Introduction to Visibility (and model fitting tips)

Euro Summer School
Circumstellar disks and planets at very high angular resolution
28 may-8 June 2007, Porto (Portugal)

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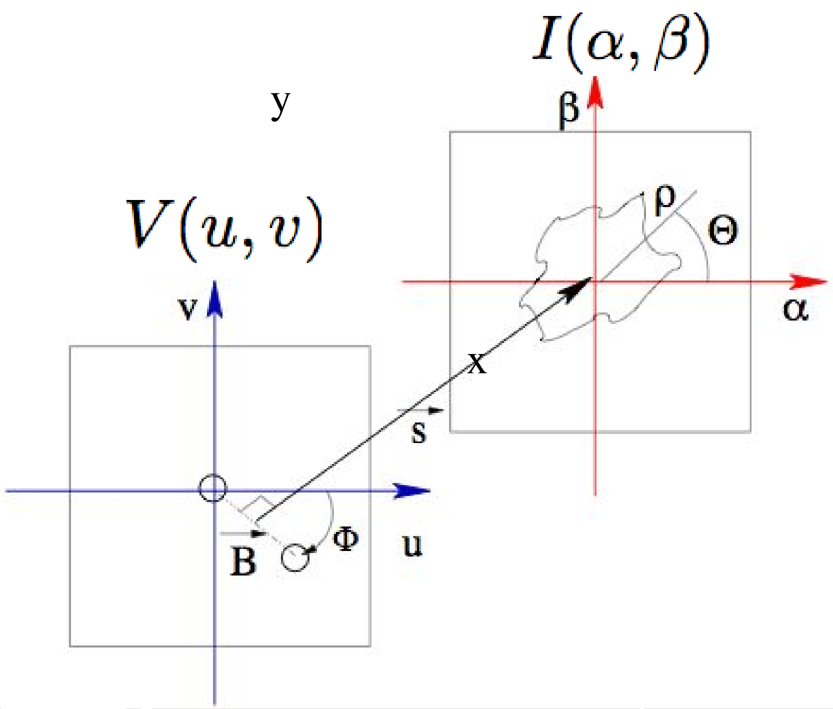
based on the presentation of J.P. Berger & D. Segransan
at the Goutelas Summer school (2006)

What is "visibility" ?

Practical application of the Van-Cittert / Zernike theorem:



The VCZ theorem links the intensity distribution of an object in the plane of the sky (in the far field) to the complex visibility measured in the array plane.

$$V(u,v) = \frac{\int\int I(\alpha,\beta)\exp^{-2i\pi(\alpha u+\beta v)}\,d\alpha d\beta}{\int\int I(\alpha,\beta)d\alpha d\beta}$$

This relation is a normalized **Fourier transform** (i.e. total flux does not matter).

Spatial frequency coordinates $u=B_x/\lambda$, $v=B_y/\lambda$

where B_x and B_y stand for projected baselines coordinates on the x and y axes of telescope

Imaging and visibility

Example : **resolved binary star** observed at the Special Astronomical Observatory (Zelentchouk)

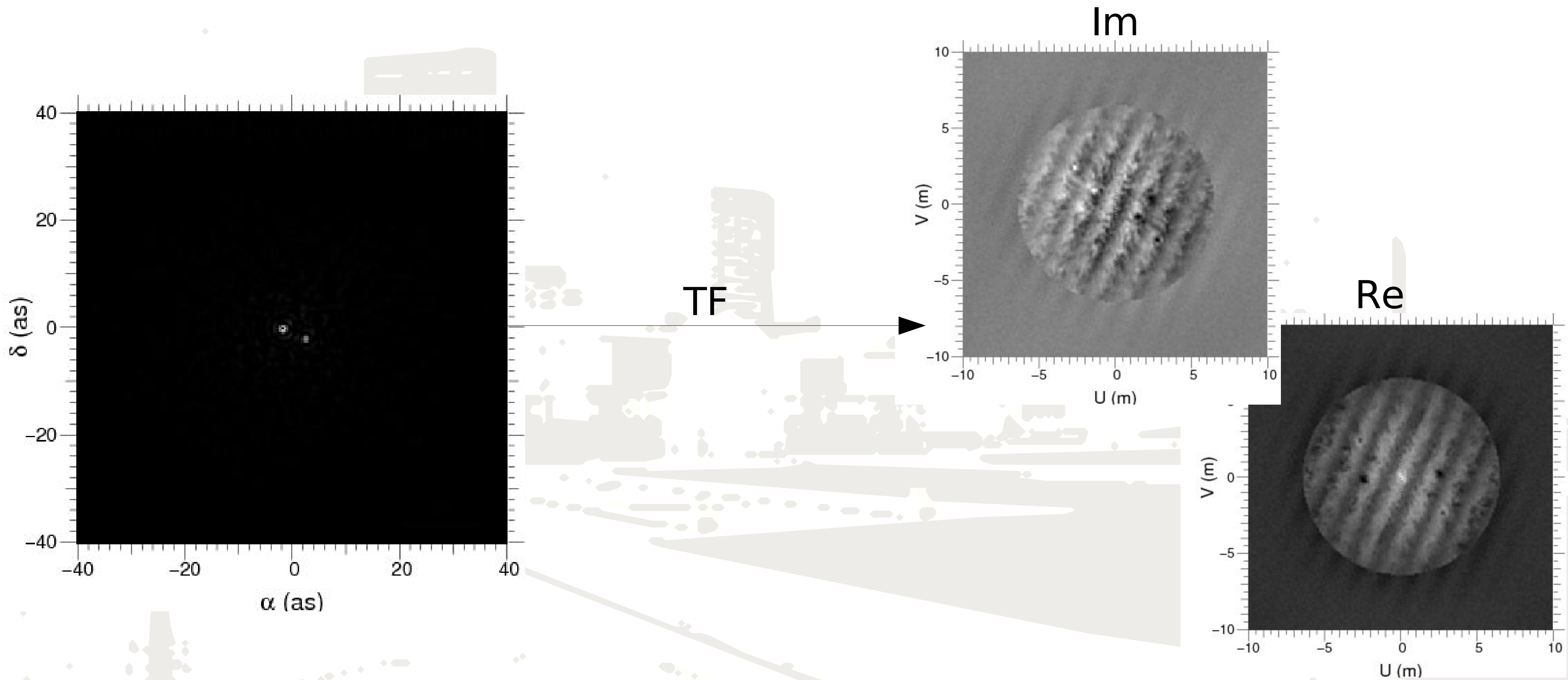


Image : I(x,y)=O*PSF

R(u,v), I(u,v) & cut-off frequency at D/λ

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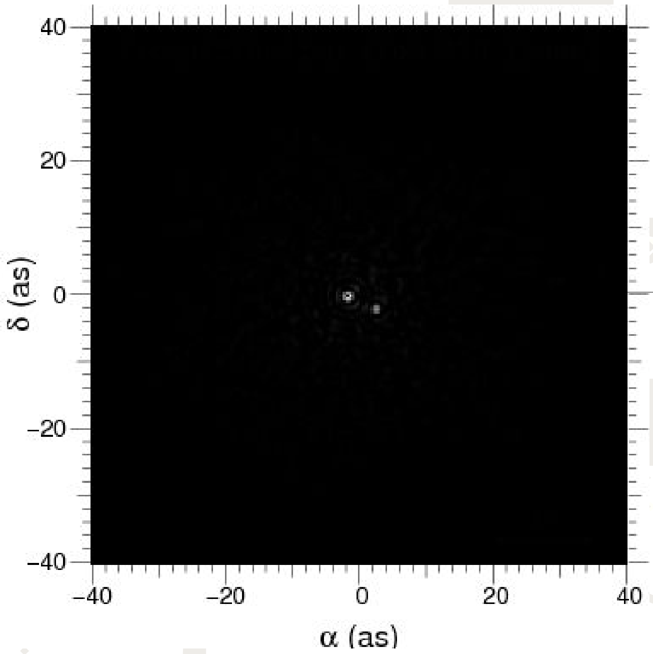
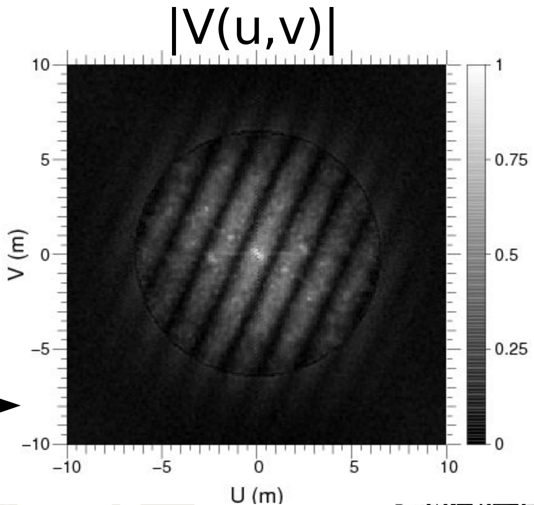


TF



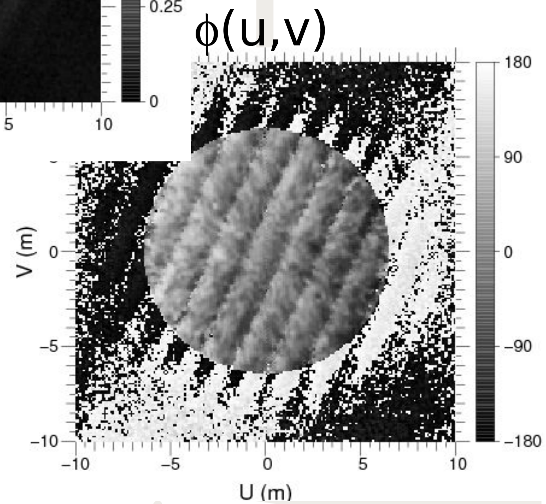


Image : I(x,y)=O*PSF

|V(u,v)|, φ(u,v) & cut-off frequency at D/λ

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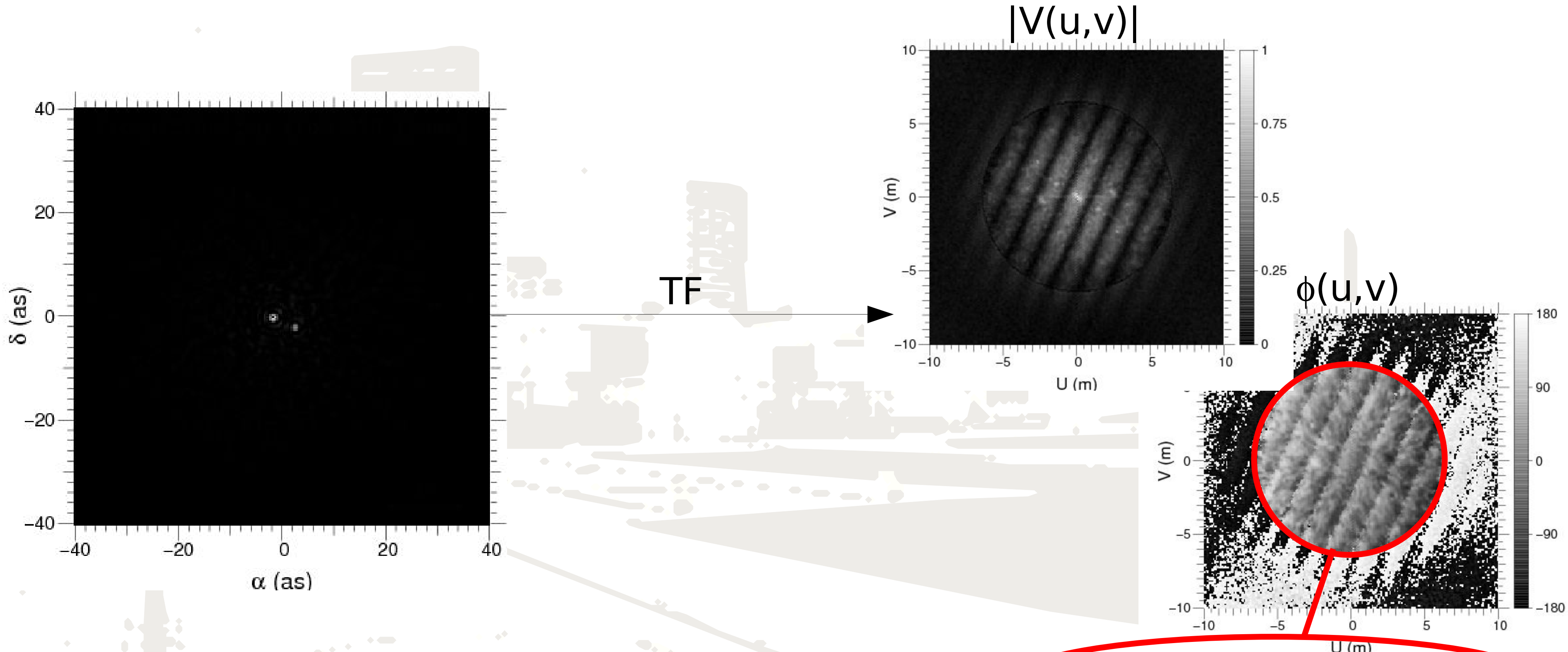
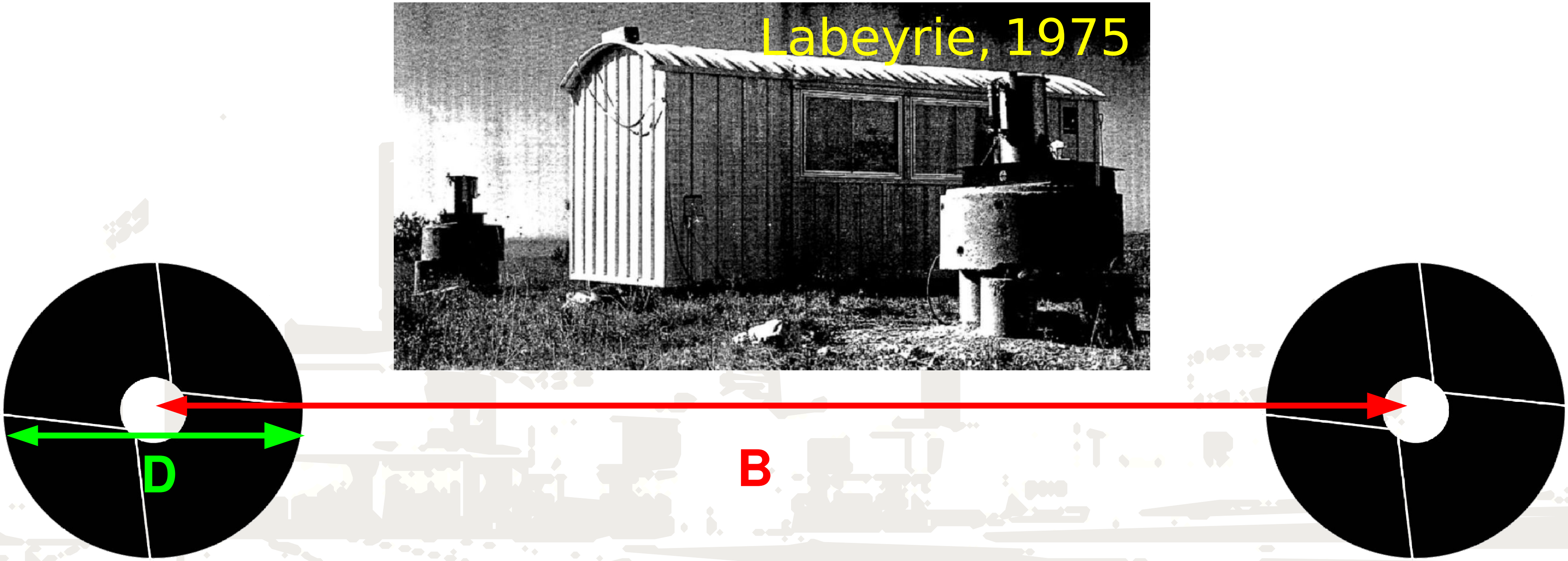


Image : I(x,y)=O*PSF |V(u,v)|, ϕ(u,v) & cut-off frequency at D/λ

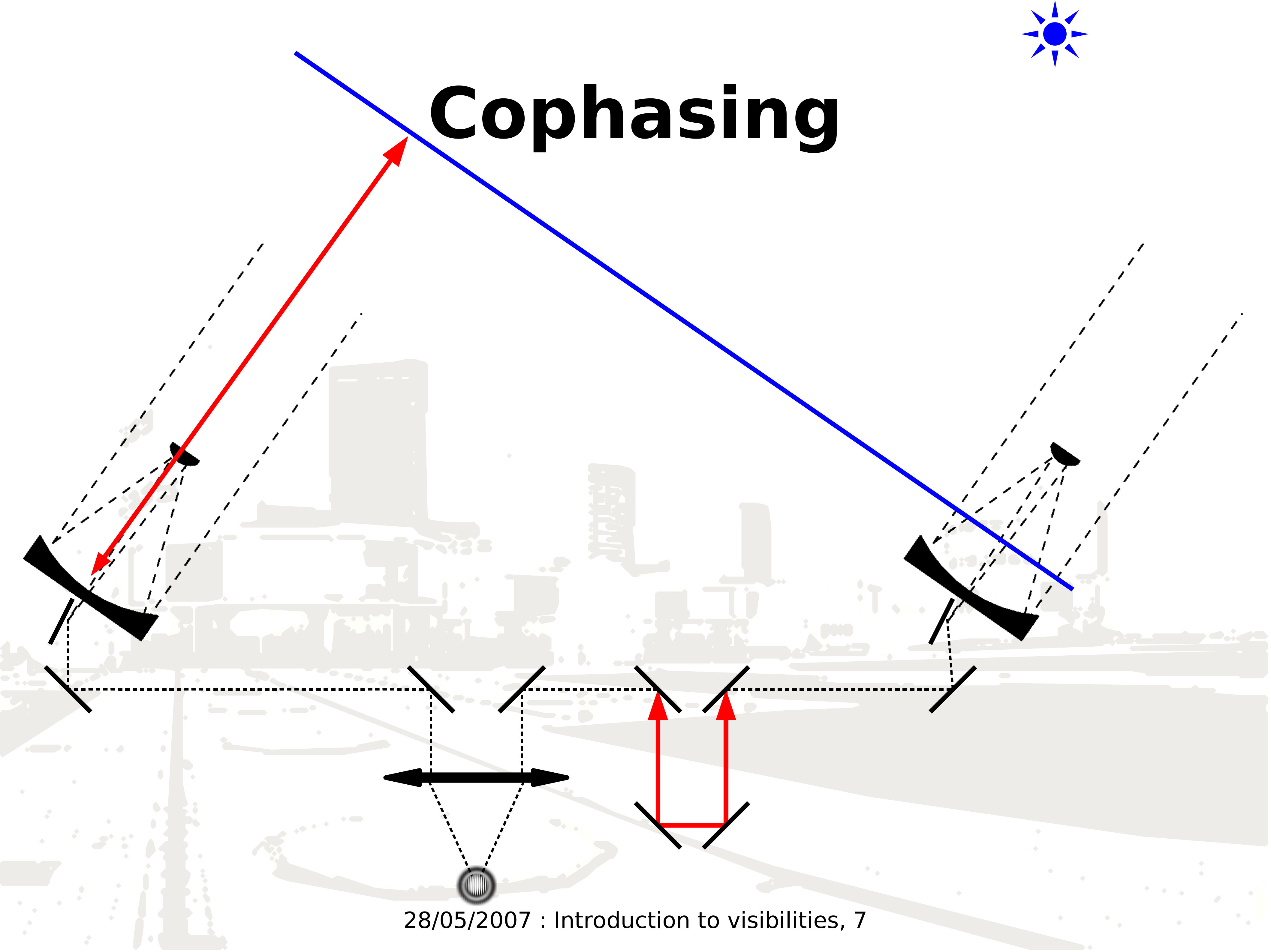
Long-baseline optical/IR interferometry



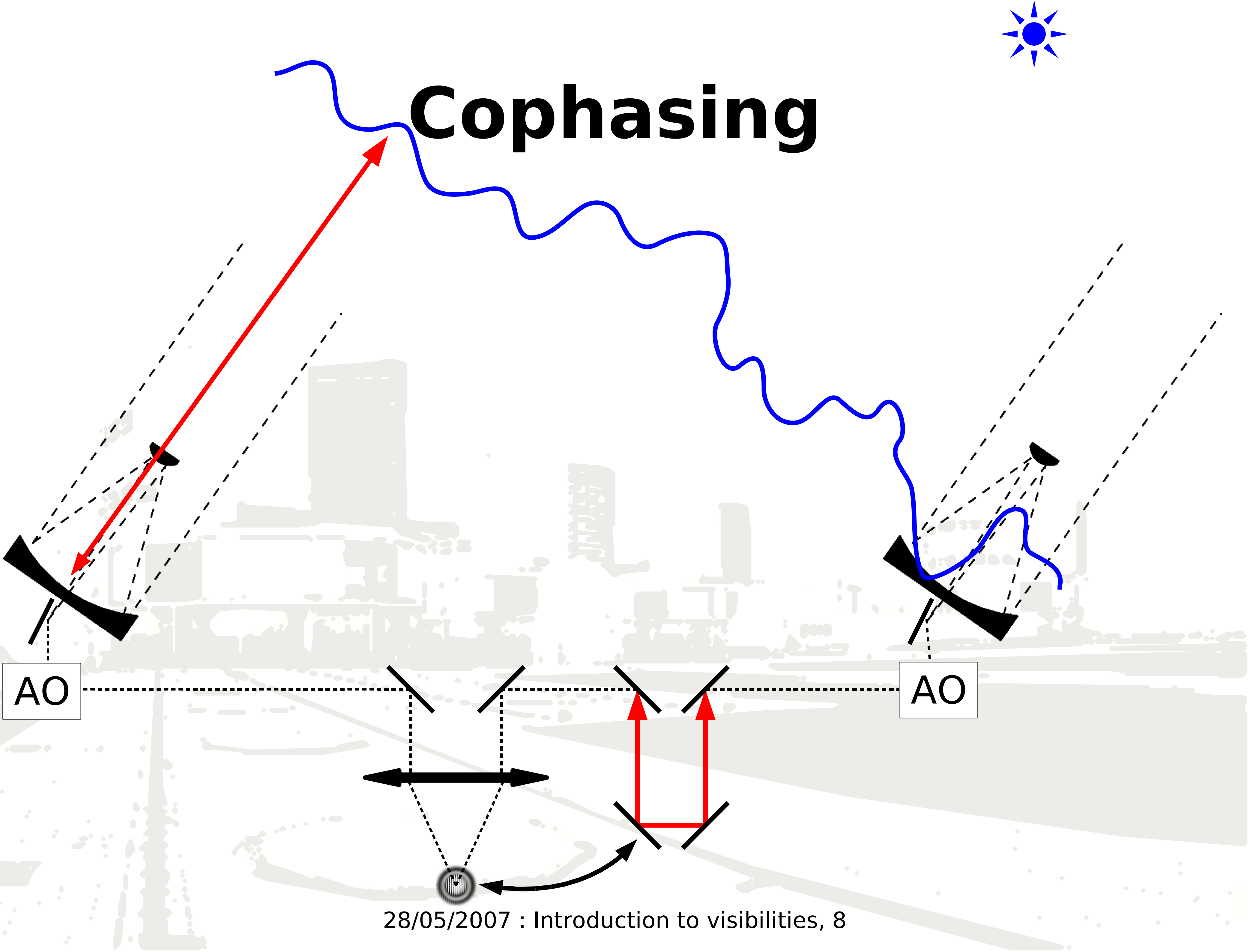
- Single-dish telescope : $0 < F_{ij} < \alpha D / \lambda$, $\alpha \sim 1$
- 2T Interferometer : $F_{ij} = 0$ and B / λ

$\Rightarrow$ Only one (or very few) spatial frequency is scanned at once by an interferometer

Cophasing



Cophasing



What visibility with interferometry ?

Example : **resolved binary star** at Special Astronomical Observatory (Zelentchouk)

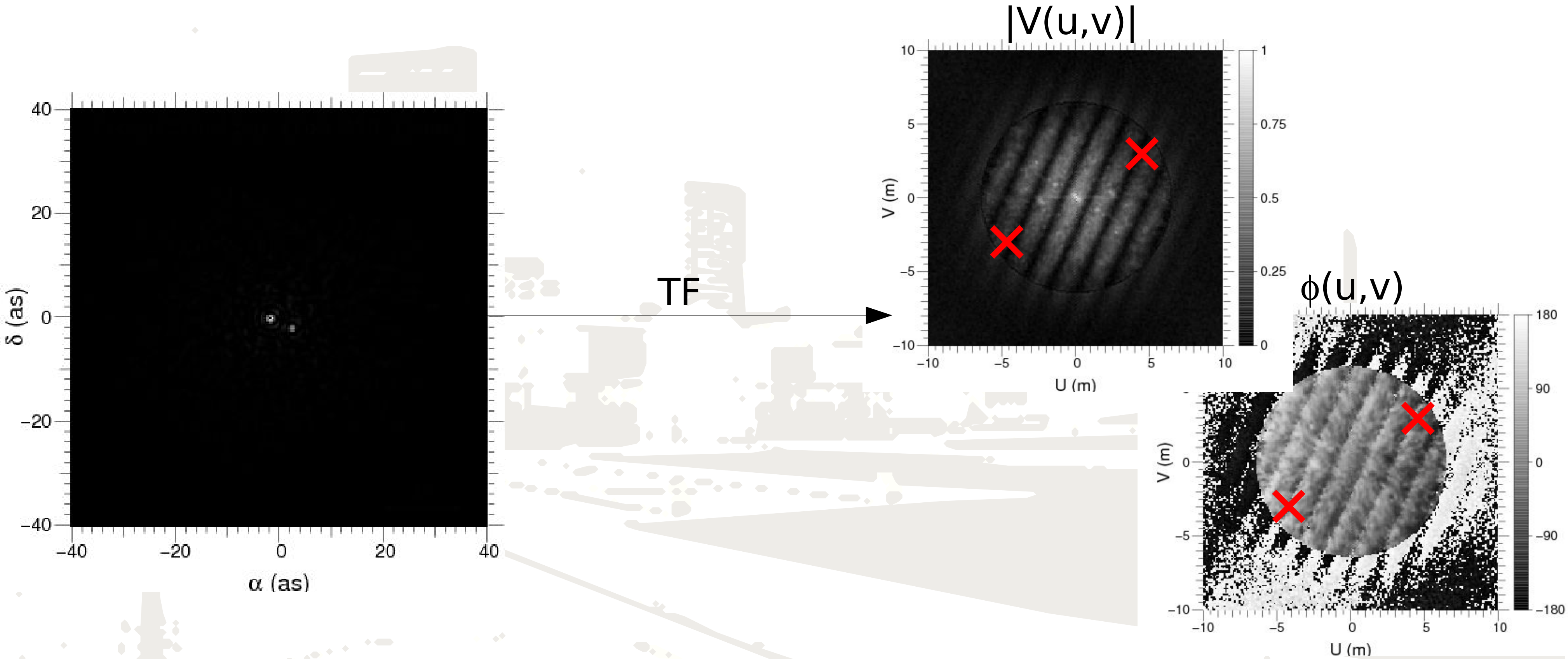


Image : $I(x,y)=O*PSF$ $|V(u,v)|$, $\phi(u,v)$ & cut-off frequency at D/λ

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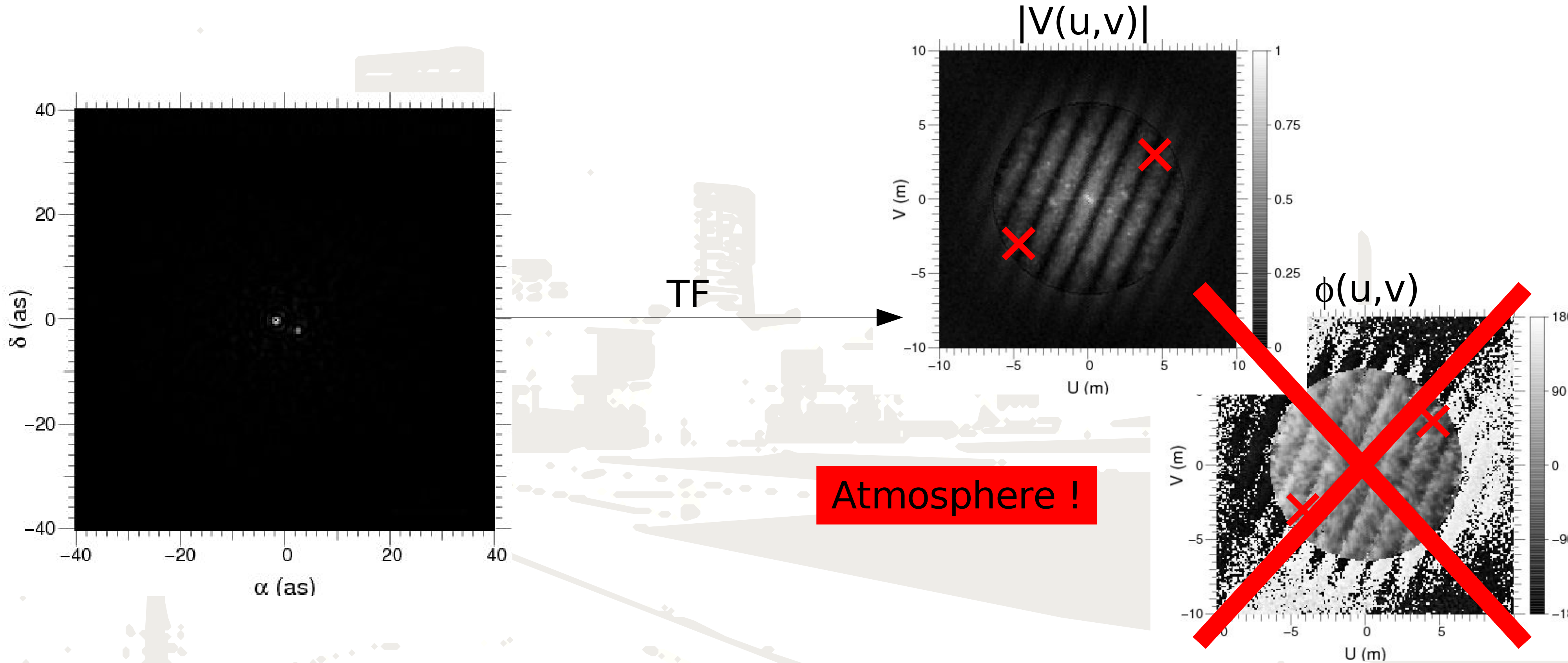

Image : I(x,y)=O*PSF |V(u,v)|, ~~ϕ(u,v)~~ & cut-off frequency at D/λ

What visibility with interferometry ?

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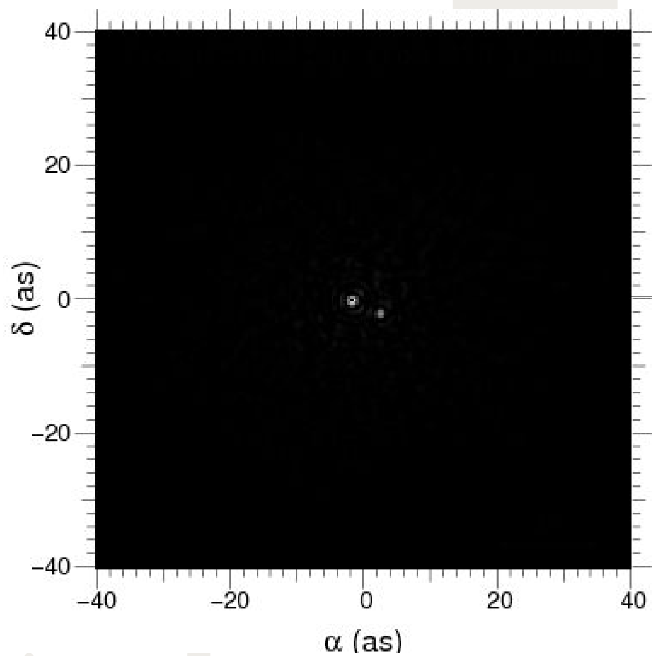
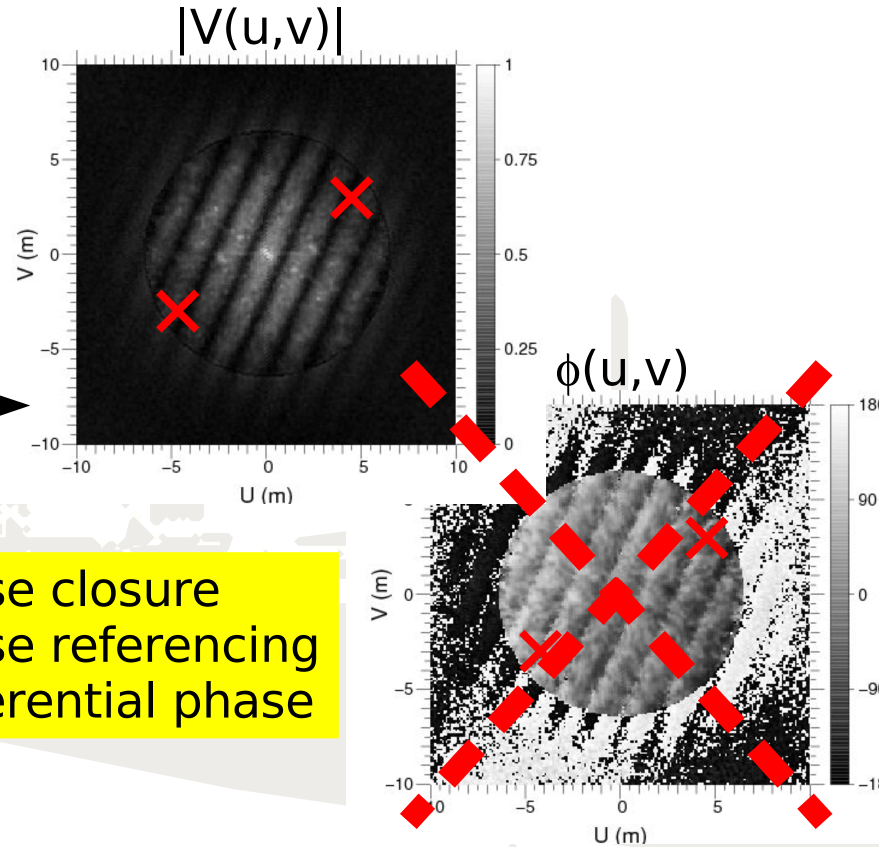




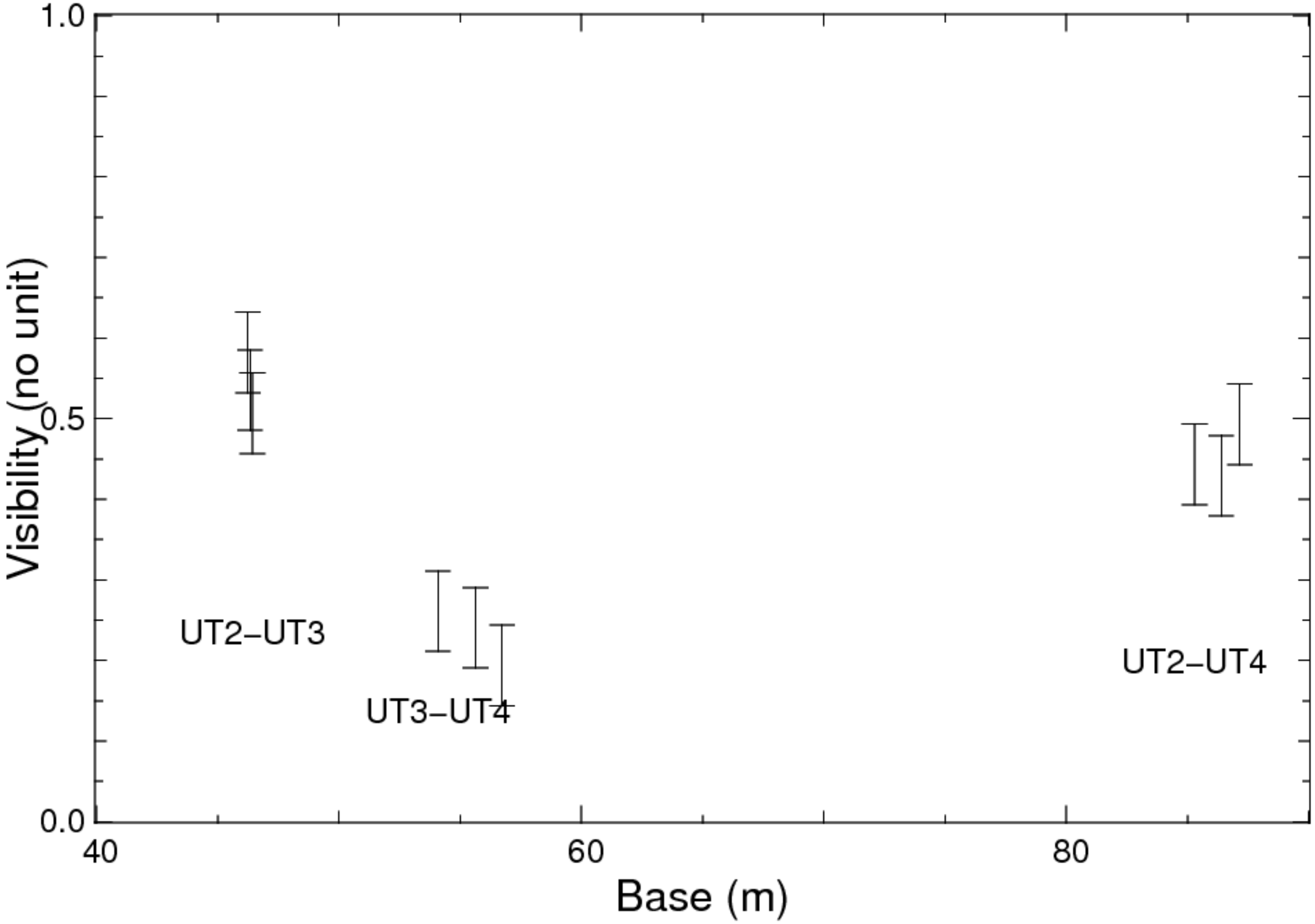
- Phase closure
- Phase referencing
- Differential phase

Image : I(x,y)=O*PSF

|V(u,v)|, ~~ϕ(u,v)~~ & cut-off frequency at D/λ

This session

is about what you can do with that ...



Simple first step : parametric analysis using basic visibility functions.

Model fitting

Basic issues of interpreting visibilities directly

Model fitting in the Fourier plane domain is attractive:

• Domain where interferometric measurements are made
=> errors easier to take into account (ex: Gaussian noise)

• When (U,V) plane sampling is poor (almost always the case)

• Is better when no imaging is possible (ex: variable source)

• Realistic in the VLTI AMBER and MIDI contexts

TENTATIVE OUTLINE

• **Modeling visibilities: principles.**

• **Some useful basic functions.**

• **Practical issues.**

• **Conclusion**

Ad-hoc modeling

Allows you to get a first idea of what you have observed!

- Use Fourier transform properties
- Use basic intensity distribution functions

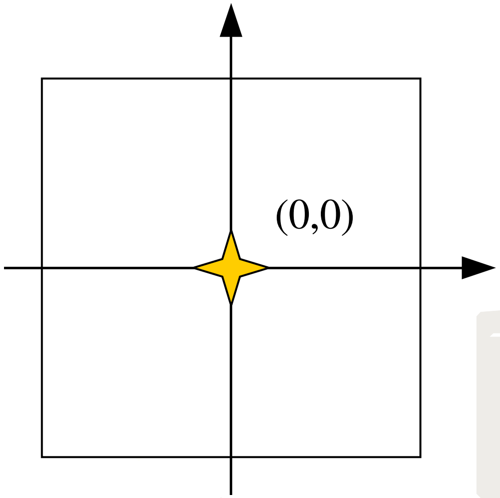
Important first step towards modelling with real physical models

Fourier transform properties:

- **Addition** $\mathrm{FT}\{f(x,y)+g(x,y)\} = F(u,v)+G(u,v)$
- **Convolution** $\mathrm{FT}\{f(x,y)\times g(x,y)\} = F(u,v).G(u,v)$
- **Shift** $\mathrm{FT}\{f(x-x_0,y-y_0) = F(u,v)\exp[2\pi i(ux_0+vy_0)]$
- **Similarity** $\mathrm{FT}\{f(ax,by)\} = \frac{1}{|ab|}F(u/a,v/a)$

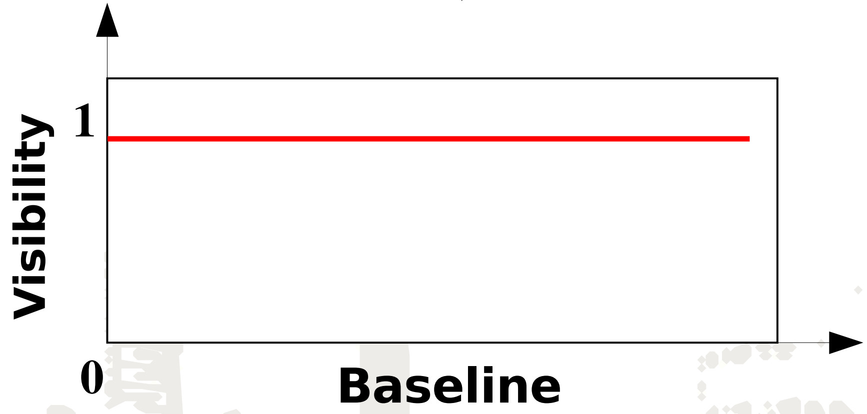
Point source function

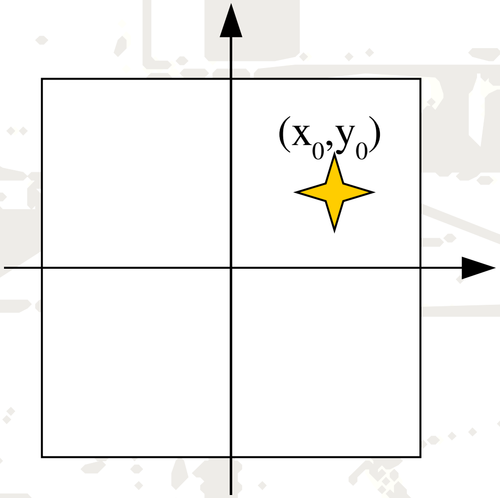
Use: Multiple stars



Centered source

$$I(x,y) = \delta(x,y) \Rightarrow V(u,v) = 1$$





Off-axis source

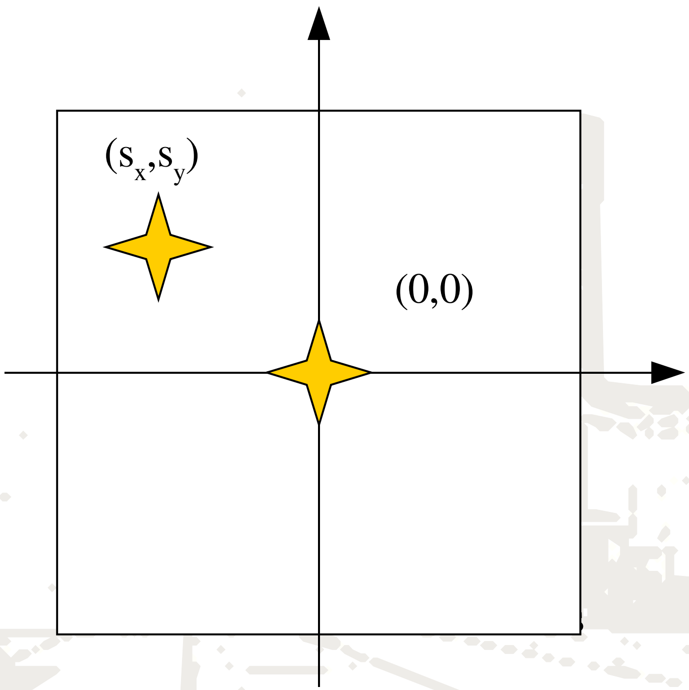
$$I(x,y) = \delta(x - x_0)\delta(y - y_0) \Rightarrow V(u,v) = \exp[-2i\pi(x_0 u + y_0 v)]$$

Amplitude =1 , linear dependence for the phase

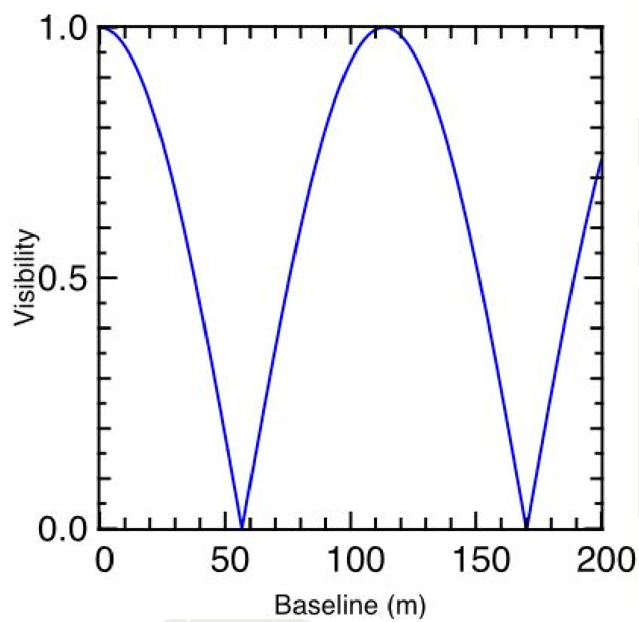
Binary star

Use: ... binary stars

$$A\delta(x,y) + B\delta(x - sx, y - sy) \text{ with } s = \sqrt{sx^2 + sy^2}$$



$$\begin{aligned} \mathrm{V(u,v)} &= \sqrt{\frac{1+r_{ab}^2+2r_{ab}\cos 2\pi \vec{L_b}\vec{s}/\lambda}{1+r_{ab}^2}} \\ \text{with } \mathrm{r}_{ab} &= \mathrm{A/B} \\ \text{with } \vec{L}_b &= \text{Baseline vector} \end{aligned}$$

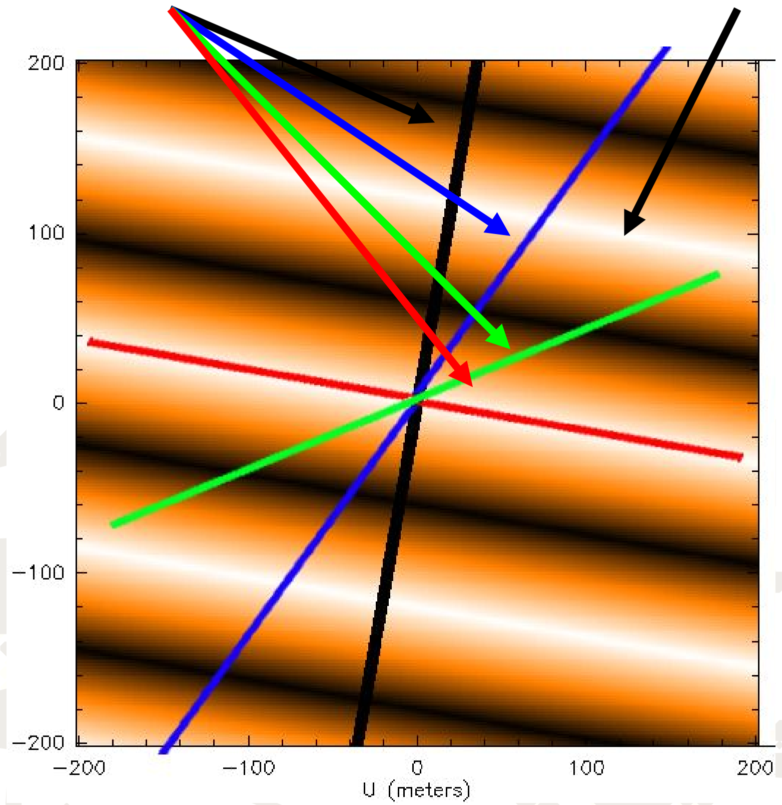


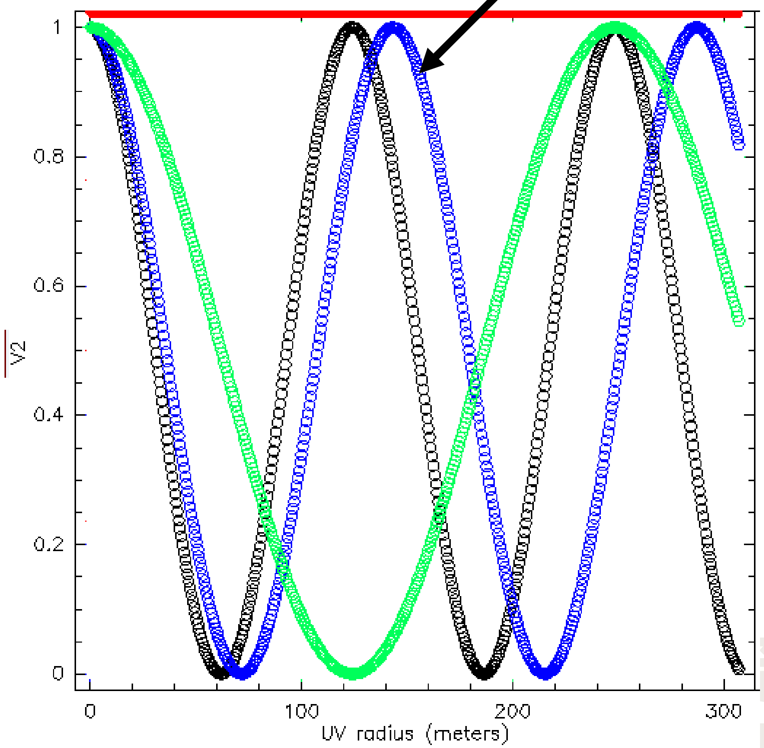
Binary star

Projection of baseline in the plane of sky

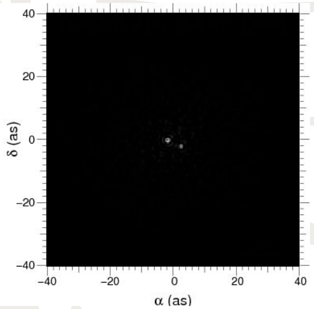
The visibility amplitude squared in (uv) plane

Squared visibility curves for three baselines as a function of baseline length





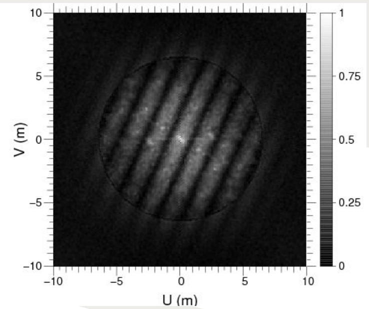
image



Remember:

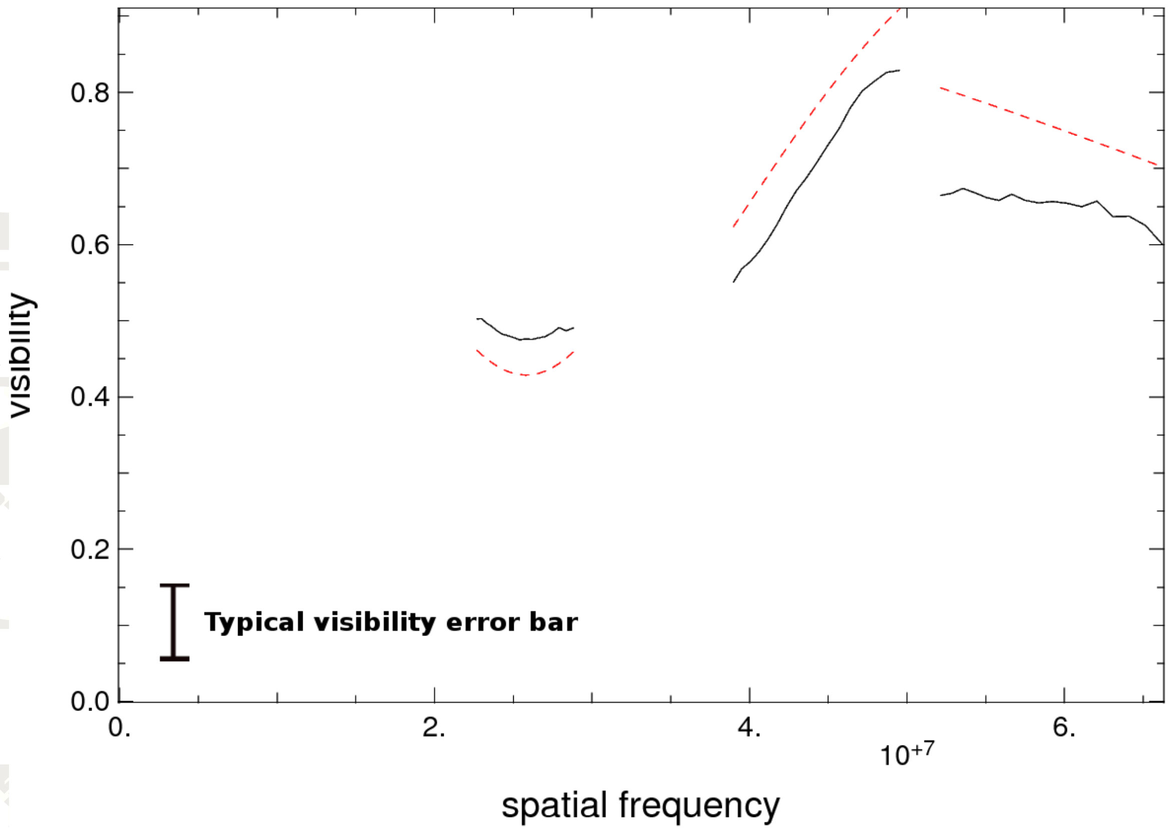
TF →

Visibility



Binary star (exemple 1)

Binary star visibility curve as a function of spatial frequency
(red = model, black = AMBER/VLTI observation)

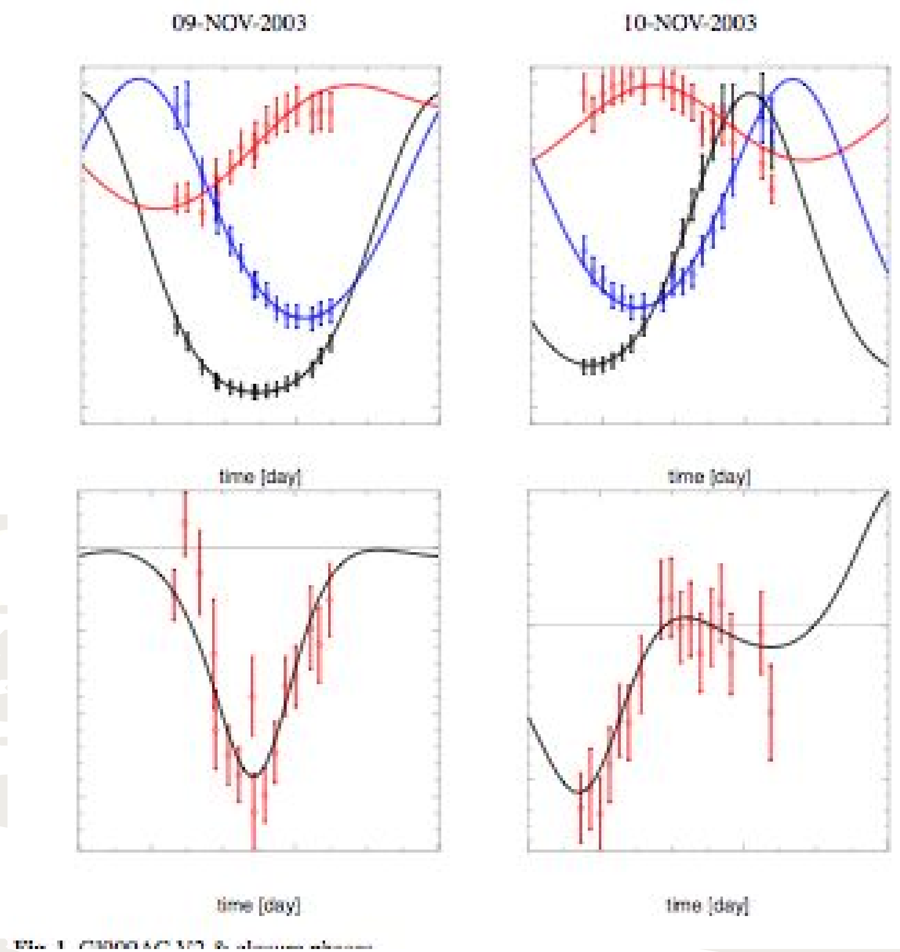


Valat et al., in prep.

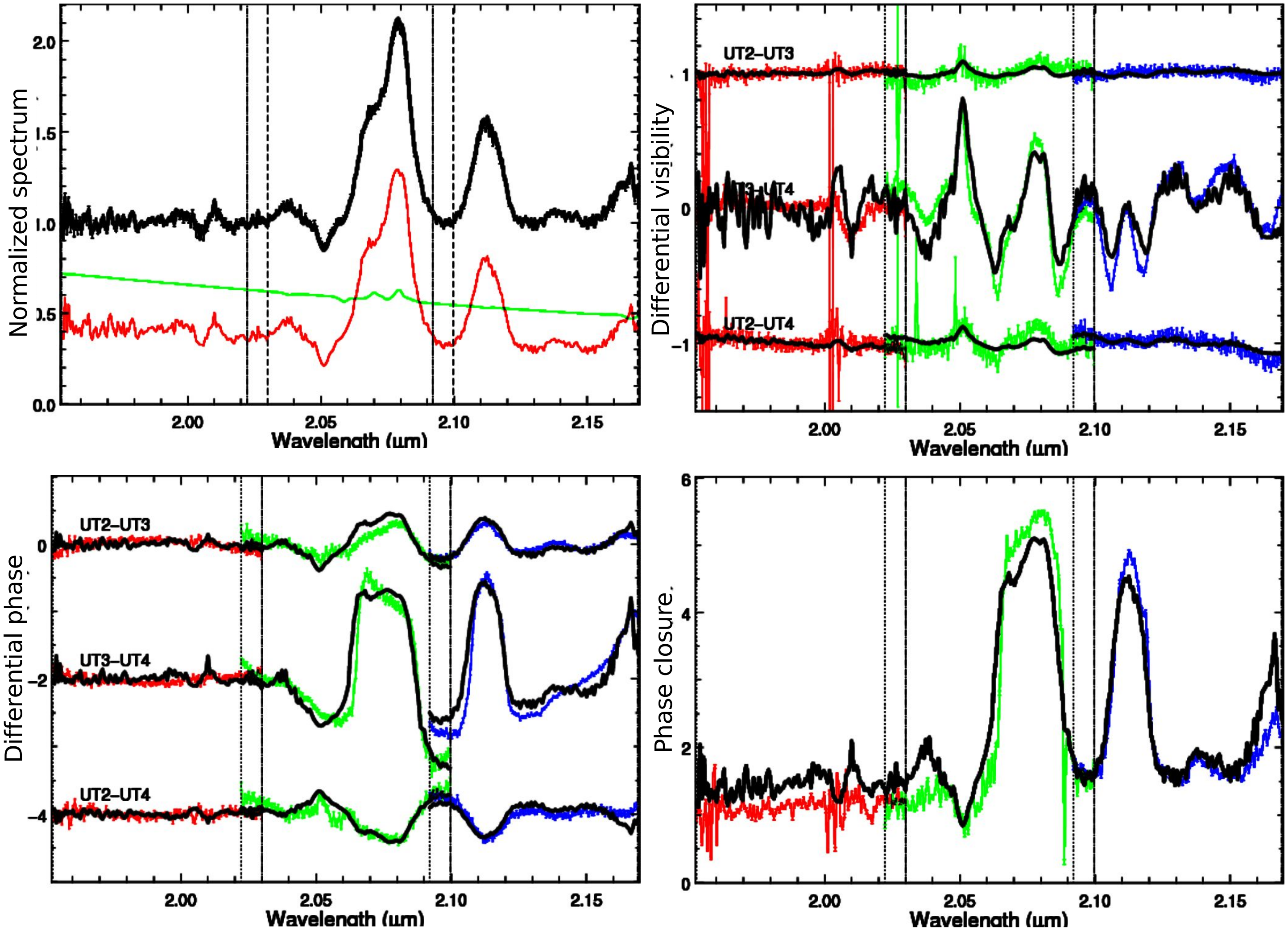
Binary star (exemple 2)

Rotation of stars along the orbit and of projected baseline makes the changes in visibilities and closure phase

(IOTA observations, Segransan 2006, Goutelas summer school)



Binary star (exemple 3)

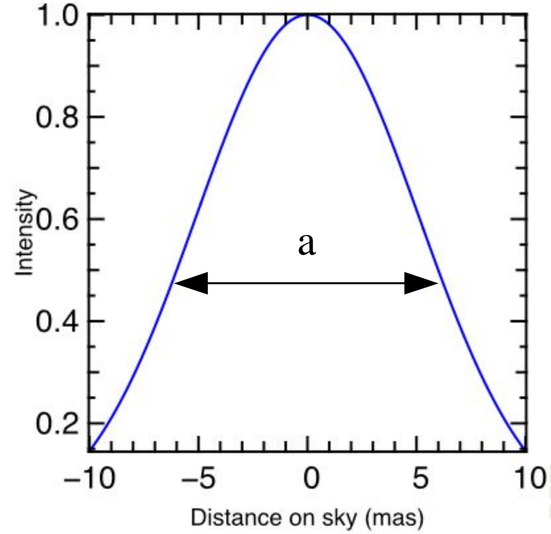


Spectrally varying flux ratio makes it working !

γ^2 Vel, Millour et al. 2007

Gaussian brightness distribution.

Use: Estimate for angular sizes of envelopes, disks, etc.

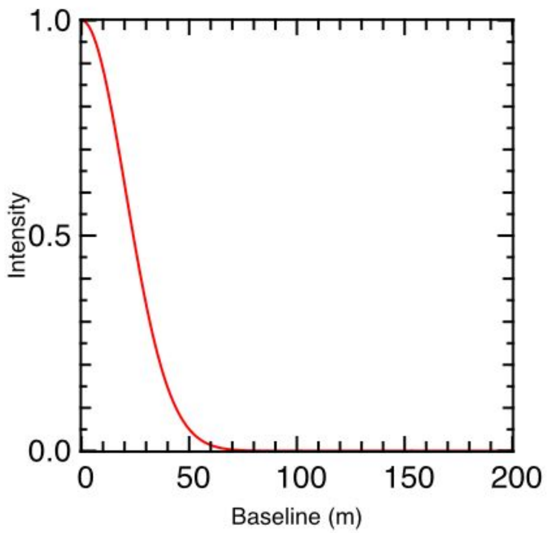


$$I(r) = \frac{I_0}{\sqrt{\pi/4\ln 2a}} \exp(-4\ln 2r^2/a^2)$$

Where a = FWHM intensity, I_0 = Peak intensity

and

$$r = \sqrt{x^2+y^2}$$

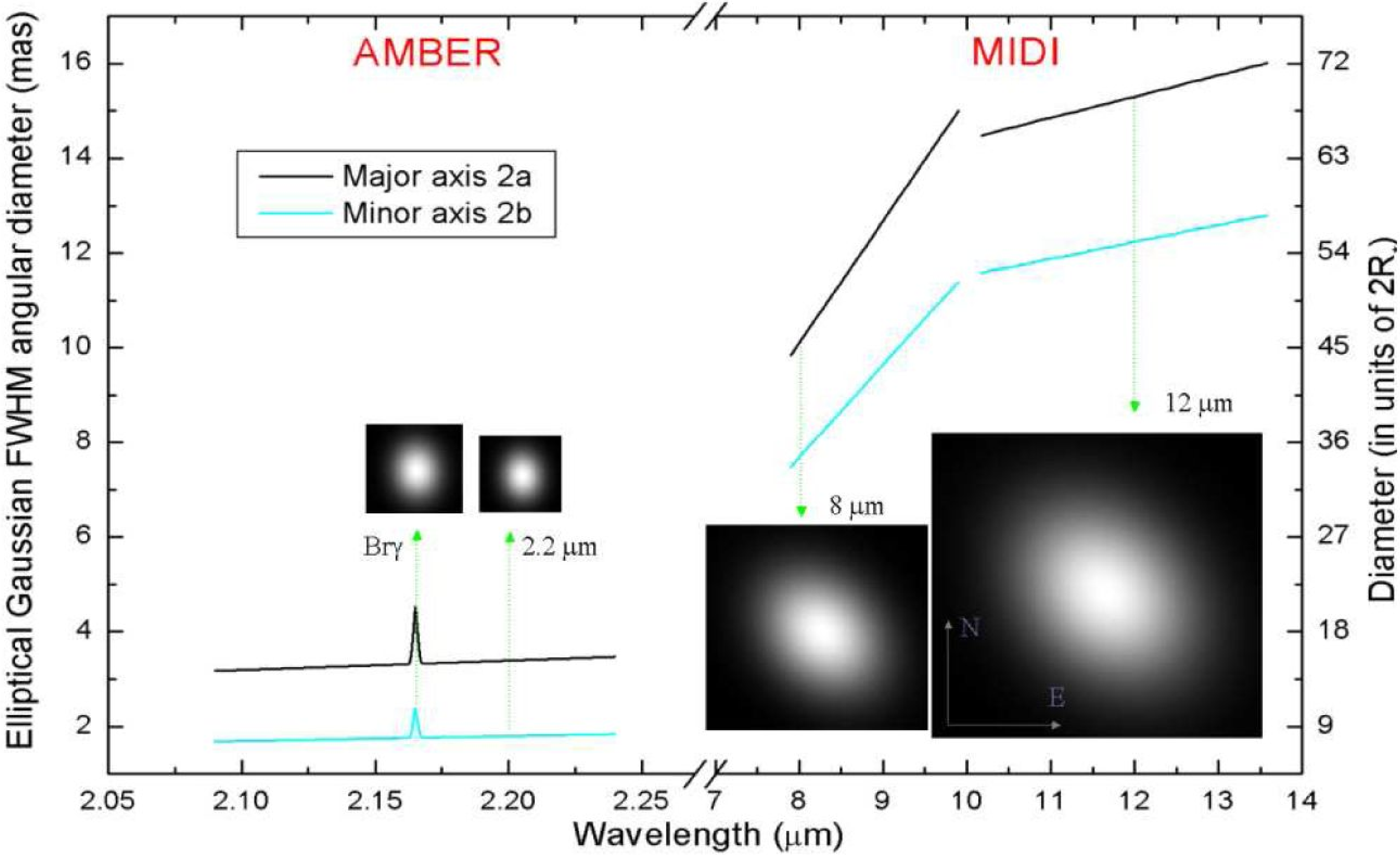


$$V(\rho) = \exp[-(\pi a \rho)^2/(4\ln 2)]$$

Where

$$\rho = \sqrt{u^2+v^2}$$

Gaussian (example)

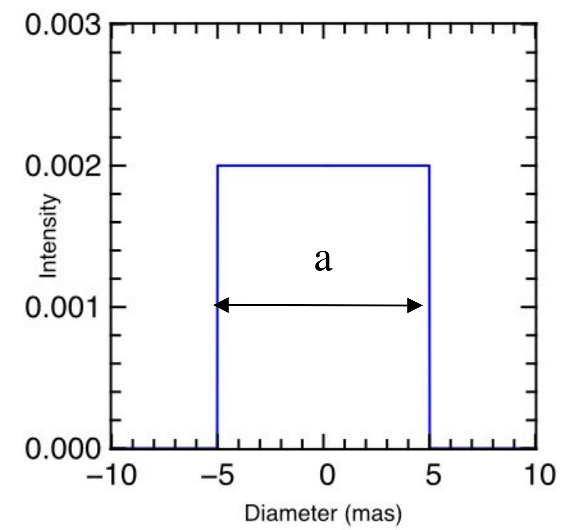


Dominiciano da Souza et al A&A 2007

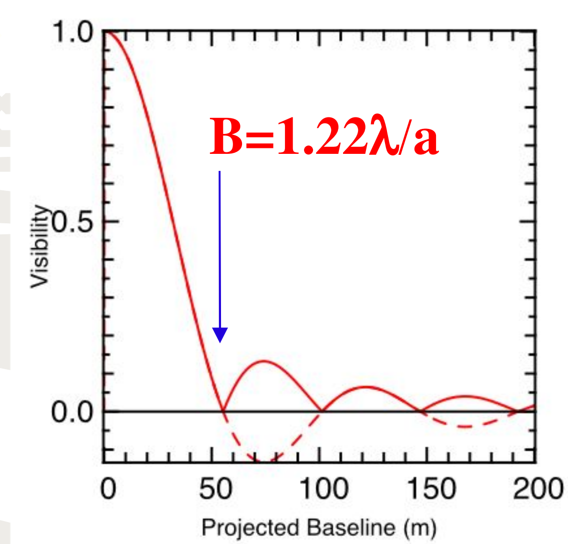
Uniform disk

Use: aproximation for brightness distribution of photospheric disk.

$$I(r) = 4/(\pi a^2), \text{if } r = \sqrt{x^2 + y^2} \leq a/2$$
$$I(r) = 0 \quad \text{otherwise}$$



$$F(\rho) = \frac{J_1(\pi a \rho)}{\pi a \rho} \text{with} \rho = \sqrt{u^2 + v^2}$$



a = diameter

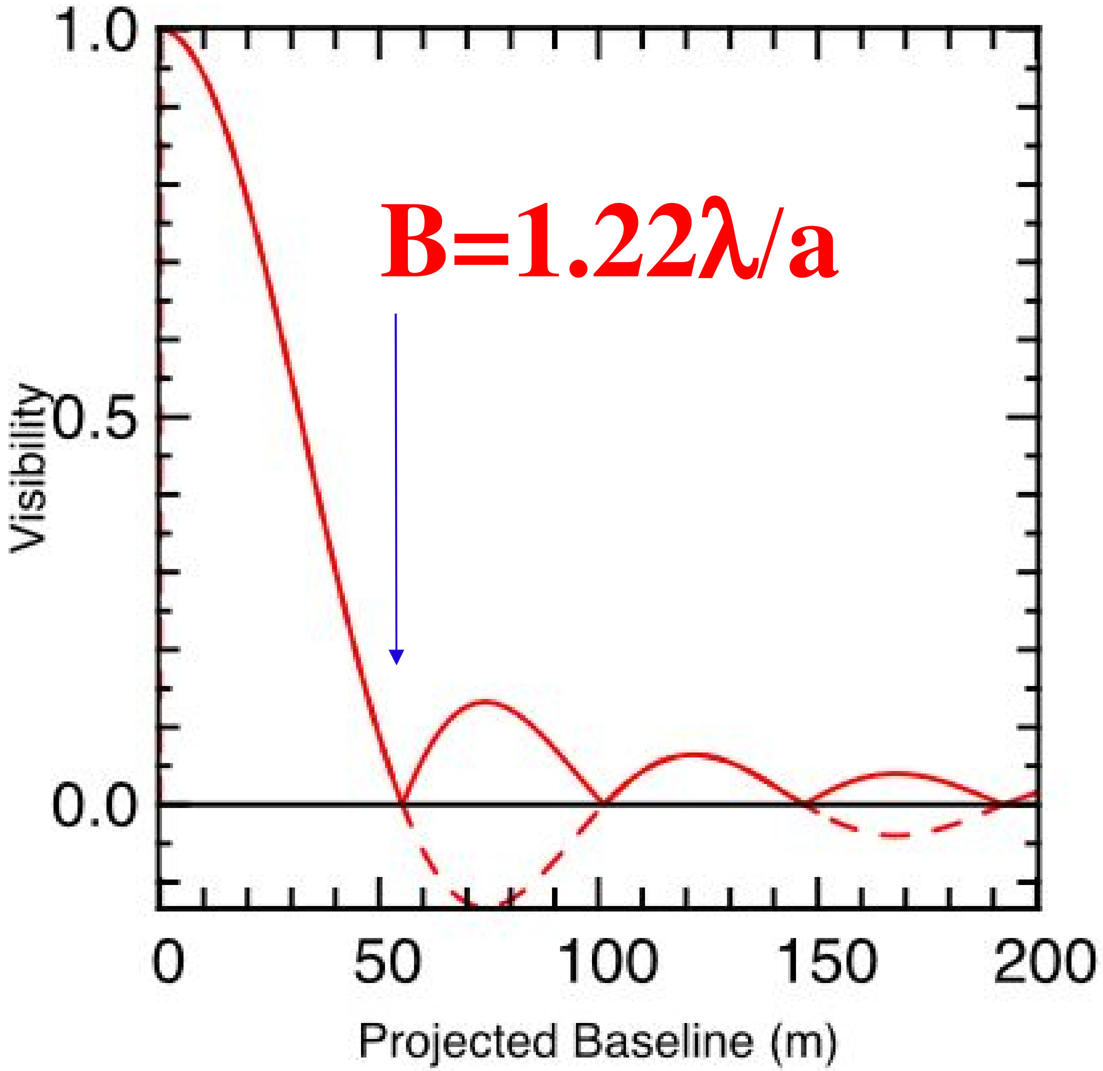
Sophistication of the model

I= f(r), limb darkening

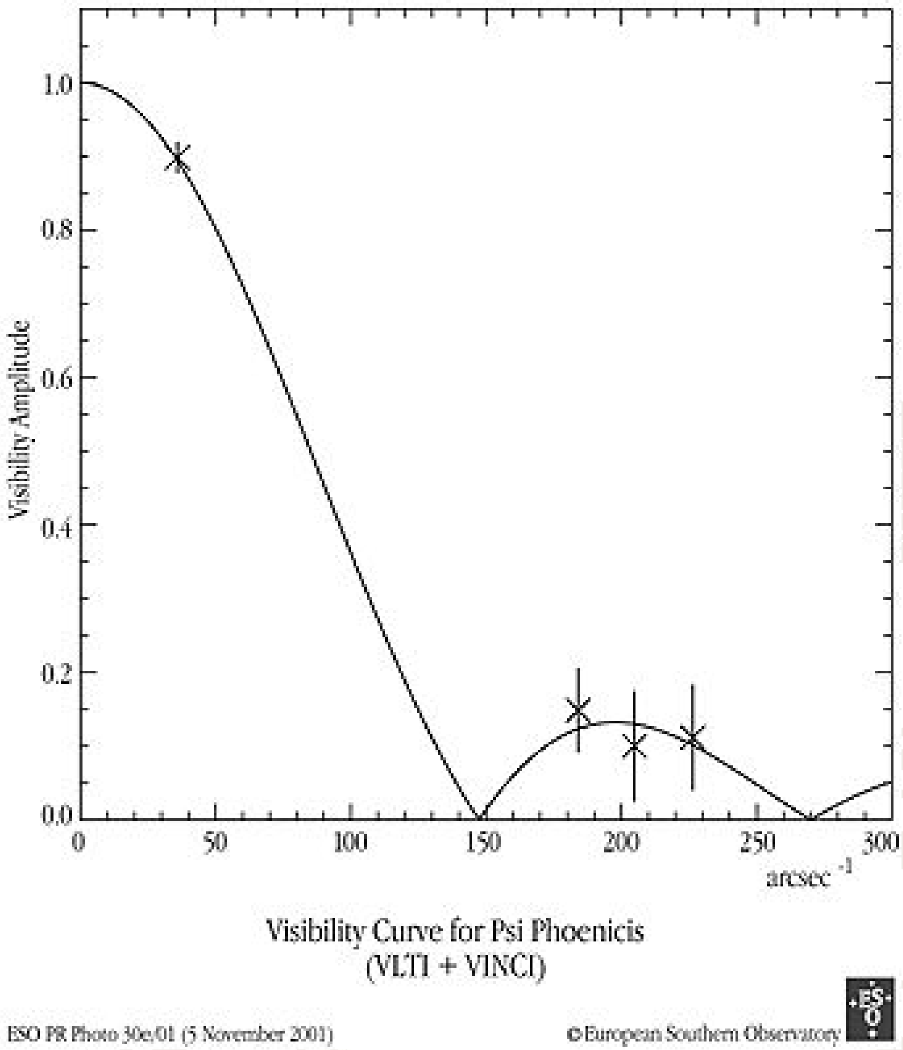
Cf Hankel transformation

Uniform disk

Use: **aproximation for brightness distribution of photospheric disk.**

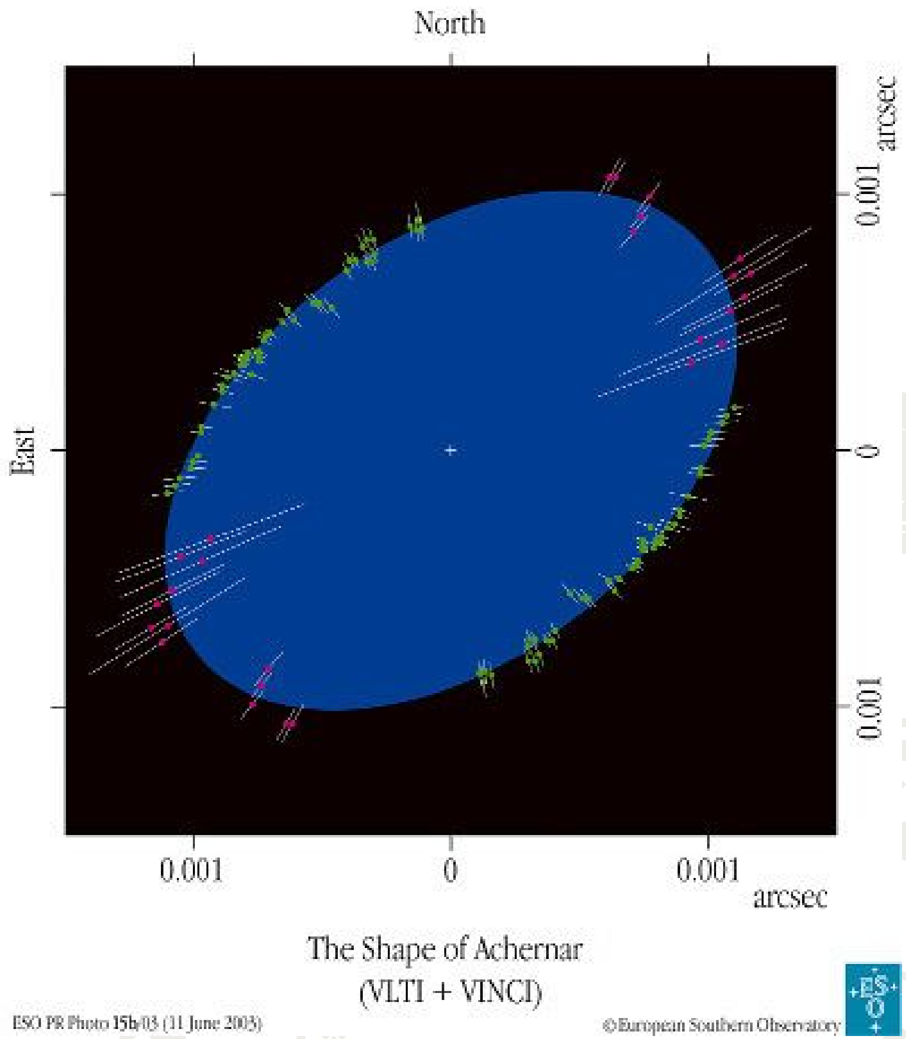


Uniform disk (example 1)



- Determination of uniform disk diameter of ψ Phe with VLTI/VINCI
- Second lobe points are the most constraining

Uniform disk (example 2)

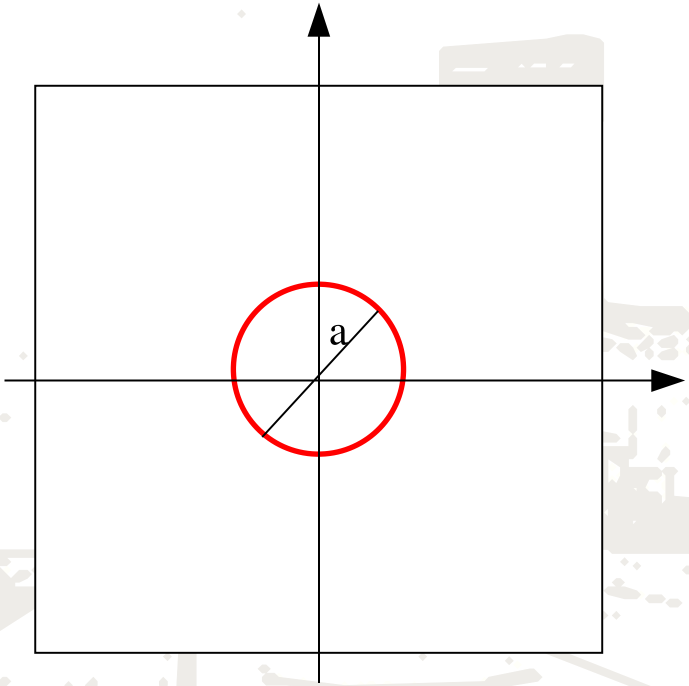


- Determination of uniform diameter of Archenar (VLTI/VINCI)
- Different positions angles shows evidence for flattening due to to fast rotation

Dominiciano da Souza et al A&A 2003

Ring

Use: complex centro-symmetric structure



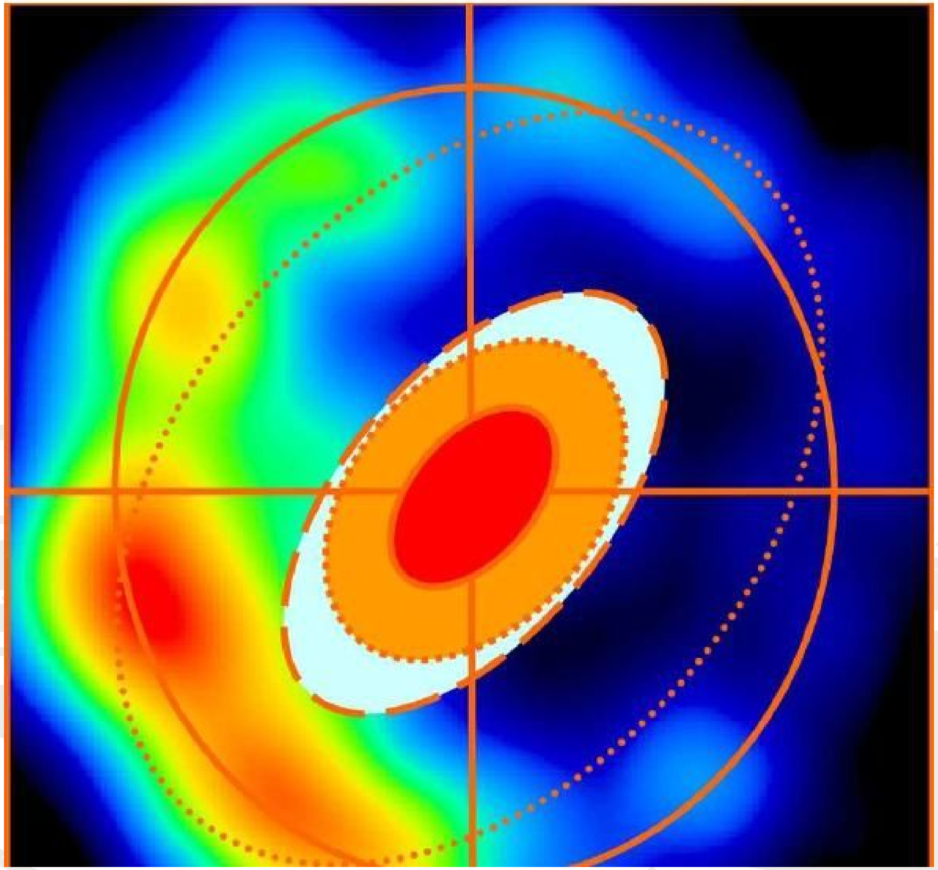
$$I(r) = 1/(\pi a)\delta(r - a/2)$$

$$V(\rho) = J_0(\pi a \rho)$$

Ring (example)

- RS Oph aspherical Nova explosion

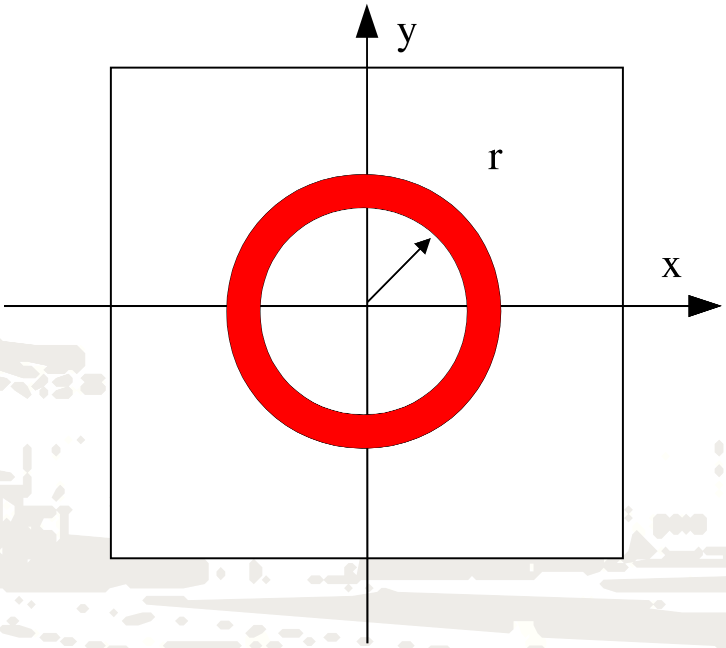
Chesneau et al., A&A 2007



Circularly symmetric object

e.g: an accretion disk made of a finite sum of annulii with different effective temperatures

Circularly symmetric component I (r)
centered at the origin of the (x,y) coordinate system.



The relationship between brightness distribution and visibility is a ***Hankel function***

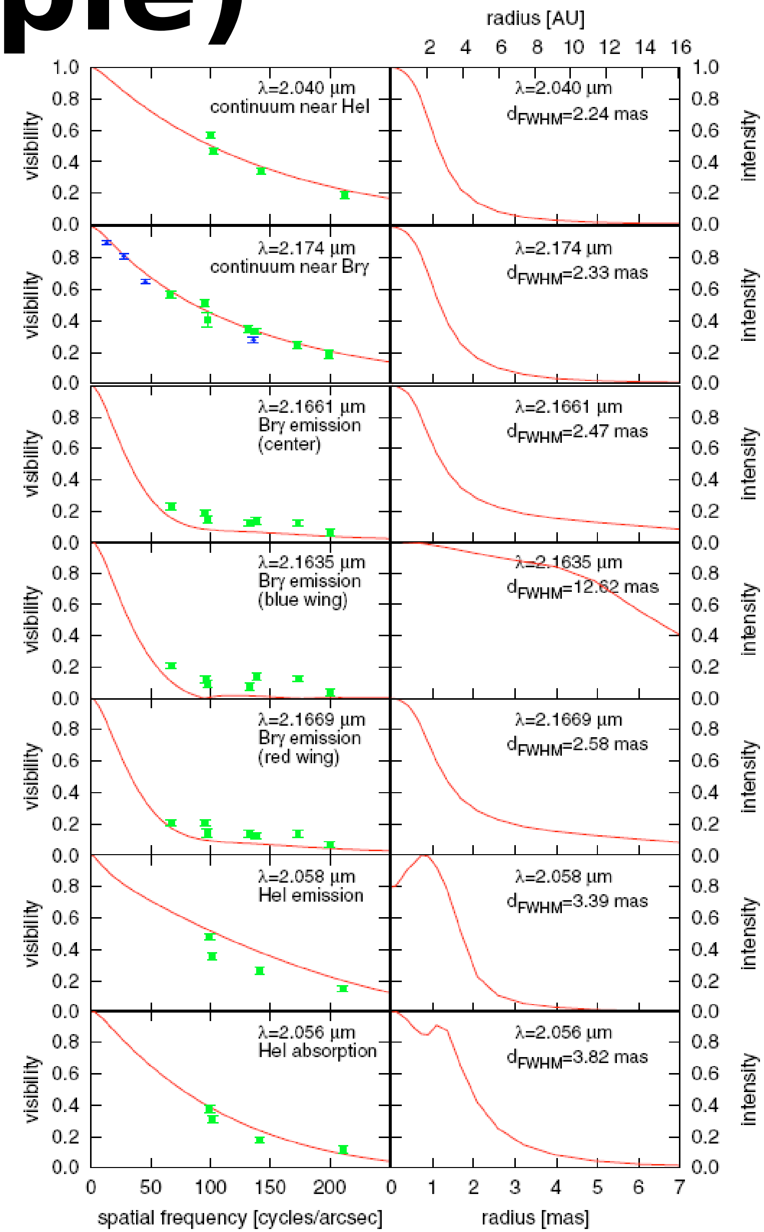
$$V(\rho) = 2\pi \int_0^\infty I(r) J_0(2\pi r \rho) r dr$$

with $r = \sqrt{x^2 + y^2}$ and $\rho = \sqrt{u^2 + v^2}$

Circularly symmetric object (example)

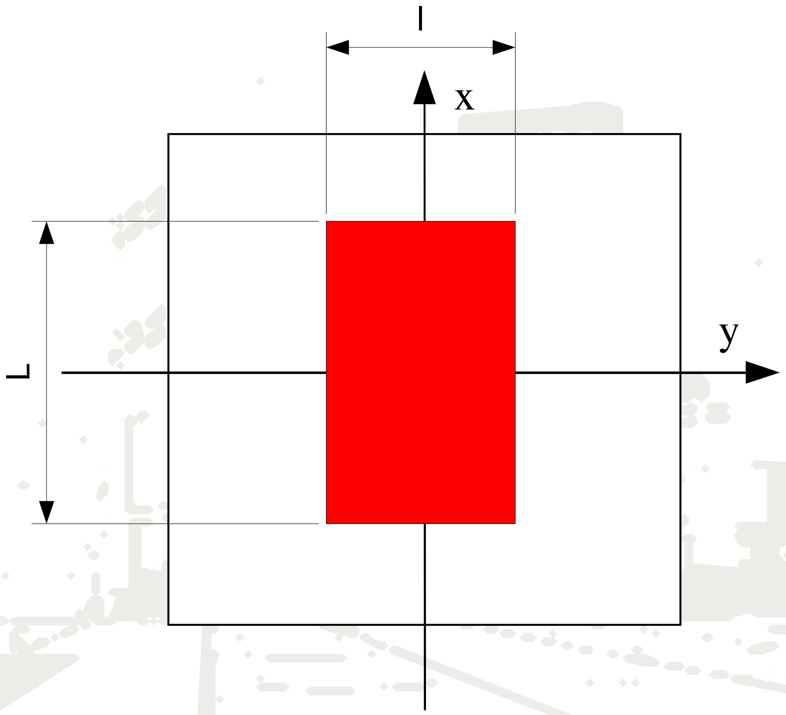
- Optically thick wind around η Car (Hillier models gives intensity profiles)

Weigelt et al., A&A 2007



Pixel

Basic brick of an image !

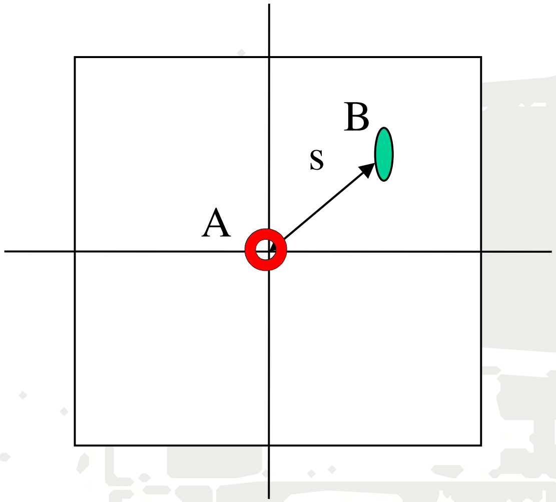


- I(x,y) = 1 if |x| < l and |y| < L
- I(x,y) = 0 otherwise

$$V = \frac{\sin(\pi x l)\sin(\pi y L)}{\pi^2 x y l L}$$

Resolved multi-structure

Use: Describing any multicomponent structure.



$$V^2(u,v) = \frac{r_{ab}^2 * V_a^2 + V_b^2 + 2r_{ab}|V_a||V_b|\cos(2\pi\vec{L_b}\vec{s}/\lambda)}{(1+r_{ab}^2)}$$

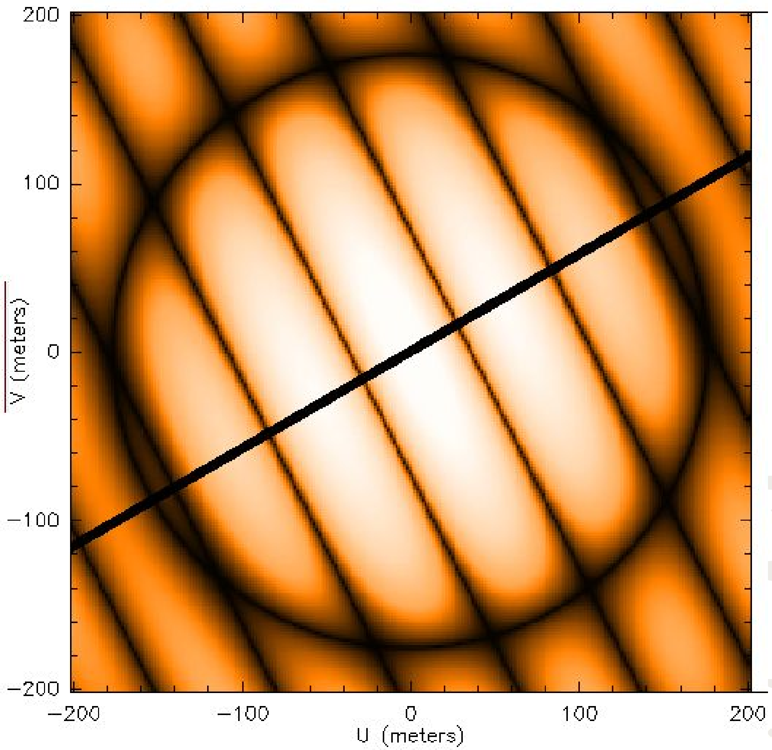
Where V_a and V_b are respectively the visibility of object A and B at baseline (u,v)

Generalization: $$V(u,v) = \frac{\sum_{i=1}^{k} F_i V(u_i,v_i)}{\sum_{i=1}^{k} F_i}$$

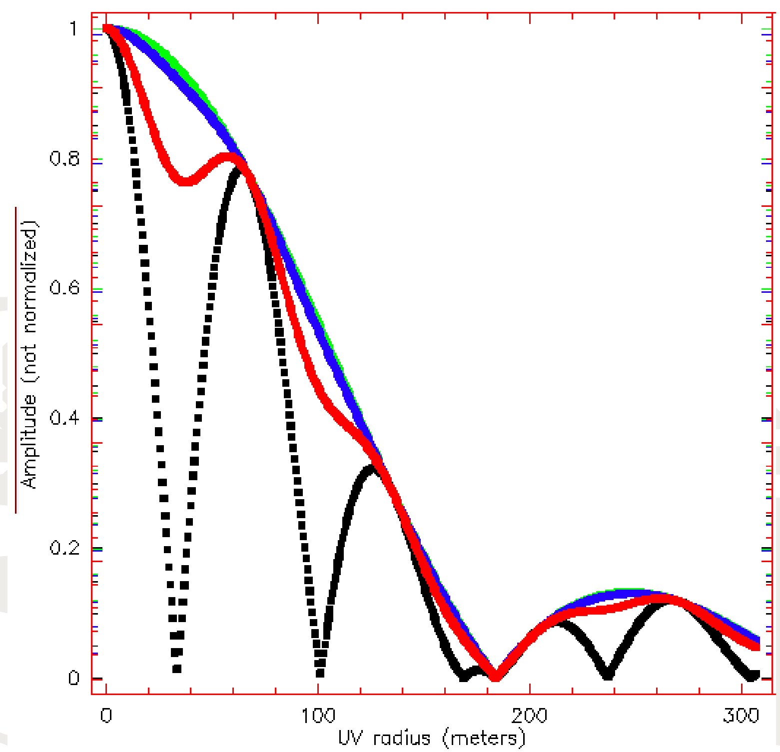
Resolved bi-structure (example)

Binary made of two resolved photometric disks: d=3mas, PA: 35deg

Visibility in (u,v) plane



Squared visibility as a function of baseline for different flux ratios



The modelling process

- Model
- Instrument / atmosphere
- Data
- Minimization

Parameters: α, β, γ, ... → $I(x, y, \alpha, \beta, \dots)$

$$V(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x,y)\exp(-2\pi i(xu+yv)))dxdy$$

Sparse sampling

$\{..., V(u_i, v_i), ...\} i = 1..n$

Observing model

$\rho(t, \lambda), \phi_\delta(t, \lambda)$

Observation

$\{..., V'(u_k, v_k), ...\} k = 1..n$

Error

$\epsilon(u, v)$

$$\chi^2 = f(V(u_i, v_i), V'(u_k, v_k), \epsilon'(u, v))$$

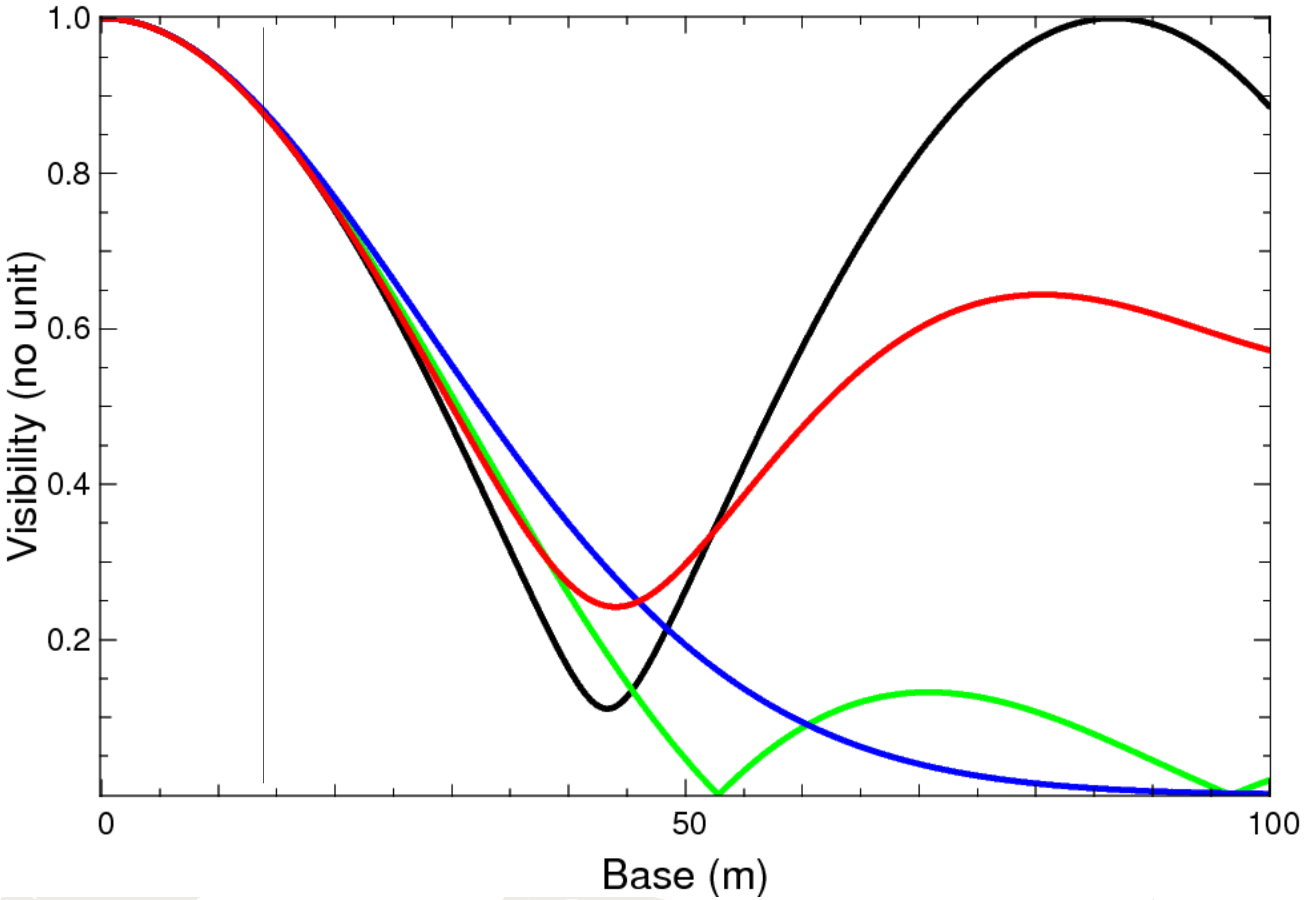
find $min(\chi^2)$

Pushing the limits

Degeneracy at small baselines

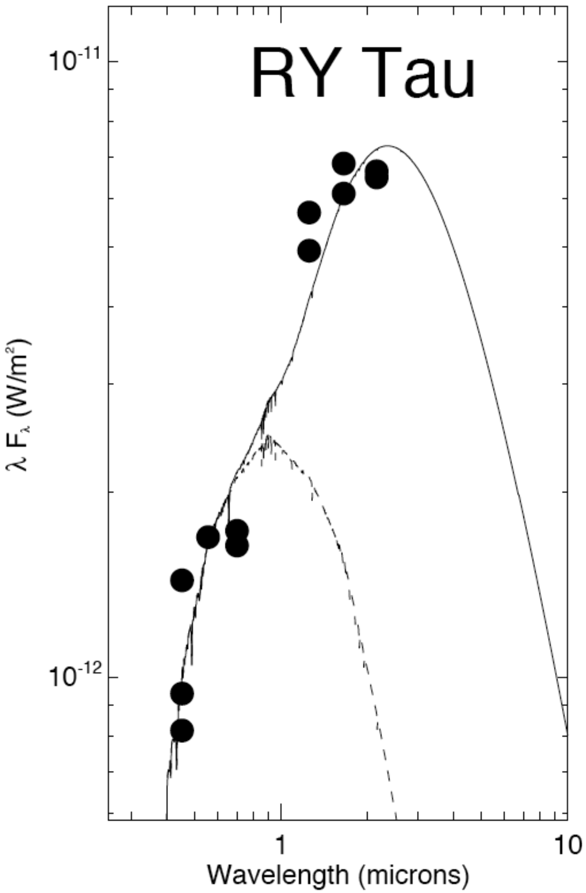
If the object is barely resolved the exact brightness distribution is not crucial the dependance is quadratic for all the basic functions: visibility accuracy is mandatory

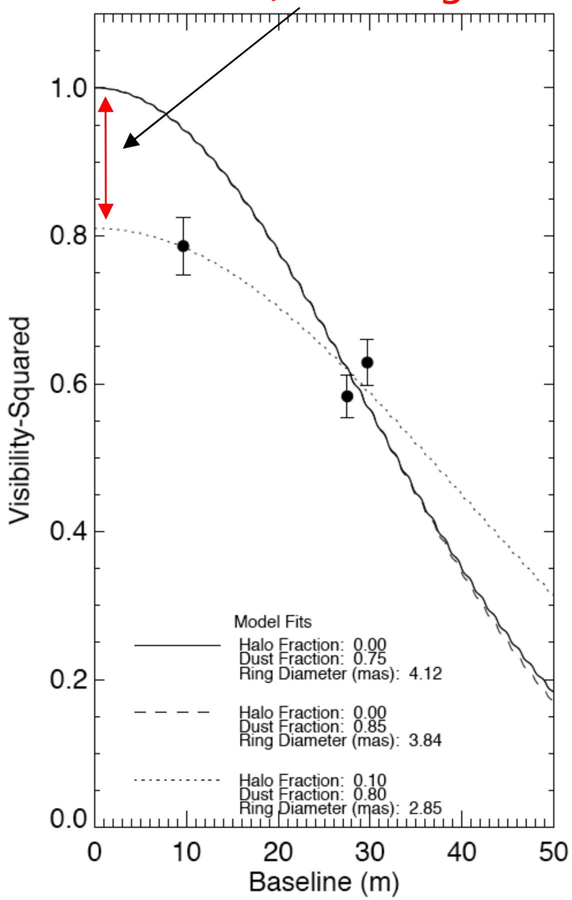
- Uniform disk (green)
- Binary (black)
- Gaussian disk (blue)
- Multiple object (red)



Detecting extended emission

Visibility drops rapidly: attributed to extended flux (10% of global emission)



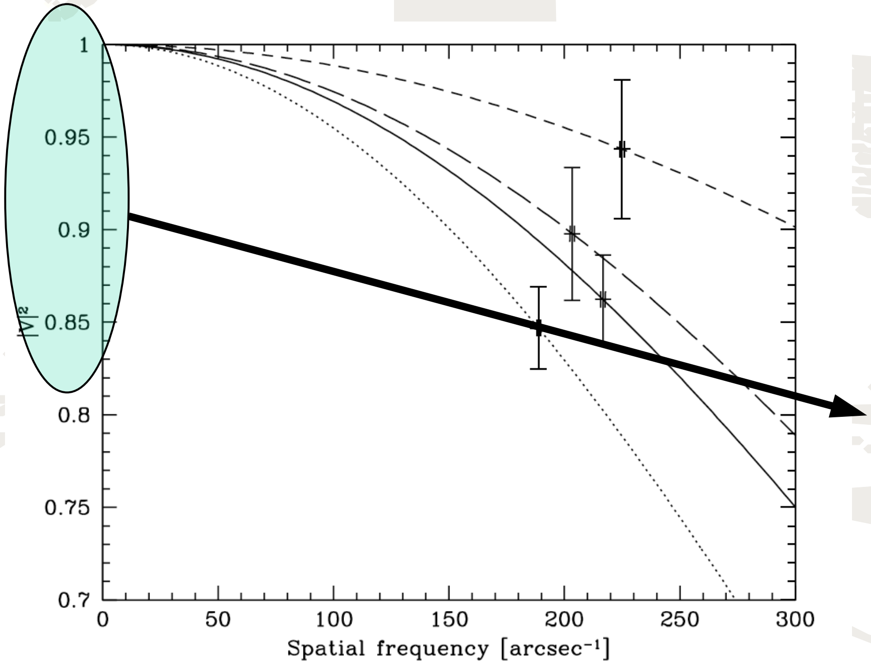


(Monnier et al ApJ 2006)

Here a simple model of extended (totally resolved) dust emission + Gaussian brings the best fit

Small diameter estimation

Model fitting can also be considered as a deconvolution process: sizes estimates or positional uncertainties can be smaller than the canonical resolution (the “beam” size”) => **super resolution**



First measurements of M dwarfs stars diameters

(Segransan et al,2003) .

Look how large visibilities are (i.e. how small the source is). No need for zero visibility measurements to retrieve diameters

Conclusion

✓Visibility study without imaging can be efficient.

✓The (u,v) coverage strategy is different from imaging. Limited allocated time means (very) limited (u,v) points.

✓Use basic models in order to prepare your observation and determine what is the more constraining configuration.

✓Visibility space is the natural place to understand the errors of the final result.

✓Always start by describing your observations in terms of basic functions. It brings quantitative information useful for further more detailed computations.