Theory of interferometric data processing

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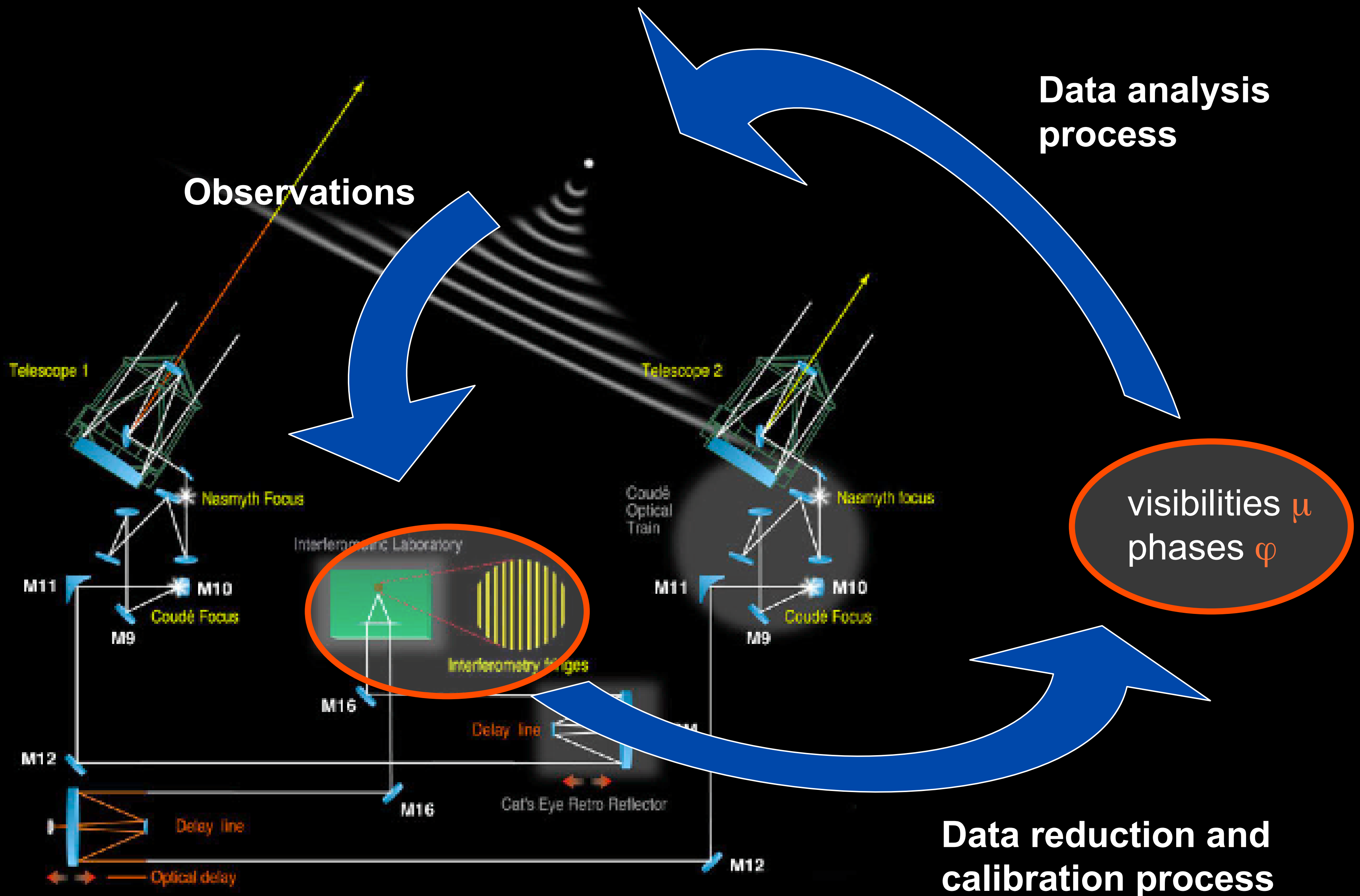
Context of this course



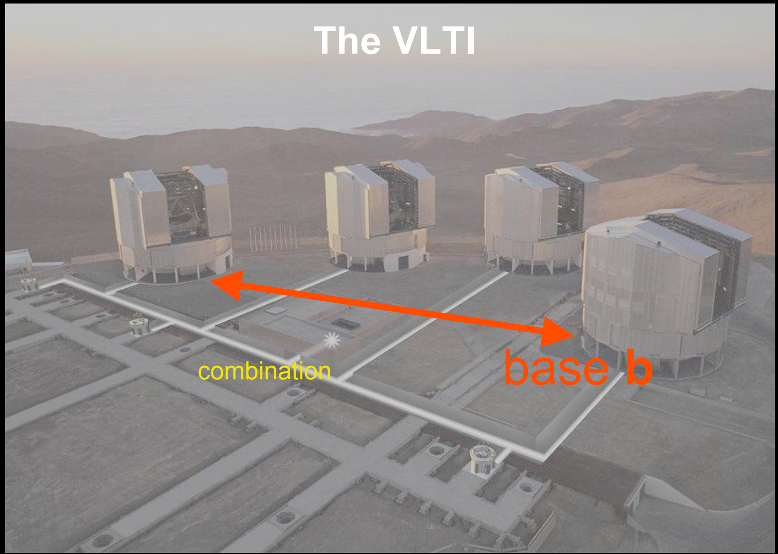
Outline

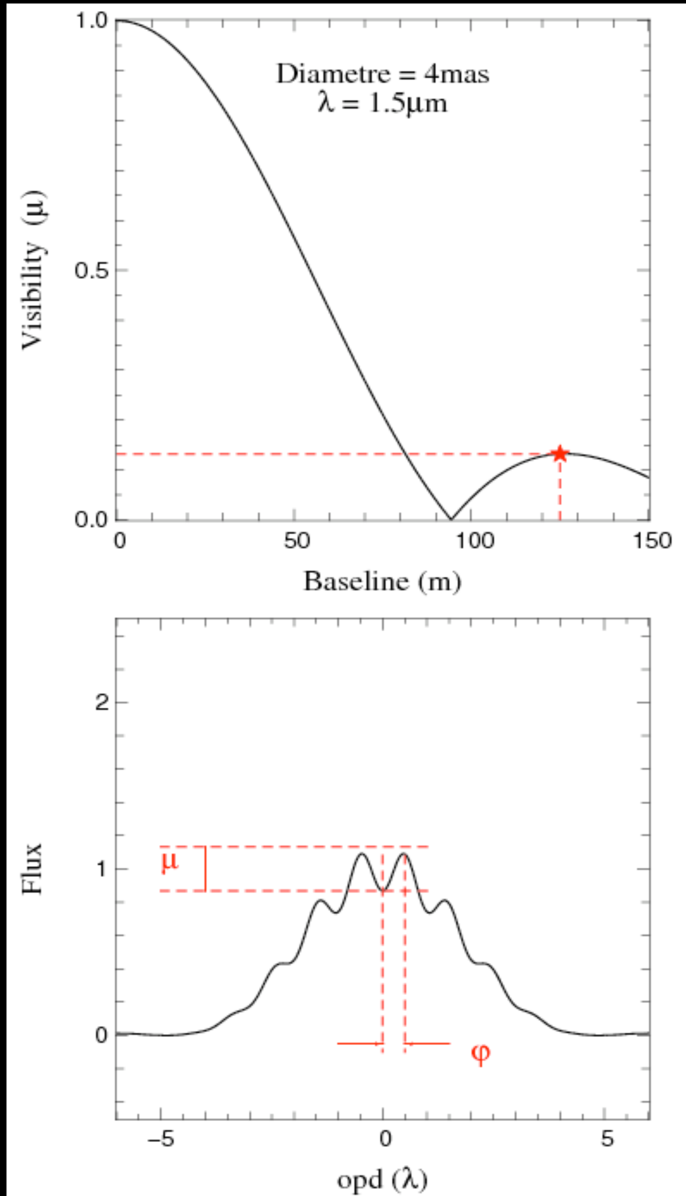
- What are we really looking for ?
- What are we fighting against ?
- Statistics of the observables
- Calibration and final errors estimate
- Data reduction of the AMBER instrument
- Data reduction of the MIDI instrument
- Conclusions

Outline

- What are we really looking for ?
  - Small recall of interferometric observables / observation
  - How do we practically form the fringes ?
  - A simple but unrealistic estimator
- What are we fighting against ?
- Statistics of the observables
- Calibration and final errors estimate
- Data reduction of the AMBER instrument
- Data reduction of the MIDI instrument
- Conclusions

Principle of interferometric observations





- Interferometric observables
  - visibility μ et phases φ
  - fonction of the target shape :

$$\mu\ e^{i\varphi} = TF\{\ objet\ \}\ (\mathbf{b}/\lambda)$$

Principle of interferometric data analysis

- Partially resolved
  - diam = 3 – 1 mas

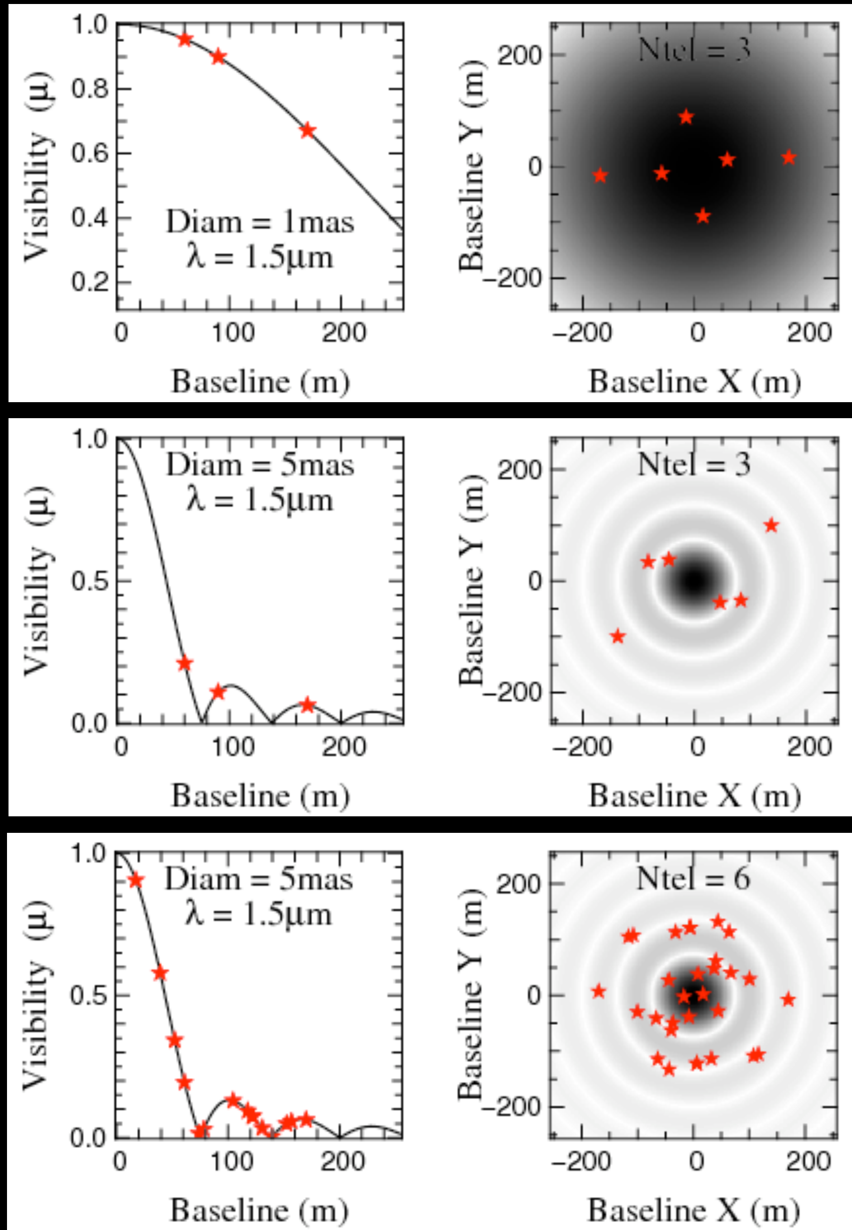
  → constraint the diameter

- Resolved
  - diam > 3 mas

  → parametric analyze of features (positions, amplitudes…)

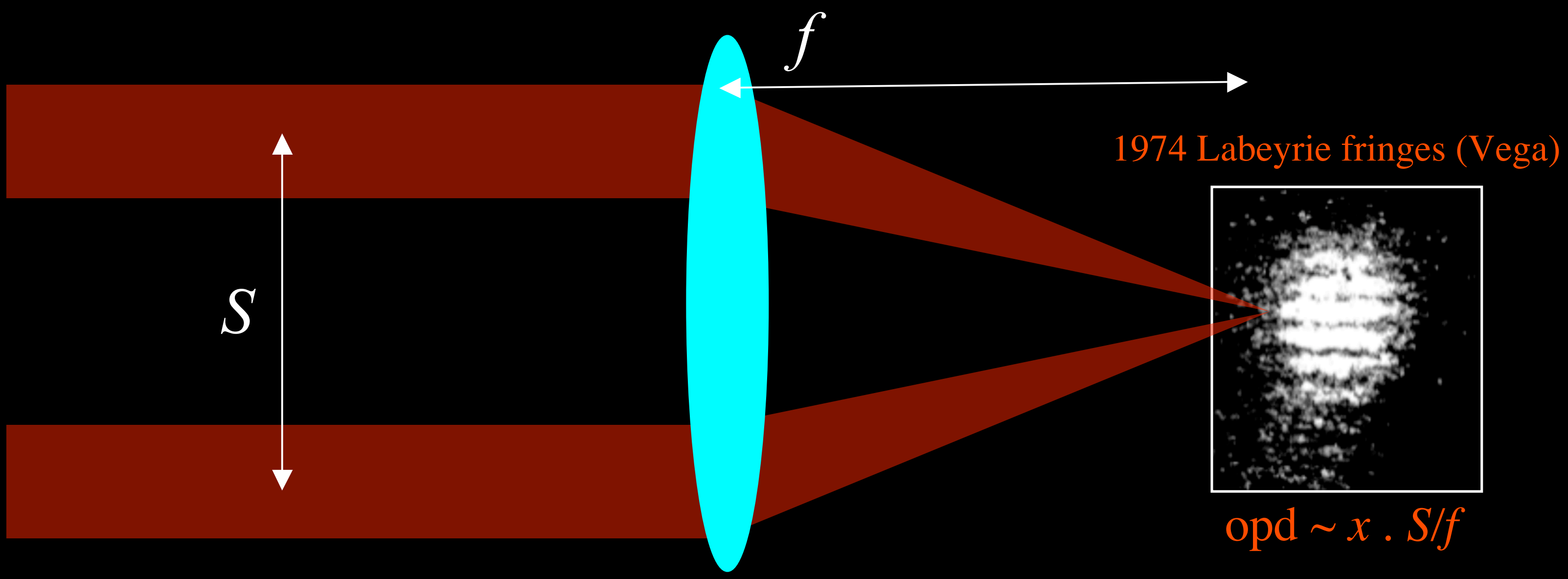
- Resolved and good uv-sampling
  - diam > 3 mas
  - a lot of telescopes/baselines

  → aperture synthesis imaging



Combination types: Spatial or Fizeau

- Overlap the beams with a lens:

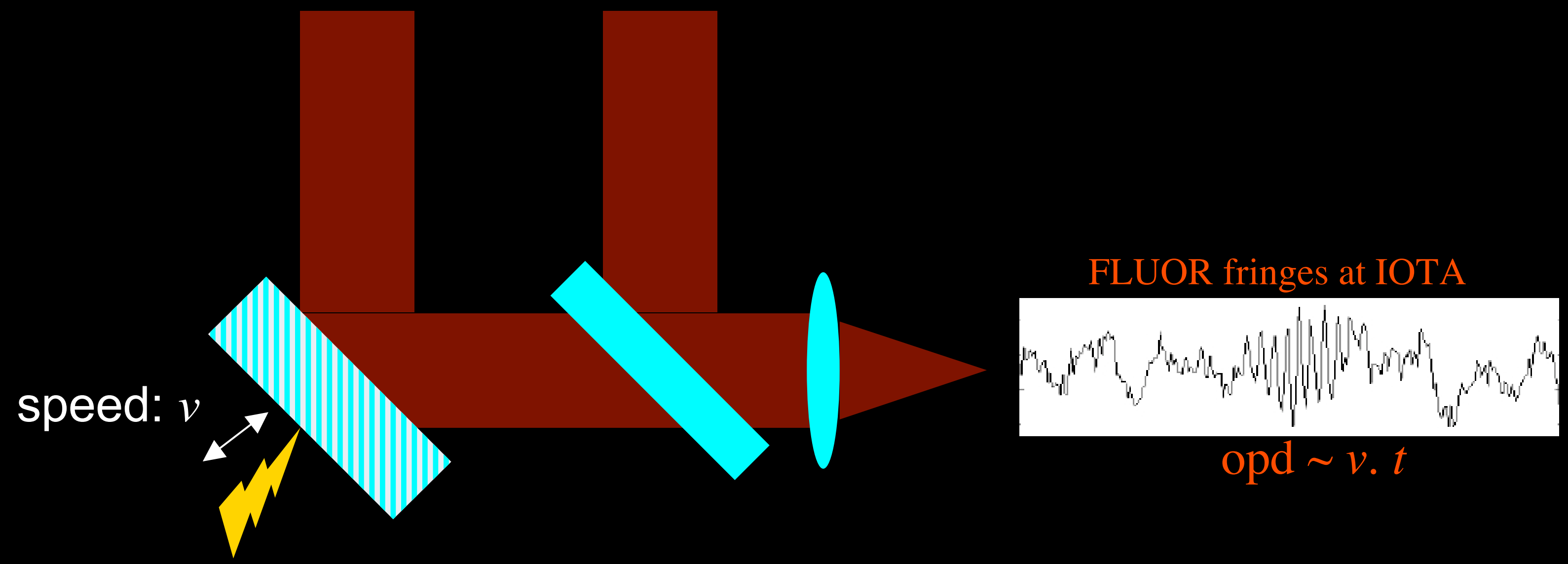


- The opd is spatially modulated:

$$I \sim \mu \,.\, \cos(2\pi\, S.f\,/\,\lambda \,.\, x + \varphi)$$

Combination types: Temporal or Michelson

- Overlap the beams with a beam-splitter:



- The OPD is modulated temporally:

$$I \sim \mu \,.\, \cos(2\pi\, v/\lambda \,.\, t \;+\; \varphi) \qquad\qquad I \sim \mu \,.\, \cos(2\pi\, opd/\lambda \;+\; \varphi)$$

Fringe size: order of magnitude…

$$I \sim \mu \,.\, \cos(2\pi \, opd/\lambda \; + \; \varphi)$$

- The fringe spacing is the wavelength of the light, so few µm in the near-IR
  - Precise instrumentation
  - Mechanical vibrations are “killers”

- When observing with a large spectral bandwidth, the fringe packet becomes small:
  - R=500 → Δ ~ 0.75mm
  - R=25 → Δ ~ 7.5 µm

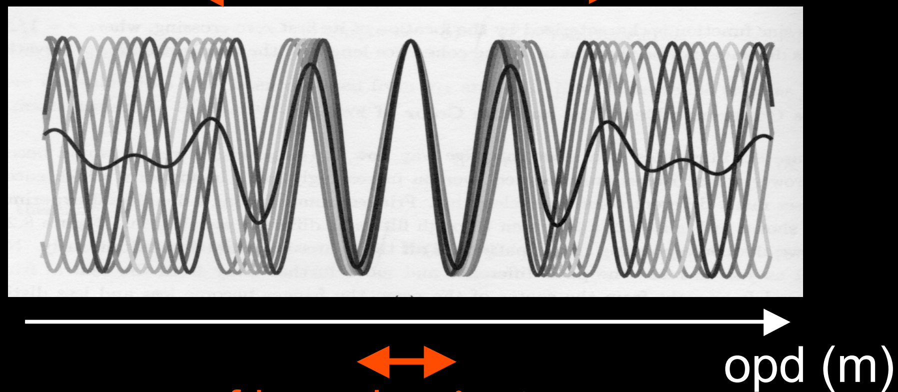
- Important to observe close to the zero-opd position, which requires a precise knowledge of:

→ the position on the star on sky

→ the internal opd of the instrument

packet size

Δ = R.λ



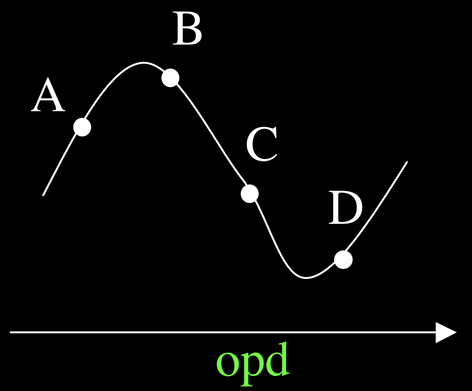
fringe size: λ~ 2µm

A simple estimator

$$I \sim \mu \,.\, \cos(2\pi\, opd/\lambda + \varphi)$$

- *A priori* no issue at all:
  - We just need to measure a modulation of amplitude μ and phase φ …
  - This can easily be done by sampling at $opd = \lambda \,.\, [0, 0.25, 0.5, 0.75]$

ABCD sampling:



$$\varphi = \frac{B - D}{A - C}$$

$$\mu^2 = \frac{(A-C)^2 + (B-D)^2}{(A+B+C+D)^2}$$

So, what are the issues ?

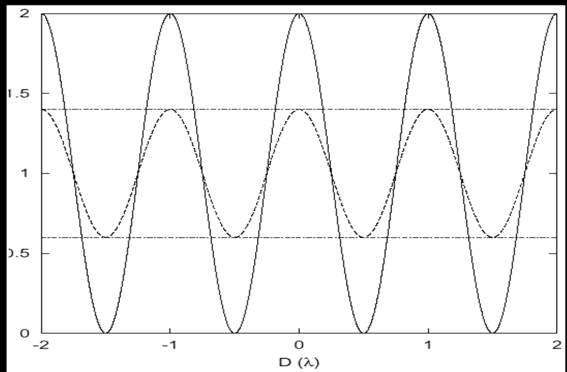
- The previous estimators:

$$I \sim \mu \,.\, \cos(2\pi\, opd/\lambda \;+\; \varphi)$$

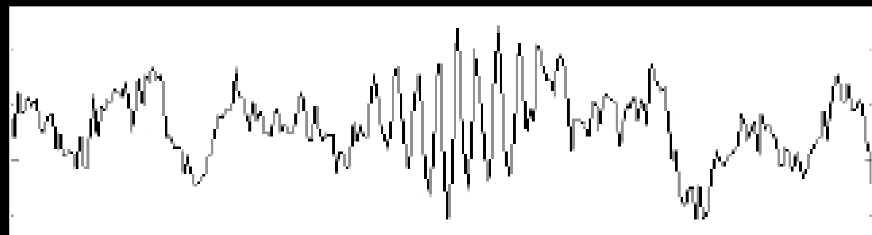
$$\varphi = \frac{B-D}{A-C}$$

$$\mu^2 = \frac{(A-C)^2 + (B-D)^2}{(A+B+C+D)^2}$$

- Work well on these data…



… but not so well on real data, even at high SNR:



Why ?

Estimators robust to noise are necessary

Outline

- What are we really looking for ?
- What are we fighting against ?
  - Additive noises and bias: sky, detector, photon…
  - Photometry unbalance
  - Atmospheric turbulence
    - description
    - how to deal with
  - Atmospheric piston
    - description
    - how to deal with
- Statistics of the observables
- Calibration and final errors estimate
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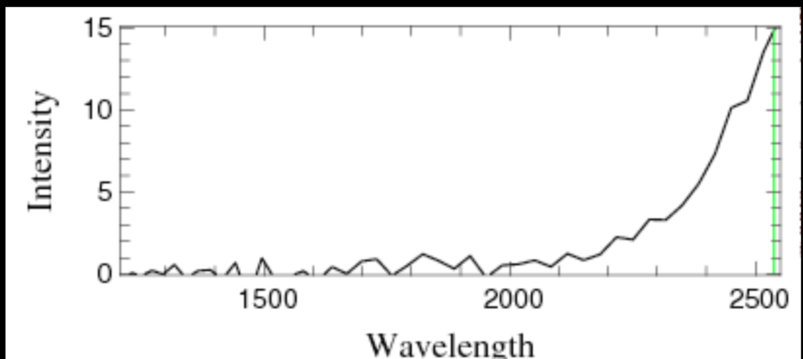
Additives noises and biases

$$I \sim \mu \cos(2\pi\, \mathrm{opd}/\lambda + \varphi) + n_b + \sigma$$

- Photon noise
- Background level: sky emission + dark current
- Detector readout noise

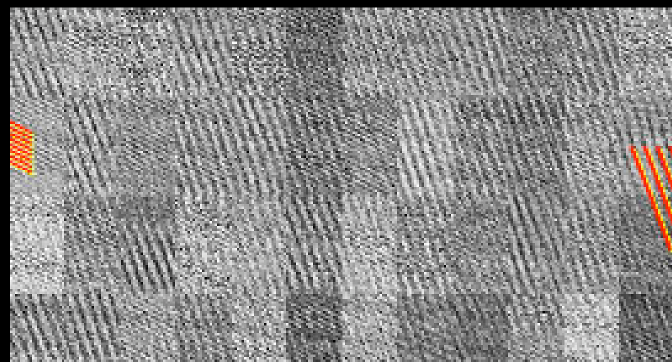
⇒ Removed by classical treatments: dark and sky exposures, chopping…

Sky intensity measured by AMBER



Sky brightness increases drastically after 2μm

AMBER dark exposures



Detector fringes induced by electromagnetic interferences (Li Causi et al. 2007).

Photometry unbalance

- Effective contrast of the fringes depends on the photometry balance between the input beams:

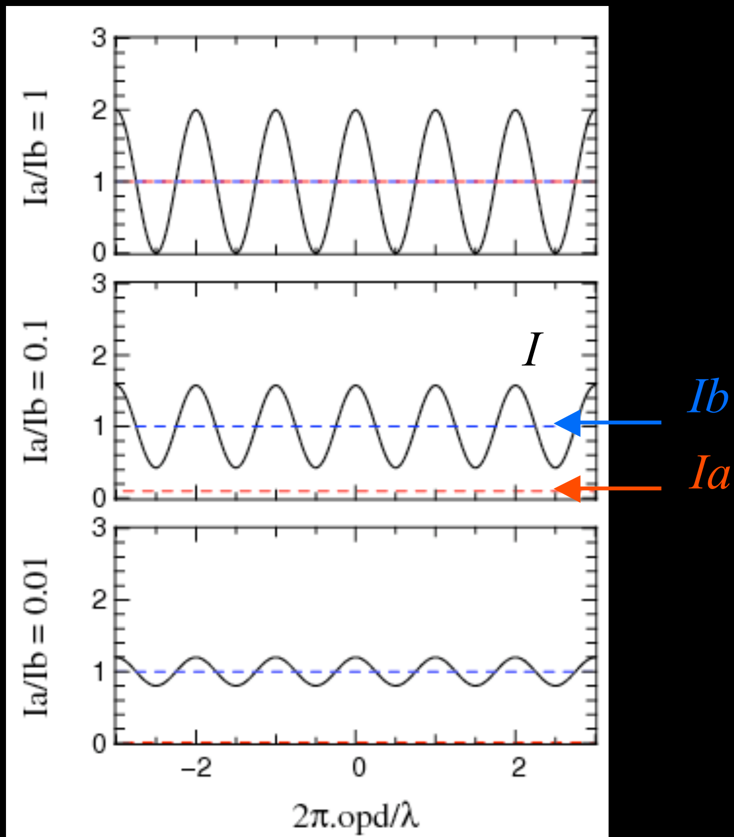
$$I \sim 2\frac{\sqrt{I_a I_b}}{I_a + I_b} . \mu \cos(2\pi \, \mathrm{opd}/\lambda + \varphi)$$

- Degrade the precision on the measure of both μ and φ

- Change the measure of μ, so should be calibrated

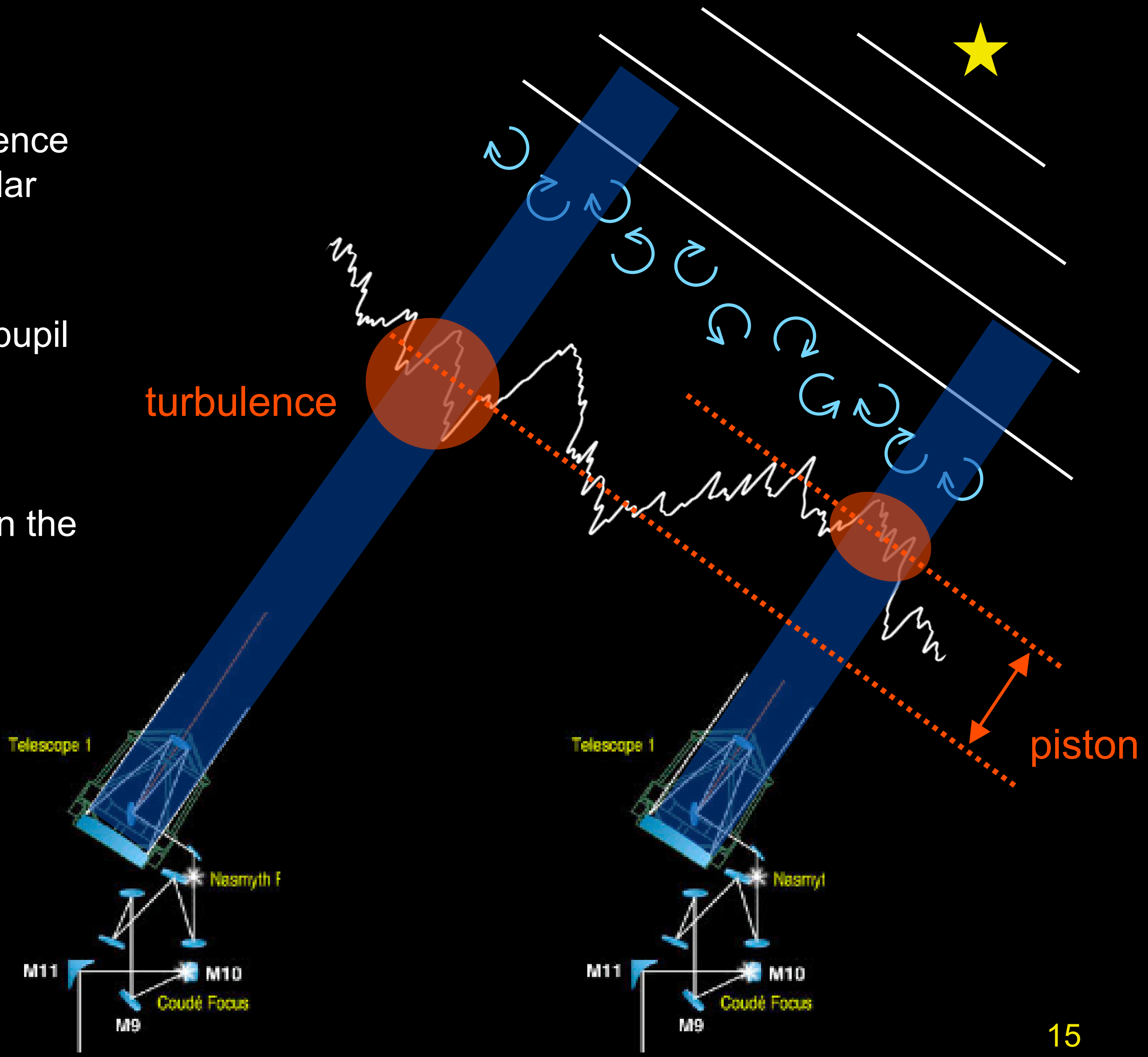
⇒ Simultaneous measures of Ia and Ib
  - Loss of flux for the fringes
  - Better accuracy

⇒ Sequence of exposures fringes, Ia, Ib:
  - Better sensitivity
  - Assume the conditions are stables

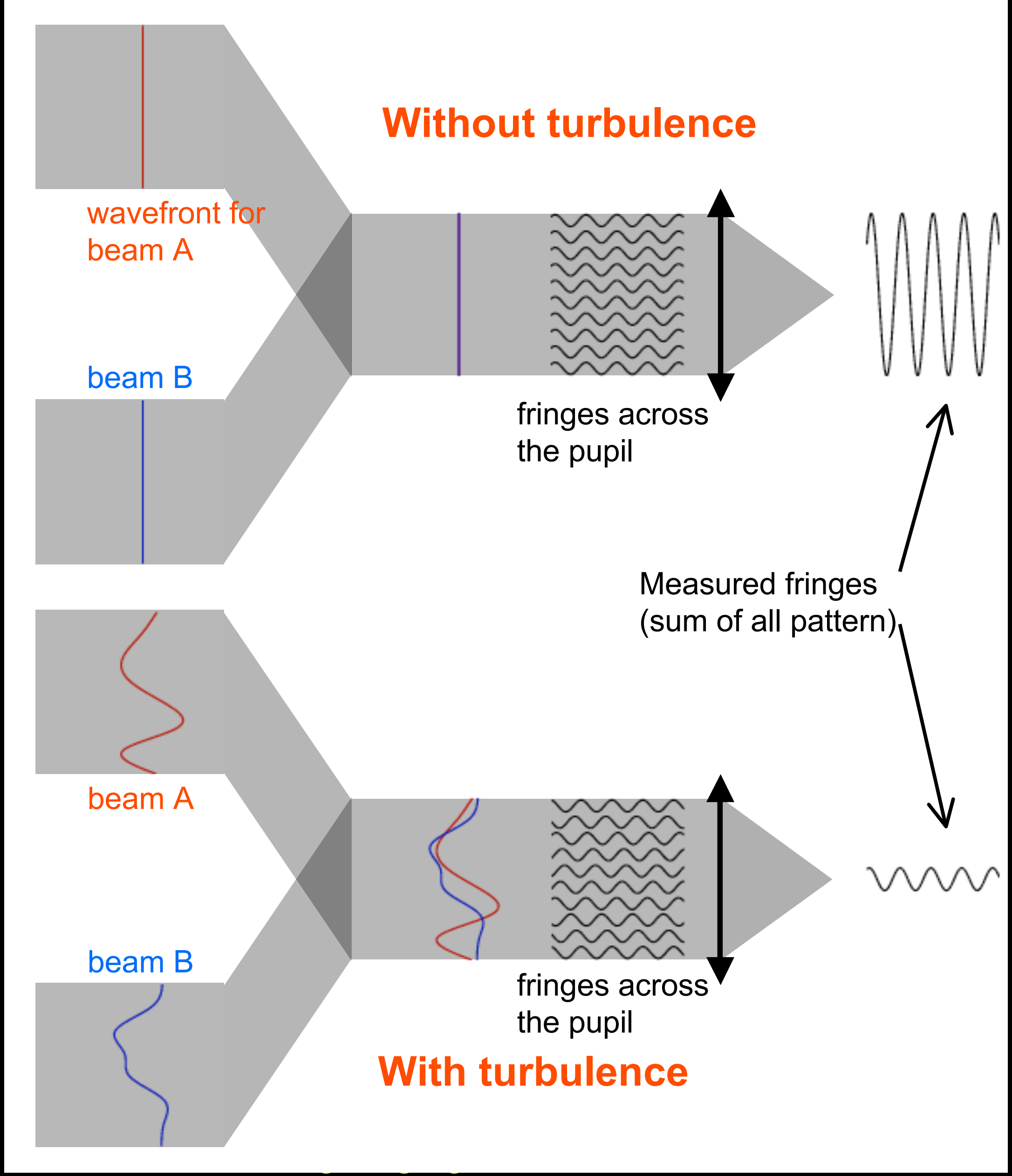


Atmospheric turbulence and piston: vocabulary

- Atmospheric turbulence cells distort the stellar wavefront
- Distortion over the pupil size is called:
  - turbulence
- Global shift between the pupils is called:
  - piston



Turbulence: fringe blurring



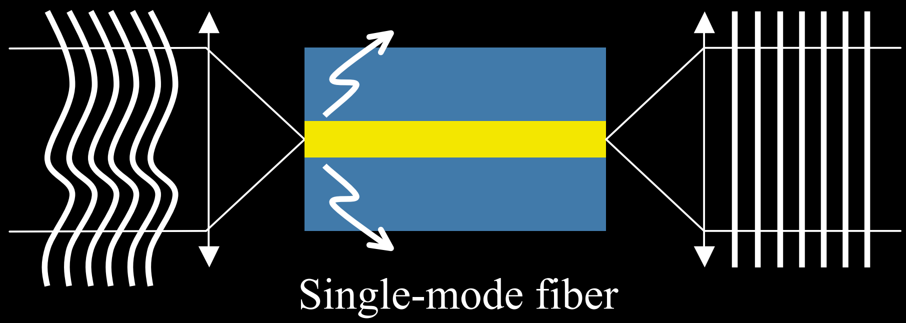
$$I \sim e^{-\sigma^2_{turb}} . \mu \cos(2\pi \, \mathrm{opd}/\lambda + \varphi)$$

- Visibility is reduced by the wavefront variance over the pupil.
  - Do nothing if the turbulence is small (IR - interferometry)
  - Reduce the telescope pupils
  - Use a perfect Adaptive Optics system (the best solution)
  - Use another technique to flatten the wavefronts

The "turbulent" visibility loss should be calibrated frequently

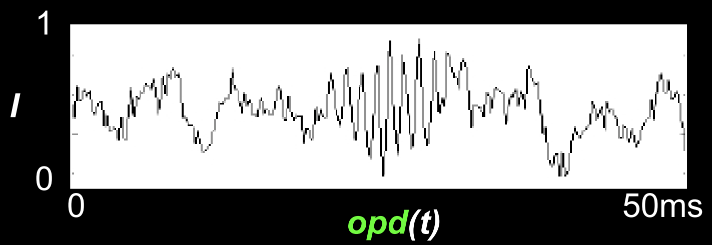
Turbulence: modal filtering

- The input wavefront is flatten by a single-mode fiber
- In fact, the “corrugated part” of the wavefront is rejected by the fiber:
  - Important flux loss if not used with Adaptive Optics or small telescopes



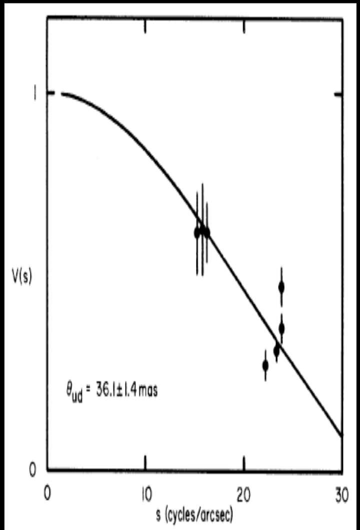
• Phase fluctuations are traded against fast intensity fluctuations…
• But these fluctuations can be measured and corrected.

$$I \sim 2\frac{\sqrt{I_a I_b}}{I_a + I_b} \,.\, e^{-\sigma^2_{turb}} \,.\, \mu \cos(2\pi\,\mathrm{opd}/\lambda + \varphi)$$



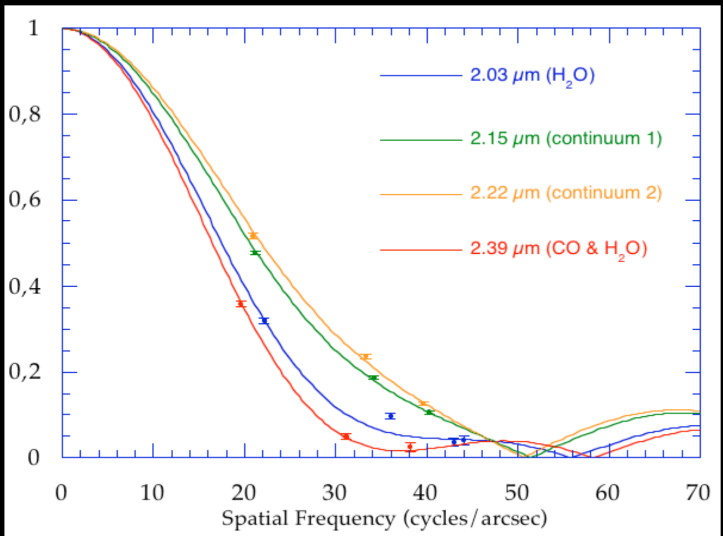
$$I \sim 2\frac{\sqrt{I_a(t) I_b(t)}}{I_a(t) + I_b(t)} \,.\, 1 \,.\, \mu \cos(2\pi\,\mathrm{opd}/\lambda + \varphi)$$

Turbulence: example of modal filtering



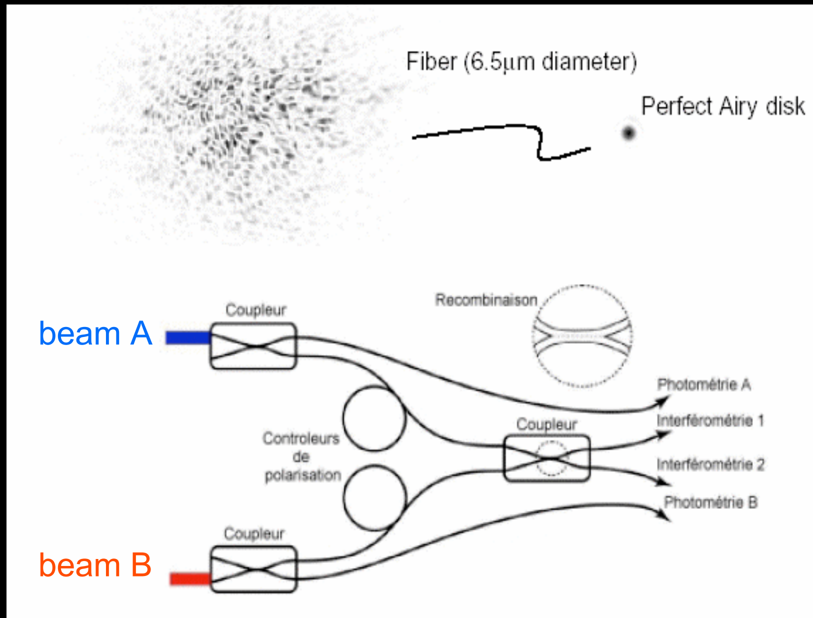
Mira observed without modal-filtering.
Ridgway et al. (1992)

And then with modal-filtering with Fluor.
Perrin et al. (2004)



- The FLUOR experiment has demonstrated the power of this technique when observing in the near-IR

Sketch of the FLUOR fibered combiner



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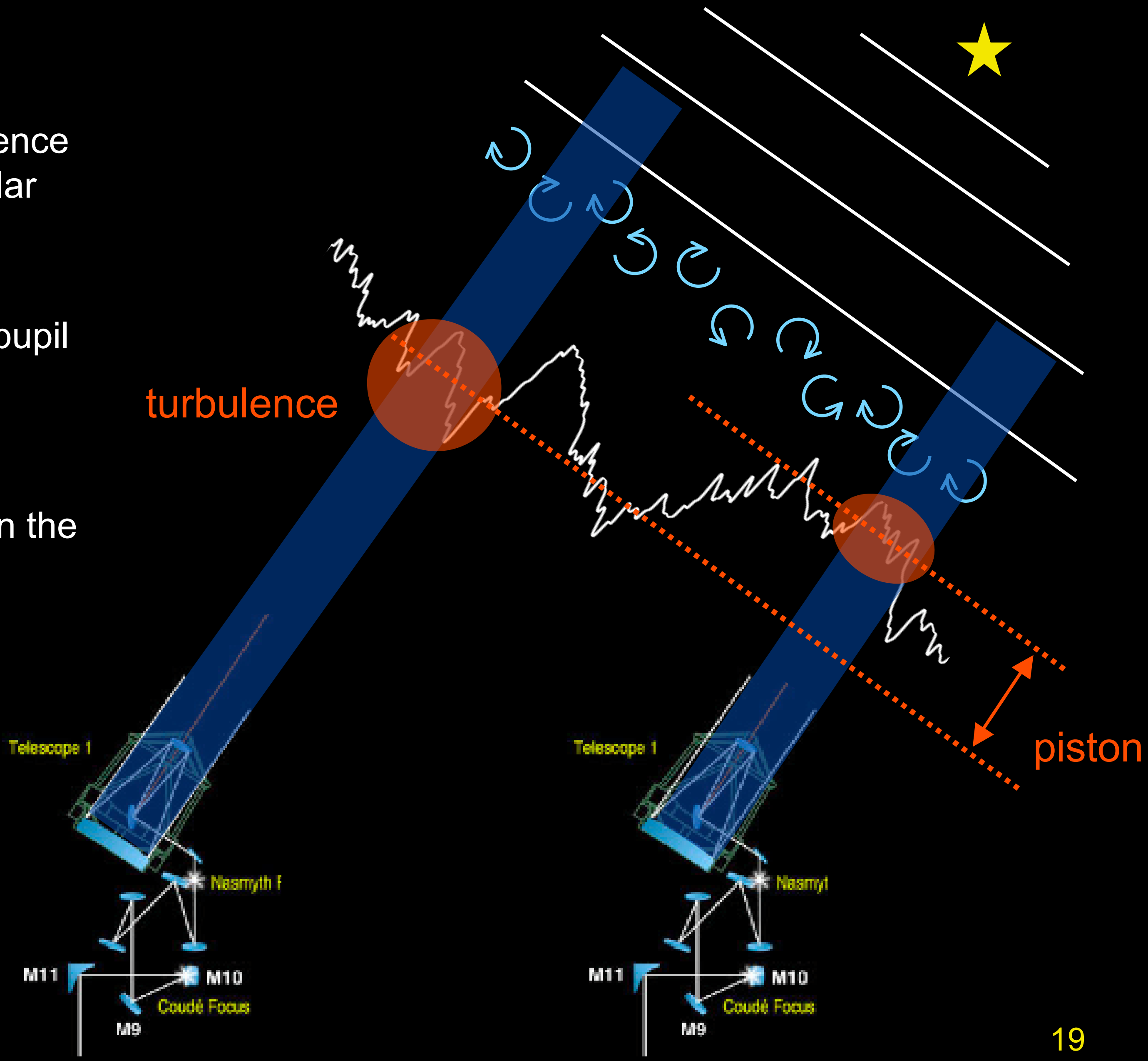


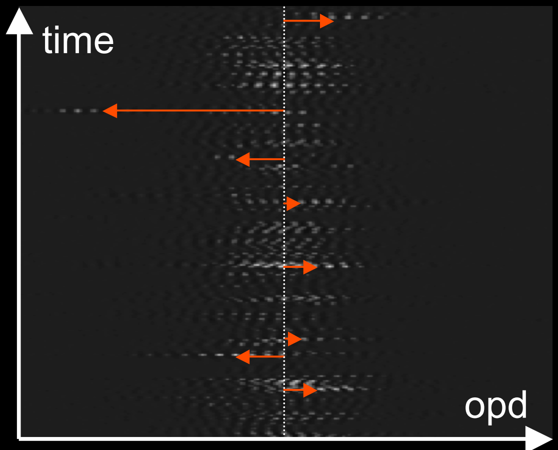
Piston: fringe motion and blurring

- Piston jitter during an exposure blur the fringes visibility:
  - use short exposure only (50ms)
  - use a fringe tracker

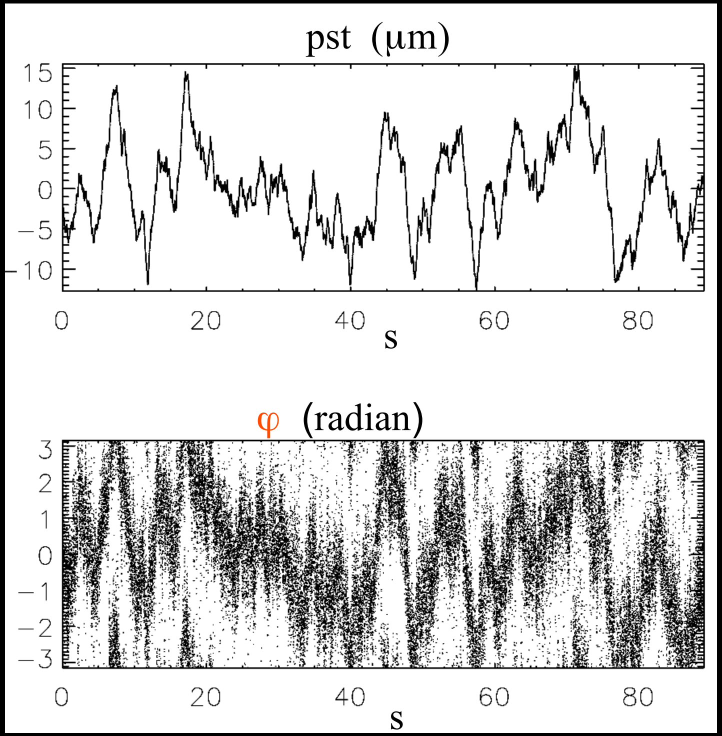
- Fringes are displaced by the averaged piston value during the exposure:
  - measured phase is meaningless

$$I \sim e^{-\sigma_{pst}^2} . \mu \cos(2\pi\,(\mathrm{opd} + \mathrm{pst})/\lambda + \varphi)$$

blurring — motion



VINCI waterfall fringes



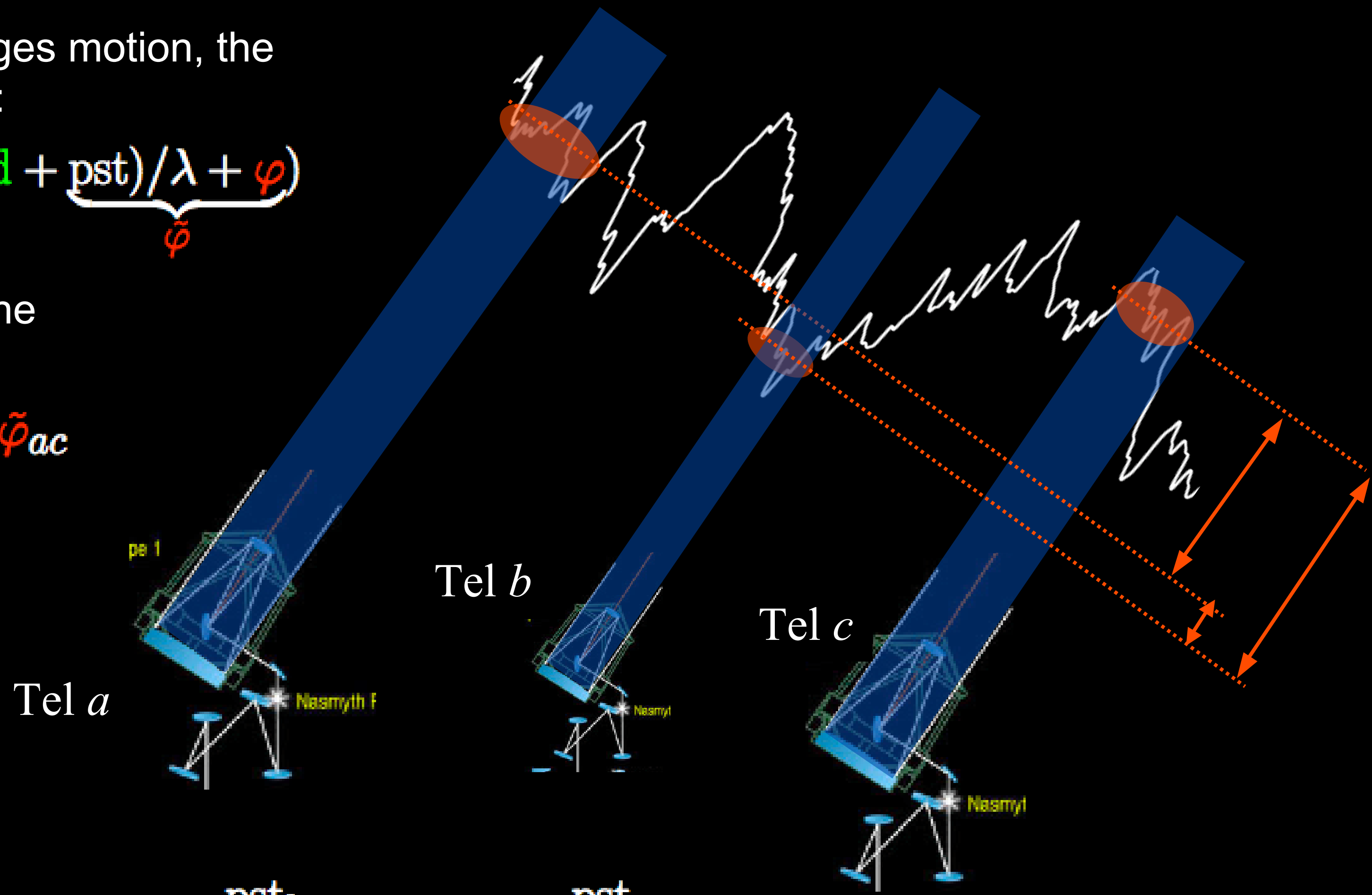
Piston: How to recover some phase information ?

- Because of the fringes motion, the measured phase is:

$$I \sim \mu \cos(2\pi\,(\text{opd} + \underbrace{\text{pst})/\lambda + \varphi}_{\tilde{\varphi}})$$

- A partial solution, the closure phase:

$$\tilde{\phi} = \tilde{\varphi}_{ab} + \tilde{\varphi}_{bc} - \tilde{\varphi}_{ac}$$



$$\tilde{\phi} = (\varphi_{ab} + \frac{\text{pst}_{ab}}{\lambda}) + (\varphi_{bc} + \frac{\text{pst}_{bc}}{\lambda}) - (\varphi_{ac} + \frac{\text{pst}_{ac}}{\lambda})$$

$$\tilde{\phi} = \varphi_{ab} + \varphi_{bc} - \varphi_{ac} + \frac{\text{pst}_{ab} + \text{pst}_{bc} - \text{pst}_{ac}}{\lambda}$$

$$\boxed{\tilde{\phi} = \varphi_{ab} + \varphi_{bc} - \varphi_{ac}}$$

Summary: real data looks like…

$$\sout{I \sim \mu \cos(2\pi\, \mathrm{opd}/\lambda + \varphi)}$$

$$I \sim 2\frac{\sqrt{I_a I_b}}{I_a + I_b} \,.\, e^{-\sigma^2_{turb}} \,.\, e^{-\sigma^2_{pst}} \,.\, \mu \cos(2\pi \frac{\mathrm{opd} + \mathrm{pst}}{\lambda} + \varphi) \,.\, \mathrm{sinc}(2\pi \frac{\mathrm{pst}}{\mathrm{R}\,\lambda}) + n_b + \sigma$$

1 2 3 4 5 6 7

1 - Photometry unbalance *(visibility loss)*

2 - Turbulence over the pupil *(fringe blurring)*

3 - Piston jitter during the exposure *(fringe blurring)*

4 - Averaged piston during the exposure *(fringe displacement)*

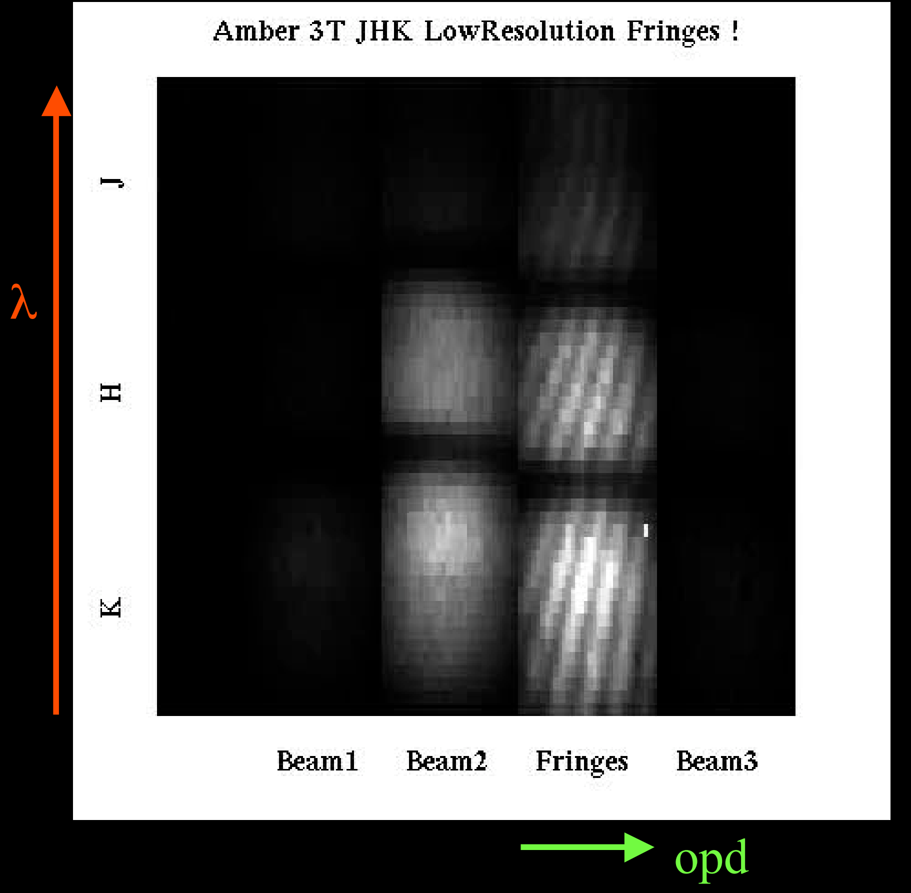
5 - Averaged piston during the exposure *(visibility loss due to the packet finite size)*

6 - Sky brightness and dark current *(additive bias and noise)*

7 - Detector readout noise and photon noise *(additive noise)*

Summary: real data look like…

Real-time AMBER raw data



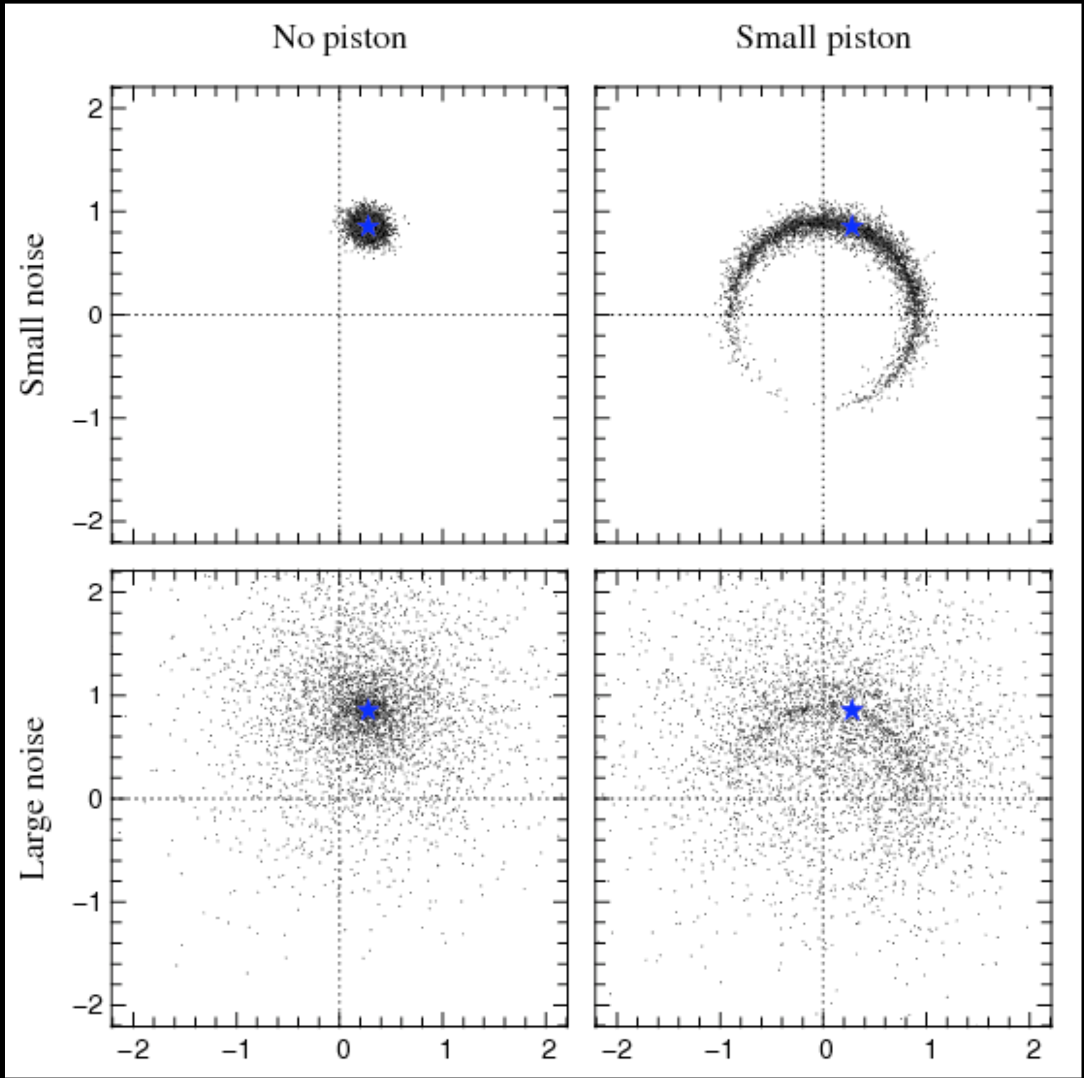
Outline

- What are we really looking for ?
- What are we fighting against ?
- Statistics of the observables
  - What do a data-set looks like ?
  - Visibility estimators
  - Phase estimators
  - Summary of the observables properties
- Calibration and final errors estimate
- Data reduction of the AMBER instrument
- Data reduction of the MIDI instrument
- Conclusions

Statistics : what do a data-set looks like ?

$$I \sim \mu \cos(2\pi\,(\mathrm{opd} + \mathrm{pst})/\lambda + \varphi) \; + n_b + \sigma$$

piston noise



- I consider only the effects of:
  - piston
  - additive noise
- The issue is to average the different measurements:

$$\nu \sim \mu e^{i\varphi}$$

- Final visibility can be obtained by
  - coherent average:

$$\tilde{\mu} = |<\nu>|$$

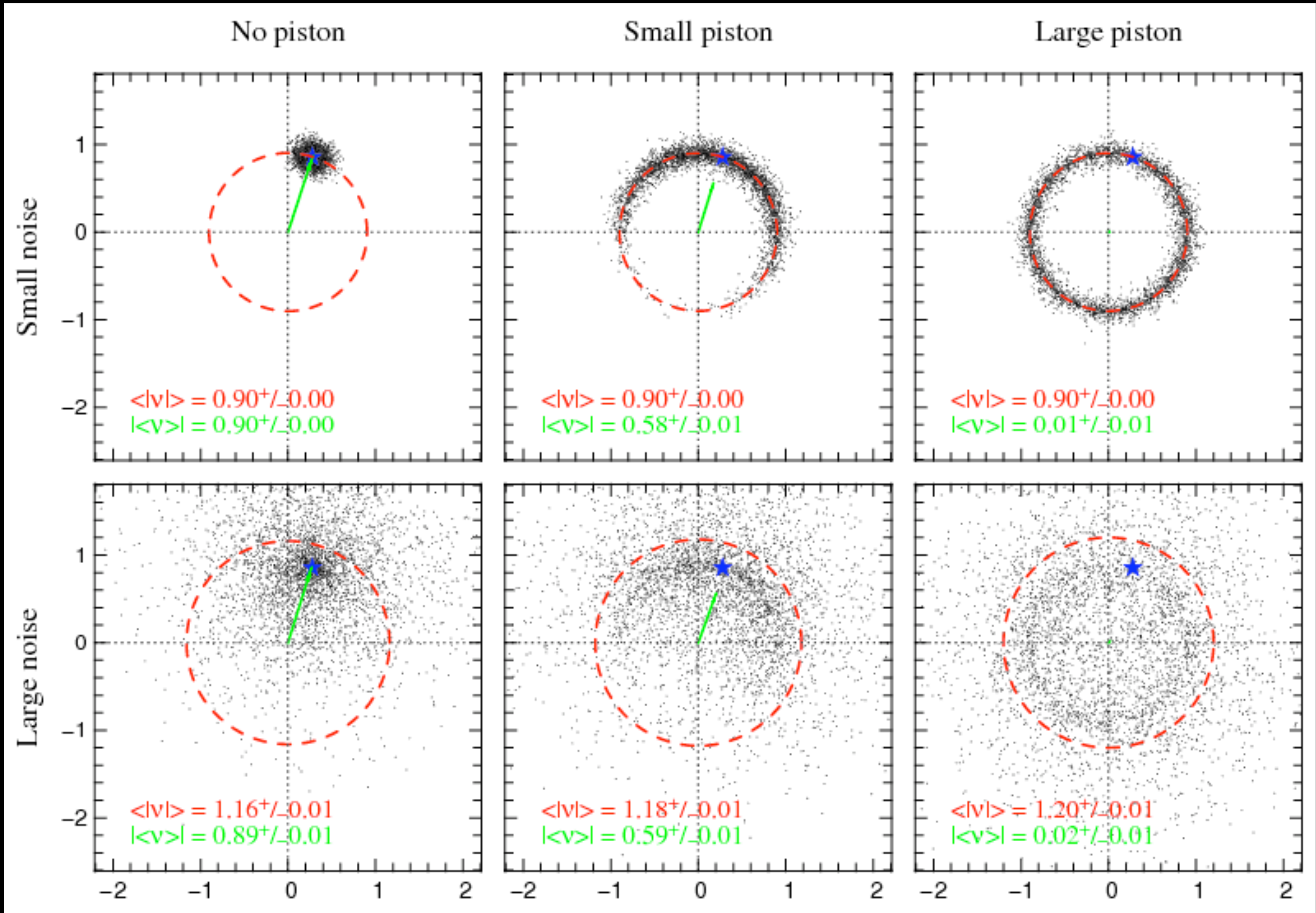
  - incoherent average:

$$\tilde{\mu} = <|\nu|> = <\mu>$$

Visibility: coherent versus incoherent average

$$I \sim \mu \cos(2\pi\,(\text{opd} + \text{pst})/\lambda + \varphi) + n_b + \sigma$$

piston   noise



un-noisy data ($\mu = 0.9$)

noisy data (piston and additive noise)

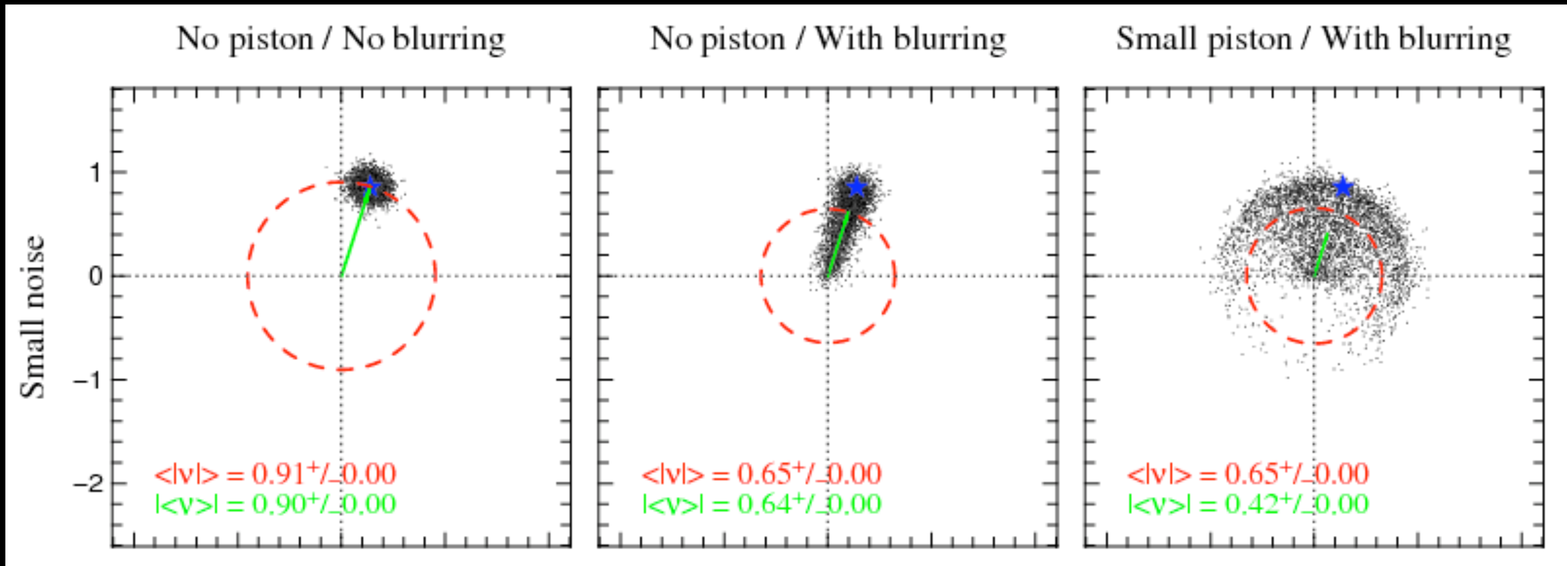
incoherent average

coherent average

Visibility: effect of the multiplicative terms !

$$I \sim e^{-\sigma_{pst}^2} . \mu \cos(2\pi\,(\mathrm{opd} + \mathrm{pst})/\lambda + \varphi) + n_b + \sigma$$

blurring piston noise



- All estimators are biased by multiplicative terms
- Non stationary phenomena (vibration, turbulence, jitter blurring…)
- Extremely hard to calibrate during the exposure
  - Assumed to be the same on the science and calibration stars

Phase: advantages and issues

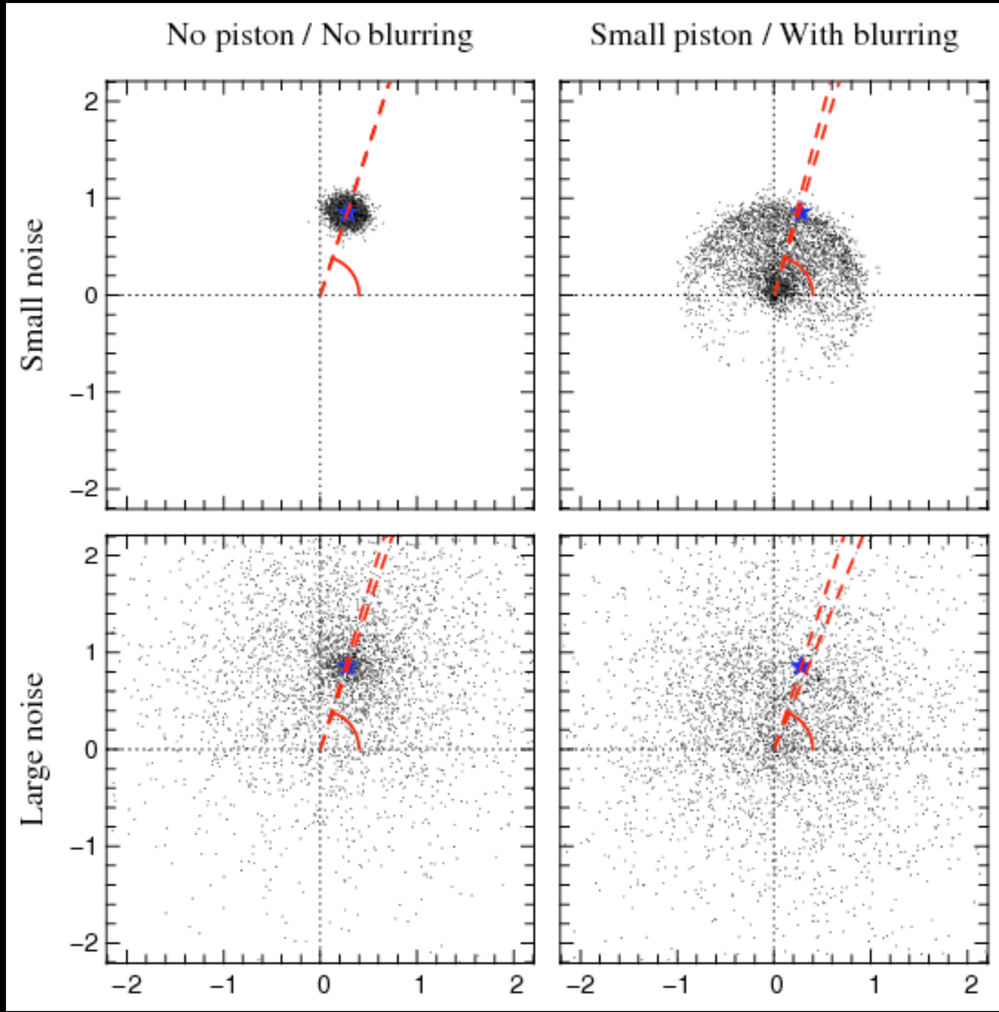
$$I \sim e^{-\sigma_{pst}^2} . \mu \cos(2\pi\,(\mathrm{opd} + \mathrm{pst})/\lambda + \varphi) + n_b + \sigma$$

blurring piston noise

- Remember that the absolute fringe phase is generally lost because of the piston:
  - closure-phase / differential phase
- Whatever the phase considered, data should be averaged coherently:

$$\tilde{\varphi} = \arg(<\nu>) = \arg(<\mu\, e^{i\,\varphi}>)$$

- Phase is not biased by multiplicative noises (photometry unbalance, fringe blurring, turbulence…)
  - much easier to calibrate
- But error bars are hard to estimate in low SNR regime…
  - bootstrapping



Observables properties : summary

- Incoherent average of the visibilities
  - insensitive to piston
  - biased by additive noises
  - biased by multiplicative noises
- Coherent average of visibilities
  - piston should be know / removed
  - not biased by additive noises
  - biased by multiplicative noises
- Differential phase / Closure-phase
  - absolute phase lost
  - not biased by noises, easier to calibrate
  - error estimation requires bootstrapping

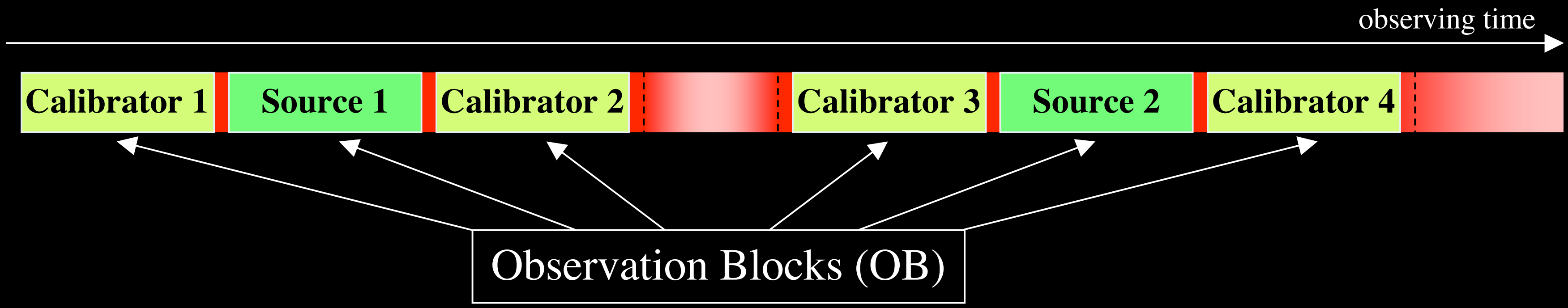
These visibility loss should be calibrated frequently

$$I \sim 2\frac{\sqrt{I_a I_b}}{I_a + I_b} . e^{-\sigma^2_{turb}} . e^{-\sigma^2_{pst}} . \mu \cos(2\pi \frac{\mathrm{opd} + \mathrm{pst}}{\lambda} + \varphi) . \mathrm{sinc}(2\pi \frac{\mathrm{pst}}{\mathrm{R}\,\lambda}) + n_b + \sigma$$

Outline

- What are we really looking for ?
- What are we fighting against ?
- Statistics of the observables
- Calibration and final errors estimate
  - Principle
  - Computing/calibrating from the transfer function
  - Examples
  - Error propagations and correlations
- Data reduction of the AMBER instrument
- Data reduction of the MIDI instrument
- Conclusions

Principle of calibration



- Why calibrate ?
  - multiplicative visibility loss
  - reference of the differential-phase / closure-phase

- How calibrate ? By measuring them on a known star:
  - same atmospheric conditions: close in time
  - same injection conditions: similar flux
  - same detector parameters: frame rate, number of frames...
  - same instrument setup: filter, spectral resolution...

About half of a night is spend on calibration stars

Computing and calibrating the transfer function

1. Measure the visibility on the science target and (at least) on one calibrator:
2. Derive the expected visibility of the calibrator (usually assuming a Uniform Disk):
3. Compute the instantaneous transfer function:
4. Compute the transfer function at the time of the science observations
   - averaging / interpolating / splining…
5. Calibrate the visibility of the science target:

1. $\tilde{\mu}^2_{sci}(t_s) \qquad \tilde{\mu}^2_{cal}(t_c)$

2. $$\mu_{theo} = 2\left|\frac{J_1(\pi.\theta.B/\lambda)}{\pi.\theta.B/\lambda}\right|$$

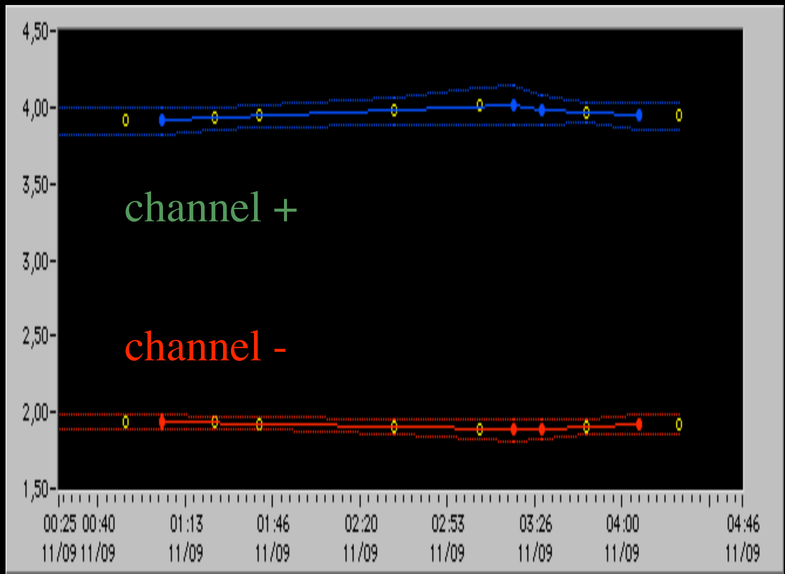
3. $$T^2(t_c) = \frac{\tilde{\mu}^2_{cal}(t_c)}{\mu^2_{theo}(t_c)}$$

4. $$T^2(t_s) = \mathrm{f}(\ T^2(t_c)\ )$$

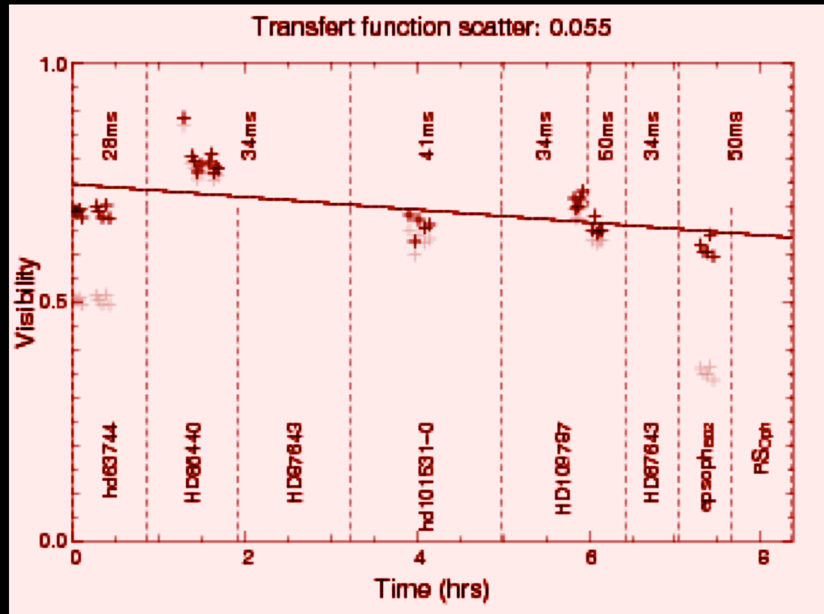
5. $$\mu^2_{sci}(t_s) = \frac{\tilde{\mu}^2_{sci}(t_s)}{T^2(t_s)}$$

Examples of transfer function (IOTA and AMBER)

Arbitrary scale



Time



- ○ time when science sources have been observed
- transfer function estimated on calibrators, with associated errors

gray: raw visibilities

black: estimated transfer function = visibilities divided by the theoretical ones

Final error bars computation

- Error sources:
  - raw visibilities
  - calibrator diameter
  - calibrator model (really a UD ?)
- Error propagation not trivial:
  - statistic / systematic errors
- Classical formula only work if:
  - the errors are really small (!)
  - the statistics are Gaussian (!)
- Otherwise: simulate the random variables distribution and compute the variance of the simulated results:
  - work with large errors

1. $\tilde{\mu}^2_{sci}(t_s) \qquad \tilde{\mu}^2_{cal}(t_c)$

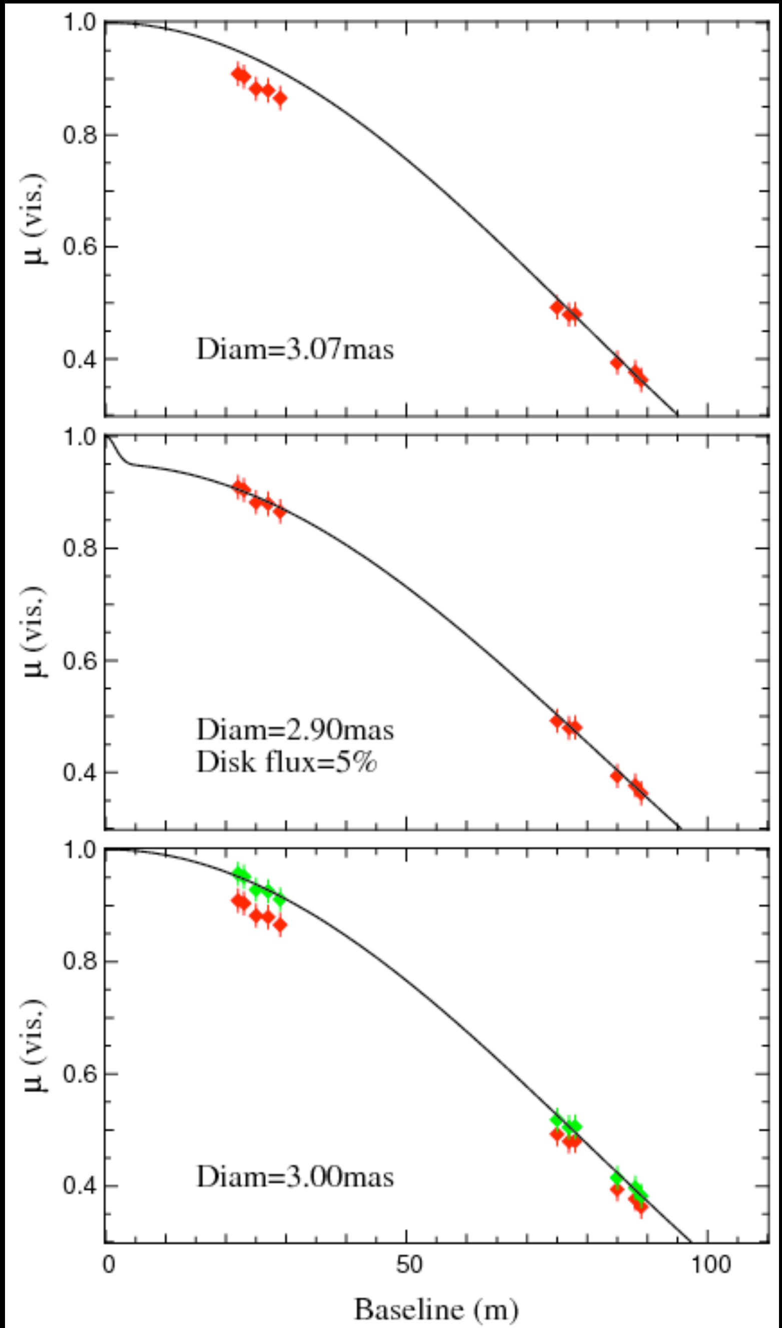
2. $\mu_{theo} = 2\left|\frac{J_1(\pi.\theta.B/\lambda)}{\pi.\theta.B/\lambda}\right|$

3. $T^2(t_c) = \frac{\tilde{\mu}^2_{cal}(t_c)}{\mu^2_{theo}(t_c)}$

4. $T^2(t_s) = \mathrm{f}(\ T^2(t_c)\ )$

5. $\mu^2_{sci}(t_s) = \frac{\tilde{\mu}^2_{sci}(t_s)}{T^2(t_s)}$

The issue of 'systematics' in data analysis



- Red are observed (and calibrated) points on a science target
- Error looks to be properly estimated since the dispersion is consistent
- UD disk model fails to fit the data set within the error bars
- A more evolved disk+UD model looks much better (great!)
- But if I multiply all points by 1.05 (green)… the data are now able to well fit a simple UD.
- Such factor is about the systematic error on the transfer function due to the calibrator size (5%)

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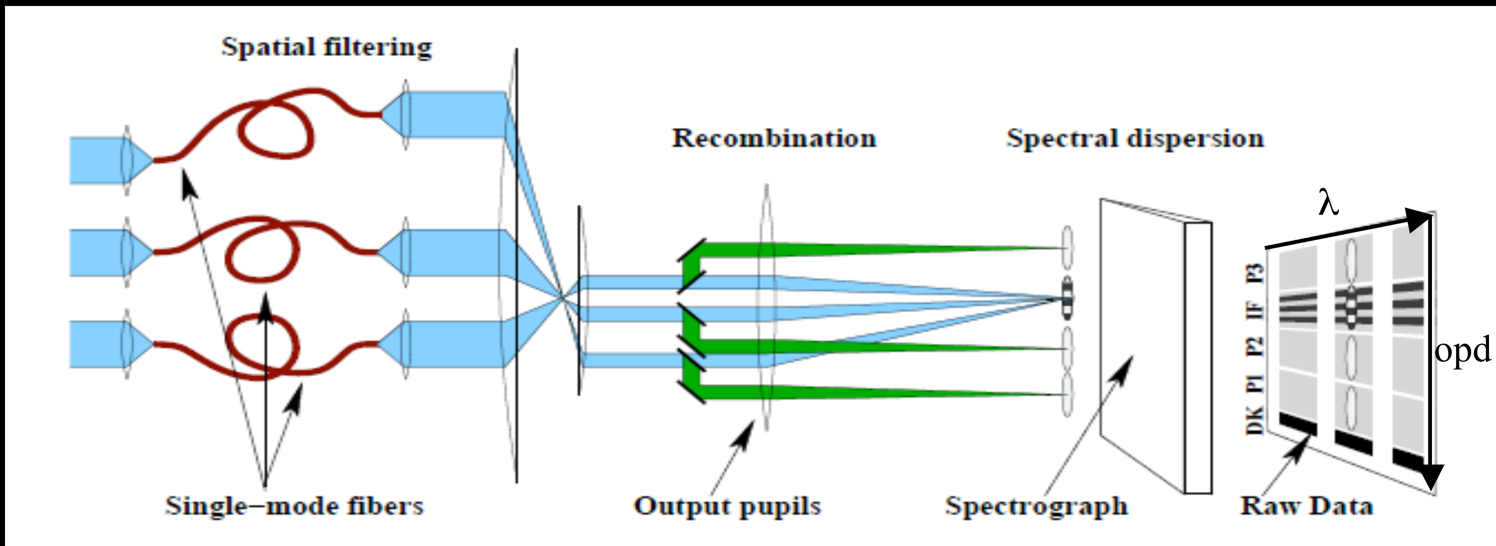
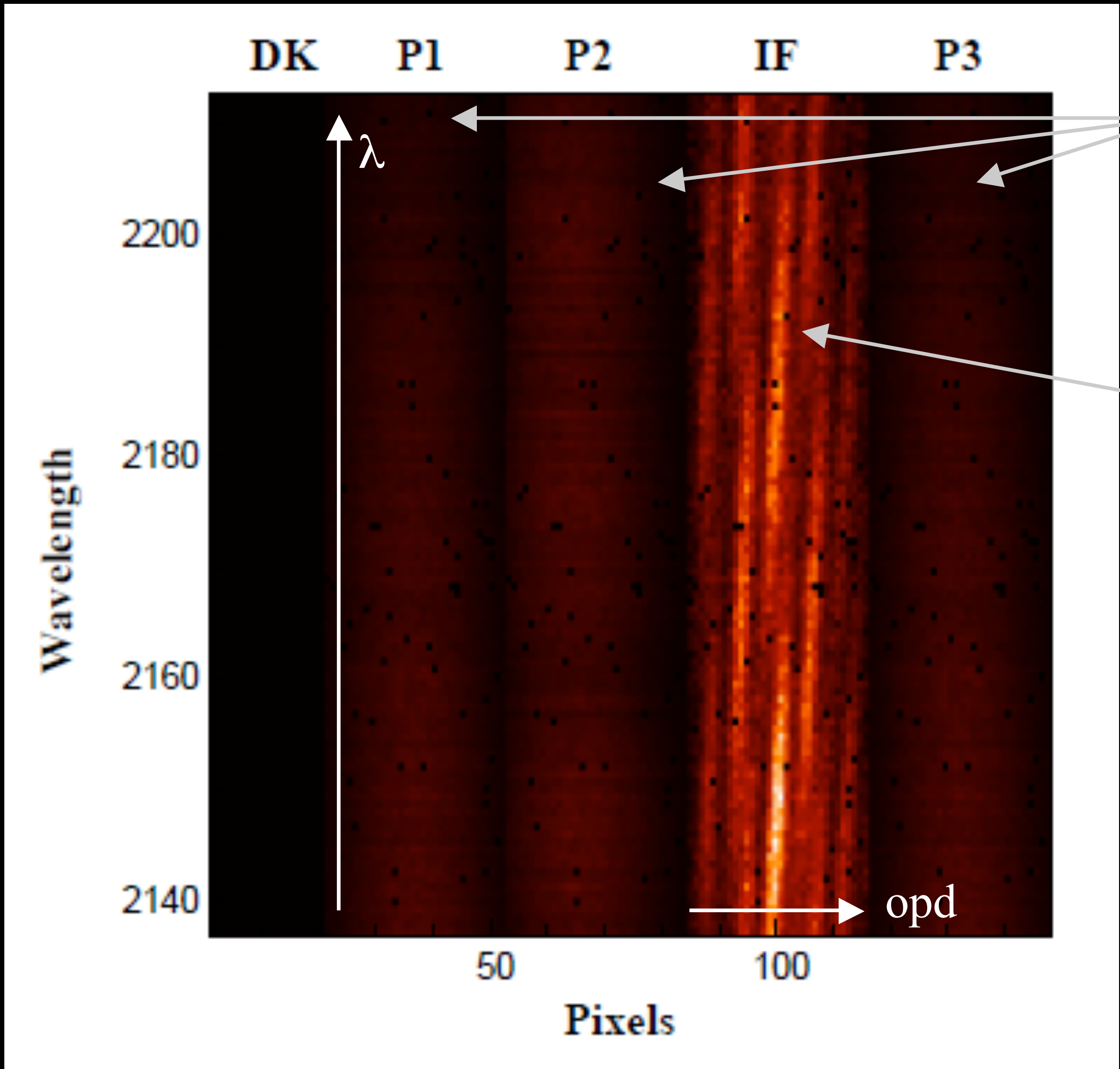
- Data reduction of the AMBER instrument
  - Description of the instrument
  - Internal calibrations
  - Data reduction work flow
  - Inspecting the data products
- Data reduction of the MIDI instrument
- Conclusions

The AMBER instrument

- Use 3 telescopes of VLTI
  - closure-phase
- Near-IR: J, H and K bands
  - Single-mode filtering
  - Simultaneous photometry monitoring
- Spectral dispersion (y-axis on detector)
  - differential visibilities / phases
- Spatial combination (opd is x-axis on detector)



AMBER: 3 fringes in a single beam and 3 photometric beams



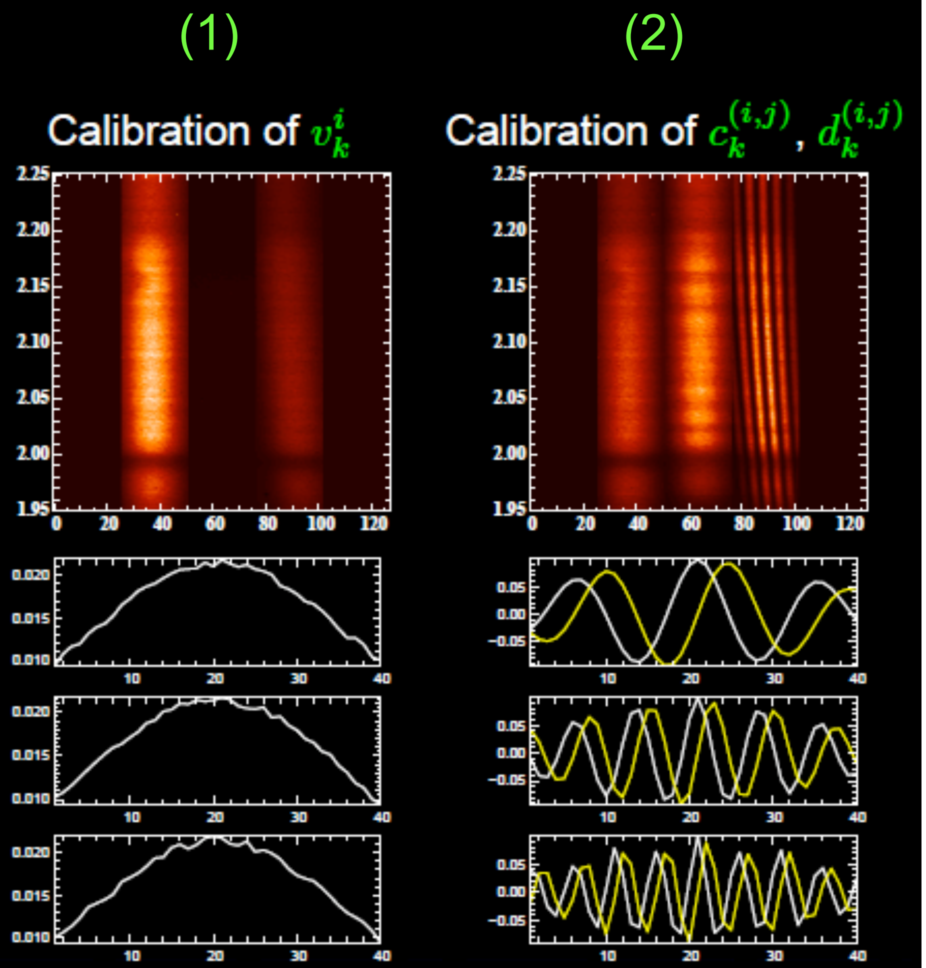
Photometric beams

Mix of 3 fringe patterns:
- 1-2
- 1-3
- 2-3

medium resolution

AMBER internal calibrations

- Need for a internal calibration:
  - relative flux in the photometric and interferometric beams
  - relative transmission in λ
  - wavelength table
  - disentangle the 3 fringe patterns by a fringe fitting technique
- Internal calibration depends
  - on setup (band, resolution…)
  - on time (unstable)
- Calibration sequence:
  - wavelength calibration
  - one beam at a time (1)
  - one pair at a time (2)



AMBER internal calibrations

The Pixel 2 Visibility Matrix (P2VM) sequence…

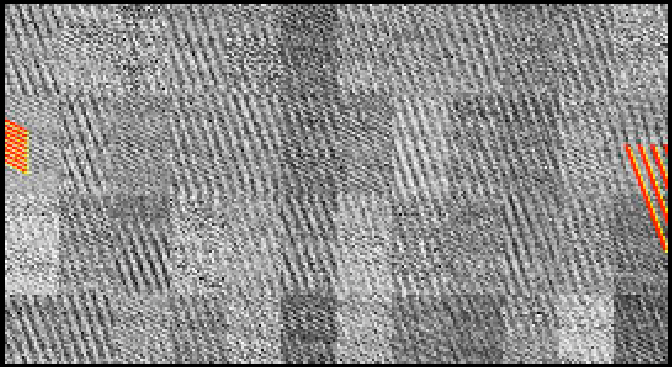
Step	Shutter 1	Shutter 2	Shutter 3	Phase γ_0	DPR key
1	Open	Closed	Closed	NO	2P2V, 3P2V
2	Closed	Open	Closed	NO	2P2V, 3P2V
3	Open	Open	Closed	NO	2P2V, 3P2V
4	Open	Open	Closed	YES	2P2V, 3P2V
5	Closed	Closed	Open	NO	3P2V
6	Open	Closed	Open	NO	3P2V
7	Open	Closed	Open	YES	3P2V
8	Closed	Open	Open	NO	3P2V
9	Closed	Open	Open	YES	3P2V

The reduced product of this sequence is called the P2VM… and allows to reduce the data

AMBER detector issues

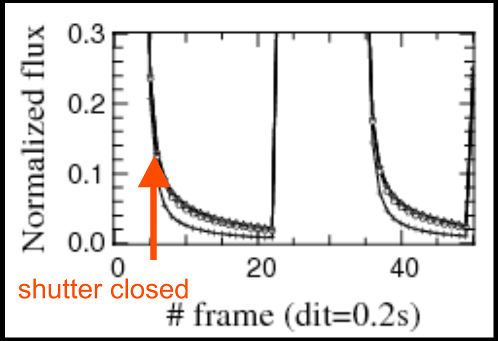
- Classical issues of IR-detector:
  - flat-field map
  - bad pixel map
- Other issues are exacerbated due to fast read-out:
  - noise structure
  - detector remanents
  - synchronizations…

Dark exposures

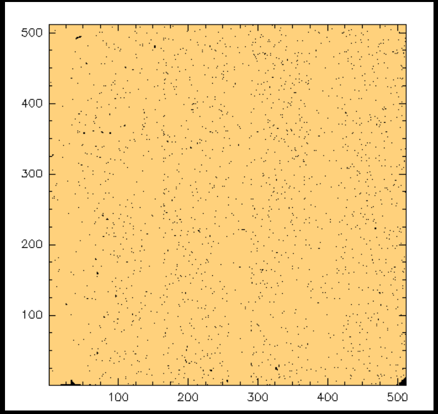


Detector fringes due to electro-magnetic interferences (Li Causi, 2007).

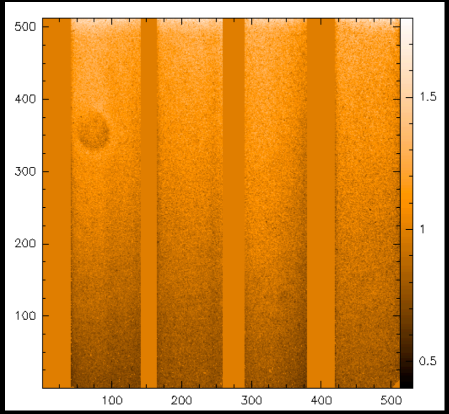
Detector remanent



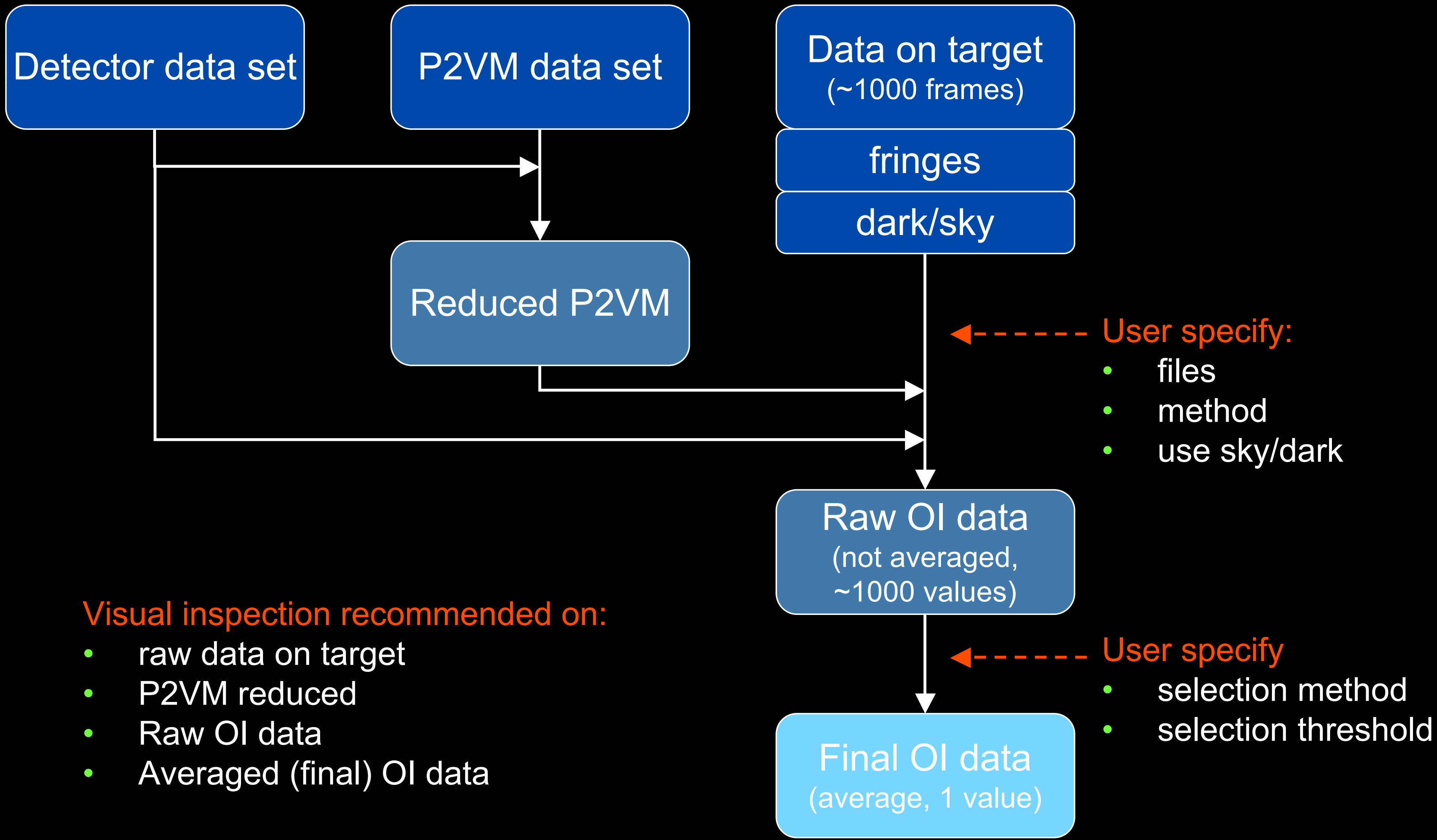
Bad pixels map



Flat field map



AMBER reduction work-flow

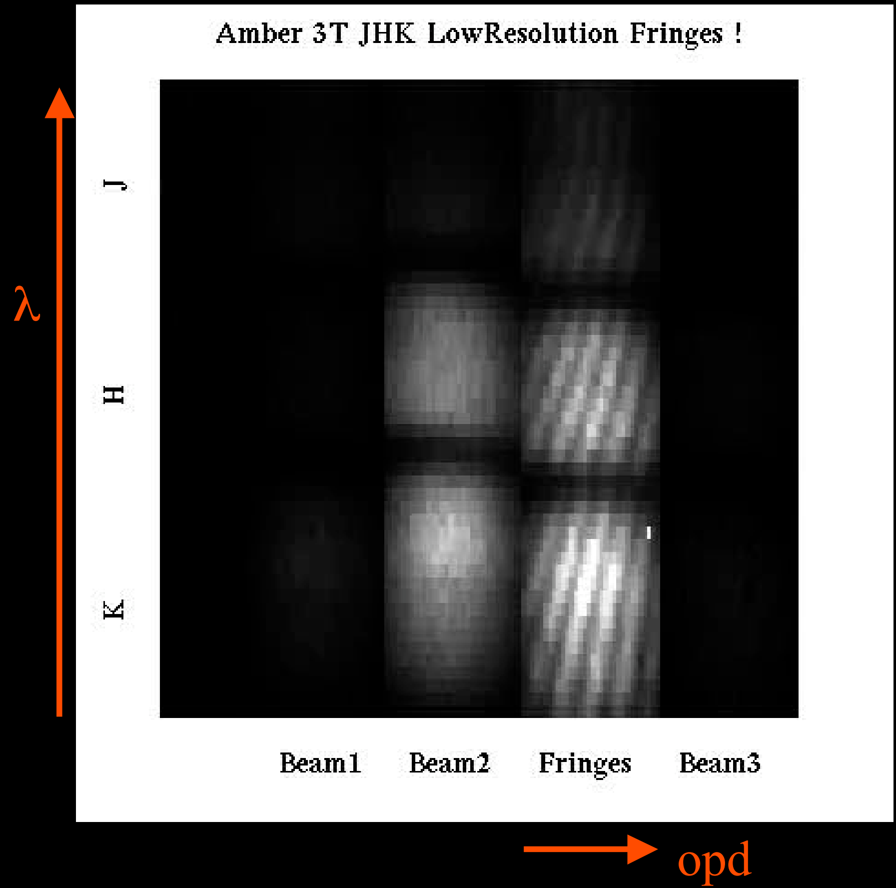


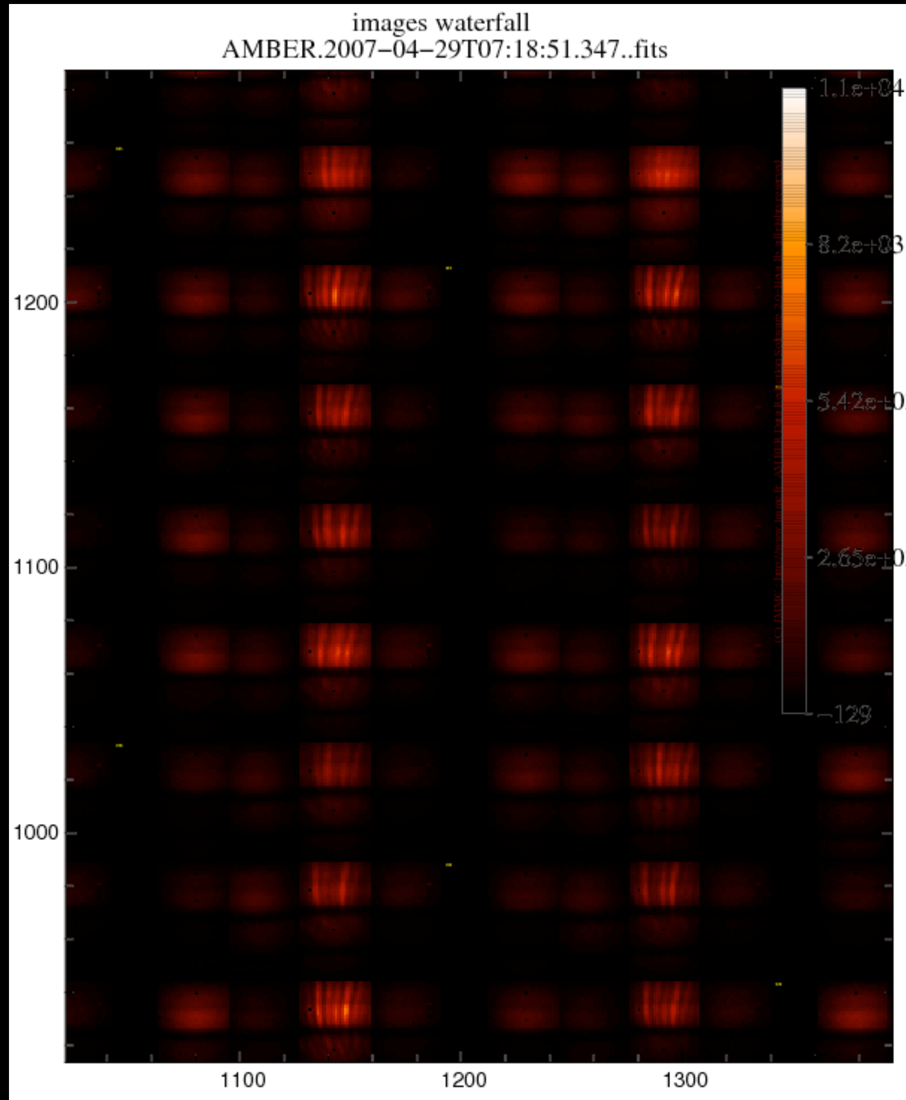
Visual inspection recommended on:

- raw data on target
- P2VM reduced
- Raw OI data
- Averaged (final) OI data

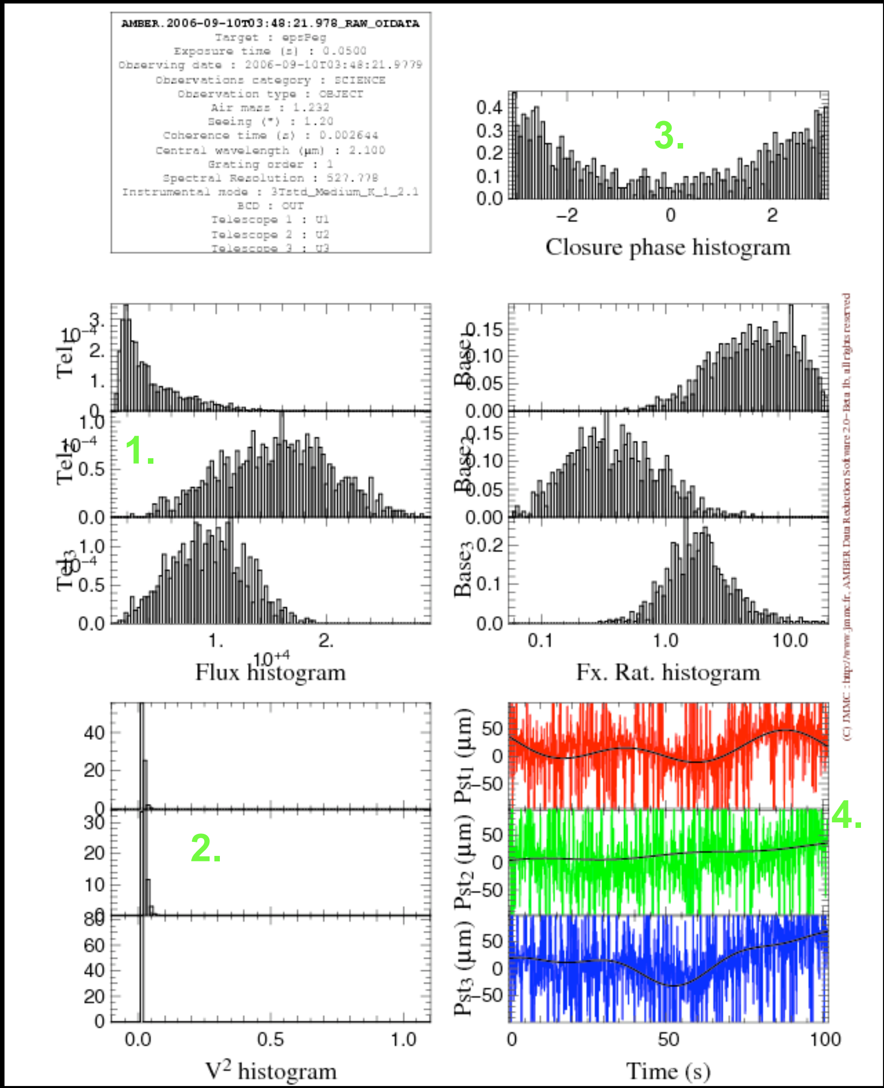
AMBER raw data inspection

Pseudo real-time AMBER raw data



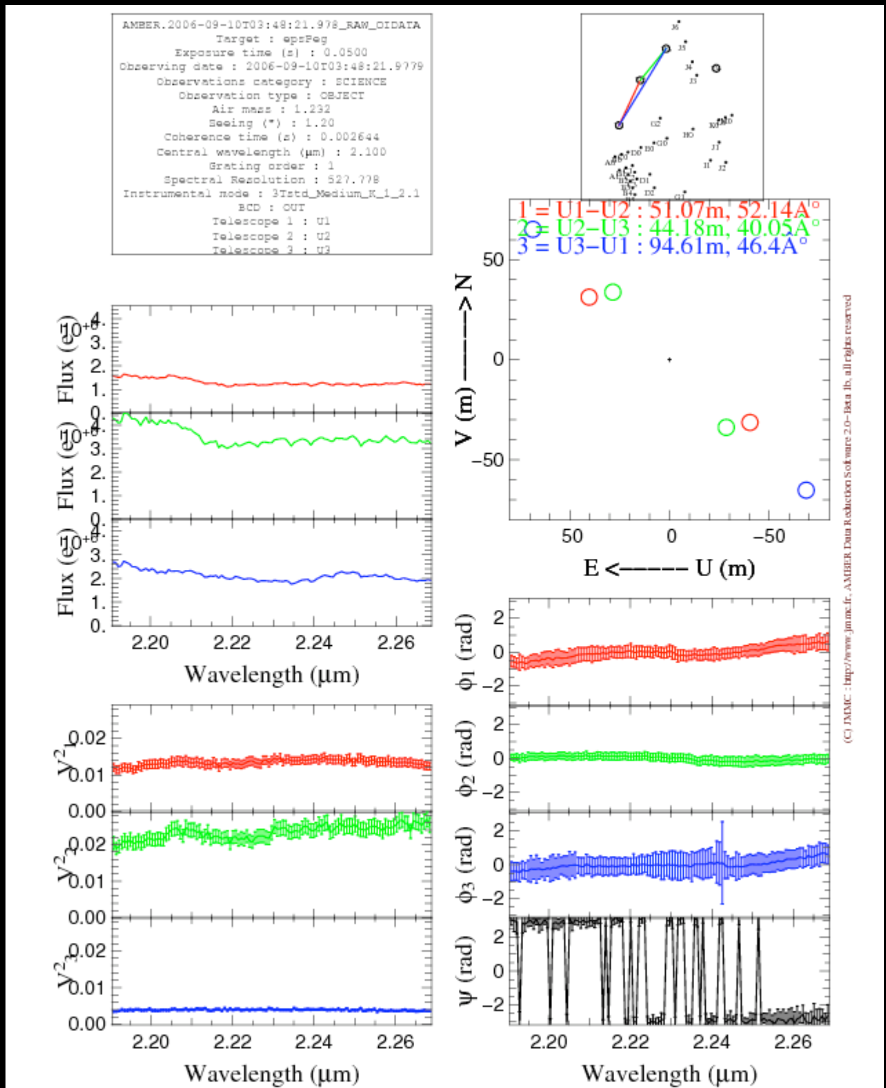


AMBER intermediate OI product inspection



- **Data set:**
  - bright super giant star
  - 3 UTs data
  - good quality
  - visibilities small
- **Observation/Star information:**
  - DIT, seeing, setup…
- **Histogram of OI observables:**
  1. flux
  2. visibilities,
  3. closure-phase
  4. piston(t)

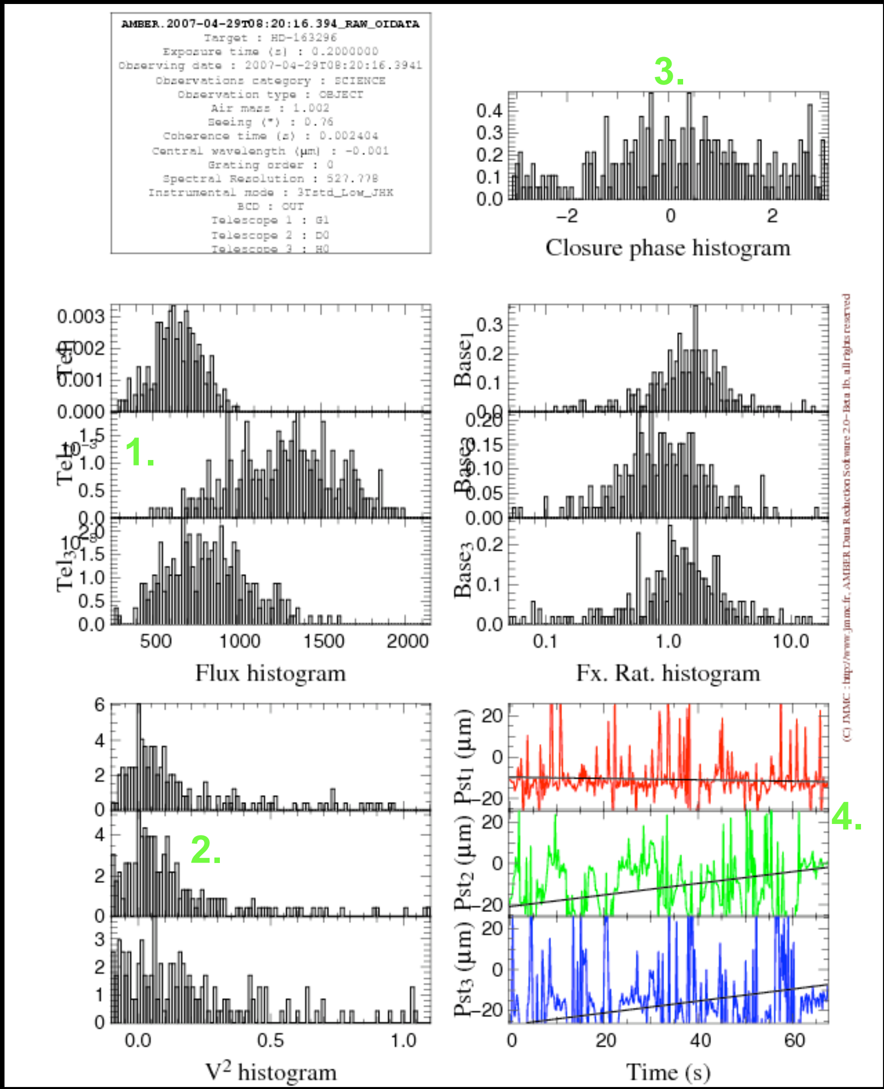
AMBER final OI product inspection



- Strongly selected data: 20% best frames sorted by SNR
- Visibilities:
  - small ($\mu^2 < 0.02$)
  - good accuracy on the visibilities
  - errors do not take into account calibration (not done yet)
- Phases:
  - differential phases are “flat”
  - closure phase is $\sim\pi$

Let’s calibrate these data and do astrophysics…

AMBER OI product: “faint target” case



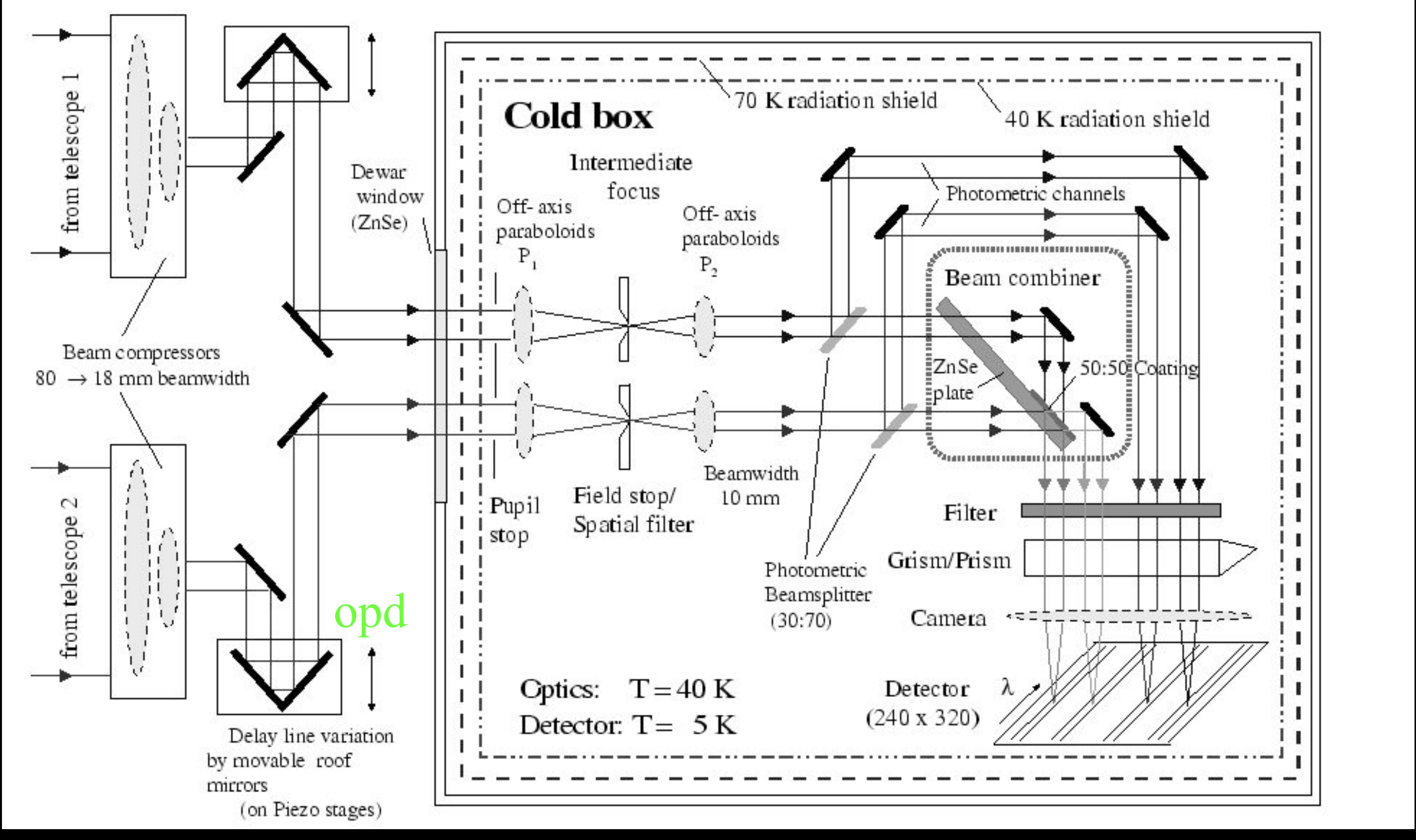
- Data set:
  - faint young star: H=5.6mag
  - 3 ATs data
  - good quality for this target
  - visibilities large (0.5)
- Histogram of OI observables:
  1. flux
  2. visibilities,
  3. closure-phase
  4. piston(t)
- What is hard in such dataset:
  - noisy !
  - visibility histograms are asymmetric
  - phase histogram is noisy and wrapped

Outline

- What are we really looking for ?
- What are we fighting against ?
- Statistics of the observables
- Calibration and final errors estimate
- Data reduction of the AMBER instrument
- ➔ Data reduction of the MIDI instrument
  - Description of the instrument
  - Data reduction work-flow
- Conclusions

The MIDI instrument

- Use 2 telescopes of the VLTI
- Thermal-IR
  → telescope chopping
- Temporal combination (opd change with time)
- Spectral dispersion

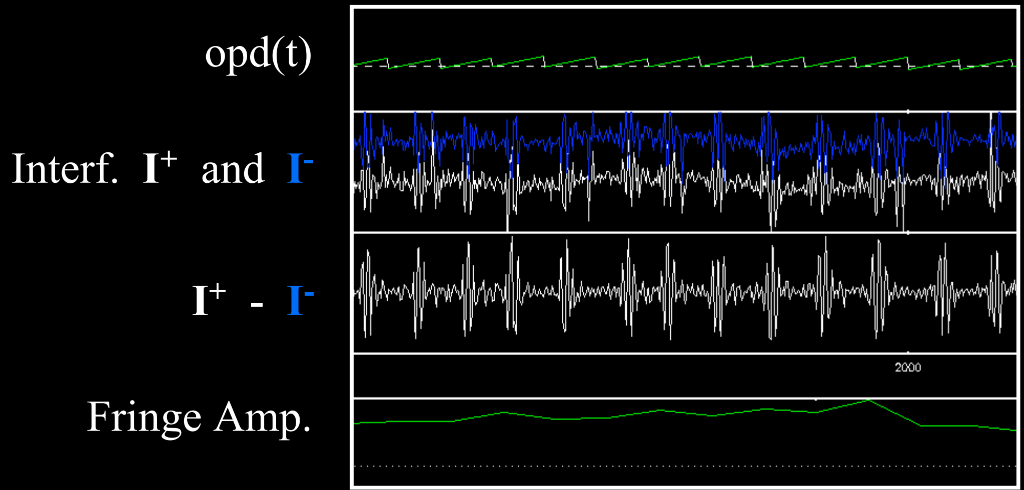


The MIDI instrument

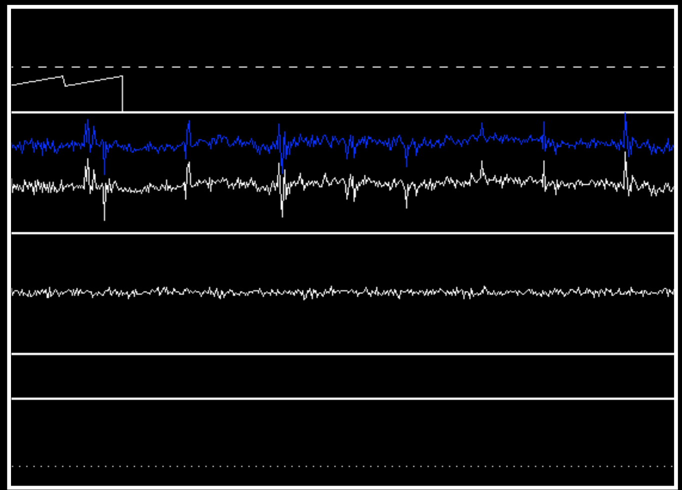
- Turbulence is smaller at 10μm, so it is less an issue than for AMBER
- Main issue is the thermal background !

Observation sequence:

- Fringe data (opd modulation)
  - HIGH_SENS (no chopping)
  - SCI_PHOT (chopping)
- Photometry (chopping on)
  - shutter A open
  - shutter B open



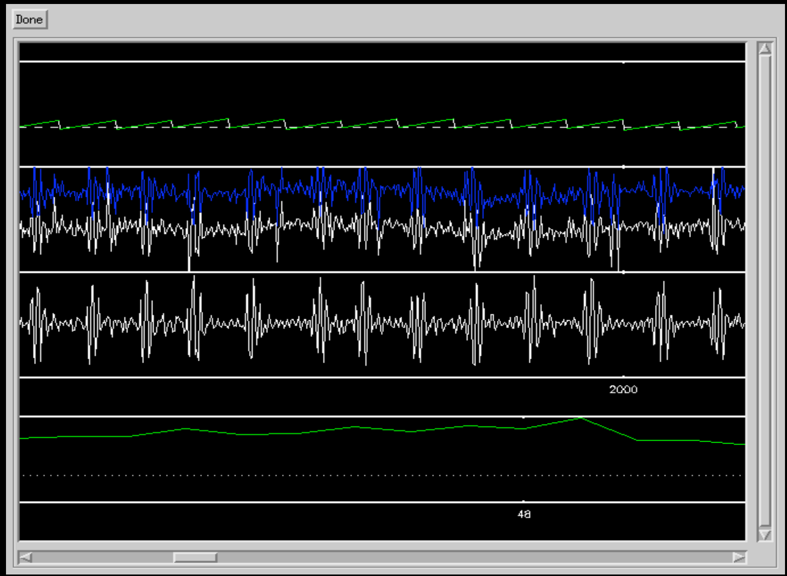
data with fringes



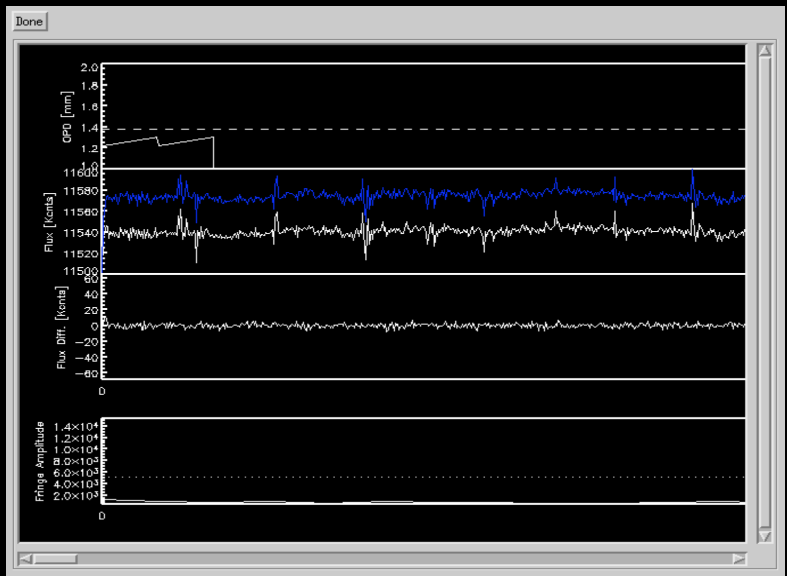
data without fringes

HIGH-SENS Principles

1.



2.



1. Observe fringes:
   - opd modulation
   - without chopping: background is removed by doing: $I = I^+ - I^-$

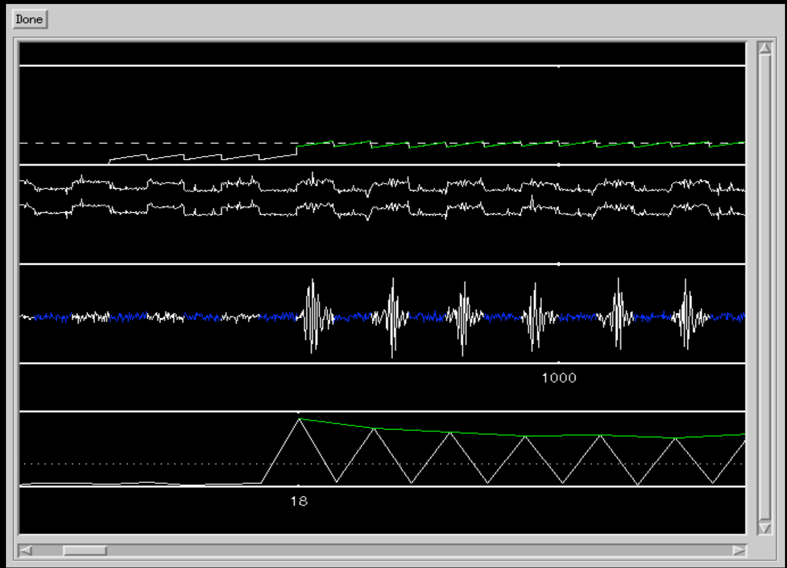
2. Observe the photometries:
   - no opd modulaiton
   - shutter in beam A and then B
   - chopping required

- Good sensitivity
- Photometry non simultaneous
  ⇒ bias in the visibilities

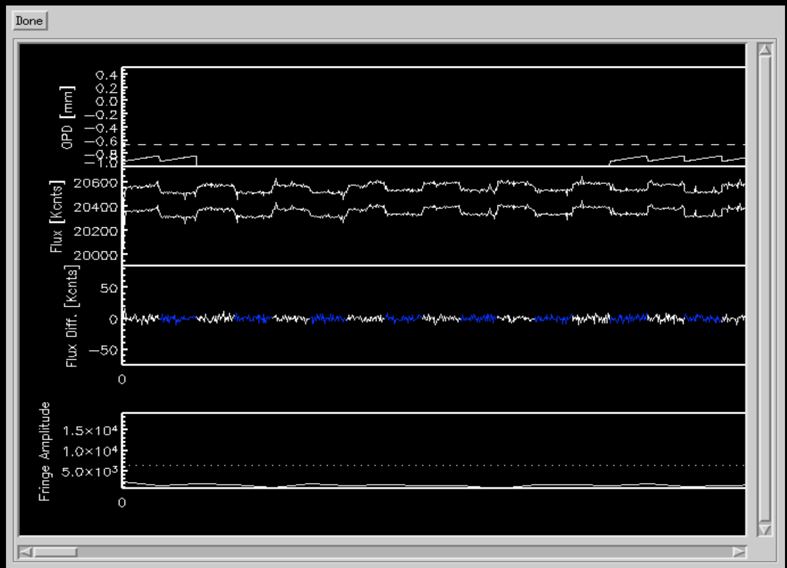
Dedicated to faint objects

SCI_PHOT Principles

1.



2.

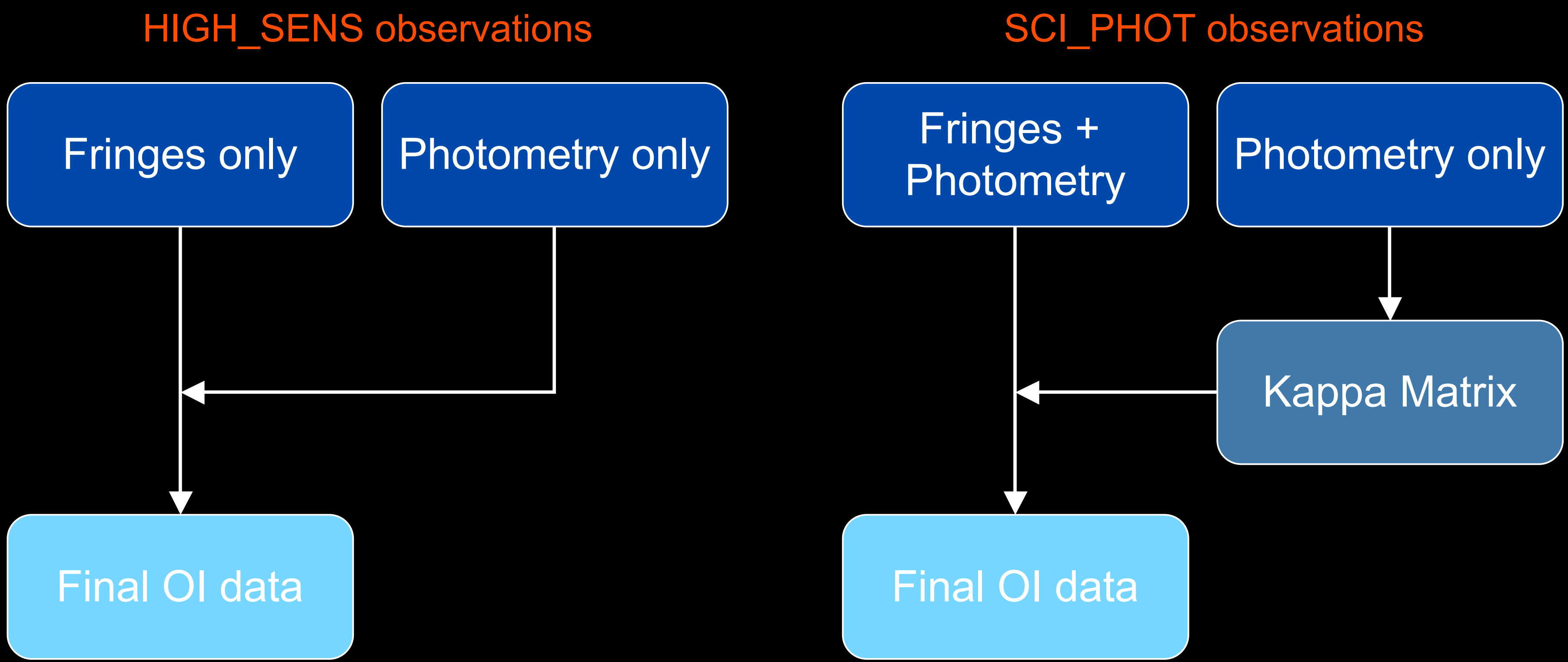


1. Observe fringes and photometry:
   - opd modulation
   - chopping required
2. Observe the photometries:
   - shutter in beam A and then B
   - chopping required
   - only used to know the splitting ratio photometry / fringes (Kappa matrix)

- Less sensitivity since the flux is split between photometry **and** fringes
- Photometry simultaneous with fringes
  - ⇒ less bias in the visibilities
  - ⇒ less photometric noise

Dedicated to bright objects

MIDI reduction work-flow



Visual inspection recommended on:

- reduced fringes
- reduced visibility set (histogram)

Several 'tuning' possible for experts:

- definition of masks
- … (I am not an expert!)

Outline

- What are we really looking for ?
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- Data reduction of the MIDI instrument
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Conclusions

- Interferometric observables are visibility and phase of the fringes
- Visibility is disturb by noises and atmospheric turbulence:
  - visibility is systematically reduced
  - therefore calibration is critical
- Absolute phase is lost but:
  - differential phase / closure-phase
  - these quantities are more robust that visibility to the turbulence
- Calibration errors should be carefully taken into account
- Data reduction is still a "research field", at least for the AMBER instrument
- Improvements are contemplated:
  - On-axis FINITO fringe-tracking (bright target)
  - Off-axis PRIMA fringe-tracking (faint target)
  - PACAM real-time logging…