The 10th VLTI School of Interferometry

Practice session I
Interferometry basics with ASPRO

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2 Our first model : the uniform disk

It is now time to play with a first model. The one already loaded in the *Example1.asprox* file is a uniform disk. It is the grounding of almost all interferometric-data analysis, and it is very often used, not only to perform stellar diameters fits, but also first-order interpretations of any extended objects.

Question : What is the visibility function of a uniform disk?

$$V(\rho) = 2\frac{J_1(\pi\rho\theta)}{\pi\rho\theta}$$

$\rho = B/\lambda$ is the spatial frequency (cycles/rad)
θ is the star angular diameter (rad)
J_1 is the first order Bessel function.

Question : Give the expression of the first value for which the visibility becomes zero?

$\theta = 1.22\ \lambda \backslash B$

This corresponds to the interferometer spatial (or angular) resolution, i.e. the smallest object that can be fully resolved.

Question : Give the same expression with the stellar diameter and wavelength expressed in mas and μm, respectively.

$\theta = 250\ \lambda \backslash B$

Now look at the **OIFits viewer** tab. Plot the visibility amplitude and phase (VISAMP and VISPHI) as a function of the baseline length (i.e. RADIUS in the software). To do so click on the three dot (...) button at the bottom-right corner, and then select the good X and Y Axes. Roughly measure on the plot the smallest baseline for which the visibility becomes zero and assume that the observations were made in the B band (i.e., at λ=0.46μm).

Question : What is our object diameter in mas?

θ = 4 mas

Imagine that you obtained a single visibility measurement (i.e. for a single baseline) and assume that the studied object looks like a uniform disk.

Question : In which case(s) can you determine an unambiguous diameter?

V>0.132
Below this value there are other lobes of the visibility that match the same values.

Now look at the phase. It is always equal to 0^o or 180^o. The jump between these two values happens when the visibility amplitude is equal to zero.

Question : Explain this phase signal.

The object is centro-symmetric
⇔
Its Fourier Transform is Real
The phase is 0 (positive number) or 180° (negative)

Question : Finally, note the baseline length corresponding to the first zero of visibility and the amplitude of the second-lobe of the visibility function?

B(V=0) ≈ 29m
Amplitude ≈ 0.132

3.1 Limb darkened disk

Question : Note the baseline length corresponding to the first zero of visibility and the second-lobe amplitude.

B ≈ 32 m
Aplitude ≈ 0.092

Now imagine you would estimate the size of this 4mas limb-darkened star by measuring the position of the first zero of visibility and using the formula determined for a uniform disk in the previous section.

Question : Does the star appears bigger or small than its real size? Explain why.

The star appears smaller although it has exactly the same size as before.
The light emission is more "concentrated" in the center due to the limb darkening.

Question : How can you discriminate between the uniform disk and the limb darkened disk?

By measuring the 2nd lobe amplitude

3.2 Gaussian distribution

Change the model to a "gaussian" (remove previous), and click on Add. Look at the new set of parameters shown in the bottom part of the window. This model has also four parameters, i.e. *flux_weight1*, *x1*, *y1* like for all models, and *fwhm1*, the full width at half maximum of the Gaussian distribution. Let's set this value to 4 mas and then click on **Ok** to close the window.

Question : What is the Fourier transform of a Gaussian distribution?

A Gaussian distribution

Question : Compare the phase signal to that of the uniform disk.

The phase is always 0

Question : What is the baseline length corresponding to a visibility of 0?

The zero is at the infinity

3.3 Ring

Open the **Target editor** again, **Remove** the Gaussian model and **Add** a Ring model instead. This time the model has 5 parameters : *flux_weight1*, *x1*, *y1*, a diameter like the uniform disk, and also a width. Let's consider the case of a 4 mas infinitely-thin ring : *diameter1* = 4, and *width1* = 0.

Question : What are the baseline length corresponding to the first zero of visibility and the amplitude of the second lobe?

B ≈ 39m
Aplitude ≈ 0.4

3.4 Comparison of these models

In the previous sections you have build four models (uniform disk, Gaussian distribution, ring, and limb darkened disk) with a characteristic size of 4 mas. Their main difference is the smoothness/sharpness of their intensity distribution.

Question : First, classify the models in term of sharpness/smoothness

Question : What is the relation between the distribution sharpness and the 2^{nd} lobe amplitude ?

3.5 Model confusion

If the interferometer largest spatial frequency (the ratio baseline length over wavelength) is not of the order of the typical size of your source, this may cause problems when you try to interpret your data.

Question : Compare the visibility obtained at 100m for a 0.9 mas uniform disk and a 0.5 mas Gaussian distribution.

Now switch to the K band using the **UV-Coverage** tab and changing the **Instrument mode**.

Question : Do the same comparison.

Typical uncertainties on the visibility measurements are of the order of a few percent.

Question : In which band(s) can you discriminate between these models and determine the object size?

Go back to the B band, and compare the visibility at 100m from the 0.9 mas uniform disk and a 0.61 mas Gaussian distribution.

Question : How can we discriminate between these two intensity distributions?

3.6 Point source and flat field

There is two additional intensity distributions that are widely used for modelling :

- the **point source** used to model a source too small to be resolved whatever by the interferometer
- the **flat field** used to represent the exact opposite, a fully resolved object.

Note that these models are useless on their own and are only used to build multi-component models described further down.

Question : What are the point source and flat field visibility functions?

In ASPRO2 point sources are modelled using the *punct* function. Flat field are not included, but can be modelled easily using a very extended uniform disk, for example with D≥250 mas.

From the sharpest to the smoothest:
Ring, Uniform disk, Limb-Darkened disk, Gaussian

Sharpness in image ⇔ higher lobes in FT
Ring = 0.4 UD = 0.13 LDD = 0.09 Gauss. = 0

0.9 mas UD ⇔V = 0.233
0.5 mas Gaussian ⇔ V= 0.372

0.9 mas UD ⇔V = V=0.958
0.5 mas Gaussian ⇔ 0.963

Only in B band as visibility are too similar in K band

The visiblity are very similar at 100m (0.23)
Having measurements at multiple baselines length can help discriminate between these models + longer baselines for 2^{nd} lobe measurements

FT(Point Source) : V = 1
FT(flat field) : V = 1 if (B=0) and V = 0 otherwise

4 Going 2D with flattened models

In the previous sections you learned about 1D functions to model interferometric measurements. Now it is time to take a look at two-dimensional intensity distributions.

4.1 Elliptical distributions

Question : Compare its visibility function to the one obtained for the 4 mas uniform disk.

> It is extacly the same! (V=0 ⇔ B=29m)
> The baseline is North-South oriented and so it ``sees'' The major-axis of the Elongated disk

Now modify the model so that the major-axis is oriented East-West.

Question : Where is the first zero of the visibility function?

> Twice as far, i.e., B=58m
> Now the baseline is oriented along the minor-axis

Question : What uniform disk diameter does this correspond to?

> 2mas

Finally, put the disk major-axis at 45°.

Question : Can you conclude on the extension measured by an interferometer?

> At 45° we found 3.1 mas, an intermediate value between the major and minor axes.
> The interferometer only measures extension along the baseline orientation

Now, let's use a new set of baselines. Load the configuration file *Example2.asprox*. Look at the UV plan in the **UV coverage** tab. The baselines in the new set all have the same length, i.e. 42m, but they have different position angles, and cover all directions.

Go to the **OIFits viewer** tab and plot the Visibility (VISAMP and VISPHI) as a function of the position angle (POS_ANGLE).

Question : Without looking at the model can you determine the major-axis position angle?

> yes, as there is some modulation, minimum is at 50°, which mean the longest dimension (the major axis) has a PA of 50°

Question : Assuming an elliptical Gaussian model what are the major and minor axes FWHMs?

> Using the FT given in the cheatsheet for the Gaussian
>
> $$\text{fwhm} \approx 1.66\sqrt{-lnV}\,\frac{\lambda}{\pi B} \Leftrightarrow fwhm = 1 \text{ and } 0.5$$

4.2 Application to geometrically thin disk

Elliptical intensity distributions are widely used to model geometrically-thin circumstellar disks. In this case the flattening is due to the projection of the disk on the sky-plane, i.e., perpendicular to the observer line of sight. At an inclination angle of 0°, i.e. pole-on, the disk is not flattened, and the elongation ratio (i.e. major-axis/minor-axis) grows with the inclination angle.

Question : Find the formula of the elongation ratio for this simple geometrically thin disk model

> Elong = 1 / cos (inclination)

Question : Conclude on the inclination angle of our object.

> 60°

5 Composed models

Imagine that your model is a weighted sum of N components :

$$I_{tot}(x,y) = \sum_{i=1}^{N} f_i \, I_i(x,y) \qquad (1)$$

where $I_i(x,y)$ is the intensity distribution of the ith component and f_i its relative flux with Σf_i=1.

Question : What is the visibility function (i.e. normalized Fourier Transform) for this model?

The Fourier transform of a weighted sum of functions is the weighted sum of their respective Fourier transforms.

5.1 Star + circumstellar disk model

Let's look at a simple star + circumstellar disk example. First, load a new observation file : *Example3.asprox*. Go to the **UV coverage** tab and look at it (again remove the annoying **Plot rise/set uv tracks** option). It is composed two perpendicular strips of baselines : one North-South and one East-West. It will allow us to probe our object along these two perpendicular orientations.

Now, open the **Target Editor** and create a two components model composed of a 0.5 uniform disk (for the star) and an elliptical Gaussian distribution (for the circumstellar disk), with a 8 mas major-axis oriented North-South and a 4 mas minor-axis. Put the flux contribution of each component to 50%.

Plot the visibility amplitude and phase as a function of the baselines lengths.

Question : Describe the visibility function.

A sum of the Fourier transform of :

- the elliptical Gaussian distribution (seen at short baselines and that gives different values for the two perpendicular strips of baselines)
- and of the uniform disk (Airy function seen at long baseline and never fully resolved)

Change the flux ratio between the two components and look at the visibility function.

Question : What has been modified? Can you explain why?

the level of the plateau between the baselines resolving the Gaussian and the ones resolving the Uniform disk have changed. This plateau allows to make a direct measure of the relative flux of the two components of our model.

Now imagine that you want to constrain the circumstellar environment extension and flattening.

Question : What baseline lengths and orientations will you choose?

Baselines shorter than the plateau : B < 20 m
Disk elongated ⇔ Baselines in many orientations.

Question : Will these set of baselines give information on the stellar surface?

No. The star start to be significantly resolved for baselines larger than 60 m.

5.2 Binaries

A second kind of very useful two components model is the binary model. We will use a single strip of North-South aligned baselines to exlpore this model. So, load the *Example1.asprox* file.

Then, open the **Target Editor** and create a model consisting of two point sources. Set their flux to 0.2 and 0.8. Look at the second component, the *x2* and *y2* parameters have been replaced by *sep2* and *pos_angle2*, i.e. polar coordinates. Choose *sep2*=5 and *pos_angle2*=0. This simulates two unresolved stars separated by 5 mas in the North-South orientation, i.e. aligned with our baselines.

Question : Describe and explain the visibility function.

The visibility function corresponding to a binary is a weighted cosine function.

Write down the amplitude of the sinusoidal modulation. Set the flux ratio to 0.1/0.9 and then to 0.5/0.5.

Question : What is the link between the flux ratio and the amplitude of the binary modulation?

If r is the flux ratio between the components (r<1), the amplitude A of the modulation is given by :

$$A = \frac{2r}{1+r}$$

Write down the modulation period in meter, and express it in B/λ units (cycles/rad). Then set the binary separation, i.e. *sep2*, to 10 mas. and do this again.

Question : Give the relation between the binary separation (in rad) and the modulation period (in cycles/rad).

- 5mas => 19m => 4.13^{e-7} cycles/rad
- 10mas => 9.5m => 2.06^{e-7} cycles/rad

The modulation period is equal to the inverse of the binary separation.

1 mas = 4.84^{e-9} rad

4.13^{e-7} = 1/5 mas 2.06^{e-7} = 1/10 mas

Now change *theta2* to 90^o : binary in the East-West orientation.

Question : Describe and explain the visibility function.

There is no modulation anymore as the baseline samples perpendicular to the binary direction. V = 1 as the interferometer ``sees” an unresolved object in the baselines orientation.

Do the same but with *theta2* equal to 30^o, 45^o, and 60^o.

Question : How does this affect the visibility function? Why?

The interferometer samples the projected separation of the binary along the baseline orientation

Finally replace the components by uniform disks and try various diameters between 0 and 2 mas, for each component, separately.

Question : What is the effect of resolving one component on the visibility function?

It is a convolution of two distributions in the image plane, so in the Fourier plane it amounts to a mutiplication of the binary pattern with the FT of a disk, i.e., a Bessel function

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5.3 Home-made models

Up to now we've been working with simple geometric models. But one might want to use home-made models such as outputs from radiative transfer codes. For that purpose ASPRO2 allow us to upload images in fits format and computes the visibility corresponding to this model and the selected interferometer and instrument configuration.

In this session, we will use a N band image of a dusty disk surrounding a massive hot star generated with the radiative transfer code MC3D developed by Sebastian Wolf.

First, load the *Example3.asprox* file to have two strips of perpendicular baselines. Then, open the **Target Editor**. In the **Model** tab, choose **User Model** and then click on the **Open** button. Go to the models sub-directory and choose the HD62623.fits file. After selecting the file you should see the image in the **Target Editor** window.

Question : what is the field of view (the size) of this image?

227 mas

Play with the zooming and the color scale.

Question : Can you explain the shape of the intensity distribution?

We see the inner rim of the dusty disk, the disk emission in N band, and some central emission which correspond to the central star.The disk is flattened which is a hint that it is not seen pole-on.

Close the **Target Editor** window, and go to the **UV Coverage** tab to select the N band. Look at the 2D Fourier transform of the image overplotted on the UV Coverage plot.

Finally, go back to the **OiFits viewer** tab to see North-South and East-West cuts of the 2D-visibility function. As before, plot VISAMP & VISPHI as a function of RADIUS.

Question : Is the visibility function closer to that of a Gaussian distribution or uniform disk?

As the object intensity distribution has no sharp edge beside the inner rim that is very small compared to the object extension, its visibility function is closer to that of a Gaussian distribution.

Question : What are the values of the visibility in the two orientations for a 20 m baseline?

V= 0.755

V= 0.618

Question : Using these values estimate the objects extension in the North-South and East-West orientations.

$$fwhm \approx 1.66\sqrt{-lnV}\,\frac{\lambda}{\pi B}$$

B = 20m λ = 12µm

fwhm ≈ 34.7 mas

fwhm ≈ 45.4 mas

Question : Assuming that the disk is geometrically thin what is the object inclination angle?

Elong = 1 / cos (incl.)

Elong = 45.4/34.7 = 1.31

Incl ≈ 40°

6 Effect of the observed wavelength

6.1 Achromatic objects observed at various wavelengths

The spatial resolution of an inteferometer strongly depends on the observing wavelength. Imagine that our North-Pole interferometer is able to observe in any photometric band between 0.46μm (B) and 12μm (N).

Question : Which band will give the highest resolution?

> Resolution ⇔ B/λ
> Smaller λ higher resolution

To change the observing wavelength go to the **UV Coverage** tab. You can select a photometric band between B and N in the **Instrument mode** list.

Question : Give the ratio between the resolutions in N and B bands

> 12/0.46 = 26

Let's assume a 0.05 uncertainty on our visibility measurement and three baselines of 50, 70 and 100 m.

Question : In which band should we observe a 1 mas star?

> B is the best (but V, R, or I are Ok)

Question : Should we observe a 5 mas star with the same configuration?

> No, it would be overresolved by the 50m baseline.
> The star should be observed in the J, H, K, or L bands.

Now, let's do a multi-wavelength observation. First load the *Example4.asprox* file. It contains a 3 telescopes (S0-S5-S16) configuration of baselines. Unlike for the other exercices the observing wavelength is not fixed to one band but ranges between 0.1μm to 10μm, the "Wide" (and fake) instrument mode of the UV Coverage panel.

Note that for this observation we use a 1 mas uniform disk model.

First plot the visibility and phase as a function of the baseline length to see the visibility of the three baselines.

Question : Explain why we obtain a large range of values for the visibility of each baseline.

> The visibility is not a function of the baseline but of the spatial frequency B/λ.
> For the same baseline length, observing at different λ leads to probing different spatial frequencies.

Now plot the visibility as a function of the spatial frequency B/λ (SPATIAL_FREQ). The different colors correspond to different wavelengths from purple for the smallest one (0.1μm) to red for the largest one (10μm).

Question : For this achromatic model case, conclude on the effect of observing at multiple wavelengths.

> This that case it is strictly equivalent to observing with different baselines lengths. It helps a lot to enhance the UV coverage.

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6.2 Examples of chromatic objects

Question : Assuming that a star and its circumstellar environment emit as black-bodies, can you conclude on their flux ratio dependence on the wavelength? Which component will dominate the visible flux? What about the mid-infrared?

The central star (hotter, but of smaller angular size) will dominate on the short wavelength, and gradually the cooler but much larger environment will take its place when the wavelength increases.

Question : How does the model depend on the wavelength?

The size of the circumstellar disk emission grows with λ. Its relative contribution to the total flux is also growing with the λ.

Before closing the **Target Editor** let's rotate our model by 90^o so that our 3 North-South baselines will make measurements along the major-axis (i.e. the equator) of this object. Now, close the **Target Editor** and go to the **OiFits Viewer** tab.

Plot the visibility and phase as a function of the spatial frequency. For chromatic objects, such plots mix-up the spatial and spectral dependency of the object. To disentangle these aspects you can draw imaginary lines between points of the same colour (i.e. wavelengths) : this will gives you information on the object spatial distribution at each wavelength.

Question : Is the object more extended in the visible (0.5μm) or the mid-infrared (10μm)?

The object is more extended at 10µm where the large circumstellar disk dominates the emission

Question : Assuming that the star is a uniform disk, estimate the stellar diameter in mas

V=0 for B/λ ≈ 158.3 ⇔ θ ≈ 1.58 mas

Question : Assuming a distance of 60pc give the stellar Radius in $R_{\odot}$.

1'' = 1au at 1 pc.
1au ≈ 107 Dsol
Rstar ≈ 0.107 x distance (in pc) x θ (in mas)
Rstar ≈ 10 Rsol

6.3 Bonus : play with the models from the AMHRA service

You can go to the AMHRA service web page, download models and load them on ASPRO2. Currently, AMHRA gives access to the following models:

- **Kinematic Be Disk** : Model of the geometry (size and shape) and kinematics (rotation and expansion) of circumstellar disks, especially of Be stars observed at high spectral resolution (R>1000). It is not adapted to our DEMO interferometer that does not have a high spectral resolution mode. But you might test it with GRAVITY (in Brγ line) or MATISSE (in Brα line)
- **Disk and stellar Continuum DISCO** : we already used it, but you might want to compute model with different paramters.
- **Evolved stars (Red Supergiants and AGBs)**: Stellar surface maps of evolved stars computed from 3D hydrodynamical simulation with CO5BOLD-OPTIM3D
- **Binary spiral model** : Phenomenological model mimicking the shock caused by the collision between the winds from massive stars (e.g. the WR and OB stars)
- **Supergiant B[e] stars with HDUST** : Grid of models for B[e] supergiants, i.e. hot stars surrounded by gaseous and dusty circusmtellar disk.
- **Limb-darkening with SAtlas** : Grid of models providing realistic intensity maps for spherically symmetric stars, showing the limb darkening effect.