Title : will be set by the publisher
Editors : will be set by the publisher
EAS Publications Series, Vol. ?, 2002

STELLAR SURFACE IMAGING USING CLOSURE PHASE

D. F. Buscher[1]

Abstract. We describe the use of interferometry to make images of features on the surfaces of cool stars. We describe observations of Betelgeuse (α Ori) and use this example to illustrate the requirements for astrophysical imaging with interferometers. We emphasise the importance of obtaining closure phase measurements and good Fourier plane coverage when imaging complex objects.

1 Introduction

The majority of optical and IR interferometric observations to date have been observations of simple objects such as binary stars or diameter measurements of single stars. There is a significant step to be taken in going from the observation of such simple objects to the observation of more complex astrophysical phenomena such as convective stellar surfaces. This step requires the astronomer to go from simple model-fitting to the making of model-independent images from interferometric data. In order to make such images, the observations need to be both quantitatively and qualitatively different from the observations required for model-fits.

When the aim is to measure a simple binary star orbit or a stellar angular diameter, the object brightness distribution can be described in terms of a small number of easily-interpreted variables such as flux ratio and separation. Fitting few-degree-of-freedom model brightness distributions to the interferometric data is practicable and extracts all the required information about the astrophysical source.

It is also in principle possible to parameterise models of more complex objects in terms of physical variables and to fit the observed data against visibilities predicted from these models. However, as the complexity of the model rises, more interferometric data required to constrain the model and the chance of parameter degeneracies (different values of model parameters giving rise to the same set of measurements) increases dramatically. This is compounded by the fact that for

[1] Cavendish Laboratory, Cambridge UK

DOI: (will be inserted later)

many complex phenomena the theoretical models are poorly understood. As a result, it becomes more and more likely that a model will fit the data but be totally wrong.

It is therefore preferable when trying to understand complex astrophysical phenomena to make model-independent images of the source morphology. In this way we can confirm the basic correctness of the models before attempting to extract model parameter information. We give an example here of model-independent imaging of stellar surface features to illustrate the principles and limitations of the method.

2 Science case for observing surface features

Without interferometry we can only directly observe the features on one star, i.e. the Sun. There are indirect methods of determining what the stellar surface looks like, for example using Doppler imaging when the star is rapidly rotating. From these indirect measurements and theoretical considerations we know that other stars have surface features quite different from the Sun. In the case of hot rapidly-rotating stars, and some pre-main sequence stars stellar prominences far larger than solar prominences have been detected (see e.g. Donati *et al.* 2000). In cooler supergiants, it has long been suspected that the convection scale size in the photosphere is comparable to the stellar radius (Schwarzschild 1975), which might lead to large hotspots on the stellar surface. Recent work by Freytag (see the movies at e.g. `http://www.astro.uu.se/~bf/movie/movie.html`) shows that the star can be distinctly non-spherical!

Imaging of these features is an important tool in understanding the convective phenomena in these stars, which are poorly constrained by conventional measurements. This is clearly of value for understanding stellar structure in these stars, but is also of wider interest: deeper understanding of these phenomena allows insight into chemical dredge up at these and later stages of stellar evolution.

For the purpose of this discussion, we will concentrate on the imaging of convective features on cool supergiants, but many of the results will apply to the imaging of other types of surface feature on stars.

3 Interferometric requirements

As with any other observational task, it is first necessary to determine plausible values for all the major experimental variables. This is in order decide which measurements are most important and also to decide on whether a measurement is possible at all given the instruments available and the physical parameters of the phenomena under consideration.

The most obvious parameter in an interferometric measurement is the angular resolution, which is related to the maximum baseline to be used. In the case of stellar surfaces, it is clear that we would like a resolution which is much finer than the angular diameter of the star we wish to observe, although later we will see that there are reasons why this can be difficult to achieve in practice. We can adopt for the moment a figure of 10×10 pixels across the stellar diameter. With a maximum resolution of order 1 milliarcsec (typical of modern interferometers), then observations on stars of angular diameter 10 milliarcseconds or larger will be appropriate.

The next most obvious parameter is the brightness of the object, since a lack of bright enough targets often limits interferometric observations. In the case of stellar surfaces it is easy to show that this is not an immediate problem. To a first approximation stars are black-body radiators, and for such sources the apparent brightness of an object can be directly inferred from its effective temperature and its *angular* diameter. The result of such a calculation for surface temperatures characteristic of M-type supergiants is that any object which subtends 10 milliarcsec or more will be brighter than a K magnitude of 0. Thus all sources which are large enough to be resolved are bright enough to do useful interferometric measurements.

The observation wavelength to choose depends in part on the kind of science we expect to do. The appearance of the atmospheres of cool stars is strongly dependent on the wavelength of observation. For late-M-type supergiants, many strong molecular bands are present at visible wavelengths and this leads to complex variations in the apparent diameter and limb darkening of the star as a function of wavelength. These features tend to be less strong at near-infrared wavelengths and so what is seen at these wavelengths can be considered as more of a "continuum" measurement. Thus near-IR observations will be more appropriate for constraining simple stellar parameters such as photospheric diameter whereas visible-wavelength observations may be more useful for probing atmospheric structure.

The observational requirement which is the most difficult to fulfil is the need for model-independent imaging. As has been argued before, this is essential in complex physical situations as exemplified by stellar convection. In order to make model independent images we need both good (u, v) plane coverage and phase information.

By "good" (u, v) coverage we mean that for a 10×10 pixel image we must measure at least 100 *independent* complex visibilities. Independent visibilities means that the points must be separated by more than $\sim \lambda/\theta_D$ in baseline space where θ_D is the angular size of the region over which we want to make the map. In this case we may want to make a map of a region few stellar diameters in size, in case there are any large stellar prominences or other atmospheric phenomena. Even taking into account Earth rotation, achieving this sort of coverage implies either using a large number ($\sim 5-10$) telescopes, or reconfiguring a small number of telescopes frequently. In the latter case we need to be aware that the source morphology is evolving on time-scales measured in weeks (Wilson *et al.* 1997), so the reconfiguration needs to be rapid.

Phase information is also important for making images of complex objects. While it is in principle possible to make model-independent images using visibility amplitude information alone, it has been found in practice that in most cases phase

Fig. 1. Upper row: Images of two interferometrists — CAH (left) and DFB (right). Lower row: hybrid images made from combining the Fourier amplitudes of CAH and the Fourier phases of DFB (left) and from combining the Fourier phases CAH and the Fourier amplitudes DFB (right). See the text for further details.

information is required to make images reliably — in some sense the information contained in Fourier phases is more important for imaging than the information contained in the Fourier amplitudes.

We can show the importance of the Fourier phase with the aid of simple numerical experiment. In this experiment we take two images such as the upper two images in figure 1 and take their Fourier transforms. If the two image intensities are $i_1(x,y)$ and $i_2(x,y)$ respectively we can denote the Fourier transforms as

$$\begin{aligned} A_1(u,v)e^{\imath\phi_1(u,v)} &= F\left\{i_1(x,y)\right\} \\ A_2(u,v)e^{\imath\phi_2(u,v)} &= F\left\{i_2(x,y)\right\} \end{aligned}$$

where $F\{g(x,y)\}$ denotes the Fourier transform of $g(x,y)$ We can then make a hybrid of the two images by combining the Fourier phases from one image with the corresponding Fourier amplitudes from the second and taking the inverse Fourier transform:

$$i_{12}(x,y) = F^{-1}\left\{A_1(u,v)e^{\imath\phi_2(u,v)}\right\}$$

We can also make the complementary pairing i_{21} which has the amplitudes from i_2 and the phases from i_1. The surprising result can be seen in figure 1, namely that the hybrid images strongly resemble the image from which their phase information is derived and bear no resemblance to the image from which their amplitude information is derived. This indicates that most of the useful information in an image is contained in its Fourier phases and not the Fourier amplitudes: the amplitudes give information about how *much* structure is in an image, but the phases tell us *where* that structure is.

Direct recovery of Fourier phase information in ground-based interferometry is made difficult because of the random phase perturbations introduced by the atmosphere. The closure phase can act as a useful substitute for the phase, and this subject is discussed further by John Monnier in these proceedings. It is worth reiterating a number of points from that discussion:

1. In order to measure a closure phase, the complex visibilities on all the baselines resulting from at least three telescopes must be measured simultaneously.

2. The closure phase is not systematically corrupted by most atmospheric and instrumental effects and is therefore a good observable which needs little or no calibration.

3. The closure phase can best be considered as a *constraint on the image* rather than as a method of indirectly deriving the Fourier phase.

4. The closure phase is either $0°$ or $180°$ for any point-symmetric image: deviations from these values are a signature of asymmetric image structure.

4 Aperture masking observations

In this section we describe interferometric observations of Betelgeuse (α Ori) which were not made using a separated-element interferometer but rather with an aperture masking technique on a single telescope (Buscher *et al.* 1990). The fact that such observations remain amongst the only model-independent images of stellar surfaces made to date is an indication of the difficulty of attaining adequate (u, v) plane coverage with separated-element interferometers with only a few telescopes.

Betelgeuse is one of the nearest late-type supergiants and has an angular diameter (somewhat wavelength dependent) of around 40-50 milliarcsec. For an observation wavelength of 800nm, we can deduce that a baseline of length 4m will resolve this star. This baseline is much *shorter* than the shortest baseline available with most separate-element interferometers. Longer baselines will provide higher-resolution information, but most of the flux from the star will be "resolved out" and we will not be sensitive to features on the scale of the stellar diameter. We will also see later that it will also be difficult to acquire fringes on the longer baselines.

A solution to the problem of making interferometric measurements on short baselines is to use an aperture mask on a single telescope. The observations described here were made on the William Herschel Telescope in La Palma, which has a primary mirror diameter of 4.2m, which is not as large as we would ideally like but is adequate for this observation. Aperture masks with linear arrays of 5

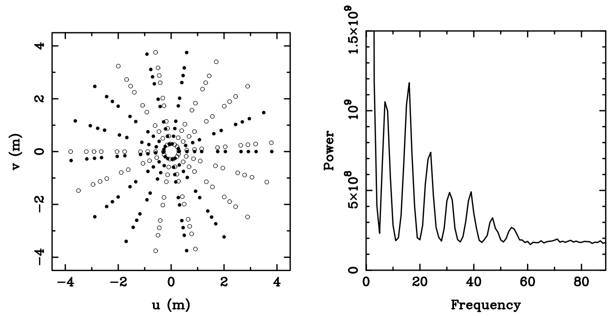

Fig. 2. (Left) The Fourier plane coverage of the Betelgeuse 710nm wavelength observations. (Right) The average power spectrum of 500 interferograms taken on Betelgeuse at 546nm wavelength. This data was taken using a 6-hole mask, but fringes are only visible on the shortest 7 out of 15 baselines because the source is resolved on longer baselines.

or 6 holes were inserted into a re-imaged pupil plane to convert the WHT into a Fizeau interferometer. The hole diameters corresponded to roughly r_0-sized patches on the primary mirror. A star seen through this arrangement consists of an Airy disc pattern crossed by sets of fringes at spatial frequencies corresponding to the spacings of pairs of holes in the mask. The mask was arranged to be non-redundant so no two pairs of holes had the same spacing: for a 5-hole mask fringes appeared at 10 different frequencies. The fringes were recorded using a CCD at frame rates of order 100Hz. Each frame of data was compressed on-chip along the direction perpendicular to the fringe intensity variation to yield a 1-d image. This compression decreased the effects of CCD read noise and increased the achievable frame rate.

To achieve 2-dimensional Fourier plane coverage the whole arrangement was rotated around the telescope axis to a number of different position angles with respect to the sky. With about 10 different position angles and 10 baselines per position angle, Fourier data at about 100 (u, v) points were obtained as shown in figure 2. These are not all independent visibility points, but the resulting image has less than 10×10 independent resolution elements in it, so the coverage is adequate.

The wavebands chosen for these observations were selected in order to isolate different spectral regions which correspond either to TiO absorption in the stellar atmosphere or continuum emission. Later analysis showed that the filters chosen did not have narrow enough bandpasses, so that there was some leakage of continuum light into the supposedly TiO-band observations and *vice-versa*.

5 Data reduction

For each position angle of the mask 1000 frames of data was analysed a frame at a time. Each frame was Fourier transformed and analysis done on the complex

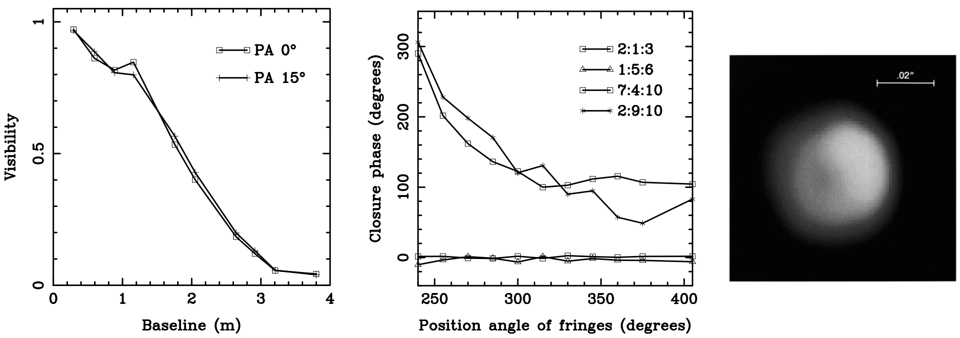

Fig. 3. A subset of the calibrated visibility (left) and closure phase (center) measurements on Betelgeuse at 710nm and the reconstructed image (right). The visibility data is plotted for two similar position angles of the mask. The closure phase data is plotted for two sets of triples containing only short baselines and two sets of triples containing long baselines. The closure phases are labelled with the indices (1-10) of the baselines making up the triple.

amplitudes at a discrete set of frequencies.

To get the fringe frequencies and amplitudes the modulus squared of the Fourier data was averaged to yield a power spectrum similar to that in figure 2. The spectrum can be seen to consist of a zero-frequency peak and a discrete set of high-frequency peaks corresponding to the fringes from different pairs of apertures. The fringe visibilities were derived from the amplitudes of these peaks, and the visibilities were calibrated by dividing by the corresponding visibilities from interferograms taken on nearby unresolved calibrator stars.

Closure phases were derived by averaging the triple product (the product of a set of complex fringe amplitudes at three different frequencies) for triplets of fringe frequencies corresponding to different triplets of apertures. The argument of the average triple product yielded a closure phase. This closure phase did not require calibration: the closure phases measured on calibrator stars were indistinguishable from zero.

6 Results and interpretation

The observations at each wavelength yielded of order 100 calibrated visibility amplitudes and 100 closure phases. Figure 3 shows a subset of these data for illustration purposes. A number of important constraints on the resulting image can be inferred purely from examination of these data:

- From the decrease in calibrated visibility with baseline it is clear that Betelgeuse is indeed resolved on the longest baselines and this is roughly consistent with a uniform disc diameter of around 50 milliarcsec.

- The closure phases measured on baseline triangles involving the longest baselines show large departures from $0°$ and $180°$, whereas the closure phases on the triangles containing only short baselines are close to zero. This means that there is significant asymmetric structure on scales comparable with the size of the stellar disc - the asymmetry cannot be due to a companion far away from the star.

- The closure phase varies over essentially $0°$ to $360°$, indicating that the asymmetric structure at this resolution is comparable to or larger than the symmetric structure at this resolution. The fraction of the flux that is unresolved at this resolution is given approximately by the visibilities on these baselines: these visibilities are of order 10%, hence the flux in the asymmetric structure is of the order of 10% of the total flux.

An additional inference which can be drawn is that the closure phase can be measured with very few systematics: the uncalibrated closure phases measured on the shortest baselines where Betelgeuse is effectively unresolved are within a degree of zero. This kind of accuracy in a conventional phase measurement would require an internal pathlength stability of better than 2nm and yet no special precautions were taken in this experiment.

These data were used as the input to an image reconstruction program designed for conventional radio VLBI imaging. It takes the measured visibility and closure phases and iteratively constructs an image which is consistent with these data and two extra constraints: image positivity and finite support. Image positivity is the constraint that there are no negative image intensities. Finite support is an assertion that the object emits no significant flux from regions more than a certain angular distance (e.g. 0.5 arcseconds) from the centre. This assertion is not always true, but for many objects there are either theoretical or observational grounds which make this a plausible constraint.

Many years of testing and experience in the radio VLBI community show that, given reasonable (u, v) coverage, this type of image reconstruction algorithm is robust and model-independent (it does not, as is sometimes assumed, require the presence of a point source somewhere in the image). In this case the image reconstruction of Betelgeuse (figure 3) produces an image which agrees with the constraints inferred from the data. It shows a stellar disc of about 50 milliarcsec diameter and a region of increased intensity on one side of the disc.

Having made a model-independent image reconstruction, one can then fit simple models to the data to get quantitative information. These model-fits show that the "hotspot" seen on the disc does indeed account for about 10% of the stellar flux. Further aperture masking observations (Wilson *et al.* 1997) show that such hotspots appear and disappear on timescales of order several weeks and observations in the infrared (Young *et al.* 2000) show that these spots have significantly lower contrast at longer wavelengths. This combination of multi-wavelength and multi-epoch measurements can be used to constrain models for the origin of these hotspots: see Bernd Freytag's poster from this winter school (`http://www.astro.uu.se/~bf/publications/leshou02_po.ps.gz`) and Young *et al.* (2000) for more details.

7 Further work

Imaging observations of supergiants with smaller angular diameters should possible if a separated-element interferometer is used. The most difficult observational requirement to meet for such observations is not the baseline but the Fourier plane coverage, since most interferometers measure only a small number of (u, v) points at any one time. The COAST interferometer, which can access 6 baselines at any one time and 9 baselines within a minute, is only just adequate to do basic model-independent imaging. Interferometers with more than 5 telescopes are clearly very desirable.

It would also be desirable to get higher angular resolution to find out more about the origin and evolution of these hotspots: the image of Betelgeuse shown above has only about two resolution elements across the stellar disc. It may at first seem that achieving such angular resolution would be easy: we would only need to use an interferometer with a 20m baseline to achieve a resolution equivalent to 5 times the number of pixels across the stellar disc. Unfortunately the very fact that we are operating at such high resolution makes the observation difficult.

The reason for this is that the signal-to-noise ratio of an interferometric measurement is generally a function of V^2N where V is the fringe visibility and N is the number of photons detected per coherence time. Thus if we were to derive a figure-of-merit representing the "effective throughput" of an interferometer, it would weight the number of photons collected with a factor proportional to V^2. On long baselines the object visibility typically falls to low values. To get 5 times the resolution across a stellar disc as was achieved in these Betelgeuse observations (which went only just past the first null of the object visibility function) we would have to observe on baselines where the maximum fringe contrast (due to the object alone, ignoring any systematic losses in fringe contrast) is about 1.6%. This corresponds to a reduction of signal-to-noise ratio by a factor of about 3600 compared to an unresolved source; over-resolving the stellar disc by this factor is equivalent in signal-to-noise terms to exchanging 8m diameter telescopes for 13cm telescopes! (This situation is ameliorated if there *is* a bright unresolved spot on the surface of the star since this will increase the fringe contrast, but one has to be careful then not to resolve the spot).

The limitation to observing fringes at such low contrast is not that one cannot eventually achieve an adequate signal-to-noise ratio by integrating for long enough but that one cannot detect the fringes in a short enough time to "phase up" the interferometer. Phasing up refers to the process by which atmospheric perturbations to the fringe position are detected and compensated. The fringe position must be inferred in a time shorter than the time taken for the atmosphere to move the fringes, a time which can range from a few milliseconds to a few seconds. If the signal-to-noise ratio which can be achieved in this integration time is not great enough to detect the fringe position then the source cannot be observed.

There are a number of ways around this limitation when observing on baselines where the source is resolved. All of these techniques rely on being able to measure higher-contrast fringes and use these to phase up the low-contrast fringes.

One technique is to measure fringes simultaneously at a longer wavelength and use these measurements to phase fringes at shorter wavelength — the lower resolution at longer wavelengths means the source is not so resolved and hence the fringe visibility is higher.

Another technique is called "baseline bootstrapping" (Armstrong *et al.* 1998). This requires an array of telescopes arranged in a "chain" of short baselines making up a longer baseline. Fringes can be measured on all the short baselines because the source is unresolved on these baselines and these measurements can be combined to phase the long baseline.

8 Conclusions

We have shown that it is both desirable and possible to make images of complex astrophysical phenomena using interferometric techniques. To make images requires better Fourier plane coverage than simple model-fitting and requires the measurement of Fourier phase information. Both requirements increase the time taken to make the observation, and there are further complications when there is a requirement to make images of heavily-resolved objects.

The upside of this is that with a model-independent image, physical interpretation of the results is less susceptible to degeneracies in relating physical models to the data. Such degeneracies are still possible when one has an image: two different models which produce the same spatial profile of emission will still be indistinguishable. However, such degeneracies are much easier to recognise and understand than those arising when, for example, two models happen to give rise to the same Fourier modulus at a given restricted set of (u, v) points. Thus model-independent imaging can be expected to lead to more robust science.

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