set of fitted parameters
  - run fit from the second best set of fitted parameters
  - Conclusion?
- Compare your result with the published one: **Kraus S.** et al, 2009, A&A. **497**-1, pp. 195-207
- Observe the behavior of the VIS2 versus the spatial frequencies:
  - infer how to improve the model

5- Additional Exercises
5.1 Theta Ori C

Theta1Ori2007Dec03_2.fits

Plot Chi2 2D, x_2, y_2



infinity of [x2, y2] giving the same Chi2 minimum
➔no single solution = **degeneracy**

[x2, y2] are correlated

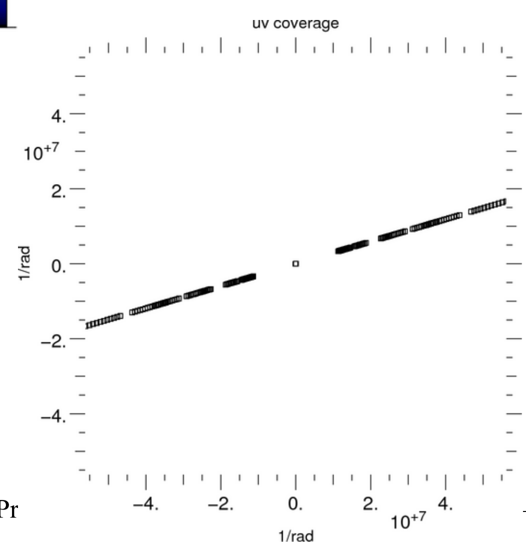
Example of a result of a fit:

flux_weight1 = 0.73235 +/- 0.395
flux_weight2 = 0.26765 +/- 0.145
x2 = -9.7938 +/- 3.7e+04 mas
y2 = -29.335 +/- 1.24e+05 mas
reduced Chi2 final= 23.59 - sigma= 0.161165

--- Correlation matrix ---

	i1	i2	x2	y2
i1	1	0.99	0.062	-0.062
i2	0.99	1	-0.023	0.023
x2	0.062	-0.023	1	-1
y2	-0.062	0.023	-1	1

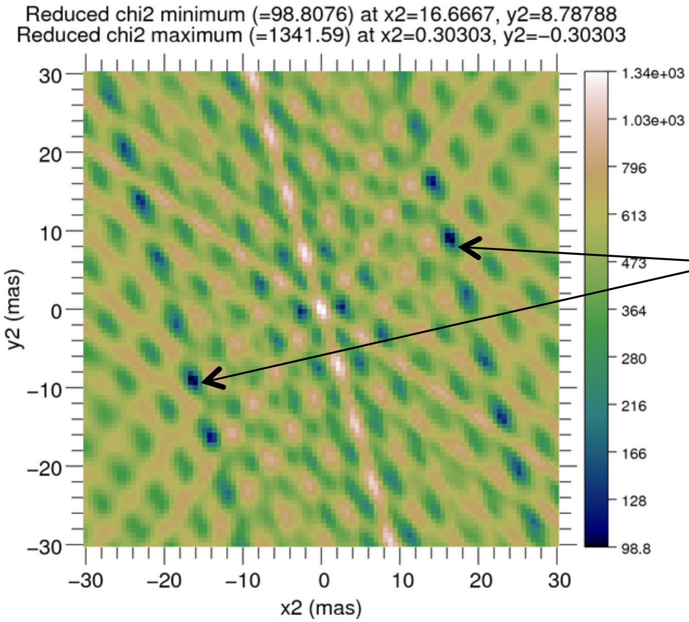
uv coverage



5- Additional Exercises
5.1 Theta Ori C

Theta1Ori2007Dec03_2.fits
+ Theta1Ori2007Dec05_2.fits

Plot Chi2 2D, x_2, y_2

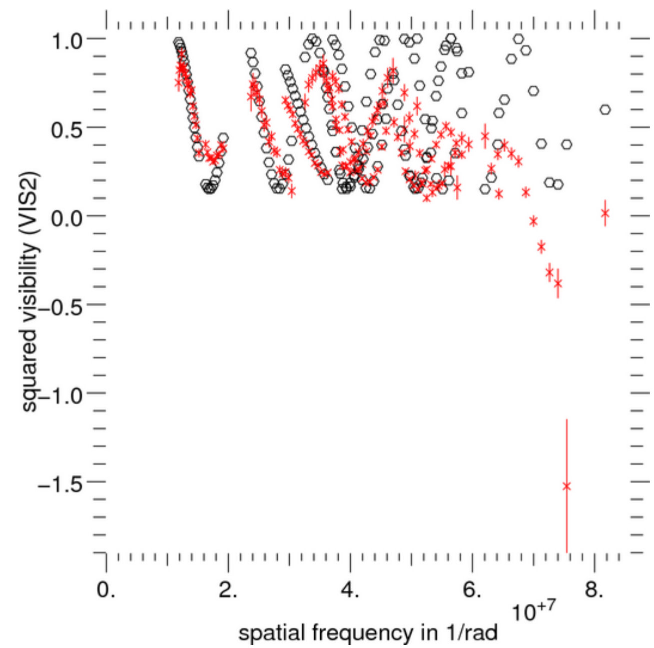


2 global minima

2 $[x_2, y_2]$ possible for initiate the fit

2 equivalent solutions

VIS2



flux_weight1 = 0.69414 +/- 0.49
flux_weight2 = 0.30586 +/- 0.216
x2 = 16.364 +/- 0.047 mas or -16.364
y2 = 9.092 +/- 0.0709 mas or -9.092
i.e. **[ρ, PA] = [18.72mas, 60.94°] or [18.72mas, 240.94°]**
reduced Chi2 final= 81.53

Kraus S. et al, 2009, A&A. 497-1, pp. 195-207

	Date	Filter	Flux ratio	PA(°)	ρ (mas)
VLTI/AMBER	2007.9233	*H+K*	0.24 ± 0.07	241.2 ± 1	19.07 ± 0.5

5- Additional Exercises
5.1 Theta Ori C

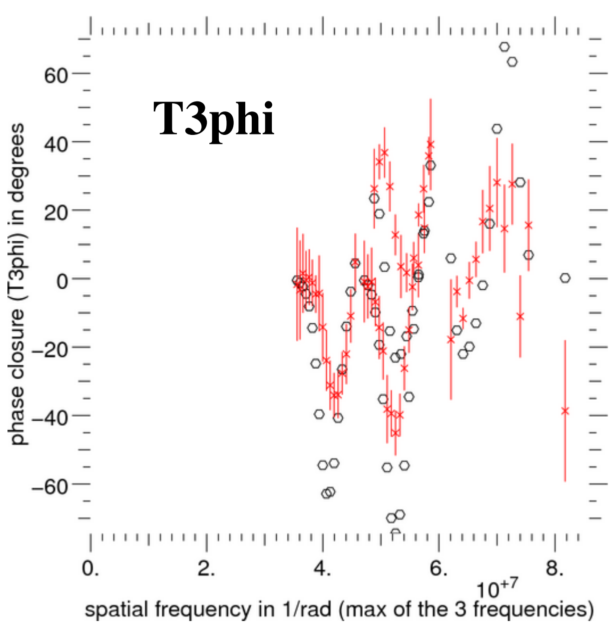
Theta1Ori2007Dec03_2.fits
+ Theta1Ori2007Dec05_2.fits

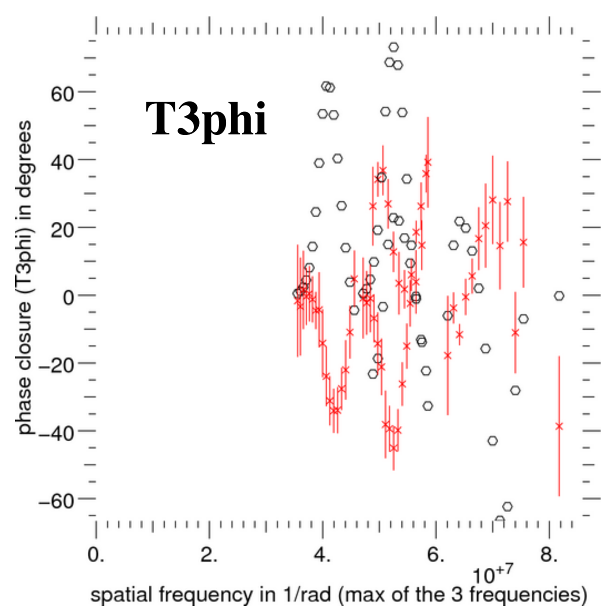
VIS2 + T3Phi

Only one best solution :
flux_weight1 = 0.6975 +/- 0.37
flux_weight2 = 0.3025 +/- 0.16
x2 = -16.372 +/- 0.0406 mas
y2 = -9.0853 +/- 0.0613 mas
i.e. [ρ, PA] = [18.72mas, 241°]
flux_ratio = 0.43
reduced Chi2 final= 62.29

Published results: **Kraus S.** et al, 2009, A&A. **497**-1, pp. 195-207

	Date	Filter	Flux ratio	PA(°)	ρ (mas)	
VLTI/AMBER	2007.9233	*H*+*K*	0.24 ± 0.07	241.2 ± 1	19.07 ± 0.5	–



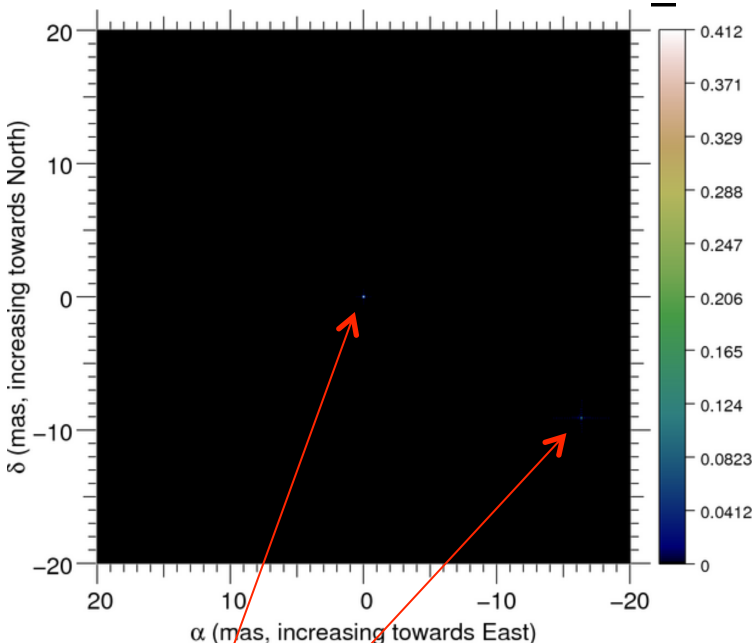


π dephasing for the bad solution

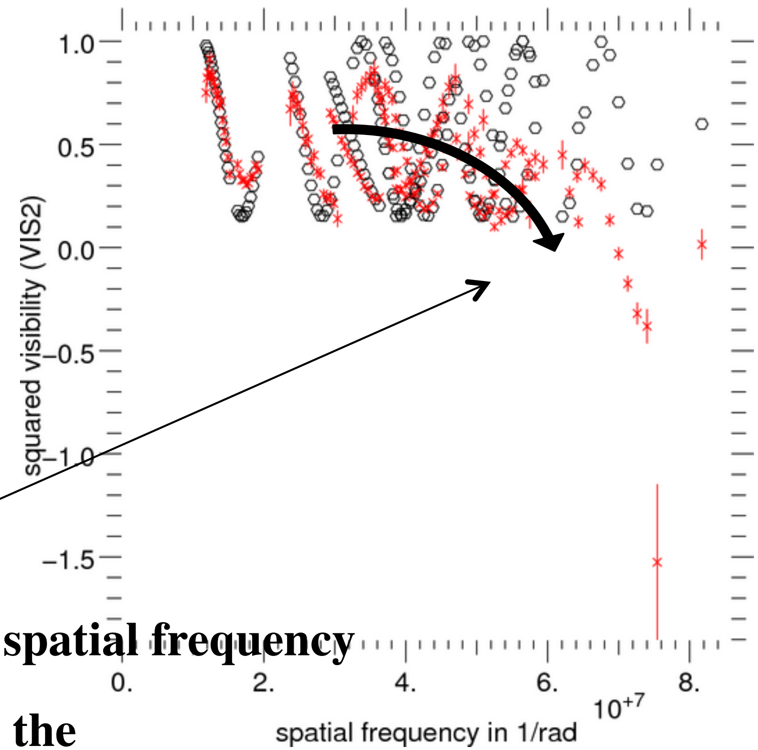
CRAL

5- Additional Exercises
5.1 Theta Ori C

Theta1Ori2007Dec03_2.fits
+ Theta1Ori2007Dec05_2.fits



VIS2 + T3Phi



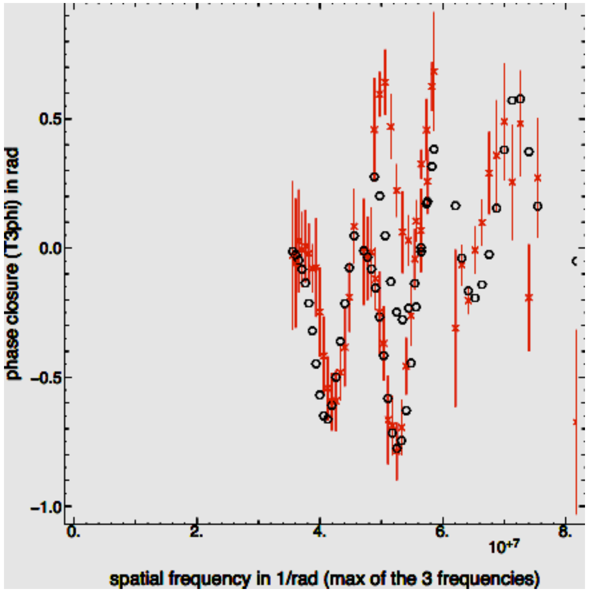
Decreasing of VIS2 with the spatial frequency

2 puncts… but the interferometer may resolve the components of the binary since resolved components give decreasing VIS2.

5- Additional Exercises
5.1 Theta Ori C

Theta1Ori2007Dec03_2.fits
+ Theta1Ori2007Dec05_2.fits

VIS2 + T3Phi



Fit with 2 disks:

diameter1 = 2.0039 +/- 0.055 mas

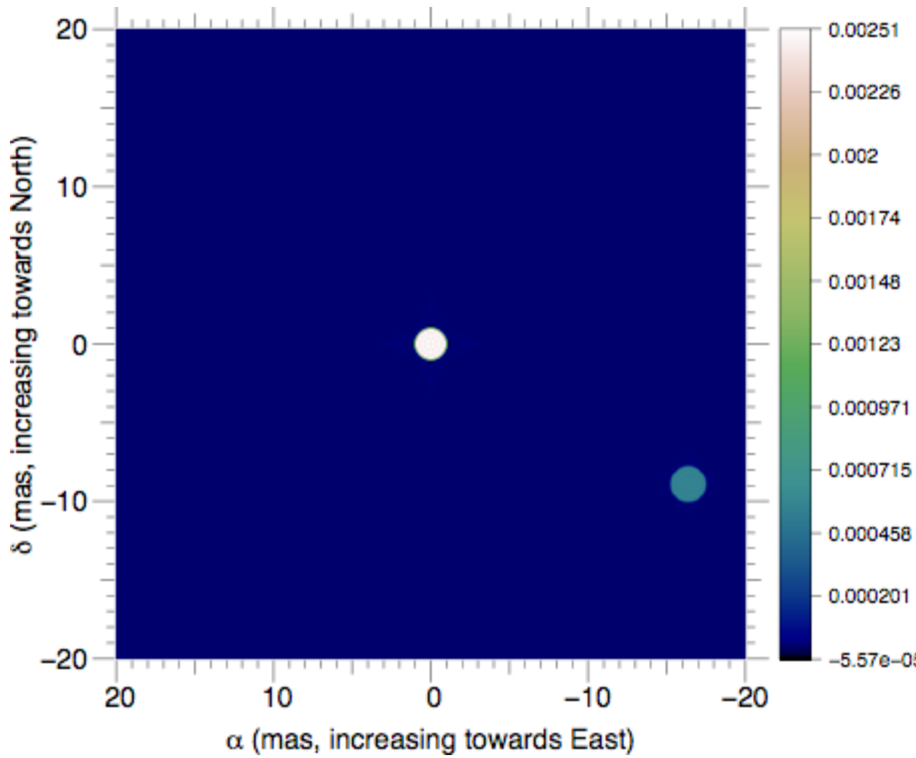
diameter2 = 2.2654 +/- 0.197 mas

Better T3phi fit

Better compatibility for the flux ratio with the published result (0.24)

5- Additional Exercises
5.1 Theta Ori C

Image of the model of Theta Ori C
provided after fitting with 2 disks
(similar results with 2 gaussians)



5-Additional Exercises
5-2 HD87643

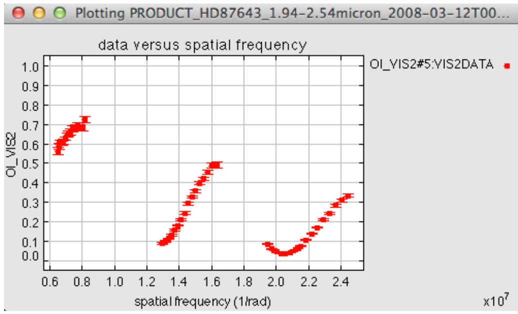
- Open a new setting and load **HD87643-bandK.fits**
  - data obtained on AMBER/VLTI in K band.

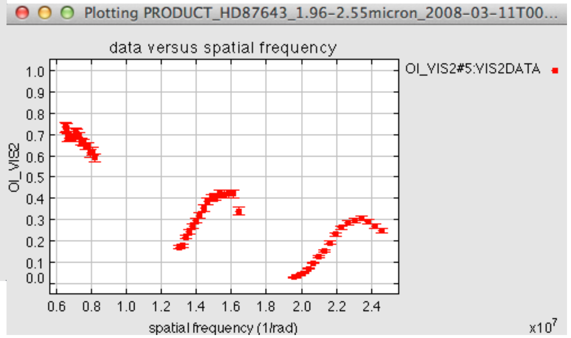
  (F.Millour et al., 2009, A&A 507, pp317-326)

- Observe the data, VIS2 and T3phi, and guess the possible shape for this object.

- Build a suitable model and find your best solution.
  - you should get chi2 < 17 at the end.

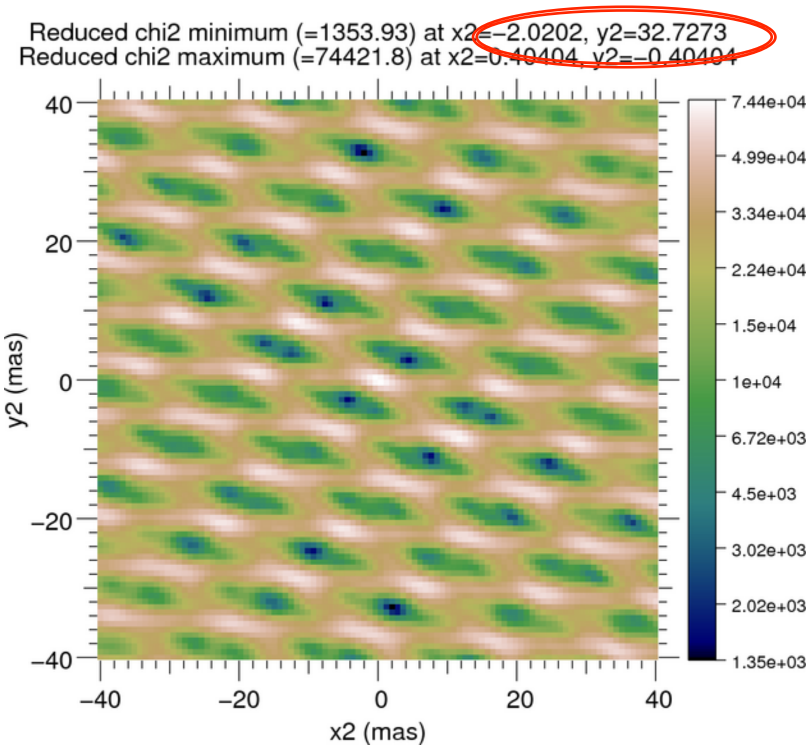
5-Additional Exercises
5-2 HD87643





periodicity and decreasing of VIS2 versus the spatial frequency
→similarity with exercise 3:
binarity with resolved components

Chi2 map



Fit VIS2 & T3phi with 2 disks

diameter1 = 5.9379 +/- 0.118 mas
diameter2 = 6.0994 +/- 0.215 mas
flux_weight1 = 0.62778 +/- 0.305 primary
flux_weight2 = 0.37222 +/- 0.181 secondary
x2 = -2.0487 +/- 0.142 mas
y2 = 33.094 +/- 0.104 mas
reduced Chi2 final= 49.92
Number of degrees of freedom = 206

5-Additional Exercises
5-2 HD87643

flux_weight1 = 0.63794 +/- 0.279 primary

flux_weight2 = 0.36206 +/- 0.159 secondary

fwhm2 = 3.7853 +/- 0.0867 mas

fwhm1 = 3.7938 +/- 0.162 mas

x2 = -1.985 +/- 0.121 mas

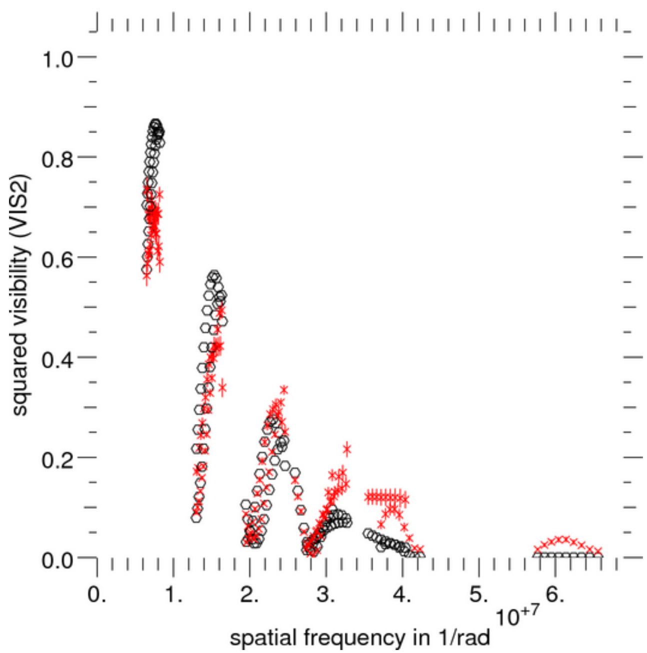
y2 = 33.098 +/- 0.0886 mas

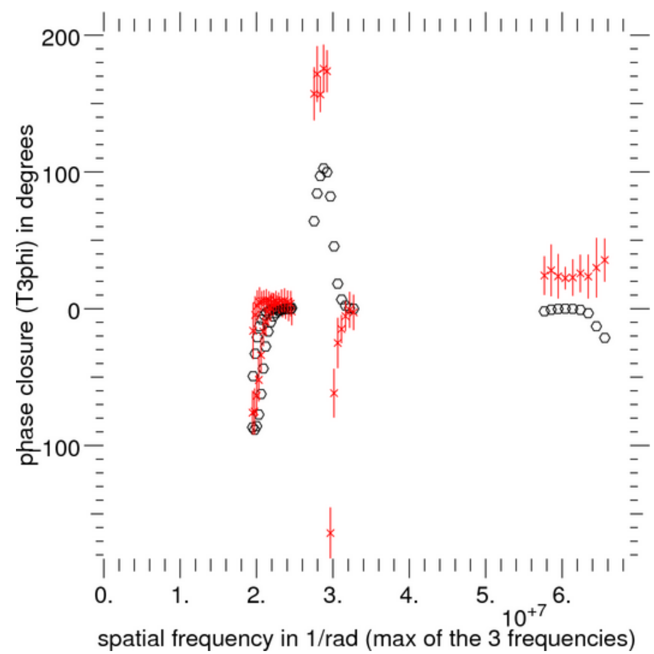
reduced Chi2: final= **40.51**

Number of degrees of freedom = 206

Fit VIS2 & T3phi with 2 gaussians

better chi2 than with 2 disks





5-Additional Exercises
5-2 HD87643

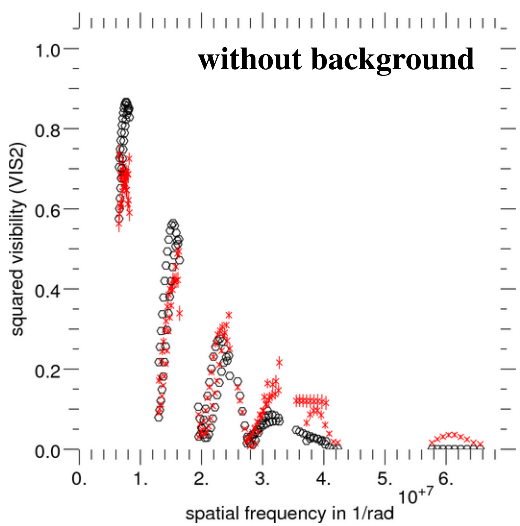
Means to decrease the visibility at low frequencies:
by adding a dirac function, i.e. a background in the image space.

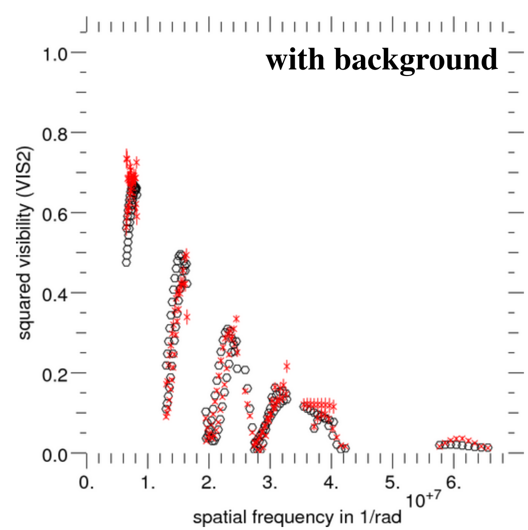
Fit VIS2 & T3phi with 2 gaussians + background

fw1 = 0.62928 +/- 0.174 primary
fw2 = 0.23253 +/- 0.064 secondary
fw3 = 0.13823 +/- 0.0389 background
fwhm1 = 3.6814 +/- 0.0551 mas
fwhm2 = 1.355 +/- 0.138 mas
x2 = -1.9868 +/- 0.0503 mas
y2 = 33.178 +/- 0.0378 mas

i.e. [ρ, PA] = [33.24mas, -3.43 = 356.57°]

reduced Chi2: final= **16.27** - sigma= 0.09877
Number of degrees of freedom = 205

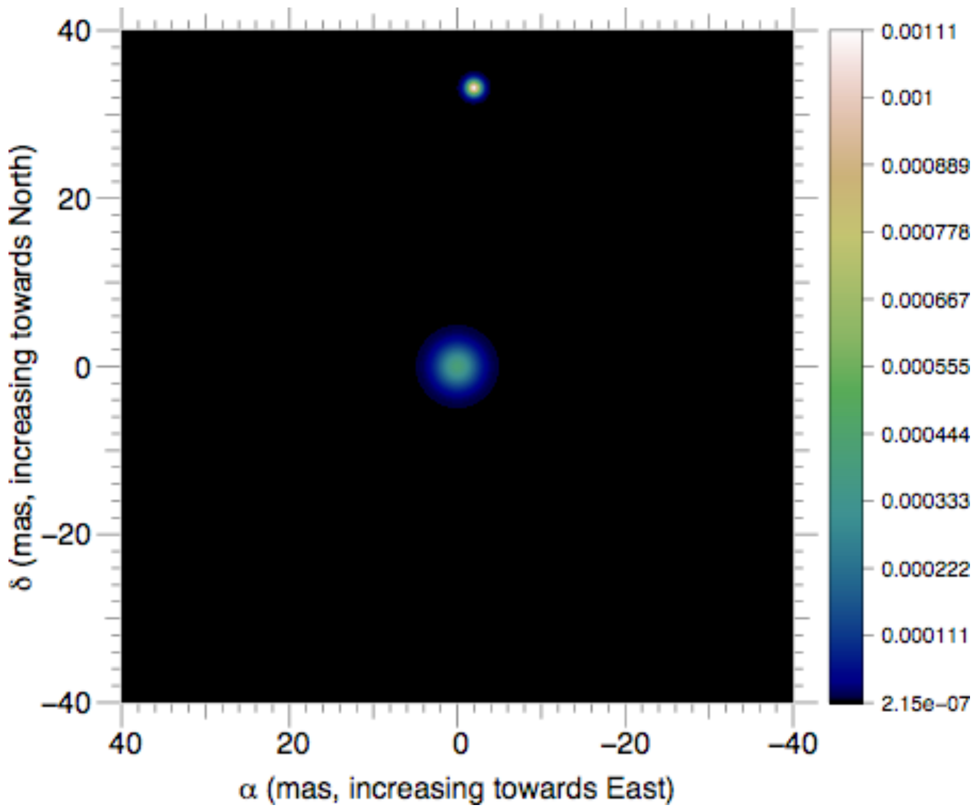




5-Additional Exercises 5-2 HD87643

Fit VIS2 & T3phi with 2 gaussians + background

Improves the fit for the frequencies < ~3.10^7 1/rad **only**
==> better model to be found for fitting the higher frequencies …



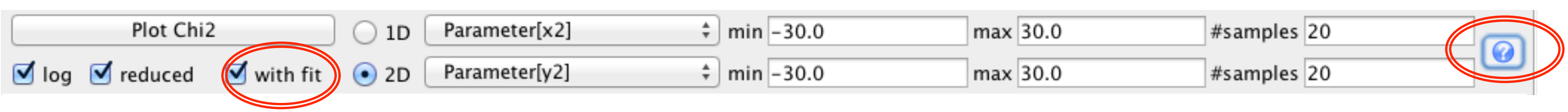
[ρ, PA] = [33.24mas, 356.57°]

Separation comparable to the one published by
F.Millour et al., 2009, A&A 507, pp317-326 : **[ρ, PA] = [33.48mas, 355.5°]**

5-Additional Exercises
5-3 2004-Interferometry Imaging Beauty Contest

- Open a new setting and load **2004-BSC1948I.fits**
  - data were simulated for the 2004-Interferometry Imaging Beauty Contest (Lawson et al, SPIE Proceedings)

- **Aim**: discover the option "with fit" of Plot Chi2
- Proceed as for Exercise 3
  - You may select directly 2 disks, but for the first step, fix the diameters value to 0 in order to explore the chi2 map with 2 puncts.
  - ***Plot Chi2 2D*** with parameters (x2, y2) as before: difficulty to find a global minimum and then the best guess for fitting.
  - You may use ***Plot Chi2 2D*** "with fit" : **read the Help**

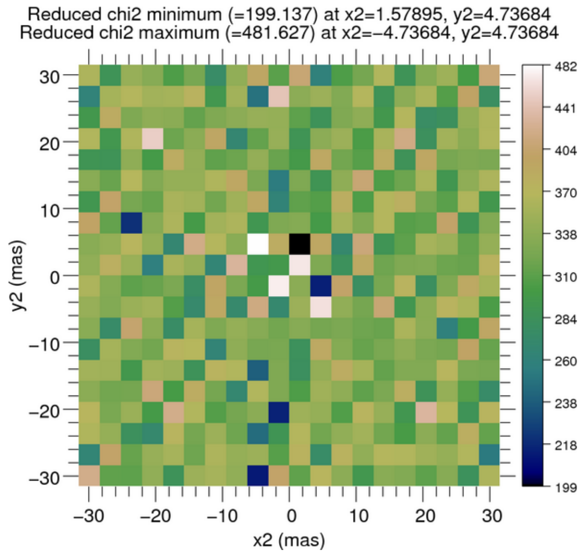


→ find the global minimum →*Run fit* from the best guess
→ make then the diameters free and *Run fit* again → the binary is resolved

5-Additional Exercises
5-3 2004-Interferometry Imaging Beauty Contest

Plot Chi2 for x2,y2 :

#samples =20

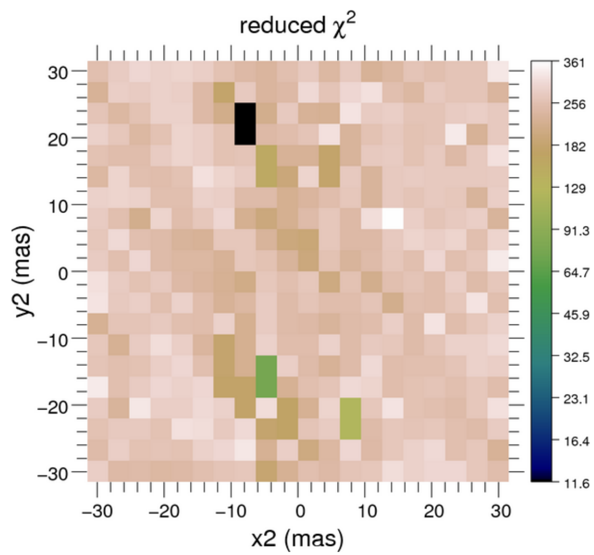


several minima
no global minimum

Plot Chi2 for x2,y2 :

#samples =20

with fit



a global minimum is detected

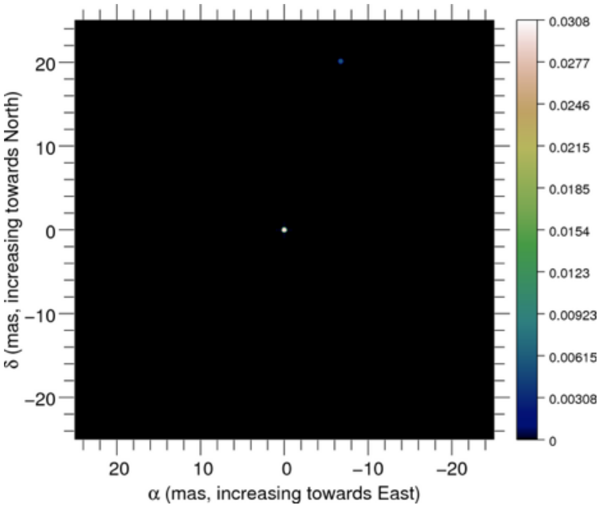
5-Additional Exercises
5-3 2004-Interferometry Imaging Beauty Contest

Fit VIS2 & T3phi with 2 puncts

reduced Chi2: final= 12.88 -
Number of degrees of freedom = 171
Final values and standard deviation for fitted parameters:
flux_weight1 = 0.16509 +/- 0.0448
flux_weight2 = 0.8349 +/- 0.226
x2 = 6.6744 +/- 0.0149 mas
y2 = -20.086 +/- 0.0213 mas

Fit VIS2 & T3phi with 2 disks

reduced Chi2: final= 0.8033
Number of degrees of freedom = 169
Final values and standard deviation for fitted parameters:
diameter1 = 0.60635 +/- 0.0458 mas
diameter2 = 0.6078 +/- 0.00663 mas
flux_weight1 = 0.14782 +/- 0.01 ➔ flux_ratio = 5.81
flux_weight2 = 0.85218 +/- 0.0577
x2 = 6.7288 +/- 0.00467 mas
y2 = -20.136 +/- 0.00636 mas
➔ rho = 21.23 mas PA = 341.52 degrees



which are compatible with those given on the Imaging Contest site :
21.2 mas sep., PA=341.6deg, Ratio 5.75, ud size of both components 0.6 mas.