Theory of Phases
in Optical Interferometry

John Young

University of Cambridge

Thanks to: Chris Haniff, John Monnier, J-P Berger

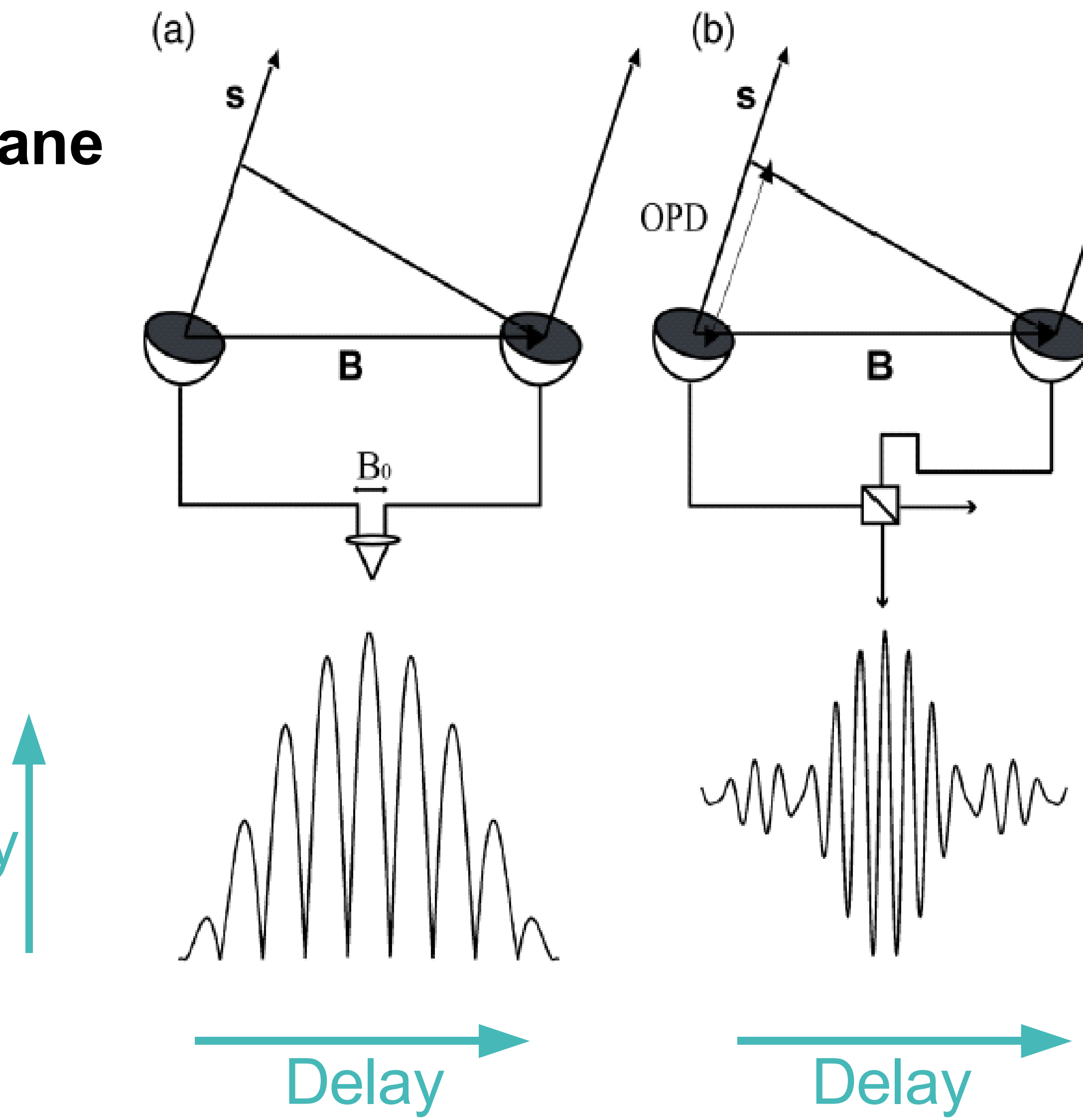
Outline of Talk

- Fringe patterns; amplitude and phase
- Impact of atmosphere
- Techniques to get phase information:
  - Phase Referencing
  - Differential Phase
  - Closure Phase
- Closure Phase
  - Definition and properties
  - Use in mapping
  - Use in model fitting

Interferometers measure fringes (revision)

Image plane

Pupil plane



Intensity

Fringe amplitude and phase depend on FT of source brightness distribution (revision)

- Recall sinusoidal interferometer output (fringes):

  $P \propto 1 + \cos(kD), \qquad D = \boldsymbol{s}.\boldsymbol{B} + d_1 - d_2$

- This is related to the Fourier Transform of the source brightness distribution:

  $P(\boldsymbol{s_0}, \boldsymbol{B}, \delta) = I_{\text{total}} + \mathrm{Re}[V(\boldsymbol{B}) \exp(-\mathrm{i}k\delta)]$,

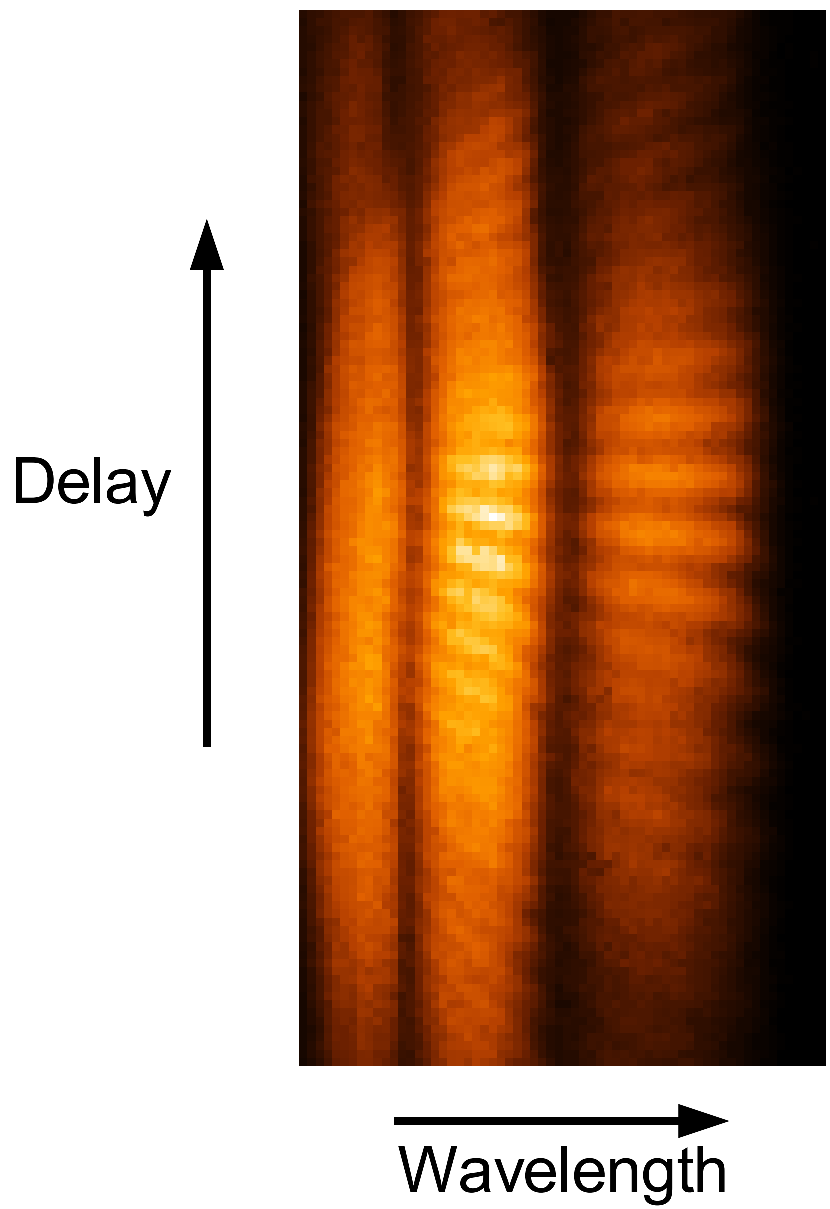
  $V(\boldsymbol{B}) = V(u,v) = \int I(\alpha, \beta) \exp(-2\pi \mathrm{i}\, (\alpha u + \beta v))\, \mathrm{d}\alpha\, \mathrm{d}\beta$

- δ is offset from expected white-light fringe delay. Measure *P* at e.g. $\delta=0$ and $\delta=\pi/4$ to obtain complex *V*

BUT....

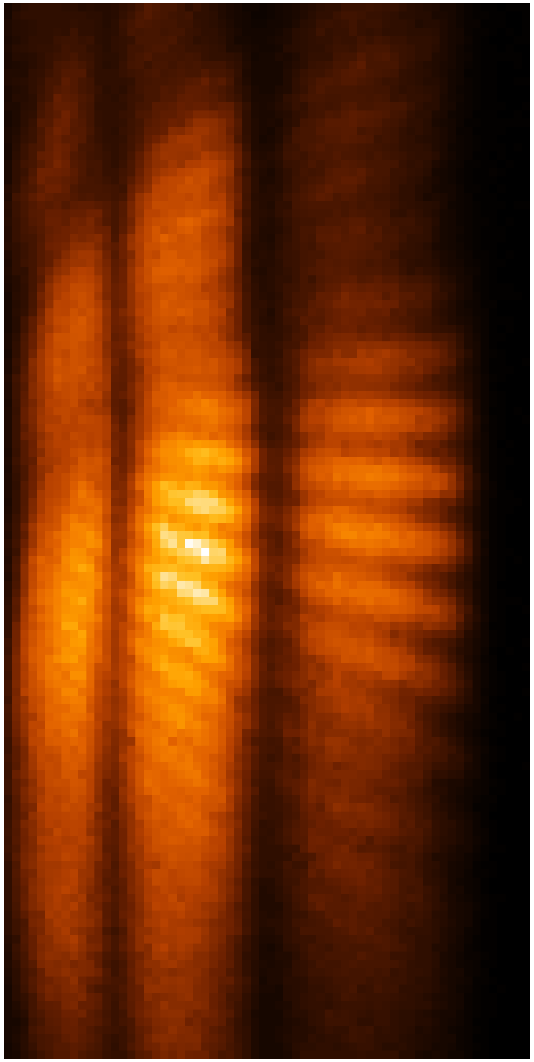
- When we observe through the Earth's atmosphere, the phase of the fringe pattern is perturbed
- We can still measure the phase, but it no longer tells us anything about the source

Actual (dispersed) fringe pattern



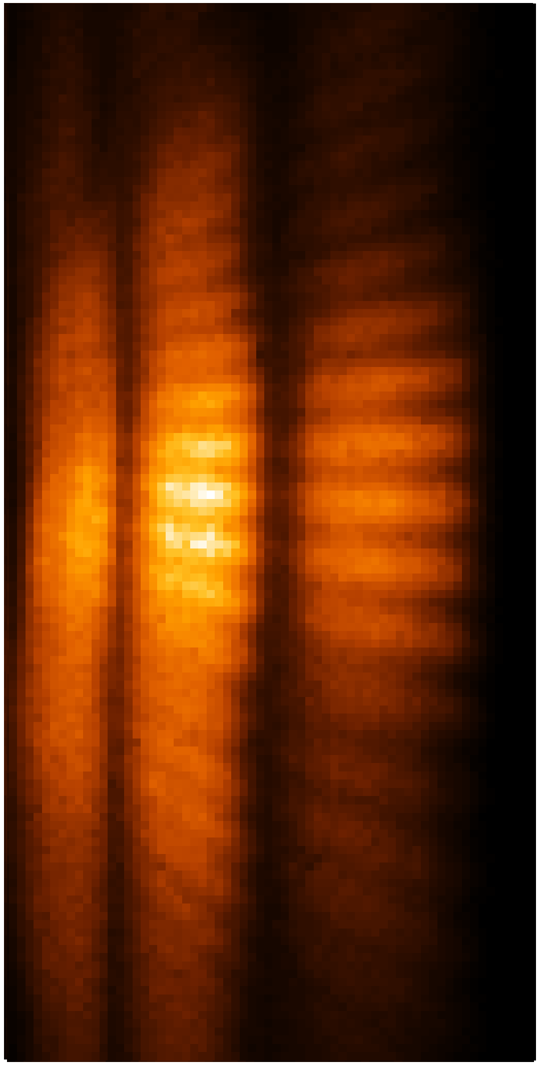
Fringe motion = phase fluctuations

- Note motions are > 1 fringe



Fringe motion = phase fluctuations

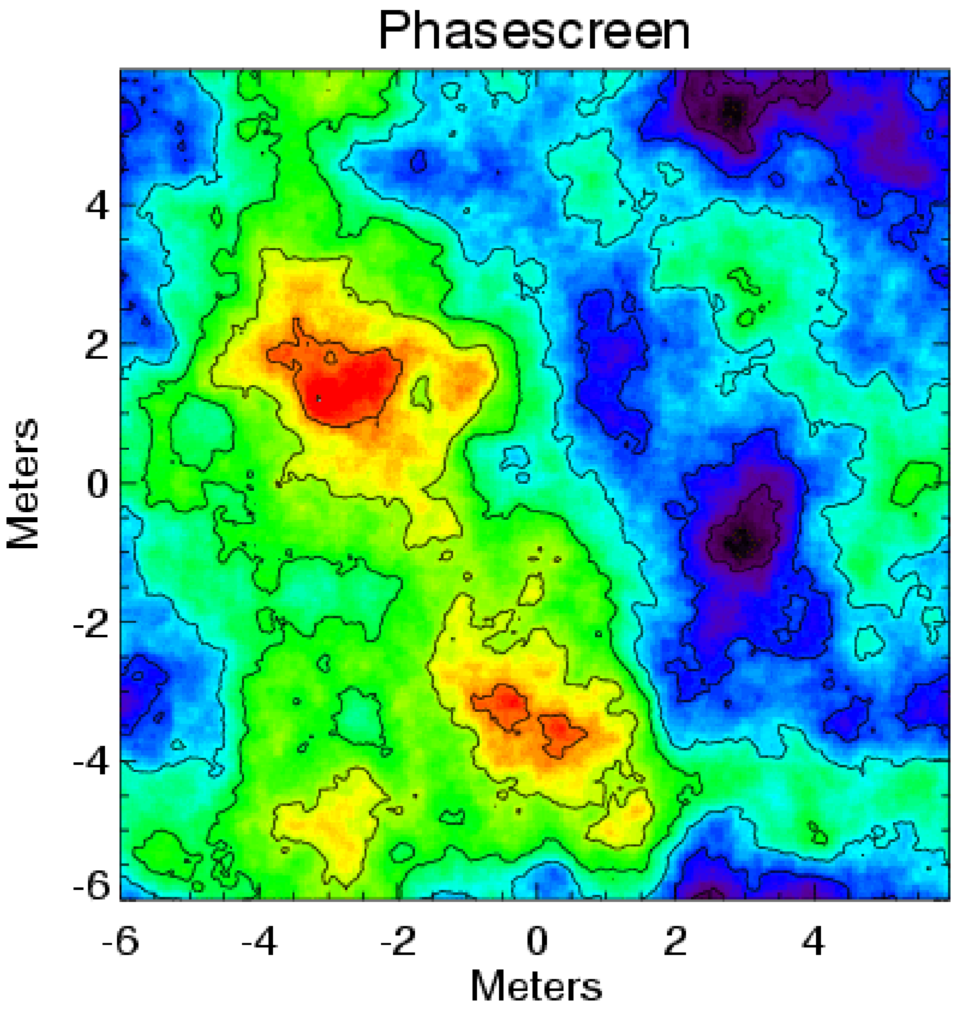
- Note motions are > 1 fringe



The atmosphere

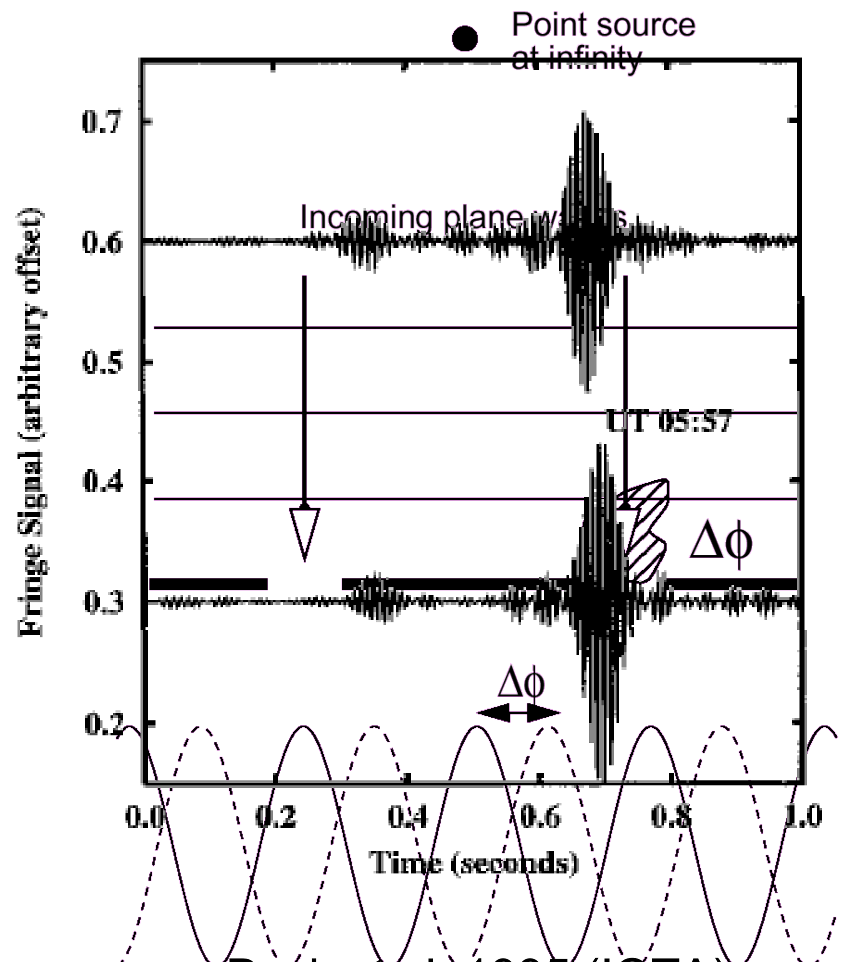
- A 12m × 12m patch of good atmosphere

- Each contour represents one radian of phase delay for light at a wavelength of 2 microns



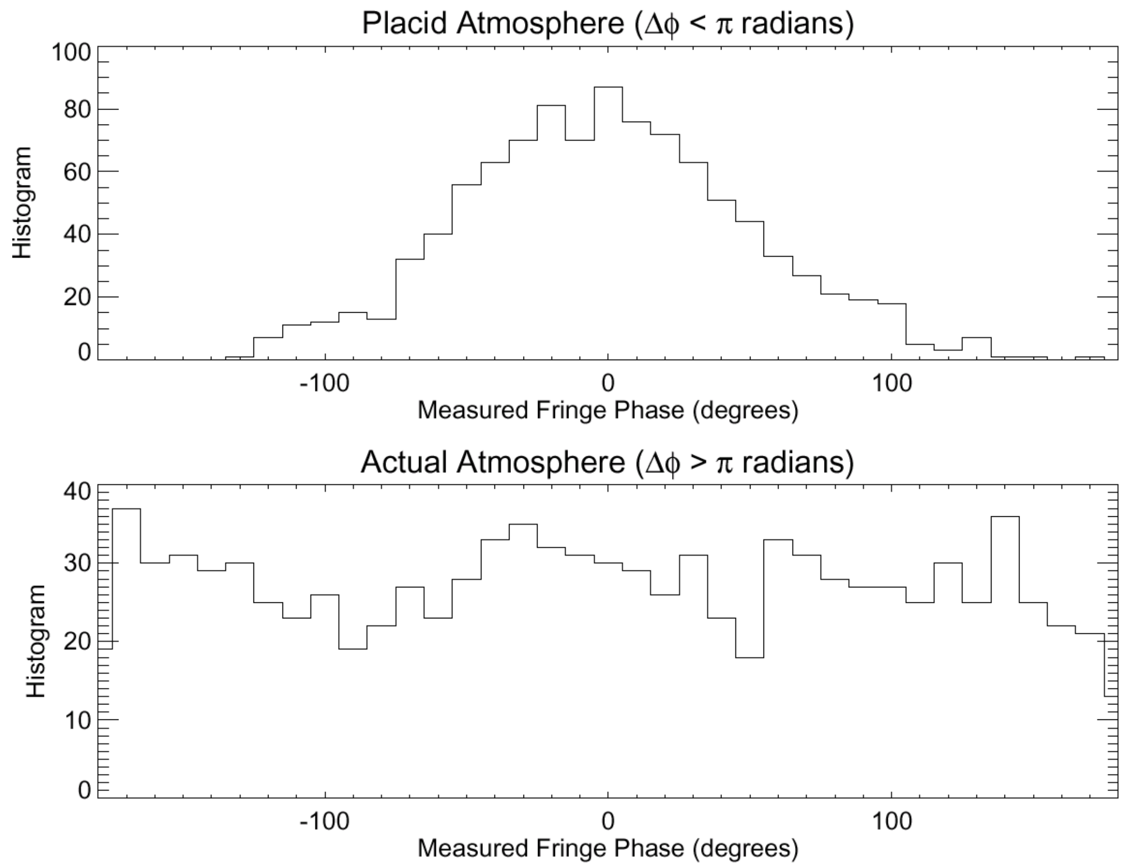
What this does

- Atmosphere introduces an unknown phase delay above each interferometric collector
- Example: suppose only one aperture affected
- Shifts output of interferometer away from expected white-light position



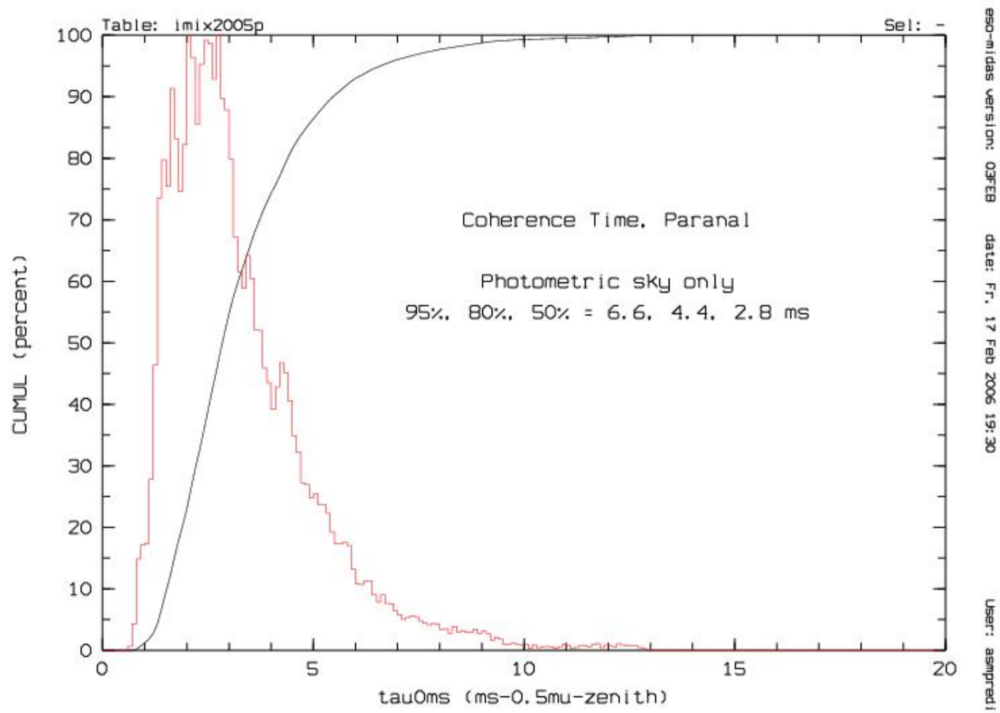
Dyck et al. 1995 (IOTA).

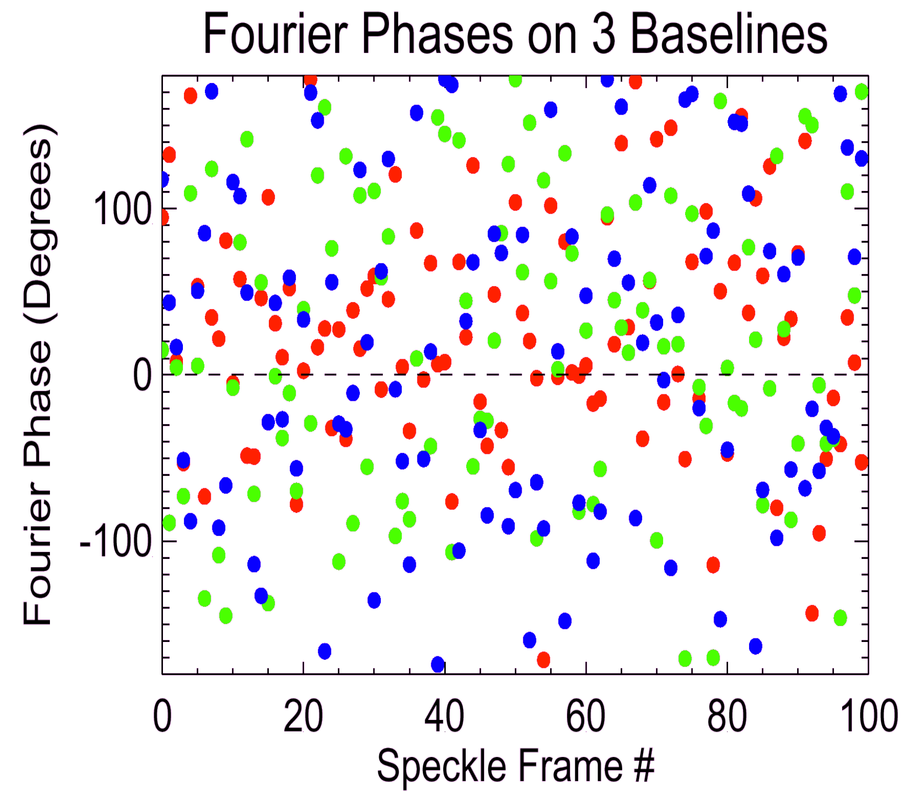
Why we can't average the phase



- Even on a good site in the near-infrared, phase excursions exceed π radians

Timescale for phase variations





- **Coherence time τ_0:**
  Interval over which RMS phase change is 1 rad
  - Scales as $\lambda^{6/5}$, so 3ms at 500nm corresponds to 18ms at 2.2μm
- Must use short exposures to avoid smeared fringes

Recap

- Interferometers measure fringe patterns:
  - Amplitude
  - Phase: location (in delay-space) of white-light fringe
- Fringe amplitude and phase are amplitude and phase of ***one Fourier component*** of the source brightness distribution
- Atmosphere perturbs measured phase by > π rad
- Timescale for phase perturbations is coherence time
  - Tens of milliseconds in NIR
  - Need short exposures even if only measuring amplitude

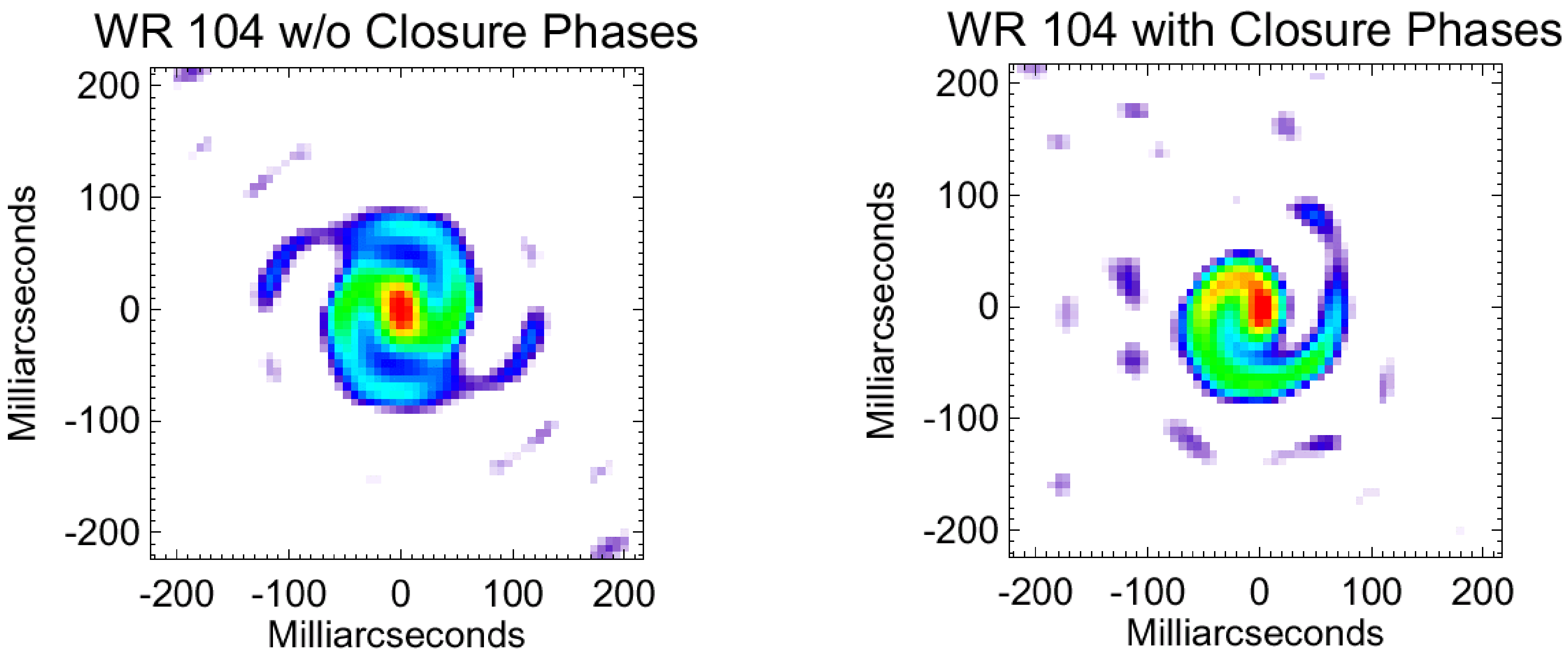
Questions?

?

Why do we care about phase?

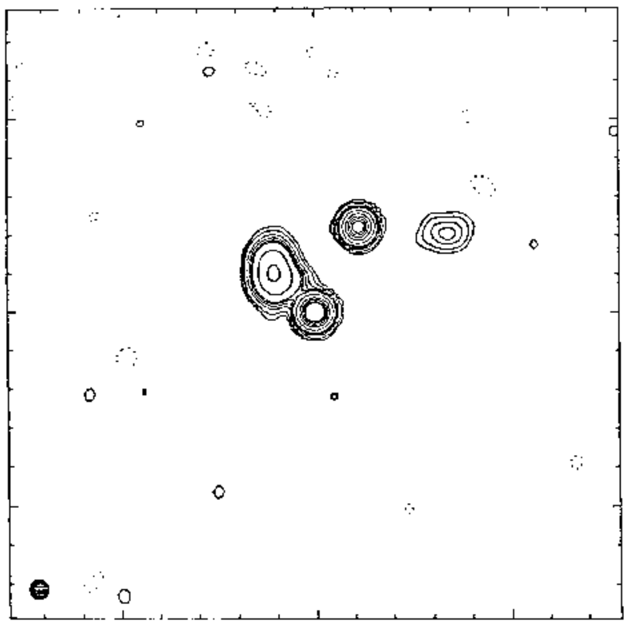
- Why don't we just measure the fringe amplitude then?
- Answer: Depends on what science you want to do. Sometimes just the amplitude is enough, often its not.
- Aside: atmosphere also affects measured visibility amplitude
- Mitigate by interleaving observations of science object and unresolved calibrator star

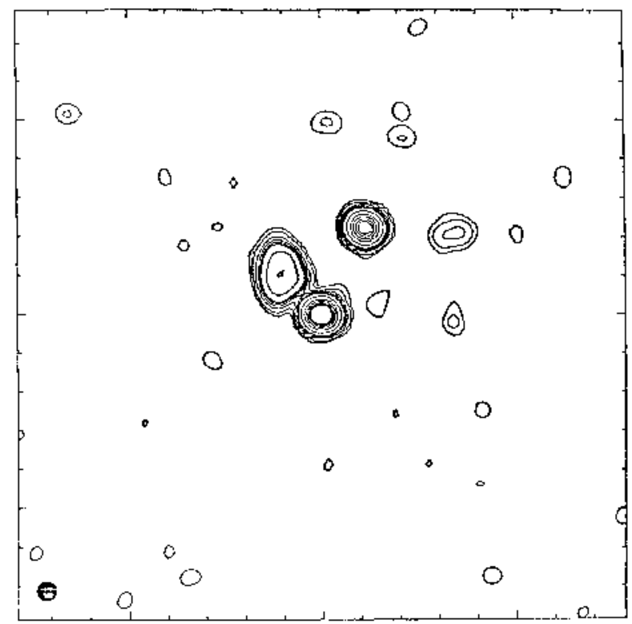
Most of information is in visibility phases (i)



- Without closure phase data, map is necessarily centro-symmetric
- With phase information, correct source brightness distribution is discovered

Most of information is in visibility phases (i)



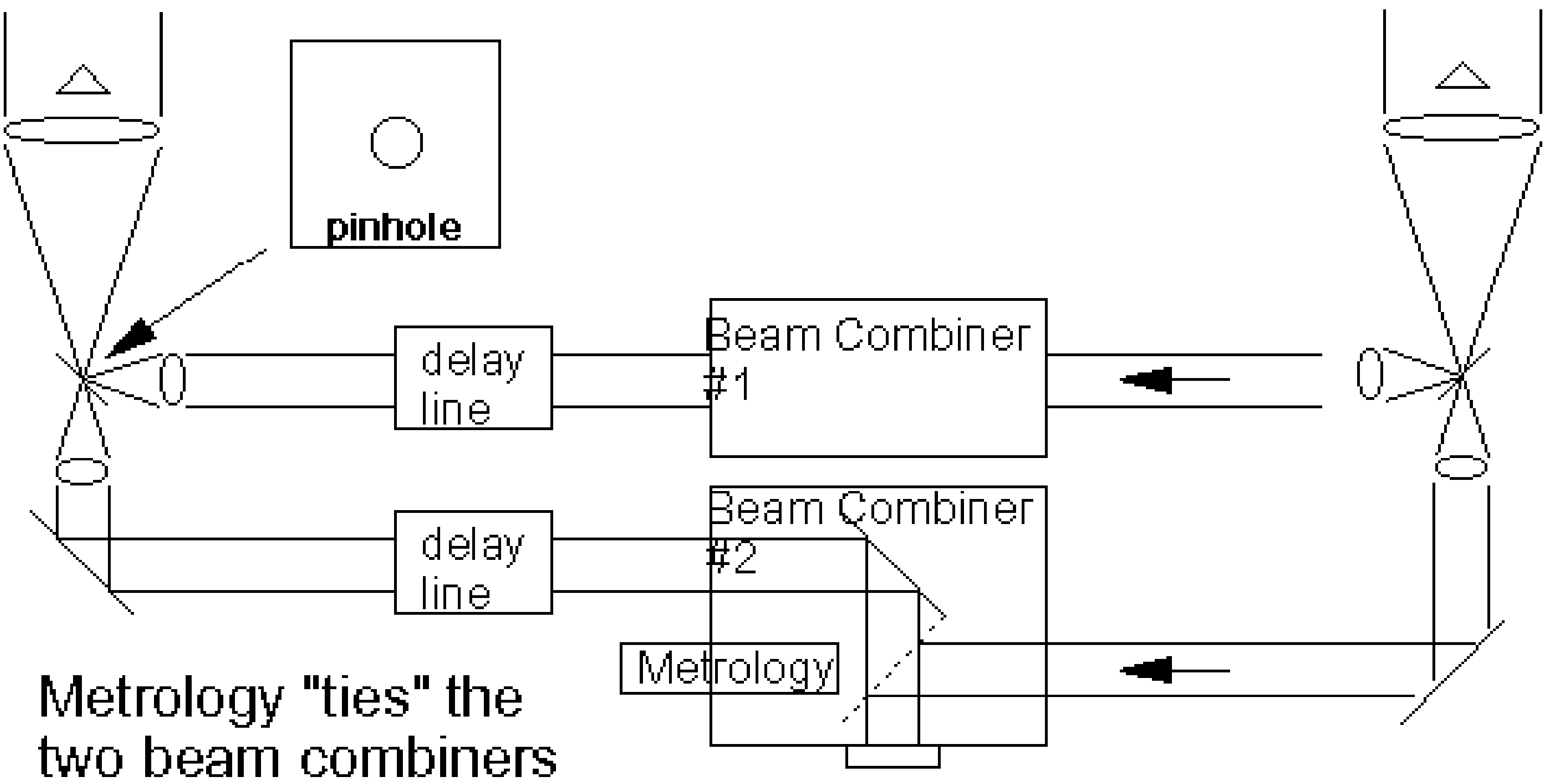


Amplitudes and phases Phases only

Techniques to recover phase information

- Closure phase (most of this talk & practical session)
- Phase referencing
- Differential phase

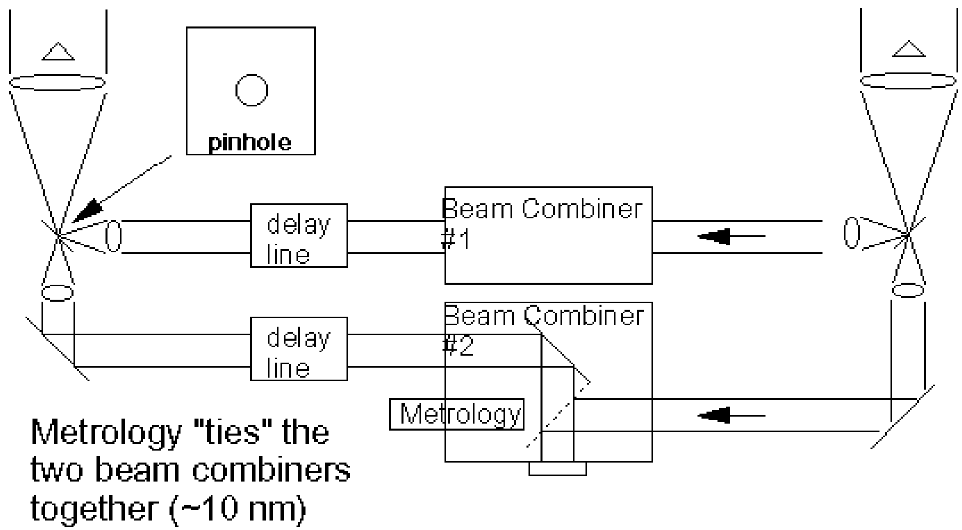
Phase Referencing
(Dual Star Interferometry)



Metrology "ties" the two beam combiners together (~10 nm)

Phase Referencing e.g. PRIMA

- Fringes on 2 sources simultaneously
- Track fringes on reference source using BC#1
- Measure amplitude and phase of science object fringes in BC#2
- Metrology system tells you phase zero-point for BC#2 – measured phase then equals true visibility phase



Differential phase e.g. AMBER/MIDI

- Extra hardware not required
- Nearby reference star not required

- Measure fringe phase as function of wavelength
- Model and remove atmospheric dispersion
  $\rightarrow \Phi_{DP} = \phi(\lambda) - \phi(\lambda_{ref})$
- Tells you photocentre shift w.r.t λ_{ref}
- Need a model for the source to interpret further

The Closure Phase

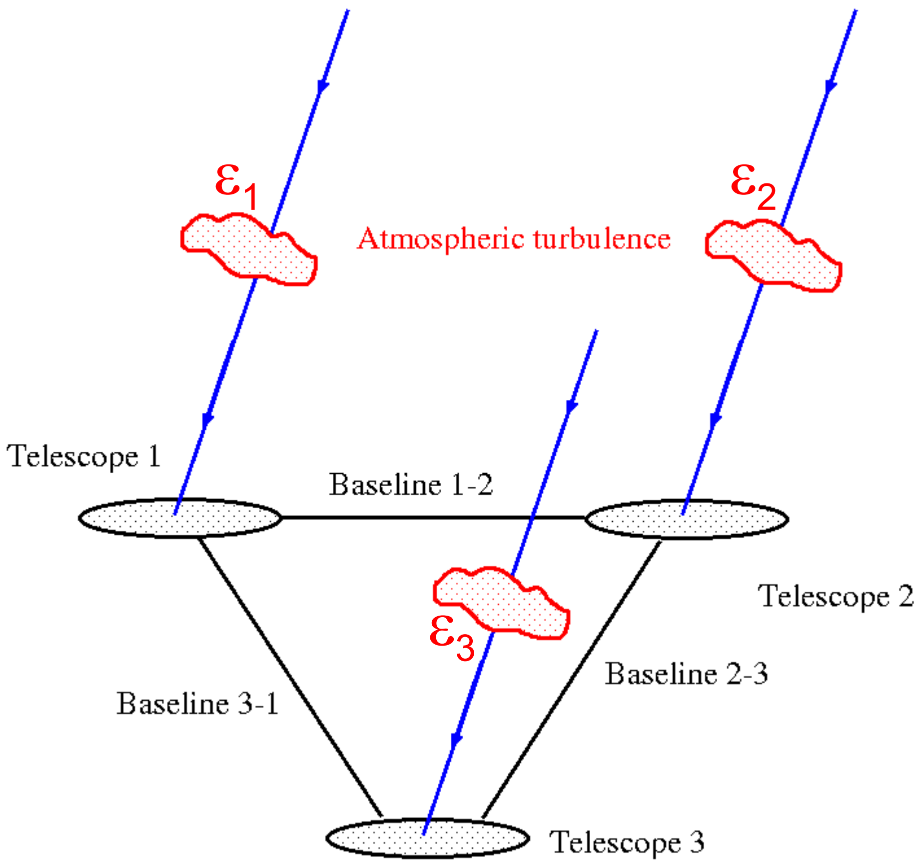
$$\Phi_{12} = \phi_{12} + \varepsilon_1 - \varepsilon_2$$

$$\Phi_{23} = \phi_{23} + \varepsilon_2 - \varepsilon_3$$

$$\Phi_{31} = \phi_{31} + \varepsilon_3 - \varepsilon_1$$

$$\Phi_{12}+\Phi_{23}+\Phi_{31} = \phi_{12} + \phi_{23} + \phi_{31}$$

- Sum of visibility phases around a closed triangle of baselines
- Telescope-dependent errors (e.g. atmosphere) cancel

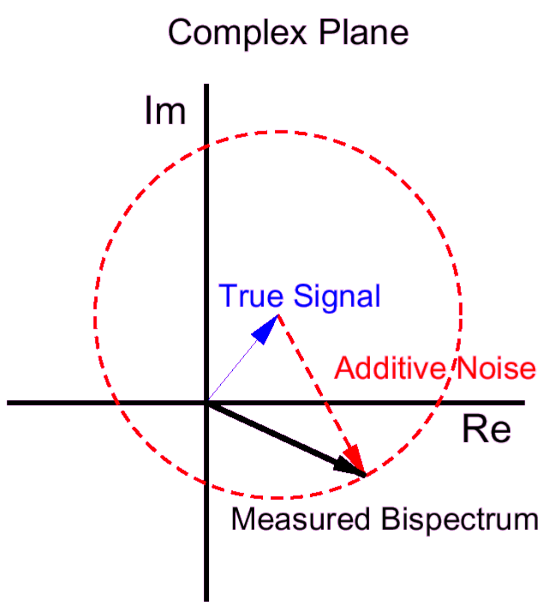


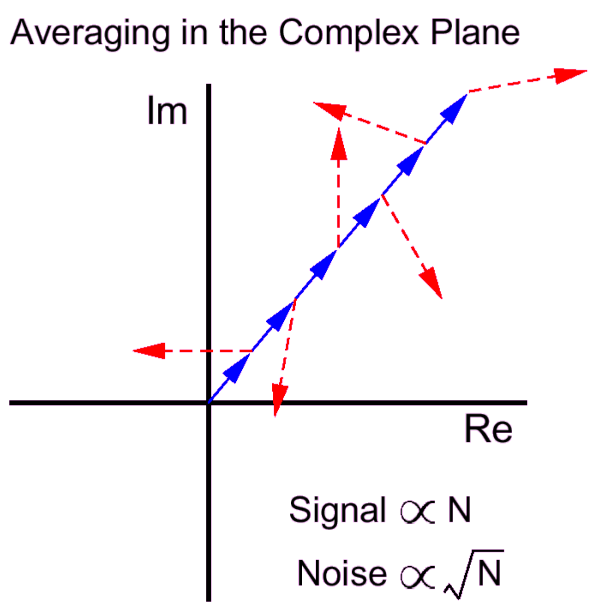
The Bispectrum

- Often more convenient to work with the **bispectrum** (a.ka. “Triple product”)
  = **product of complex visibilities** around a closed triangle of baselines
- Argument of bispectrum is the closure phase

Bispectrum measurements can be averaged

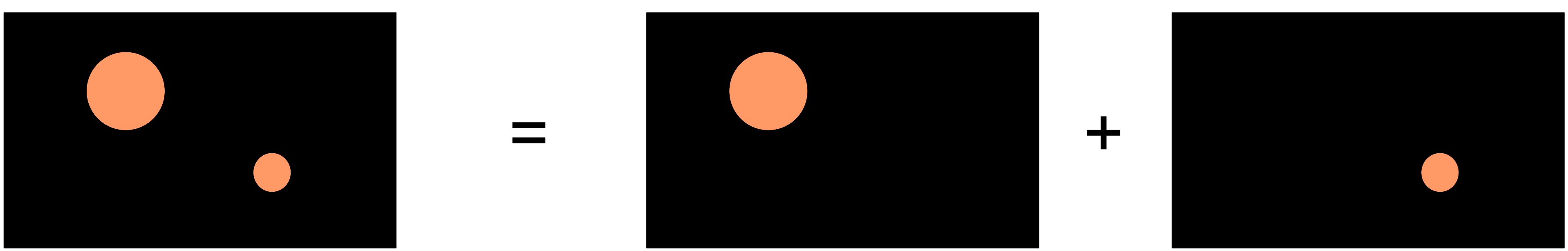
- Successive bispectrum measurements can be averaged in the complex plane
- Average is useful even if SNR of individual measurements is low





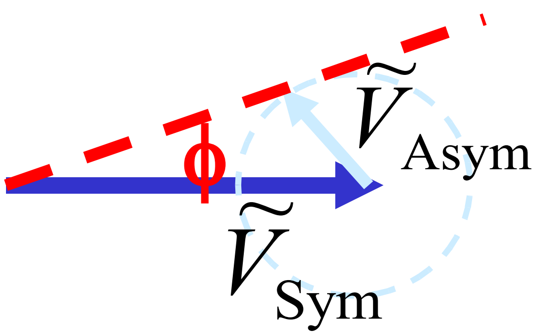
Recap on Visibility Functions

- FT is linear → V(component1 + component2)
  = V(component1) + V(component2)
  - Remember V is complex
  - Use this to predict bispectrum and hence closure phase for complicated sources



What does a closure phase measure?

- Insensitive to source position
  - Unlike visibility phase
- Point-symmetric sources have CP of 0 or 180°
  - Common examples: symmetric disc, equal binary
- CP measures fraction of asymmetric flux
  - On the angular scale to which you have resolved the source
  - $|\,\mathrm{CP}/\mathrm{rad}\,| \approx F_{\mathrm{asymm}} / F_{\mathrm{symm}}$



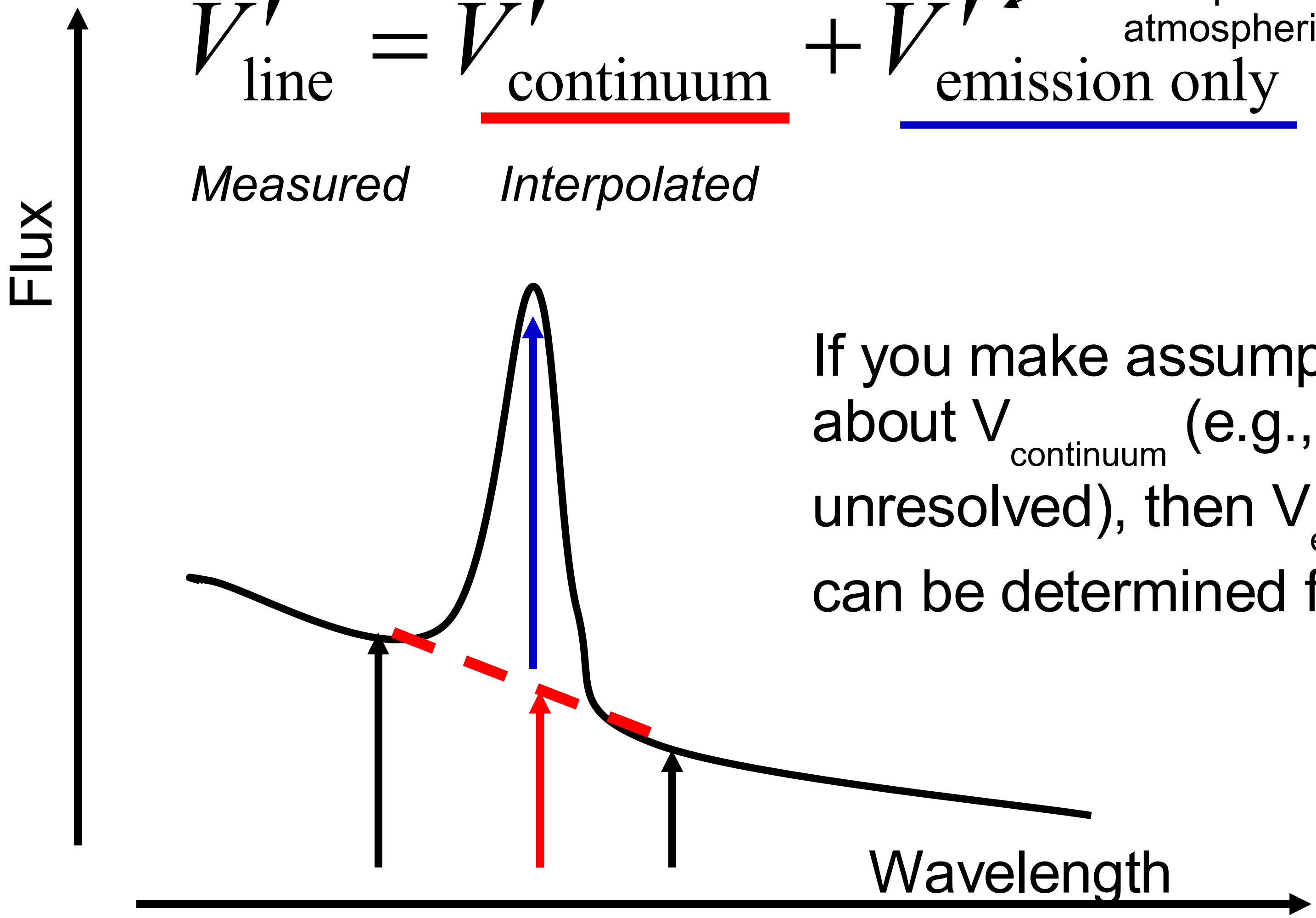
  - With enough closure phases, you can discover the nature of the asymmetry

Differential Phase Example

$$\widetilde{V}'_{\text{line}} = \widetilde{V}'_{\text{continuum}} + \widetilde{V}'_{\text{emission only}}$$

Measured *Interpolated*

Prime indicates corrupted by atmospheric piston



If you make assumption about $V_{\text{continuum}}$ (e.g., unresolved), then V_{emission} can be determined fully

Recap: closure phase

- Bispectrum is product of complex visibilities around closed triangle of baselines
  - Closure phase is argument of bispectrum
- Bispectrum is “good observable” in presence of atmosphere
  - Can be averaged over many coherence times
- Closure phase measures fraction of asymmetric flux, on scale at which you have resolved the source

Questions?

?

Important Properties of Closure Phases

- More robust to calibration error than visibility amplitude
  - Atmospheric turbulence generally does not bias measurement
  - Reasonable hope of measurement error reducing as $\sqrt{N}$
  - There can be biases due to chromatic effects (same for visibility amplitude)
- Sensitive to asymmetries in brightness distribution
  - Bispectrum *real* for point-symmetry ($\Phi_{CP} = 0$ or $180°$)
  - Must resolve object to have significant signal
  - Critical for validating model fits to visibility amplitude data
  - Necessary for imaging (if no phase referencing)

Model Fitting with Closure Phases

- Conventional techniques used to fit model for source brightness distribution to measured visibility amplitudes and closure phases/bispectra
  - Least-squares
  - Bayesian: minimise negative log posterior probability

    $$L = \text{Prior} + \Sigma\,(D_i - M(\boldsymbol{a}))^2/(2\sigma_i^2)$$

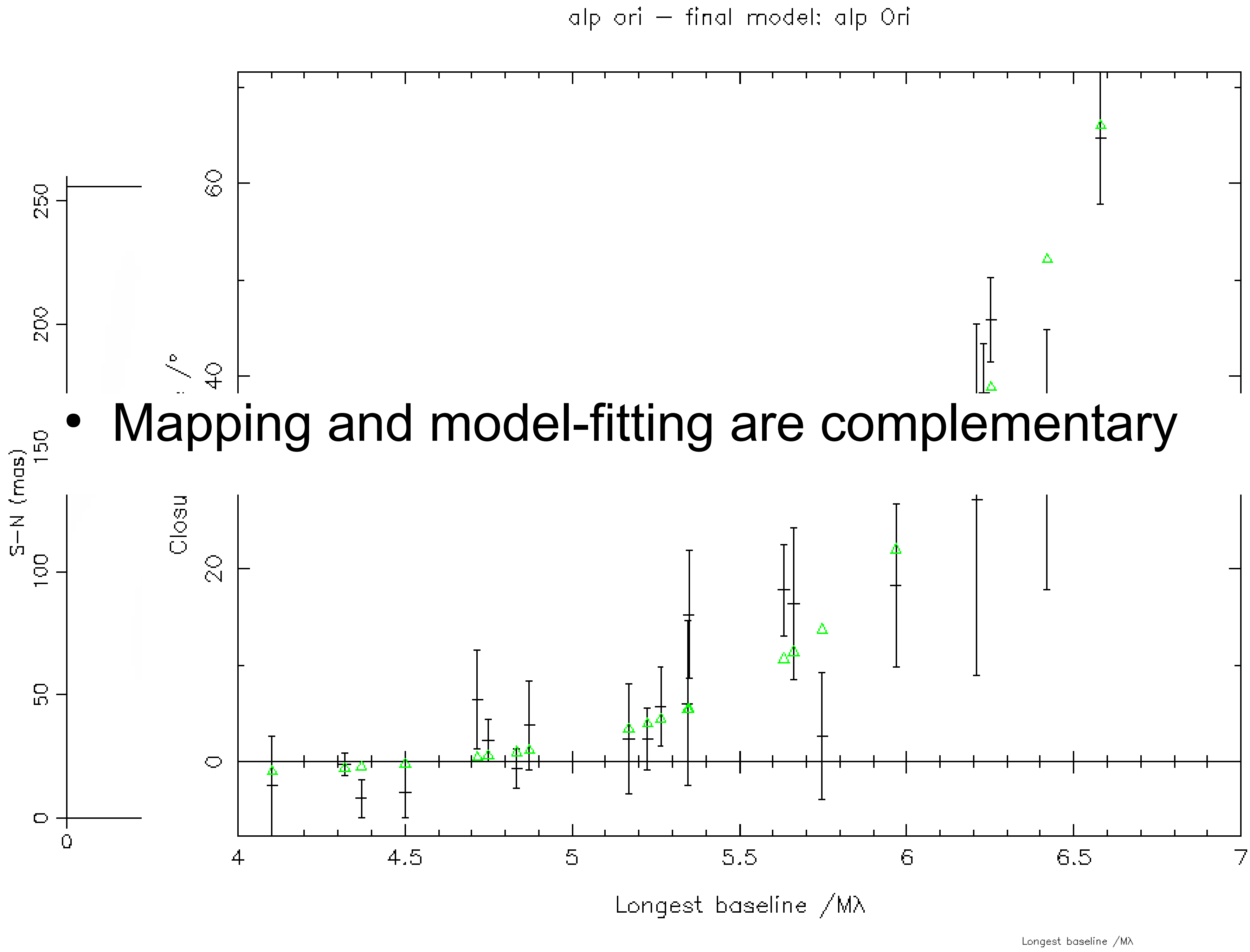
    by varying vector ***a*** of model parameters
    - Here D_i is Squared Visibility or Closure Phase ($D_i - M$ calculated modulo 2π)
- Model is either:
  - Sum of uniform discs, elliptical Gaussians etc.
  - Output of radiative transfer code

Mapping with Closure Phases: (a) Iterative Deconvolution

- Closure phases used as a constraint in an iterative scheme
  - Assign some phases consistent with the closure phases using a <u>model</u>
  - Fourier Invert
  - Deconvolve (e.g. CLEAN)$\rightarrow$ new model
  - Start over with new model
  - Unless procedure has <u>converged</u>

Mapping with Closure Phases: (b) Fitting with Regularization

- Fit model consisting of pixel values to visibility amplitude and closure phases
- No unique solution, so constrain using prior knowledge:
  - Positivity
  - Limited Field of View
  - Regularization term to favour “simple” solutions
    - e.g. Maximum Entropy: compressed range of pixel values
  - Note that iterative schemes incorporate prior knowledge implicitly



- Mapping and model-fitting are complementary

Concluding Remarks

- Phase information important for unambiguous interpretation of Fourier plane data
- Several techniques for obtaining phase information in phase-unstable conditions
  - Closure phase
  - Differential phase
  - Phase referencing (see talk on PRIMA)
- Closure phases/differential phases can be used in mapping and model-fitting