An introduction to the theory of interferometry

ONTHEFRINGE School

Active Galactic Nuclei at the highest angular resolution: theory and observations

Torun, Poland,
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27th August 2007

Outline

- Image formation with conventional telescopes
  - The diffraction limit
  - Incoherent imaging equation
  - Fourier decomposition

- Interferometric measurements
  - Fringe parameters
  - The van-Cittert Zernike theorem

- Science with interferometers
  - Interferometric imaging
  - Useful rules of thumb
  - Interferometric sensitivity

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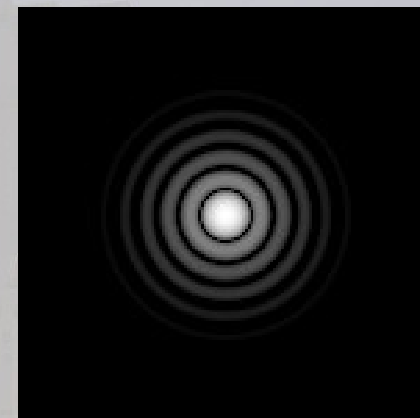
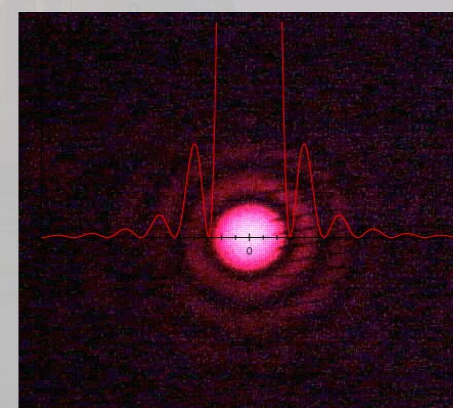
Characterizing source structure

How do we understand image formation?

- Consider a perfect telescope in space observing an unresolved point source:
  - This produces an Airy pattern with a characteristic width: $\theta = 1.22\lambda/D$ in its focal plane.





Even in perfect conditions, imaging is imperfect.

What does a more complex target look like?

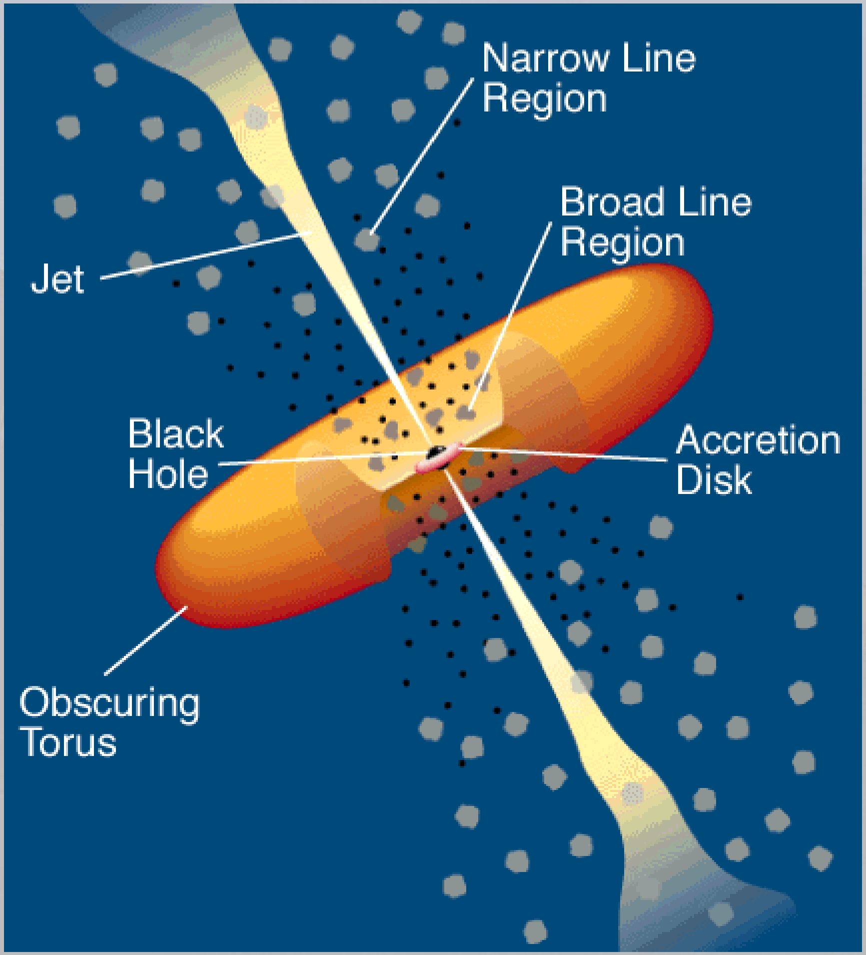
- Image formation (under incoherent & isoplanatic conditions):
  - Each point in the source produces a displaced Airy pattern. The superposition of these limits the detail visible in the final image.

- But what causes the Airy pattern?
  - Interference between parts of the wavefront that originate from different regions of the aperture.
  - In this case, the relative amplitude and phase of the field at each part of the aperture are what matter.
  - The further apart these parts of the aperture the narrower the PSF.

Conventional imaging is interferometric.

Why do you care?

- Because even nearby AGN are very small compared to the diffraction limit of the telescope observing them.
  - 10 pc (~obscuring torus scale) at 400 Mpc subtends an angle of ~5 mas.
  - 0.1 pc (~BLR cloud scale) at 400 Mpc subtends an angle of 0.05 mas



Imaging of these structures is challenging!

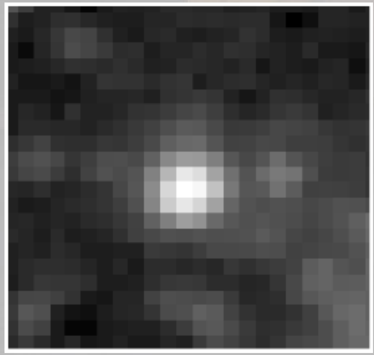
A mathematical description of imaging

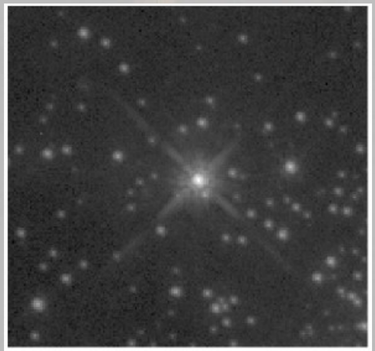
- The fundamental relationship for incoherent space-invariant (isoplanatic) imaging is:

$$I(l, m) = \int\int P(l-l', m-m') \, O(l', m') \, dl' \, dm' ,$$

i.e. the observed brightness distribution is the true source brightness distribution convolved with a point-spread function, P(l, m).

Note that here l and m are angular coordinates on the sky, measured in radians.





An alternative representation of the sky

- This convolutional relationship can be written alternatively, by taking the Fourier transform of each side of the equation, to give:

$$I(u, v) = T(u, v) \times O(u, v) \ ,$$

where *italic* functions refer to the Fourier transforms of their roman counterparts, and *u* and *v* are now spatial frequencies measured in radians^{-1}.

- The essential properties of the target are encapsulated in its Fourier spectrum, $O(u, v)$.
- The essential properties of the imaging system are encapsulated in a complex multiplicative transfer function, $T(u, v)$.
- This is just the Fourier transform of the PSF.

An aside on the Transfer function

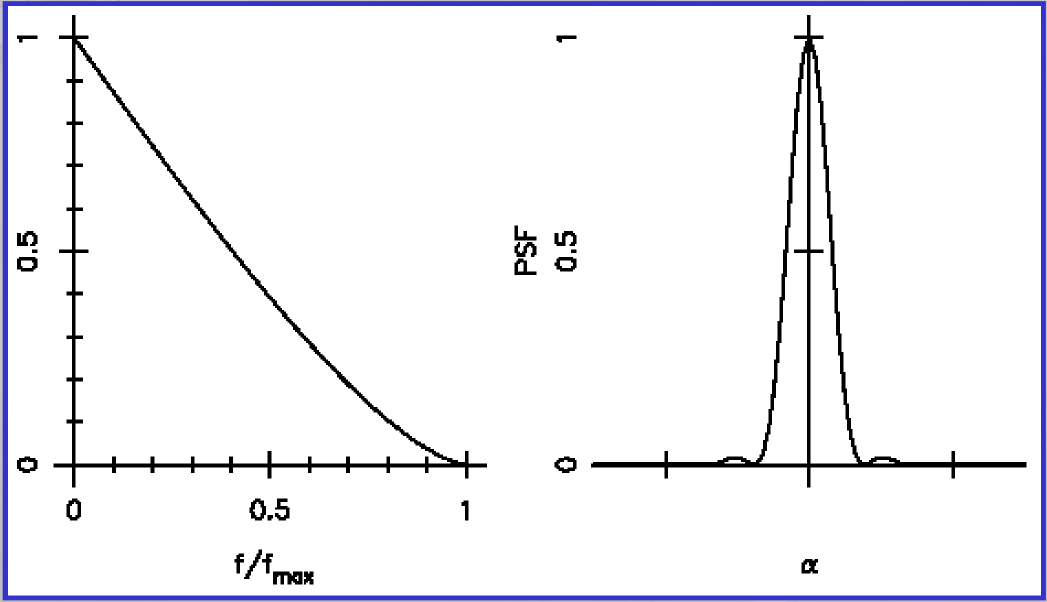
- In general the transfer function is obtained from the auto-correlation of the complex aperture function:

$$T(u, v) = \int\int A^*(x, y)\, A(x+u, y+v)\, dx\, dy \,,$$

  where x and y denote co-ordinates in the aperture. In the absence of aberrations A(x, y) is equal to 1 where the aperture is transmitting and 0 otherwise.
- Some key features of this formalism worth noting are:
  - For each spatial frequency, u, there is a physical baseline, B, in the aperture, of length $\lambda\ u$.
  - Different shaped apertures measure different Fourier components of the source.
  - Different shaped apertures give different PSFs:
    - Important, e.g., for planet detection.

The example of a circular aperture

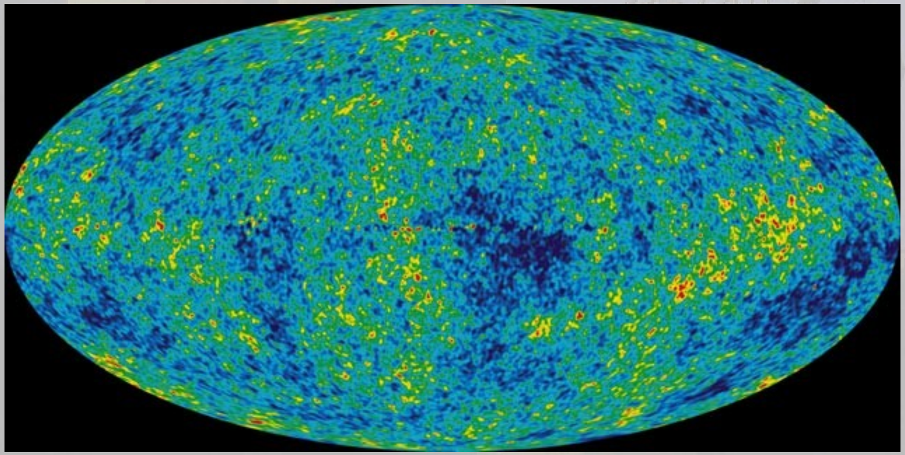
- For a circularly symmetric aperture, the transfer function can be written as a function of a single co-ordinate: $T(f)$, with $f^2 = u^2+v^2$.
- $T(f)$ falls smoothly to zero at $f_{max} = D/\lambda$.
- The PSF is the familiar Airy pattern.
- The full-width at half-maximum of this is at approximately λ /D.

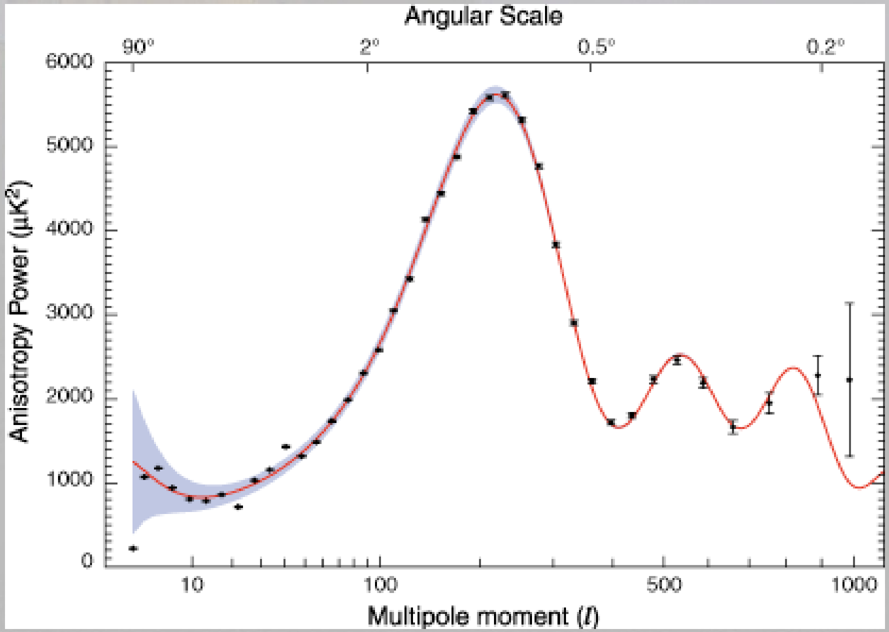


So, even perfect telescopes remove the high-frequency content of the source.

What should you learn from all this?

- Decomposition of an image into a series of spatially separated compact PSFs.
- The equivalence of this to a superposition of non-localized sinusoids, i.e. Fourier components.





What should you learn from all this?

- Decomposition of an image into a series of spatially separated compact PSFs.
- The equivalence of this to a superposition of non-localized sinusoids, i.e. Fourier components.
- The action of an incoherent imaging system as a filter for the Fourier spectrum of the source.
- The association of each Fourier component (or spatial frequency) with a distinct physical baseline in the aperture that samples the light.
- The form of the point-spread function as arising from the relative weighting of the different spatial frequencies measured by the pupil of the imaging system.

If you can measure the Fourier components of the source, you can do your science.

Questions?

Quiz 1

1. Assume you have a representation of the sky in terms of a PSF and a complete list of strengths and locations at which to lay them down. What happens to the “map” if:
   - You lose the first quarter of the list of strengths and locations? *(Assume that the list is ordered by decreasing distance from the center of the map, i.e. most distant first.)*
   - You lose all information about the PSF between position angles 0° and 90°?
   - You are given a smoothed version of the PSF.

2. Assume you have a representation of the sky in terms of a complete list of Fourier components and a list of strengths and phases for each of these. What happens to the “map” if:
   - You lose the first quarter of the list of strengths and phases. *(Assume that this list is ordered by decreasing spatial frequency, i.e. highest spatial frequency first.)*
   - The amplitude data in the list is scaled inversely with the modulus of the spatial frequency, so that the amplitudes at higher spatial frequencies are multiplied down?
   - The phases of the last quarter of the list are randomized?

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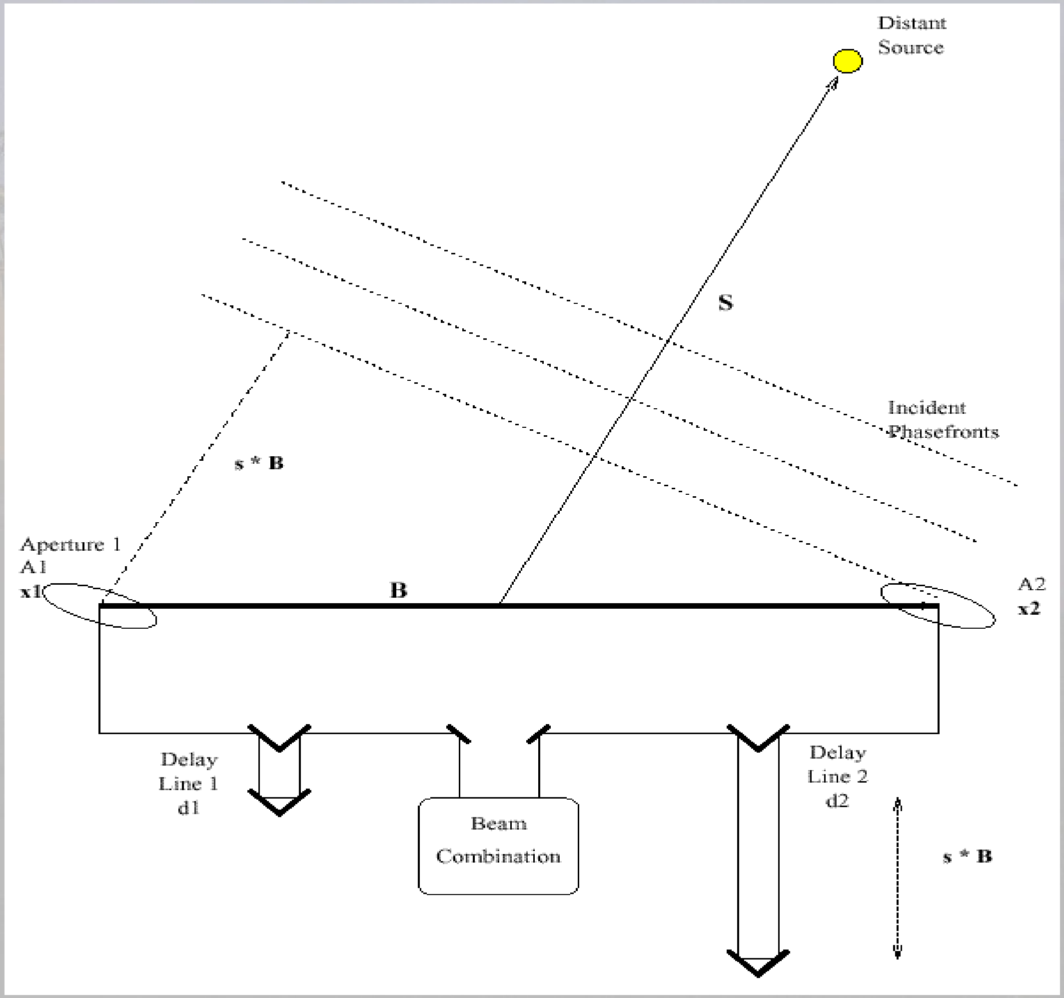
What/how do interferometers measure?

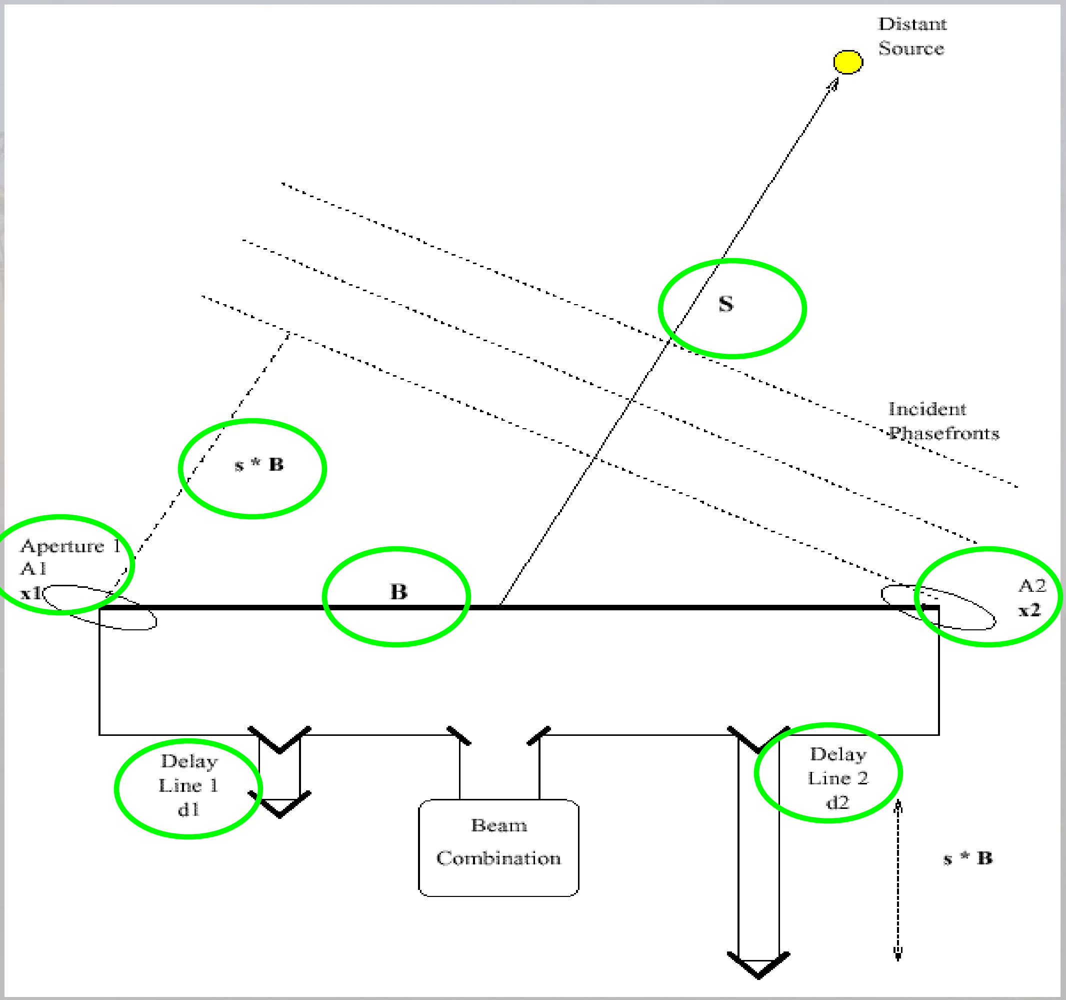
A two element interferometer - functions

- Sampling of the radiation (from a distant point source).
- Transport to a common location.
- Compensation for the geometric delay.
- Combination of the beams.
- Detection of the resulting output.



A two element interferometer - nomenclature

- Telescopes located at x_1 & x_2.
- Baseline $B = (x_1 - x_2)$:
  - Governs sensitivity to different angular scales.
- Pointing direction towards source is S.
- Geometric delay is $\hat{s}.B$, where $\hat{s} = S/|S|$.
- Optical paths along two arms are d_1 and d_2.



The output of a 2-element interferometer (i)

- At combination the E fields from the two collectors can be described as:

  $\psi_1 = A\exp(ik[\hat{s}.B+d_1])\exp(-i\omega t)$ and $\psi_2 = A\exp(ik[d_2])\exp(-i\omega t)$.

- So, <u>summing</u> these at the detector we get a resultant:

$$\Psi = \psi_1 + \psi_2 = A\left[\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])\right]\exp(-i\omega t)\,.$$

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$$\Psi = \psi_1 + \psi_2 = A\left[\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])\right]\exp(-i\omega t)\ .$$

- Hence the time averaged intensity, $\langle \Psi \Psi^* \rangle$, will be given by:

$$\langle \Psi \Psi^* \rangle \propto \langle\, [\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])] \times [\exp(-ik[\hat{s}.B + d_1]) + \exp(-ik[d_2])]\,\rangle$$

$$\propto 2 + 2\cos(\,k\,[\hat{s}.B + d_1 - d_2]\,)$$

$$\propto 2 + 2\cos(kD)$$

  Note, here $D = [\hat{s}.B + d_1 - d_2]$. This is a function of the path lengths, d_1 and d_2, the pointing direction (i.e. where the target is) and the baseline.

The output of a 2-element interferometer (ii)

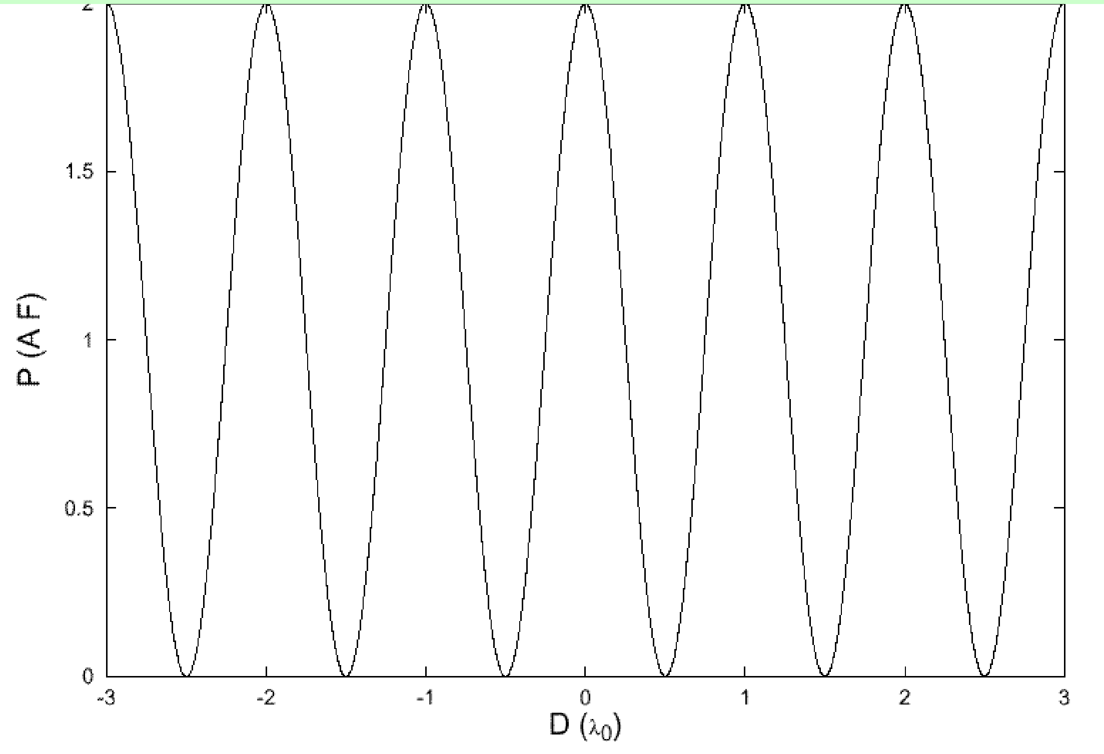
Detected power, $P = \langle \Psi \Psi^* \rangle \propto 2 + 2\cos(k\,[\hat{s}.B + d_1 - d_2])$
$\propto 2 \times [1+\cos(kD)]$, where $D = [\hat{s}.B + d_1 - d_2]$

- The intensity varies co-sinusiodally with kD, with $k = 2\pi/\lambda$.
- Adjacent fringe peaks are separated by

  $\Delta\, d_{1\ \mathrm{or}\ 2} = \lambda$ or

  $\Delta\, (\hat{s}.B) = \lambda$ or

  $\Delta\, (1/\lambda) = 1/D$.

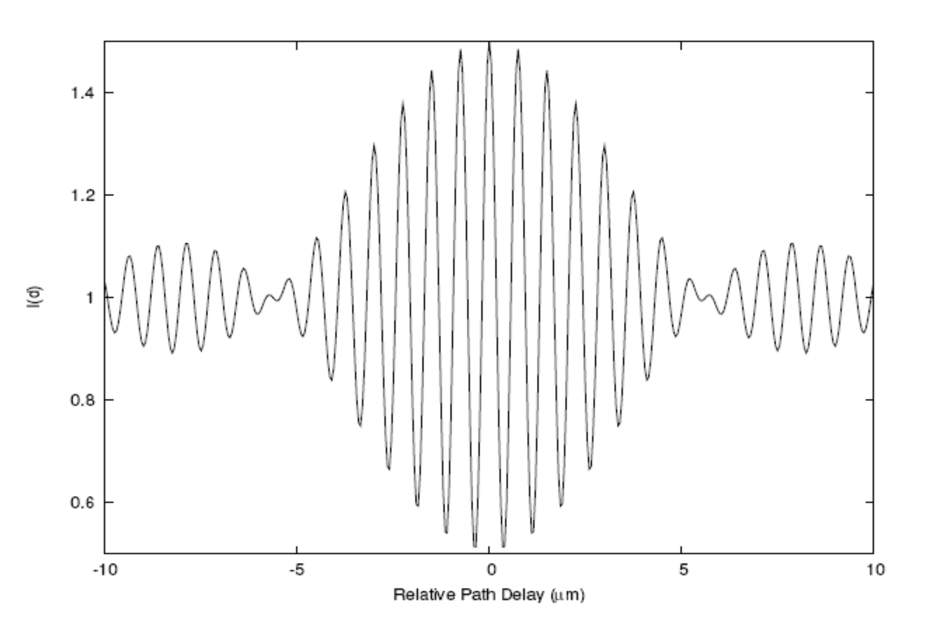


So what do we actually measure?

- From an interferometric point of view the key observables are the contrast and location of these modulations in intensity.
- In particular we can identify:
  - The fringe visibility at D=0:

$$V = \frac{[I_{max} - I_{min}]}{[I_{max} + I_{min}]}$$

  - The fringe phase:
    - The location of the white-light fringe as measured from some reference (radians).



In fact, the fringe amplitude and phase actually measure the amplitude and phase of the Fourier transform of the source brightness distribution at a single spatial frequency.

Questions?

Quiz 2

1. Check that you understand the "periodicity" of the interferometer output as a function of:
   - The wavevector, $k = 2\pi / \lambda$.
   - The baseline, B.
   - The pointing direction, s.
   - The optical path difference between the two interferometer arms.

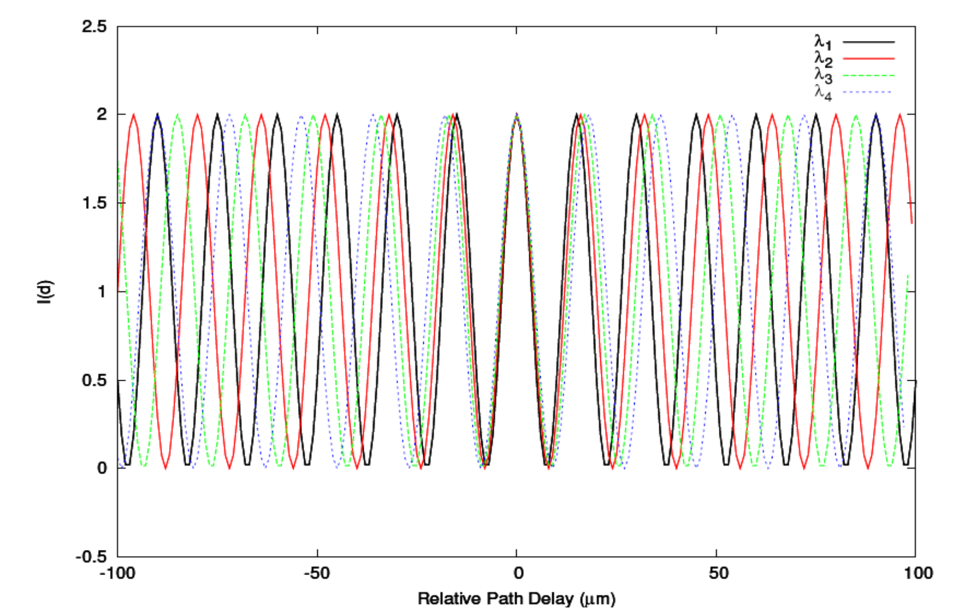
- How rapidly does the geometric delay change during the night? *(Only an approximate answer is needed – assume a 100m baseline interferometer observing a source at 45° elevation).*
- Imagine an interferometer with a 100m baseline is observing a target at the zenith at a wavelength of 1 micron. If the source could be moved by some small distance in the sky, how far would you have to move it to see the interferometer response change from a maximum to a minimum and then back to a maximum again?

What happens with polychromatic light?

- We can integrate the previous result over a range of wavelengths:
  - E.g for a uniform bandpass of $\lambda_0 \pm \Delta\lambda/2$ (i.e. $\nu_0 \pm \Delta\nu/2$) we obtain:

$$P \propto \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} [2+2\cos(kD)]\, d\lambda$$

$$= \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} 2\,[1+\cos(2\pi D/\lambda)]\, d\lambda$$



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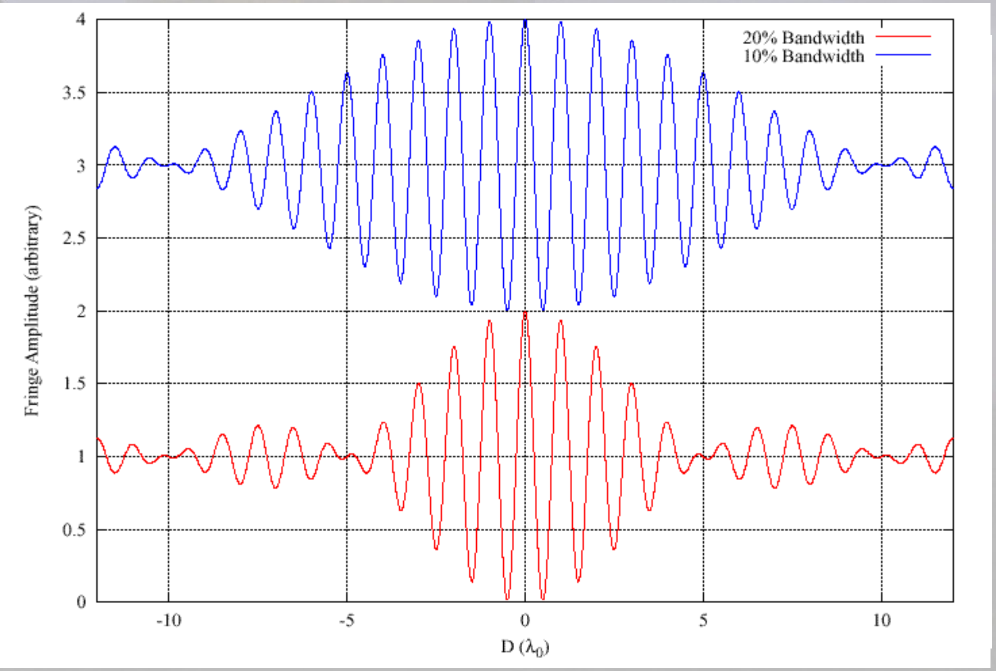
$$= \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} 2\,[1+\cos(2\pi D/\lambda)]\,d\lambda$$

$$= \Delta\lambda\left[1+\frac{\sin \pi D\Delta\lambda/\lambda^2_0}{\pi D\Delta\lambda/\lambda^2_0}\cos k_0 D\right]$$

$$= \Delta\lambda\left[1+\frac{\sin \pi D/\Lambda_{coh}}{\pi D/\Lambda_{coh}}\cos k_0 D\right]$$



So, the fringes are modulated with an envelope with a characteristic width equal to the coherence length, $\Lambda_{coh} = \lambda^2_0/\Delta\lambda$.

Key ideas regarding the interferometric output (i)

- The output of the interferometer is a time averaged intensity.
- The intensity has a co-sinusoidal variation – these are the “fringes”.
- The intensity varies a function of (kD), which itself can depend on:
  - The wavevector, $k = 2\pi / \lambda$.
  - The baseline, B.
  - The pointing direction, s.
  - The optical path difference between the two interferometer arms.

- If things are adjusted just so, then the interferometer output can remain fixed: in that case there will be no fringes.

Key ideas regarding the interferometric output (ii)

- The response to a polychromatic source is given by integrating the intensity response for each color.
- This alters the interferometric response and leads to modulation of the fringe contrast:
  - The desired response is only achieved when $k\,[\hat{s}.B + d_1 - d_2]) = 0$.
  - This is the so called white-light condition.
- This is the primary motivation for matching the optical paths in an interferometer and correcting for the geometric delay.

- The narrower the range of wavelengths detected, the smaller is the effect of this “coherence envelope”:
  - This is usually quantified via the coherence length, $\Lambda_{coh} = \lambda_0^2/\Delta\lambda$.

Quiz 3

1. You are observing with a 100m baseline interferometer at a wavelength of 1 micron. Due to unknown causes, the optical paths in the interferometer are unmatched by 5 microns. By how much are the maximum and minimum outputs of the interferometer altered if the fractional bandwidth of the light being collected is:
   a) 1 percent?
   b) 5 percent?
   c) 10 percent?

- What are the coherence lengths of light passed by the standard J, H and K band near-infrared filters? *(Only approximate answers are needed.)*
- Assume you are observing with a 100m baseline interferometer at a wavelength of 1 micron with a 10 percent fractional bandwidth. If you are looking at a target at an elevation of 45°, how far away can another target be such that the geometric delay for the secondary target is no more than a coherence length different from that of the primary target?

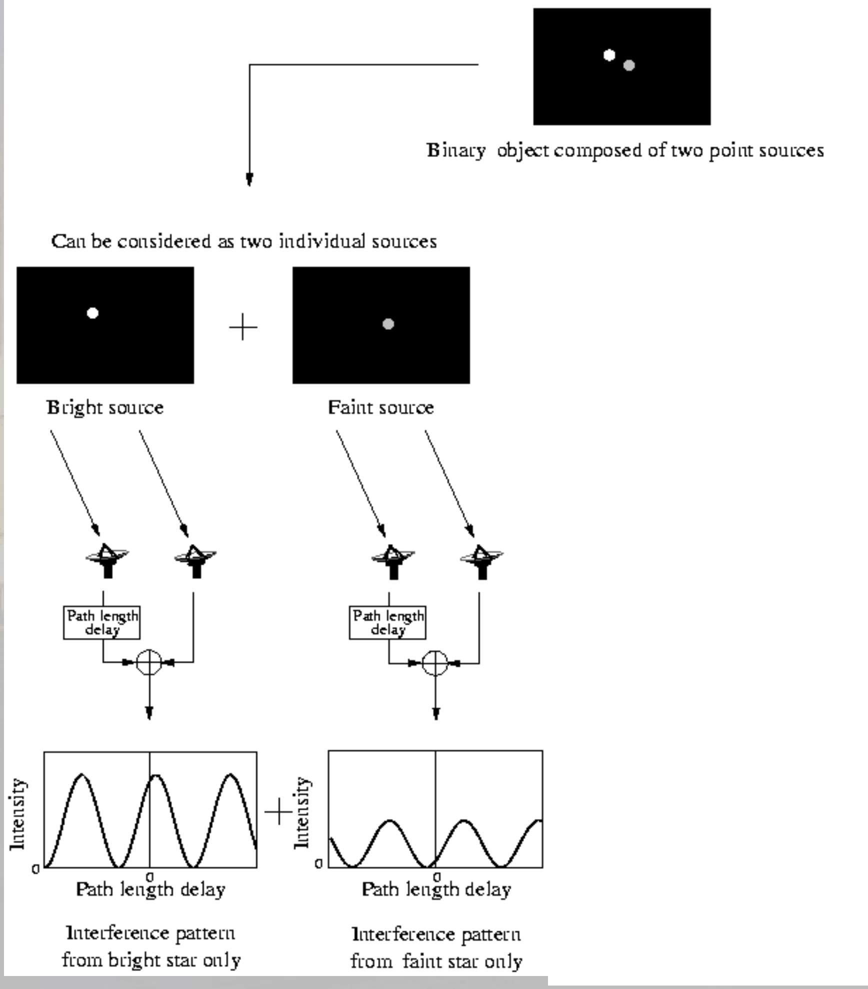
  What does this tell you about the maximum field of view of an interferometer given its baseline and its fractional bandwidth?

Timeout

- We measure an intensity.
- It usually varies co-sinusoidally.
- The modulation encodes information about the Fourier spectrum of the target.

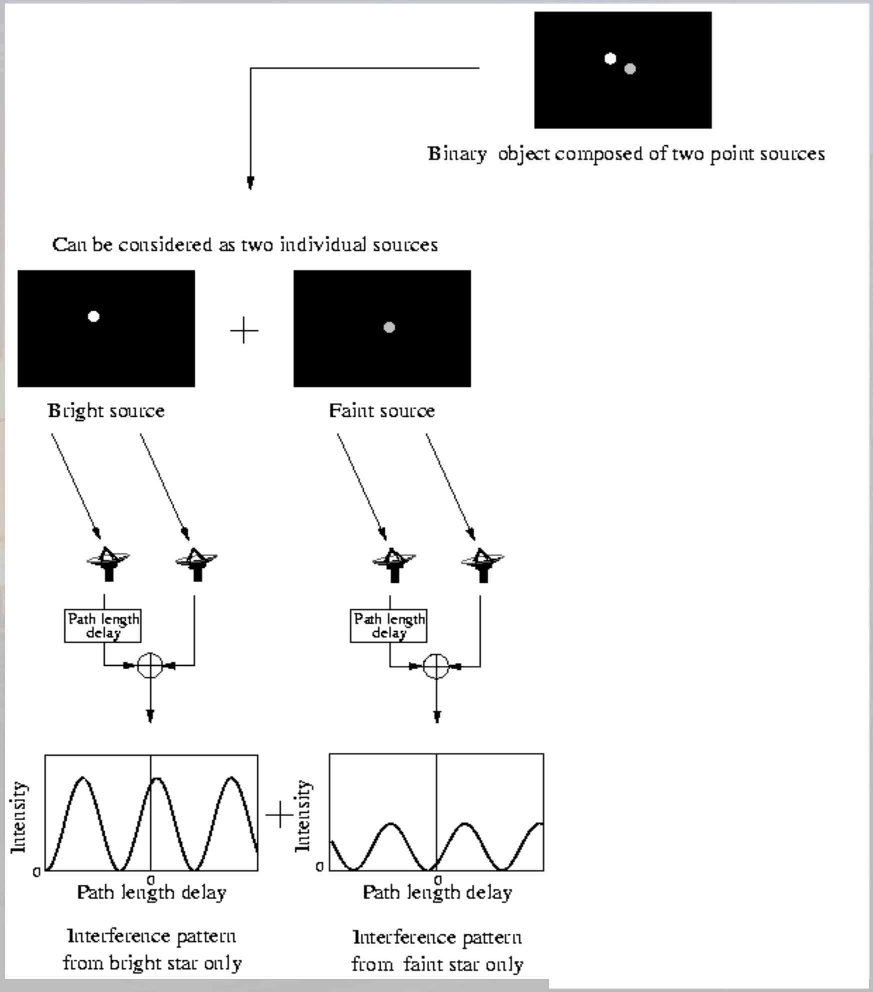
Exactly how does this work?

Heuristic operation of an interferometer



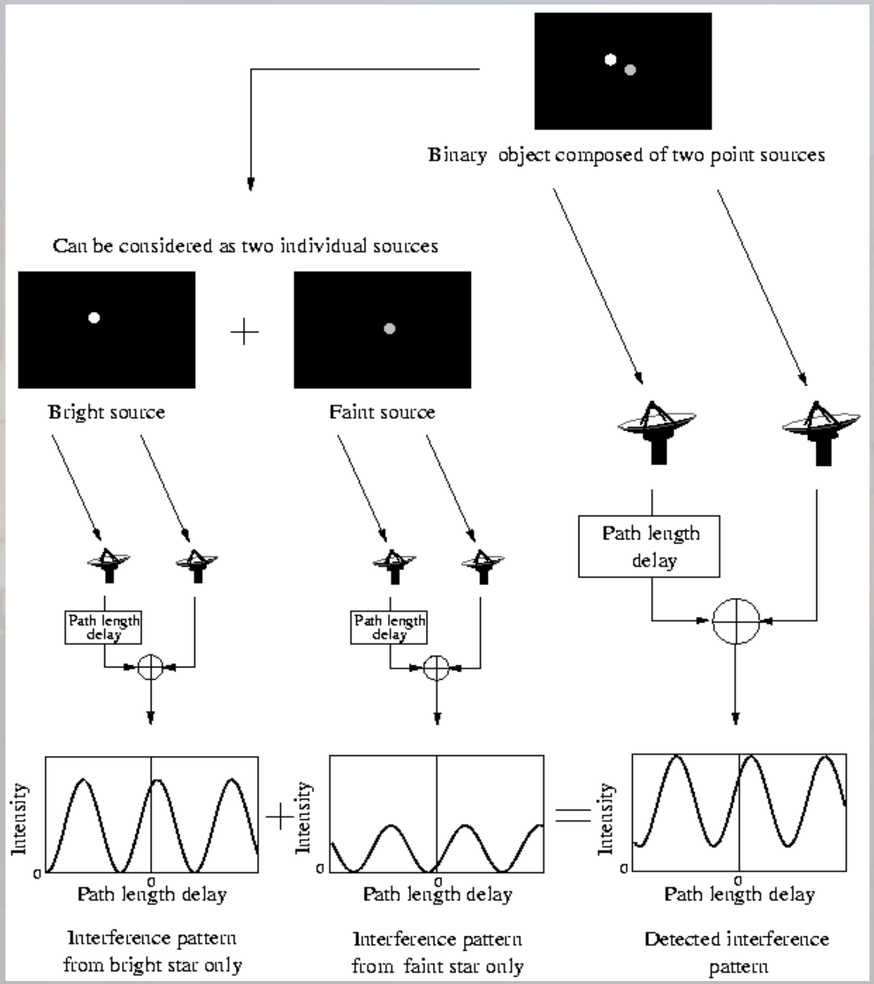
- Each unresolved element of the source produces its own fringe pattern.
- These have unit visibilities and phases that are associated with the location of that source element in the sky:
  - This is the basis for astrometric measurements with interferometers.

Heuristic operation of an interferometer



- The observed fringe pattern from a distributed source is just the intensity superposition of these individual fringe pattern.
- This relies upon the individual elements of the source being "spatially incoherent".

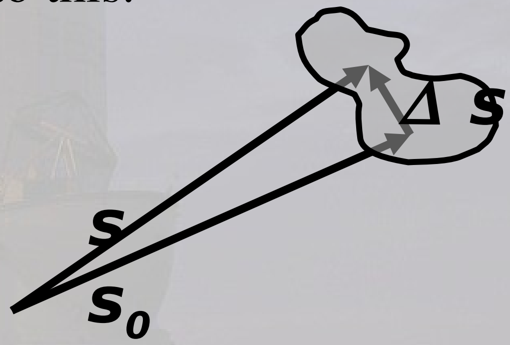
Heuristic operation of an interferometer



- The resulting fringe pattern has a contrast that is reduced with respect to that from each source individually.
- The positions of the sources are encoded (in a scrambled manner) in the resulting fringe phase.

A formal relationship between I_{meas} and I_{sky}

- Consider looking at an incoherent source whose brightness on the sky is described by $I(\hat{s})$. This can be written as $I(\hat{s}_0+\Delta s)$, where $\hat{s}_0$ is a vector in the pointing direction, and Δs is a vector perpendicular to this.



- The detected power will be given by:

$$
\begin{aligned}
P(s_0, B) &\propto \int I(s)\left[1+\cos kD\right] d\Omega \\
&\propto \int I(s)\left[1+\cos k(s.B+d_1-d_2)\right] d\Omega \\
&\propto \int I(s)\left[1+\cos k([s_0+\Delta s].B+d_1-d_2)\right] d\Omega \\
&\propto \int I(s)\left[1+\cos k(s_0.B+\Delta s.B+d_1-d_2)\right] d\Omega \\
&\propto \int I(\Delta s)\left[1+\cos k(\Delta s.B)\right] d\Omega'
\end{aligned}
$$

Heading towards the van Cittert-Zernike theorem

- Consider now adding a small path delay, δ , to one arm of the interferometer. The detected power will become:

$$P(s_0, B, \delta) \propto \int I(\Delta s)\left[1+\cos k(\Delta s.B+\delta)\right] d\Omega'$$

$$\propto \int I(\Delta s)\, d\Omega' + \cos k\delta \cdot \int I(\Delta s) \cos k(\Delta s.B)\, d\Omega'$$

$$- \sin k\delta \cdot \int I(\Delta s) \sin k(\Delta s.B)\, d\Omega'$$

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- We now define something called the complex visibility V(k,B):

$$V(k, B) = \int I(\Delta s) \exp[-ik\Delta s.B]\, d\Omega',$$

so that we can simplify the equation above to:

$$P(s_0, B, \delta) \propto \int I(\Delta s)\ \ d\Omega' + \cos k\delta\, \mathrm{Re}[V] + \sin k\delta\, \mathrm{Im}[V]$$

$$P(s_0, B, \delta)\ =\ I_{total} + \mathrm{Re}\left[V \exp[-ik\delta]\right]$$

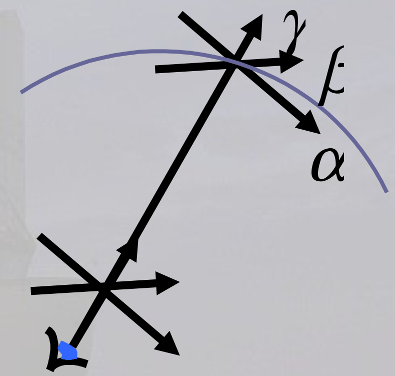
What is this V that we have introduced?

- Lets assume $\hat{s}_0 = (0,0,1)$ and Δs is $\approx (\alpha, \beta, 0)$, with α and β small angles measured in radians.

$$V(k,B) = \int I(\Delta s) \exp[-ik\Delta s.B]\, d\Omega'$$

$$= \int I(\alpha,\beta) \exp[-ik(\alpha B_x + \beta B_y)]\, d\alpha\, d\beta$$

$$= \int I(\alpha,\beta) \exp[-i2\pi(\alpha u + \beta v)]\, d\alpha\, d\beta$$



So, the complex quantity V we introduced is the Fourier Transform of the source brightness distribution.

- *u* (= B_x/λ) and *v* (= B_y/λ) are the projections of the baseline onto a plane perpendicular to the pointing direction.
- *u* and *v* are the spatial frequencies associated with the physical baselines, with units of rad^{-1}:
  - These are the same *u* and *v* we introduced when we took the Fourier transform of the incoherent imaging equation.

How this all fits together

- We can put this all together as follows:
  - Our interferometer measures: $P(s_0, B, \delta) = I_{total} + \mathrm{Re}\left[V \exp[-ik\delta]\right]$
  - So, if we make measurements with, say, two values of δ (= 0 and λ /4), this recovers the real and imaginary parts of the complex visibility, V.
  - Since the complex visibility is nothing more than the Fourier transform of the brightness distribution, we have our final results:

The output of an interferometer "measures" the Fourier transform of the source brightness distribution.

The measurement is at a single spatial frequency, dependent on B.

If B is large the interferometer probes small scale structures, if short, it probes larger structures.

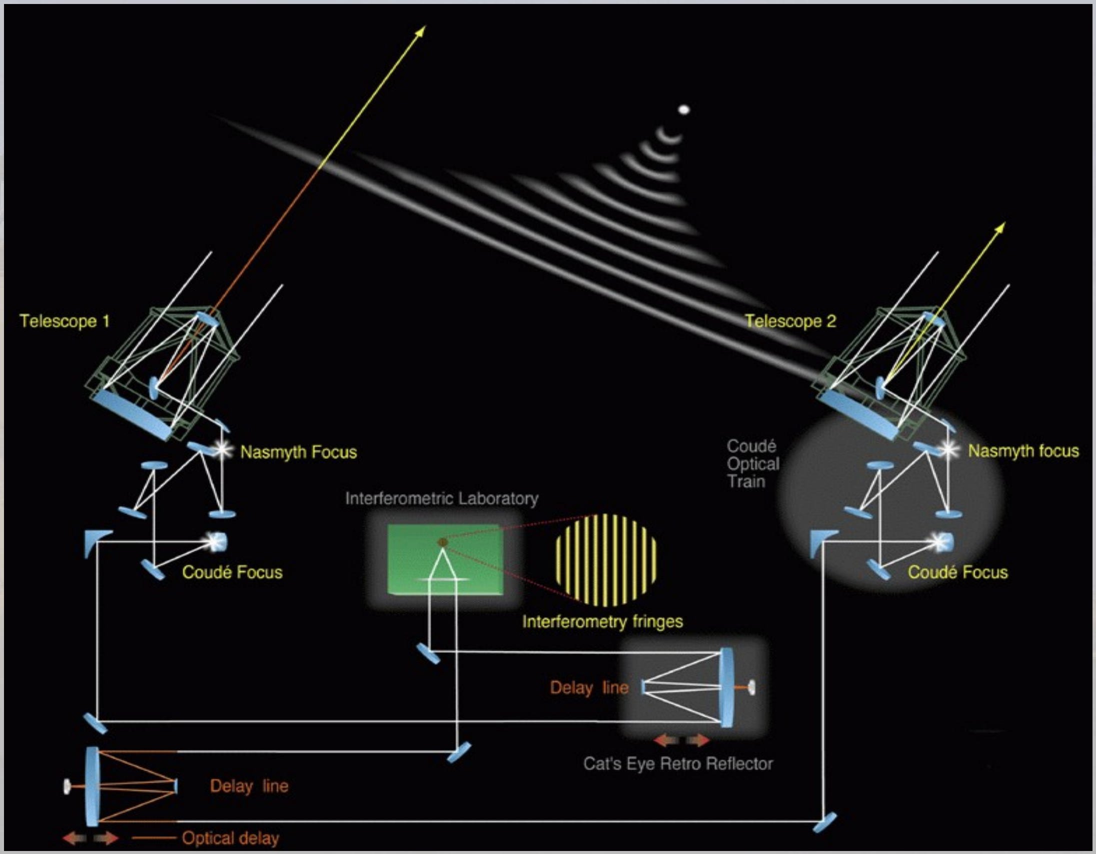
Questions?

Quiz 4

- Check that you understand how introducing two values of δ (= 0 and $\lambda/4$), allows one to recover the Real and Imaginary parts of the complex visibility.
- What happens to the interferometer output, $P(s_0, B, \delta)$ if δ is changed smoothly from zero to 2λ? Sketch it. *(You will need to assume values for I_{total} and V – let I_{total} equal 1.0 and let V (don't forget it is complex) have an amplitude of 0.5 and a phase of 45°.)*
- Assume you are using an interferometer with a baseline of 100m, oriented in an E-W direction at a latitude of 45°, and observing a target at declination 15°. If the source is observed at ± 3 hours around transit, what values do *u* and *v* (the projected baseline components) take? Sketch them on a plot of the *uv* plane.

Are we heading in the correct direction?

- Telescopes sample the fields at r_1 and r_2.
- Optical train delivers the radiation to a lab.
- Delay lines assure that we measure when $t_1=t_2$.
- The instruments mix the beams and detect the fringes.



- We measure the fringes.
- We interpret the measurements of the Fourier spectrum of the target.

“Science” with interferometers

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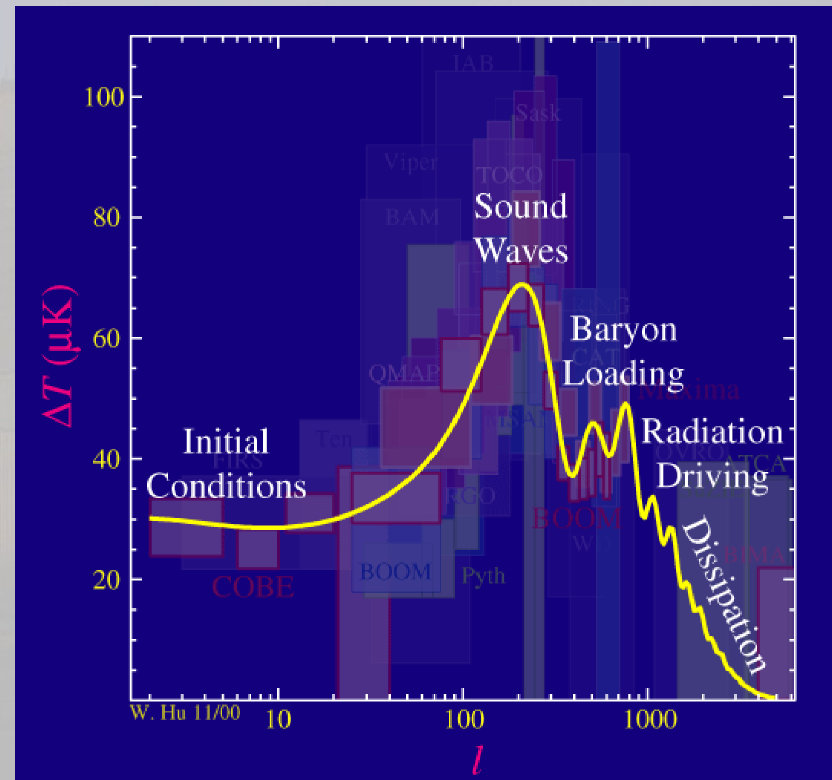
Planning interferometric science

What types of questions are pertinent here?

- Which bits of the Fourier spectrum of the target, *V(u,v)*, are the ones you wish to measure?
- How easy will it be to measure these parts?
- How easy will it be for you to interpret these measurements?
- What do we do with the measurements of *V(u, v)* if we wish to make a map of the sky?
- How faint can you go?



Where are the science prizes?

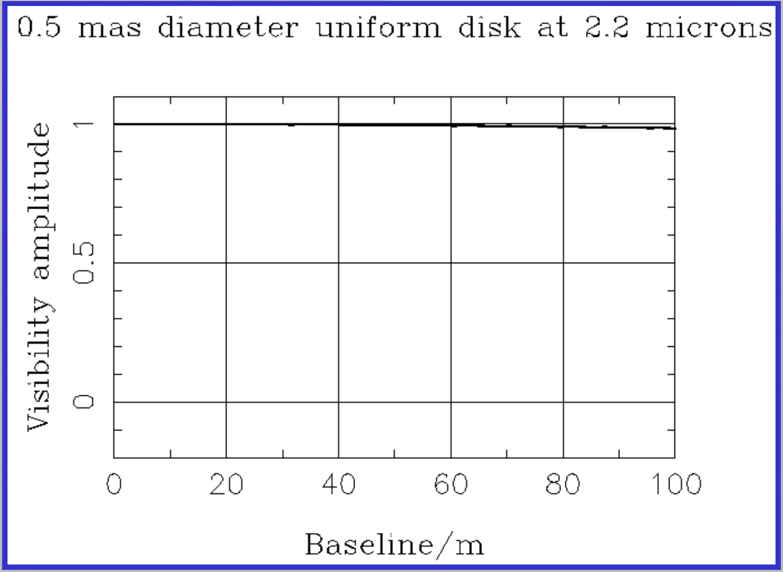
Visibility functions of simple 1-d sources (i)

$$V(u) \propto \int I(l)\, e^{-i2\pi (ul)}\, dl.$$

Point source of strength A_1 and located at angle l_1 relative to the optical axis.

$$V(u) = \int A_1 \delta(l-l_1)\, e^{-i2\pi (ul)}\, dl \div \text{total flux}$$
$$= e^{-i2\pi (ul_1)}.$$

- The visibility amplitude is unity $\forall\, u$.
- The visibility phase varies linearly with u ($= B/\lambda$).
- Sources such as this are easy to observe since the interferometer output gives fringes with high contrast.

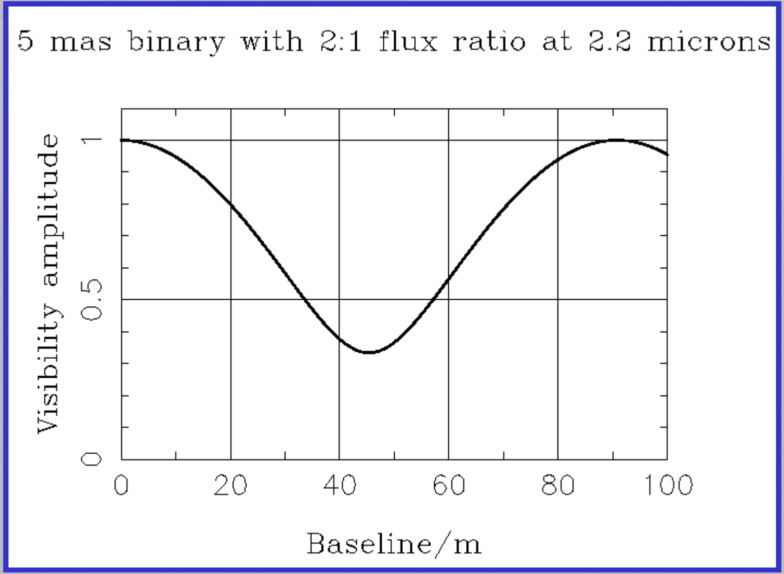


Visibility functions of simple sources (ii)

A double source comprising point sources of strength A_1 and A_2 located at angles 0 and l_2 relative to the optical axis.

$$V(u) = \int [A_1\delta(l) + A_2\delta(l-l_2)]\, e^{-i2\pi(ul)}\, dl \div \text{total flux}$$
$$= \propto [A_1 + A_2 e^{-i2\pi(ul_2)}] .$$

- The visibility amplitude and phase oscillate as functions of u.
- To identify this as a binary, baselines from $0 \rightarrow \lambda/l_2$ are required.
- If the ratio of fluxes is large the modulation of the visibility becomes difficult to measure, i.e. the contrast of the interferometric fringes is similar for all baselines.



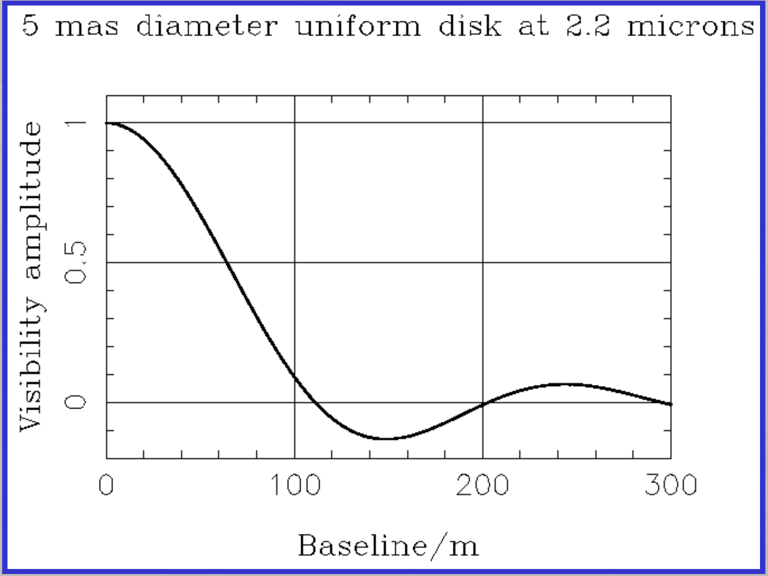
Visibility functions of simple sources (iii)

A uniform on-axis disc source of diameter θ .

$$V(u_r) \propto \int^{\theta/2} \rho \; J_0(2\pi \rho u_r)\, d\rho$$

$$= 2J_1(\pi \theta u_r) \div (\pi \theta u_r) .$$

- The visibility amplitude falls rapidly as u_r increases.



- To identify this as a disc requires base-lines from $0 \rightarrow \lambda/\theta$ at least.
- Information on scales smaller than the disc correspond to values of u_r where $V \ll 1$, and is difficult to measure. This is because the interferometer output gives fringes with very low contrast.

How does this help planning observations?

- Compact sources have visibility functions that remain high whatever the baseline, and produce high contrast fringes all the time.
- Resolved sources have visibility functions that fall to low values at long baselines, giving fringes with very low contrast.

$\Rightarrow$ Fringe parameters for fully resolved sources will be difficult to measure.

- To usefully constrain a source, the visibility function must be measured adequately. Measurements on a single, or small number of, baselines may not be enough for unambiguous interpretation.

- Imaging – which necessarily requires information on both small and large scale features in a target, will generally need measurements where the fringe contrast is both high and low.

Questions?

Quiz 5

1. Plot the visibility amplitude and phase for a 5mas separation binary (see slide 42) as a function of projected baseline length up to 100m assuming you are taking observations at 1 micron and that the binary has:
   a) a flux ratio of 2:1,
   b) a flux ratio of 100:1,
   c) a flux ratio of 1000:1.

   (Remember, $V = V(u)$ and $u = B/\lambda$).
2. Assuming observations at 1 micron, what baseline lengths are needed to resolve main sequence stars of spectral types O, A, G, and M, if their apparent magnitudes are:
   a) 0,
   b) 6,
   c) 12.

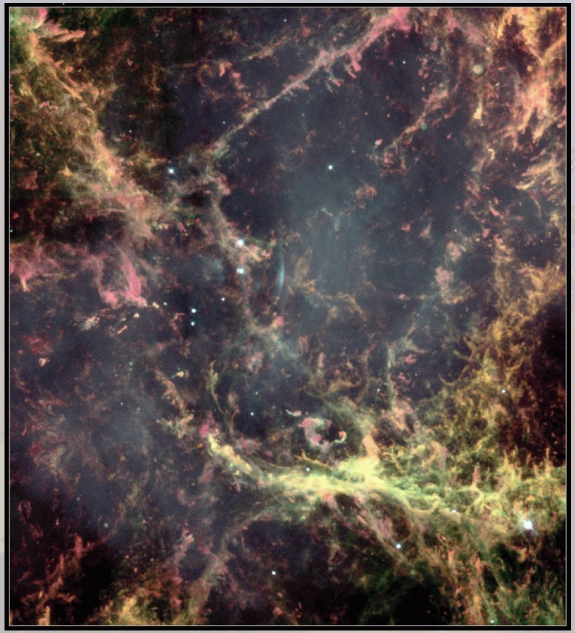
   What do these results tell you about observing faint targets with an optical interferometer?

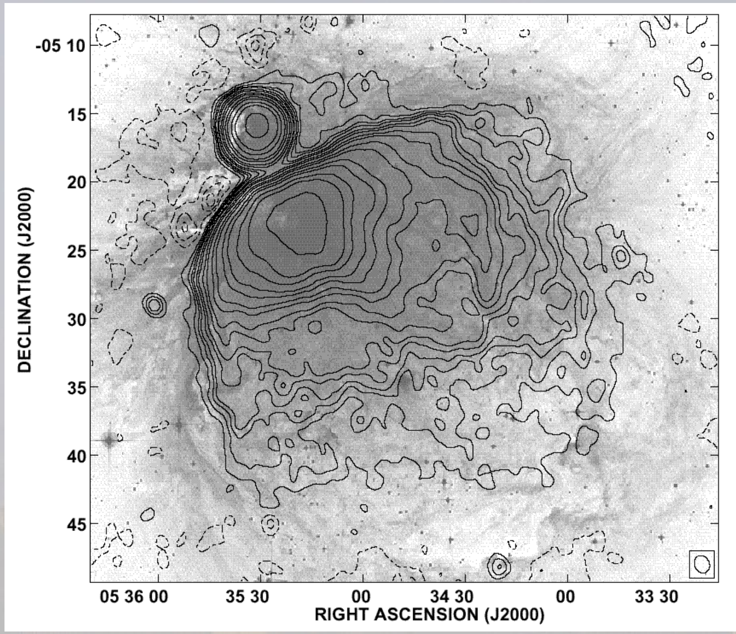
   (You will need to look up the absolute magnitudes and physical radii of these stars in a textbook.)

Some final thoughts and warnings

- What are the real prospects for imaging AGN with interferometers?
- Aren’t interferometers only good for “bright-source” astrophysics?
- Other mistakes and misunderstandings.

Making maps with interferometers – the evidence





- Optical HST (left) and 330Mhz VLA (right) images of the Crab Nebula and the Orion nebula. Note the differences in the:
  - Range of spatial scales in each image.
  - The range of intensities in each image.
  - The field of view as measured in resolution elements.

Making maps with interferometers – the “rules”

- The number of visibility data ≥ number of filled pixels in the recovered image:
  - $N_{tel}(N_{tel}-1)/2 \times$ number of reconfigurations ≥ number of filled pixels.
- The distribution of samples in the Fourier plane should be as uniform as possible:
  - To aid deconvolution of the interferometric PSF.
- The range of interferometer baselines, i.e. B_{max}/B_{min}, will govern the range of spatial scales in the map.
- There is no need to sample the visibility function too finely in the Fourier plane:
  - For a source of maximum extent θ_{max}, sampling very much finer than $\Delta u \sim 1/\theta_{max}$ is unnecessary.

How maps are actually recovered

- The measurements of the visibility function are secured and calibrated.
- These can be represented as a sampled version of the Fourier transform of the sky brightness:

$$V_{meas}(u, v) = V_{true}(u, v) \times S(u, v).$$

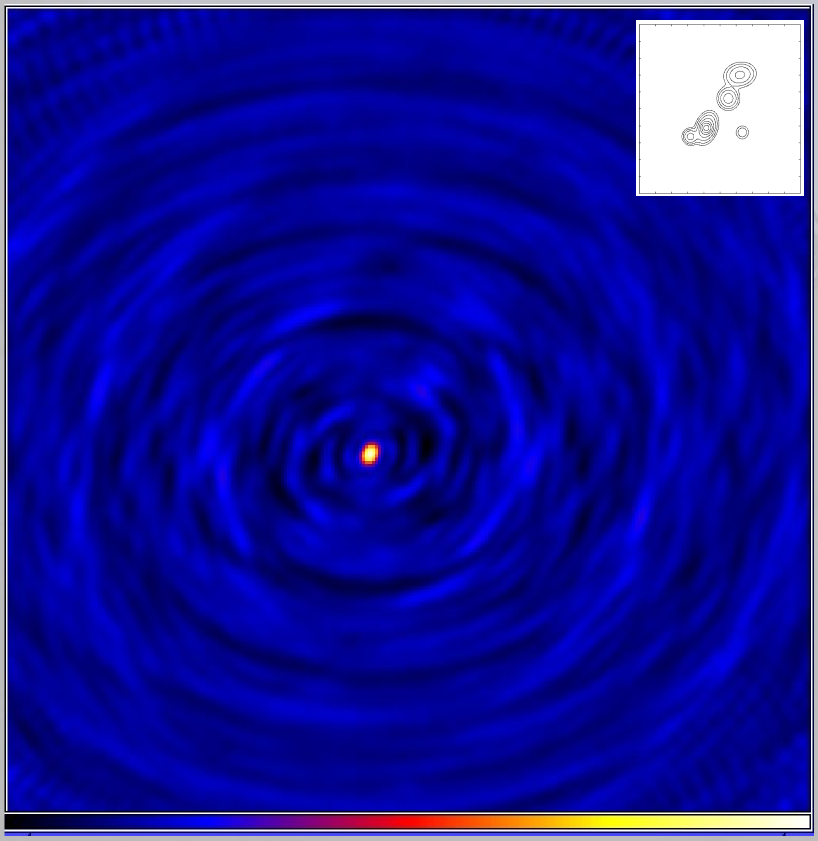
- These data are inverse Fourier transformed to give a representation of the sky, similar to that of a normal telescope, albeit with a strange PSF:

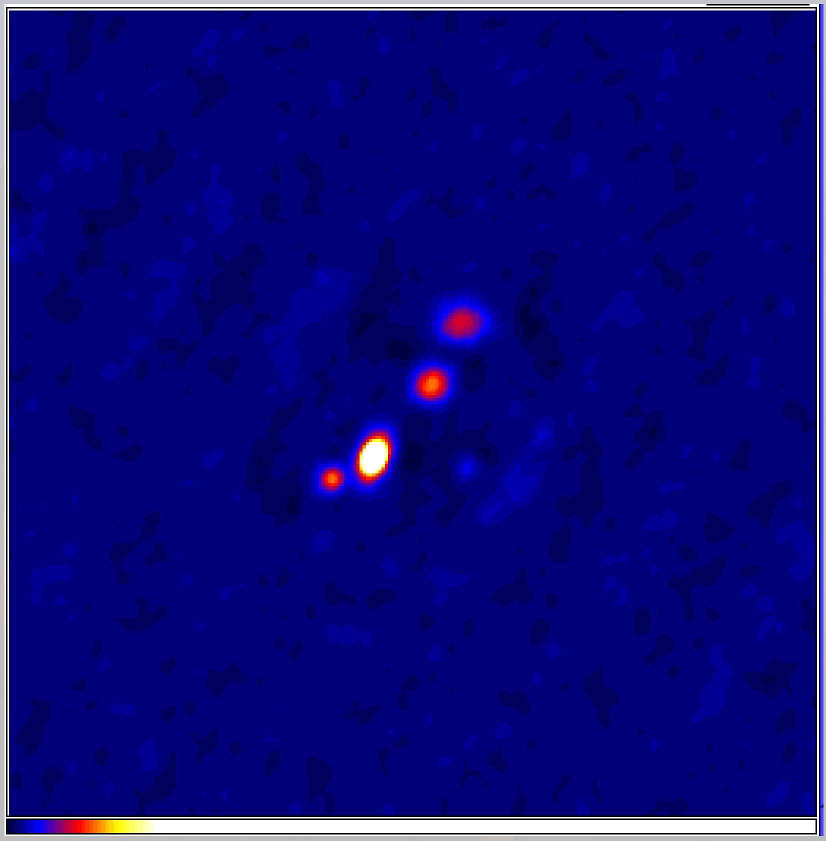
$$\int\int S(u, v)\, V_{meas}(u, v)\, \mathrm{e}^{+i2\pi\,(ul + vm)}\, \mathrm{d}u\, \mathrm{d}v = \mathrm{I}_{norm}(\mathrm{l}, \mathrm{m}) * \mathrm{B}_{dirty}(\mathrm{l}, \mathrm{m}),$$

  where $\mathrm{B}_{dirty}(\mathrm{l},\mathrm{m})$ is the Fourier transform of the sampling distribution, or the so-called dirty-beam, and the image is usually referred to as the “dirty” map.

- While the interferometer PSF is generally far less attractive than an Airy pattern, it’s shape is completely determined by the samples of the visibility function that are measured.

The effect of the sampling distribution





- Interferometric PSF's can often be horrible. Correcting an interferometric map for the Fourier plane sampling function is known as deconvolution and is broadly speaking straightforward.

Other useful interferometric rules of thumb

- The FOV will depend upon:
  - The field of view of the individual collectors. This is often referred to as the primary beam.
  - The FOV seen by the detectors. This is limited by vignetting along the optical train.
  - The spectral resolution. The interference condition OPD < $\lambda^2/\Delta\lambda$ must be satisfied for all field angles. Generally ⇒ FOV $\leq [\lambda/B][\lambda/\Delta\lambda]$.

- Dynamic range:
  - The ratio of maximum intensity to the weakest believable intensity in the image.
  - $> 10^5$:1 is achievable in the very best radio images, but of order several × 100:1 is more usual.
  - DR $\sim [S/N]_{per\text{-}datum} \times [N_{data}]^{1/2}$

- Fidelity:
  - Difficult to quantify, but clearly dependent on the completeness of the Fourier plane sampling.

What does sensitivity depend on?

A: Whether a guide star is present.

B: Whether a bright enough guide star is present.

C: Whether a bright enough guide star is close enough.

D: How long the AO system stays locked for.

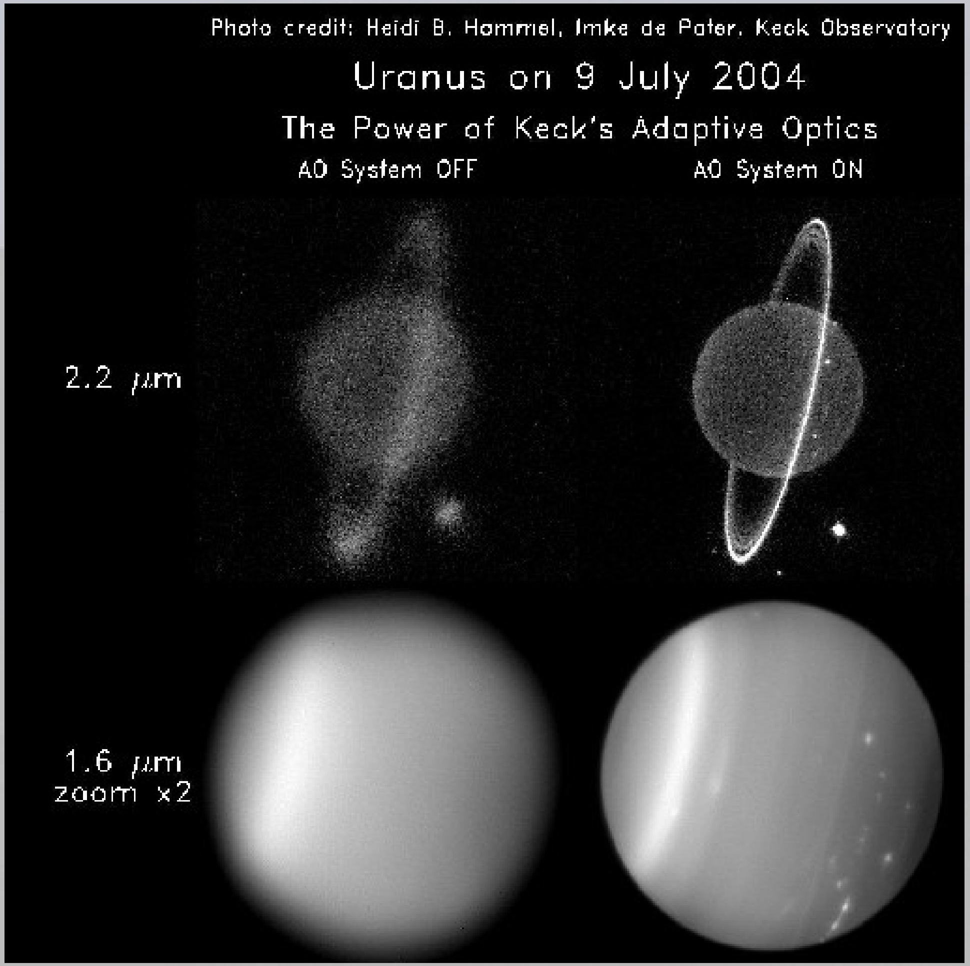
E: How large my telescope is.

F: How sensitive my detector is.

G: How long an exposure time I can sustain.

H: All of the above.

I: None of the above.



How do we assess interferometric sensitivity?

- The "source" has to be bright enough to:
  - Allow stabilisation of the interferometer against any atmospheric fluctuations.
  - Allow a reasonable signal-to-noise for the fringe parameters to be build up over some total convenient integration time. This will be measured in minutes.
- Once this achieved, the faintest features one will be able to interpret reliably will be governed by S/N ratio and number of visibility data measured.
- In most cases the sensitivity of an interferometer will scale like some power of the measured fringe contrast × another power of the number of photons detected while the fringe is being measured.

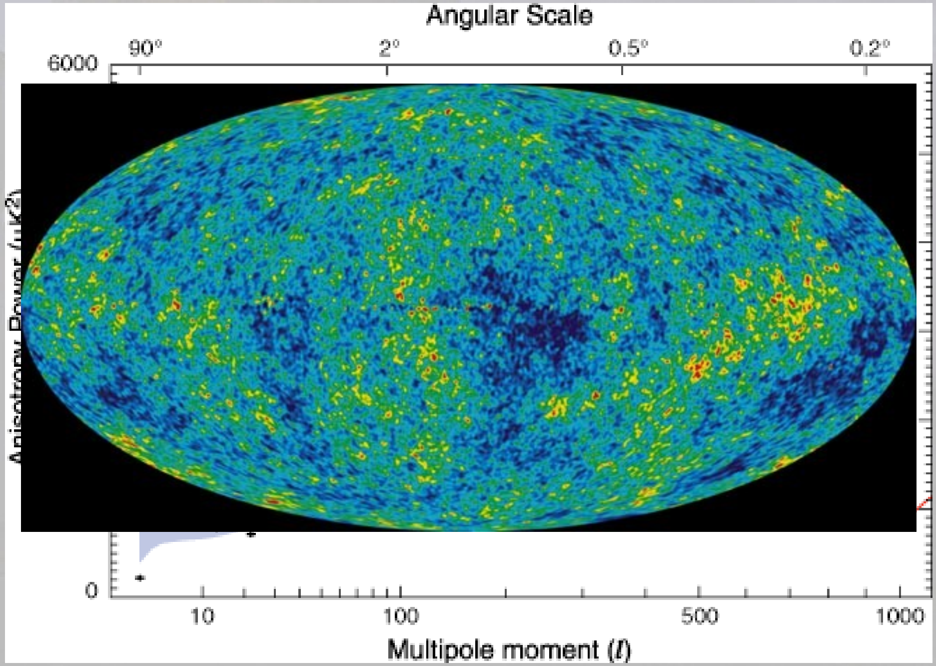
This highlights fringe contrast <u>and</u> throughput as both being critical.
This also highlights the difficulty of measuring resolved targets where V is low.

Quiz 6

- Why might the total integration time for an measurement of the visibility function with an interferometer be limited to a few minutes? *(You may wish to consider the rotation of the Earth and the how much u and v have to change so that you are measuring an independent value of the Fourier transform of the target.)*
- What are the typical spatial and temporal scales associated with the atmosphere for observations made at 1 micron? How do these compare to the equivalent scales at centimetric radio wavelengths (e.g. at the VLA site in New Mexico) and at sub-mm wavelengths (e.g. at the ALMA site in Chile). *(You will need to bug the lecturers for this information or do some literature sleuthing to find the answers.)*
- If the S/N for some interferometric observation of an unresolved target (V=1) is X, how much brighter (in magnitudes) must a target be to be measured with the same S/N if it is sufficiently resolved that its visibility function, V, is equal to 0.1? *(Assume that you are working in the regime where the S/N scales like $V\times\sqrt{(flux)}$.)*

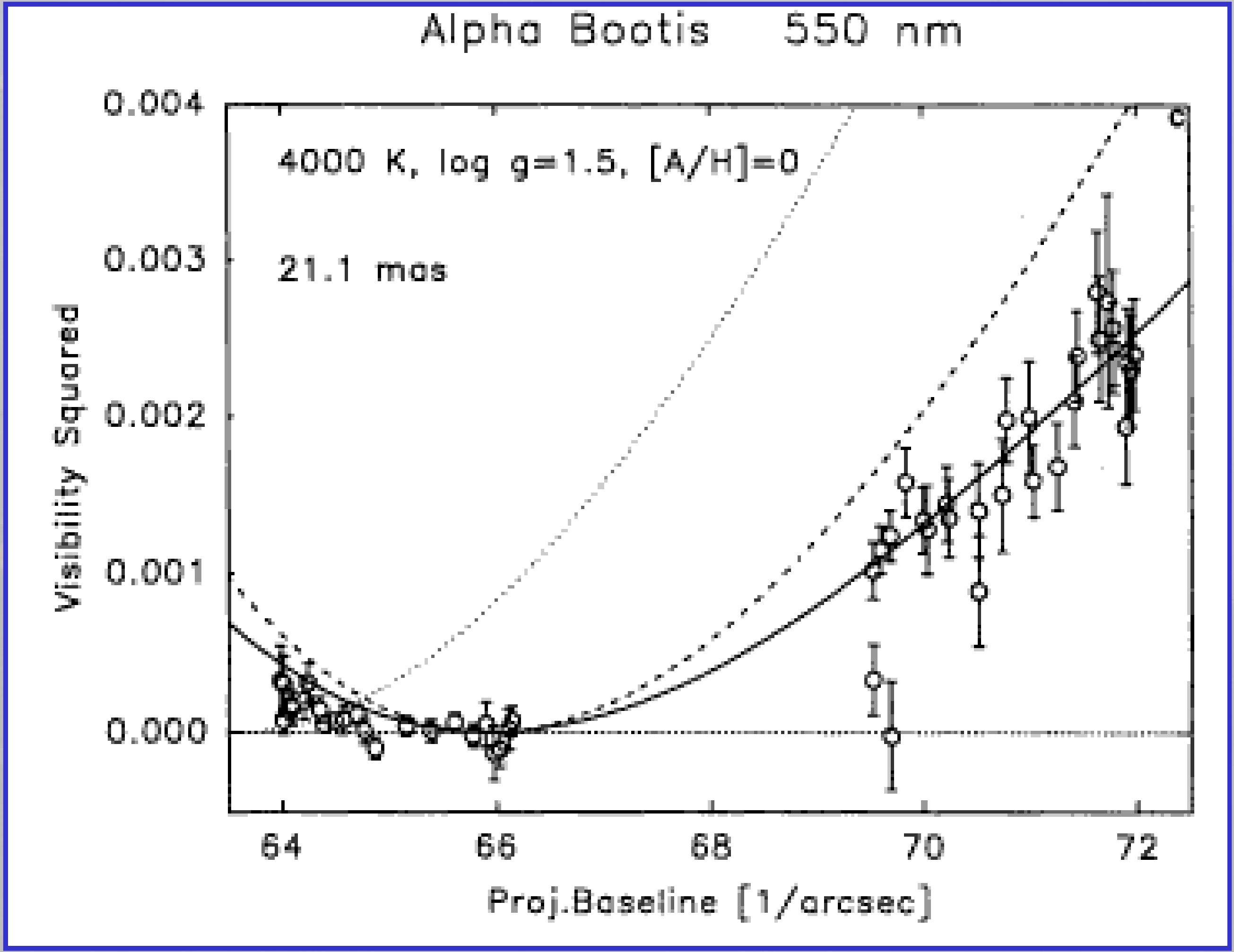
Closing thoughts

- Once we have measured the visibility function of the source, we have to interpret these data. This can take many forms:
  - Small amount of Fourier data:
    - Model-fitting.
  - Moderate amount of Fourier data:
    - Model-fitting.
    - Rudimentary imaging.
  - Large amount of Fourier data:
    - Model-fitting.
    - Model-independent imaging.

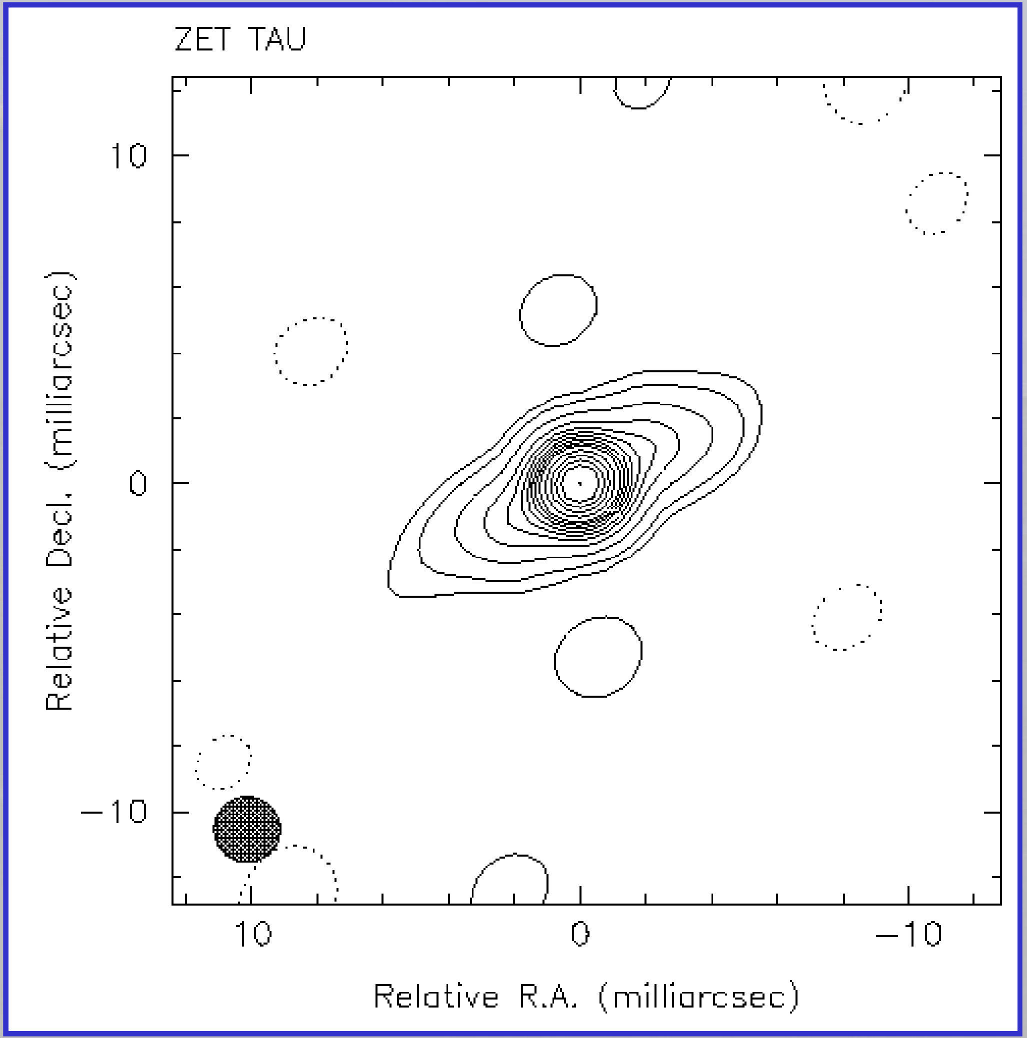


- Don’t forget that making an image is not a requirement for doing good science:

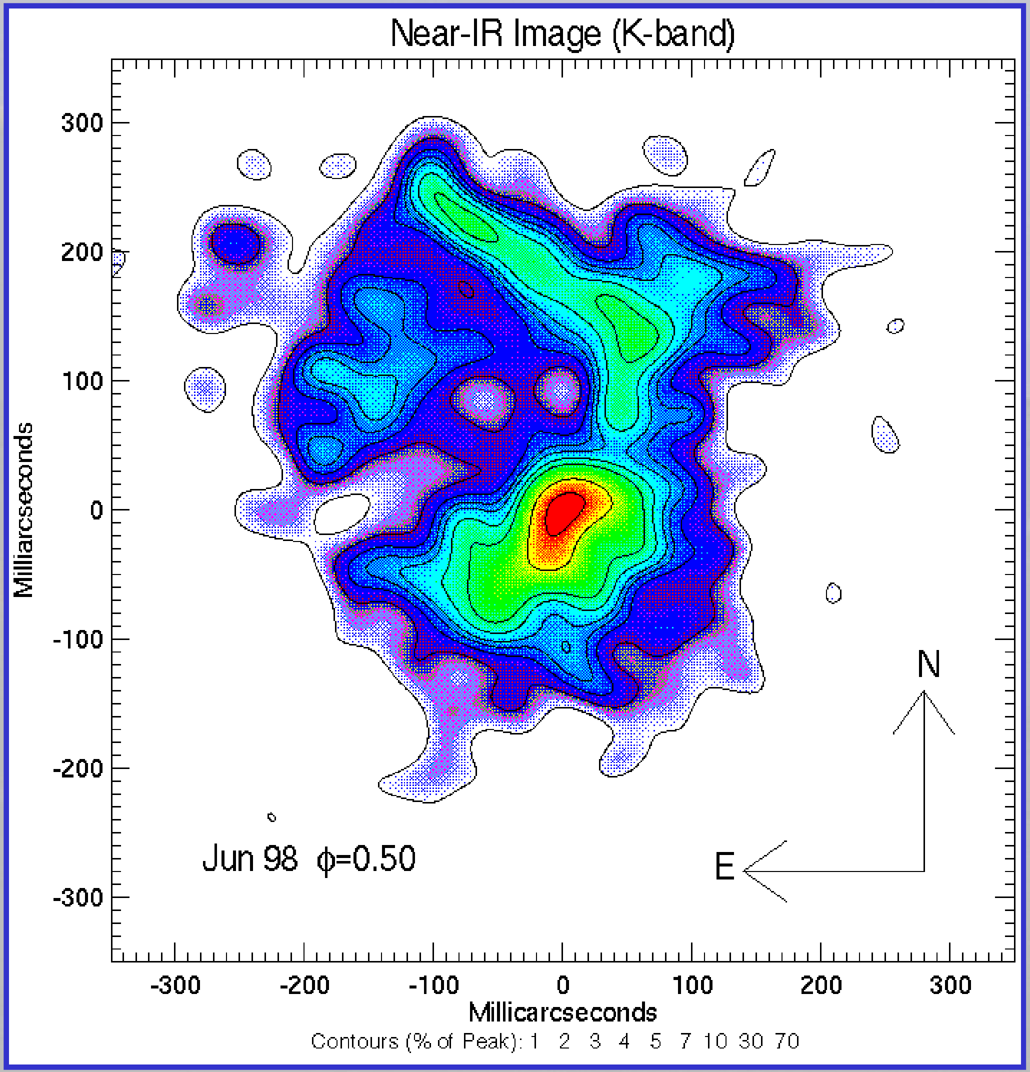
Interferometric science – 2 telescopes



Interferometric science – 5 telescopes



Interferometric science – 21 telescopes



Key lessons to take away

- Alternative methods for describing images:
  - Fourier decomposition, spatial frequencies, physical baselines.

- Interferometric measurements:
  - Interferometer make fringes.
  - The fringe amplitude and phase are what is important.
  - More precisely, these measure the FT of the sky brightness distribution.
  - A measurement with a given interferometer baseline measures a single Fourier component (usually the square of V and its phase are measured).

- Science with interferometers:
  - Multiple baselines are obligatory for studying a source reliably.
  - Resolved targets produce fringes with low contrast – these are difficult to measure well.
  - Once a number of visibilities have been measured, reliable interpretation can take multiple forms – making an image is only required if the source is complex.