Simulations of Imperfect PRIMA Fringe Sensing Units and Calibration Strategies

PRIMA = **P**hase-**R**eferenced **I**maging and **M**icro-arcsecond **A**strometry

ESPRI = **E**xoplanet **S**earch with **PRI**MA

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Summary:

The PRIMA instrument at ESO/VLTI is scheduled for commissioning in late 2008. A key subsystem consists of two fringe sensing units. They employ polarized and dispersive optics to measure differential phases in five narrow K band channels. The differential phases are used to correct the differential delays, which are the primary observables used to determine relative proper motions, relative parallaxes, and planetary orbits. Since small systematics will appear in the differential phases for real optical components, we present in this poster:

- **a closed mathematical form for the differential phase, including small systematic offsets and random errors**
- **Monte Carlo simulations to understand how the small systematic offsets and random errors affect the differential phases and**
- **that delay-line stepping can be used to eliminate the effects of small systematic offsets and random errors**

Introduction:

PRIMA will have two operational modes: phase-referenced imaging and dual-star narrow-angle astrometry[1]. The **ESPRI** consortium, which is constructing the instrument in collaboration with ESO, will use guaranteed time to perform an astrometric exoplanet survey with positional errors on the order of 10-30 µas. PRIMA should detect a wide range of objects and provide mass estimates without lower limits.

PRIMA consists of the following subsystems – two star separators (**STSs**), a differential laser metrology system (**PRIMET**), two fringe sensor units (**FSUs**), and four differential delay lines (**DDLs**) in vacuo – that are used in conjunction with the existing auxiliary telescopes (**ATs**) and main delay lines (**MDLs**).[1] One STS is required for each AT. One FSU is used for the program star (almost always bright), while the other is used for the reference star (almost always faint).

Instrumental and environmental systematic offsets and random errors are summarized in the error budget.[2] Calibration of systematic errors in the presence of random errors is key for the success of PRIMA. We show how stepping can be used to fit and remove the effects of small systematic offsets in the presence of random errors, leading to an algorithm that will ultimately be used in the astrometric data reduction software.[3]

PRIMA Mathematical Model:

The response of a classical interferometer is $I(\kappa) = I_0(\kappa) + |I_{12}(\kappa)| \cos[\Phi(\kappa)]$, where $\Phi(\kappa)$ is the observed phase, κ is the wavenumber, $I_0(\kappa)$ is the uncorrelated flux and $|I_{12}(\kappa)|$ is the correlated flux magnitude.

For **narrow-angle dual-star astrometry**, each star is observed by independent classical interferometers (i = A and B) through approximately the same isoplanatic patch. The astrometric observable is the **differential delay** between the two stars. When the fringe tracking is in error (e.g. for the faint star), the differential delays are corrected using the **differential phases** between the interferometers.

The PRIMA FSUs measure the ***ABCD* pixel** values simultaneously (4 discrete points at the central fringe, separated by 90º) instead of dithering along the central fringe. The differential phase can then be expressed in terms of the cross-flux phasor

$$I_x(\kappa) = [X_B X_A + Y_B Y_A] + j[-X_B Y_A + Y_B X_A] = |I_x(\kappa)| e^{j\Delta\Phi(\kappa)} ,$$

where $X_i(\kappa) = A_i(\kappa) - C_i(\kappa)$ and $Y_i(\kappa) = B_i(\kappa) - D_i(\kappa)$ are proportional to the real and imaginary parts of the visibilities measured by interferometer A and B. The **differential phase** for an ideal PRIMA, averaged over a scan, is then given as

$$\langle \Delta\Phi(\kappa) \rangle = \langle \Phi_A(\kappa) - \Phi_B(\kappa) \rangle = \tan^{-1} \frac{\langle I_{x,img}(\kappa) \rangle}{\langle I_{x,real}(\kappa) \rangle} .$$

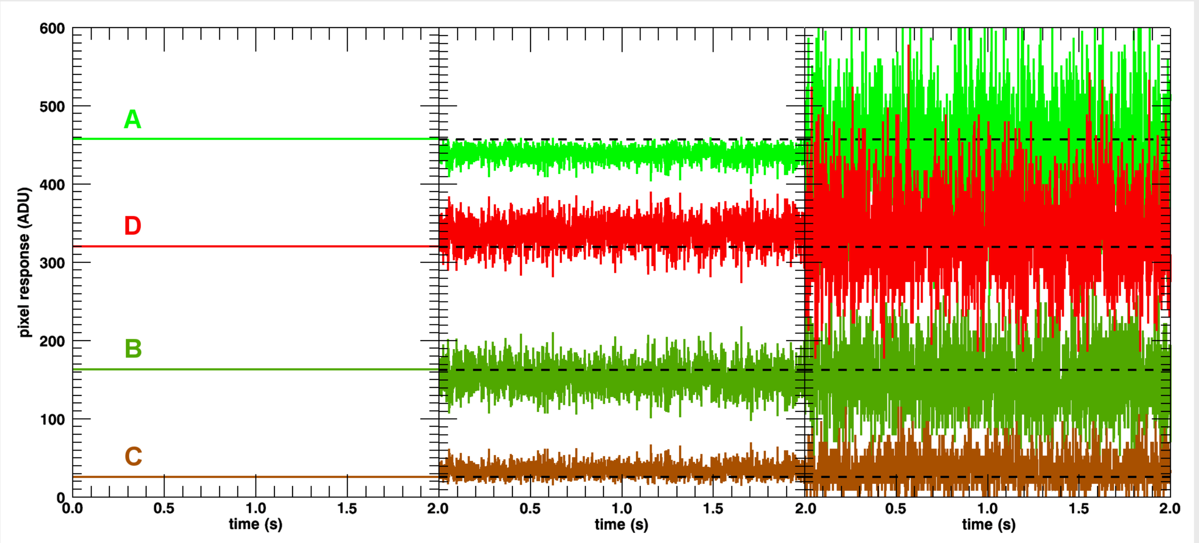

Figure 1: The FSU response to a 6th magnitude star for all four *ABCD* at κ = 1/2.1 μm^{-1}. The observing conditions are: 2 s of data using a raw integration time of 1 ms, instrumental throughput = 0.003, ADU gain = 8.9, removed sky background and no read noise. ***Left:*** No systematic offsets and random errors. ***Middle:*** With the systematic offsets (standard deviations):
$\sigma_{\Delta g0}$= 3 % (uncorrelated gain offsets),
$\sigma_{\Delta g12}$= 0.5 % (correlated gain offsets),
$\sigma_{\Delta\varphi}$= 3º (phase offsets) and a random error
σ_j= 5º (phase jitter, atmosph. turbulence).
Right: Same as the middle, but with Poisson noise added.

Imperfections of real PRIMA lead to **relative (un)correlated pixel gains** (Δg_0 and Δg_{12}, a few percent) **and phases** ($\Delta\Phi$, a few degree) that are not exactly 90º. When the systematic offsets are included we obtain for the **pixel response** (Figure 1)

$$\begin{aligned}
A_i(\kappa) &\approx [1+\Delta g_{A0,i}] I_{0,i} + [1+\Delta g_{A0,i}+\Delta g_{A12,i}] I_{12,i} \cos[\Phi_i + \Delta\varphi_{A,i}] \\
B_i(\kappa) &\approx [1+\Delta g_{B0,i}] I_{0,i} + [1+\Delta g_{B0,i}+\Delta g_{B12,i}] I_{12,i} \sin[\Phi_i + \Delta\varphi_{B,i}] \\
C_i(\kappa) &\approx [1+\Delta g_{C0,i}] I_{0,i} - [1+\Delta g_{C0,i}+\Delta g_{C12,i}] I_{12,i} \cos[\Phi_i + \Delta\varphi_{C,i}] \\
D_i(\kappa) &\approx [1+\Delta g_{D0,i}] I_{0,i} - [1+\Delta g_{D0,i}+\Delta g_{D12,i}] I_{12,i} \sin[\Phi_i + \Delta\varphi_{D,i}] .
\end{aligned}$$

Monte Carlo Simulations:

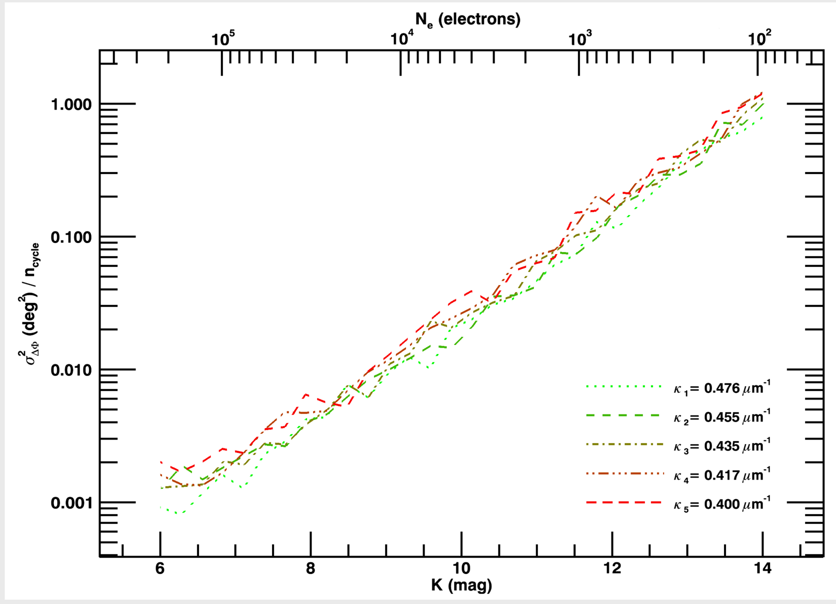

Figure 2: Monte Carlo simulations of the differential phase variance versus the K magnitude of the faint star, Poisson noise only. As orientation, the upper abscissa shows the total number of electrons detected at all four *ABCD* over 2 seconds of integration at each wavenumber channel (throughput = 0.003). The K magnitude of the bright star was 6. The variances were always calculated from n_{cycle} = 75 two second averages for a 2.5 min scan. No sky background and read noise is included.

The (un)correlated gain offsets and phase offsets in Figure 1 are drawn from their own zero-main **Gaussian distribution** and they are constant here, but they could very slowly over time scales of minutes to hours.

To verify the math and fully understand the error propagation for the differential phase we done a lot of Monte Carlo simulations. One example is shown in Figure 2. As expected, the relationship is linear and independent of wavenumber.

With these kinds of simulations we surmise that **first order approximations** are sufficient for the data reduction.

Stepping:

How can the effects of (un)correlated gain and phase offsets be calculated and removed? If this effects are not calibrated, they will introduce systematic errors into the differential phases and ultimately degrade the quality of the differential delays. If one simultaneously scans the DDLs over the fringe packets of both stars through several discrete steps, keeping the differential delay fixed, **sinusoidal variations** in the cross-flux phasor parts will appear (Figure 3).

The average coefficients, $I_{x,img,0}(\kappa)$ and $I_{x,real,0}(\kappa)$, from the sinusoidal fitting, can then be used to calculate the (disturbed) differential phases

$$\Delta\Phi'(\kappa) = \tan^{-1} \frac{I_{x,img,0}(\kappa)}{I_{x,real,0}(\kappa)} = \Delta\Phi(\kappa) + \Delta\beta(\kappa) ,$$

where $\Delta\beta(\kappa)$ are the first order calculated systematic differential phase errors, which can be measured by a separate calibration. This removes the small systematic offsets.

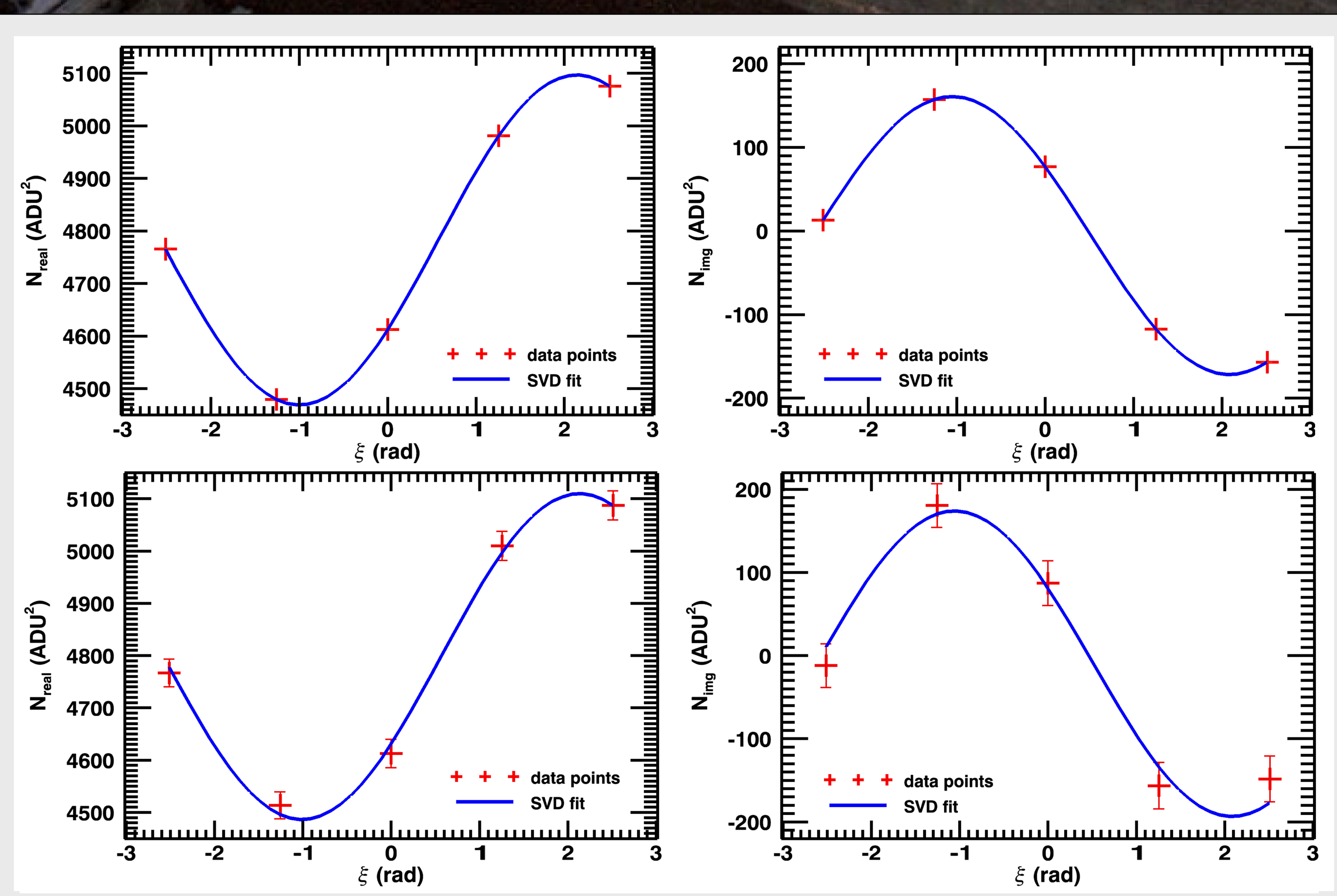

Figure 3: Cross flux components of a 6th and 10th magnitude star with uncorrelated gain offsets of $\sigma_{\Delta g0}$= 3 % at κ = 1/2.1 μm^{-1}. The **top** two plots correspond to the real (left) and imaginary (right) components with no Poisson noise. The **bottom** two correspond to the same quantities, including Poisson noise. Each step is 2 s in duration. There are five evenly spaced steps over 300º. The stepping cycle is repeated over an ≈ 2.5 minute scan. Each data point corresponds to the average of the step state over the scan. The curve corresponds to the SVD (Singular Value Decomposition) fit.

1. Quirrenbach, A. Henning, T., Queloz, D. et. al., in *New Frontiers in Stellar Interferometry*, ed. W.A. Traub, *Proceedings of the SPIE* 5491, 424.
2. Tubbs, R.N., van Belle, G., Launhardt, R., Quirrenbach, A., Elias II, N.M., Henning, T., Queloz, D., 2008, *These Proceedings*.
3. Elias II, N.M., Köhler, Stilz, I., Reffert, S., Geisler, R., Quirenbach, A., de Jong, J., Delplancke, F., Tubbs, R., Henning, T., Queloz et. al., D. 2008, *These Proceedings*.