Theory (and practice) of interferometric data processing

Christian Hummel

(with contributions by Guy Perrin, Eric Tatulli, and Gilles Duvert)

“Black boxes”

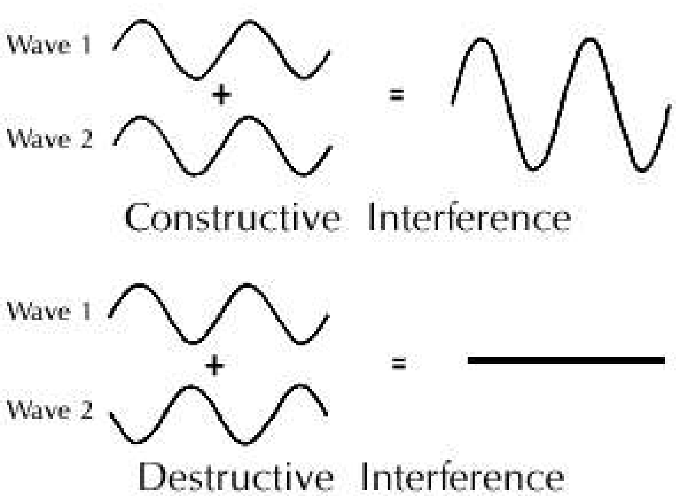
- Interferometry inherently more complicated
- Reduction in the radio dominated by a few packages
- ESO made interferometry available to the community in service mode
- Demand that software packages allow analysis of intermediate results

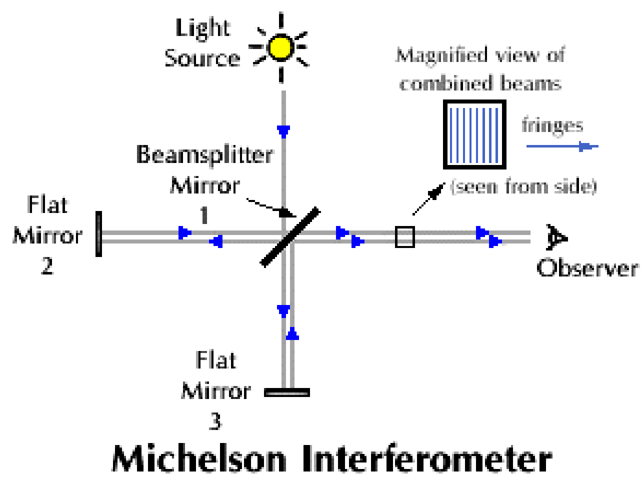
Overview

- Fringe detection and estimators
- Detector bias and noise
- Challenges of atmospheric turbulence
- Wave front filtering in the NIR
- Back ground compensation in the MIR
- Calibration and error estimates
- Interferometric imaging

A reminder: what is interference ?

- Interference is the addition (superposition) of electric fields in electro-magnetic waves such as visible light or radio waves, resulting in a new wave pattern
- Interference can be constructive or destructive depending on the relative phase offset between the two waves
- Detectors integrate the intensity of light, i.e. the squared field amplitude
- If the waves are mutually coherent during the integration, low recorded intensity results from destructive interference and high intensity from constructive interference.





Courtesy of Lu Rarogiewicz

Stellar interferometer

Van Cittert – Zernike Theorem:

The complex degree of coherence measured by an interferometer baseline is equal to a single spatial frequency of the Fourier transform of the object brightness distribution.

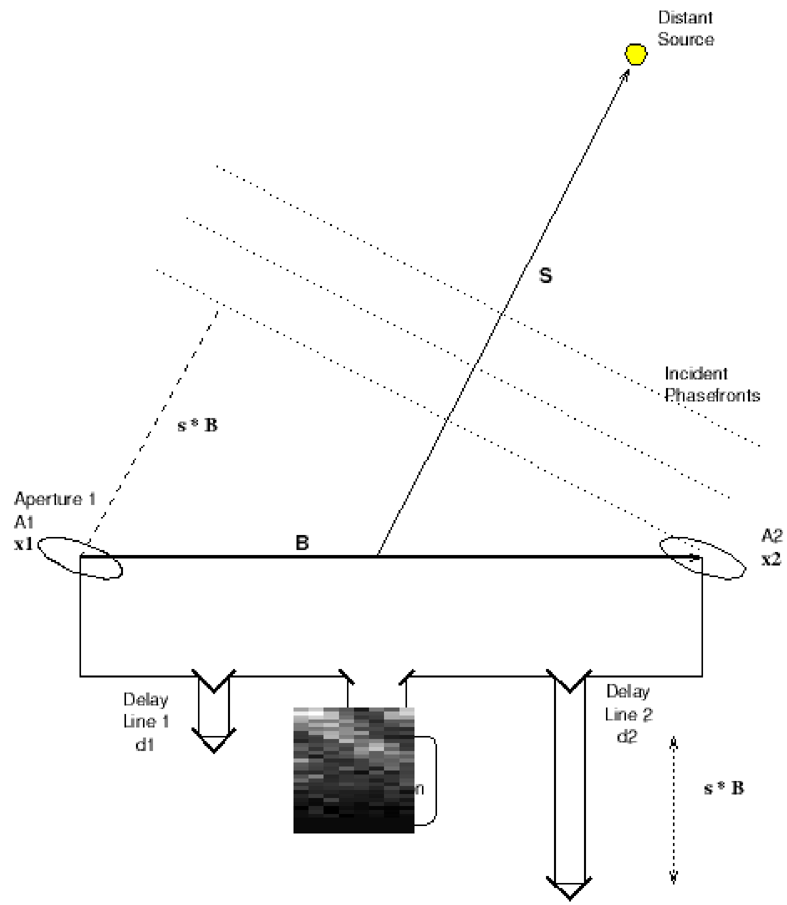
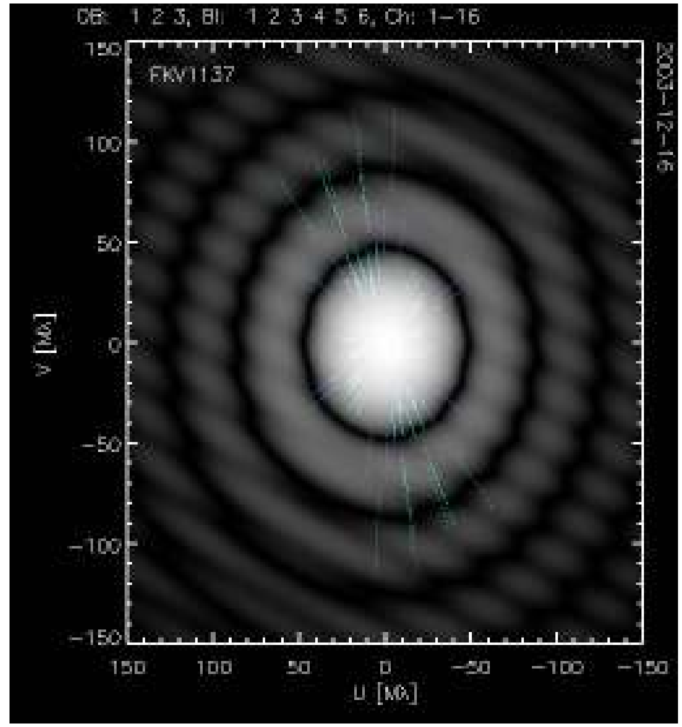


Figure 2.1: Idealized Interferometer.

Aperture Synthesis

Earth's rotation changes a baselines orientation and projected length relative to the source.

Thus, with a multi-element interferometer, an aperture much larger than that of a single telescope can be synthesized.




```
function mapvis,map,u,v
;
; Compute the visibility for a map at a single coordinate u and v[lambda].
; Map positions are in mas. map.x corresponds to RA and increases towards
; East, i.e. left.
;
RAD=180/!pi
MAS=1/3600000.d0
;
arg=2*!pi*(u*map.x+v*map.y)*MAS/RAD
;
return,total(map.i*complex(cos(arg),sin(arg)))
;
end
                                                            1,1           Top
```

Overview

- Fringe detection and estimators
  - Michelson, Fizeau
  - temporal, spatial
- Detector bias and noise
- Challenges of atmospheric turbulence
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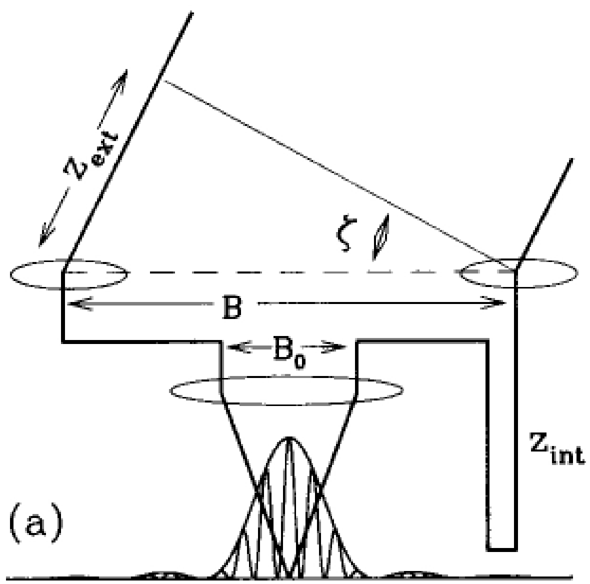
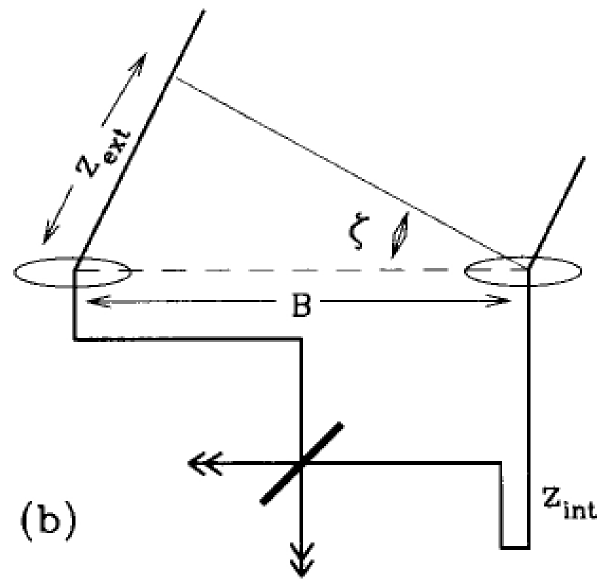
- Interferometric imaging

Beam combiner designs

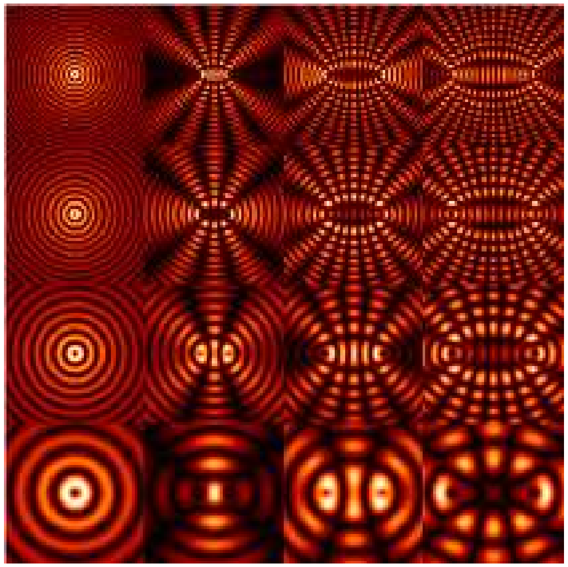
“Fizeau”
(Image plane)



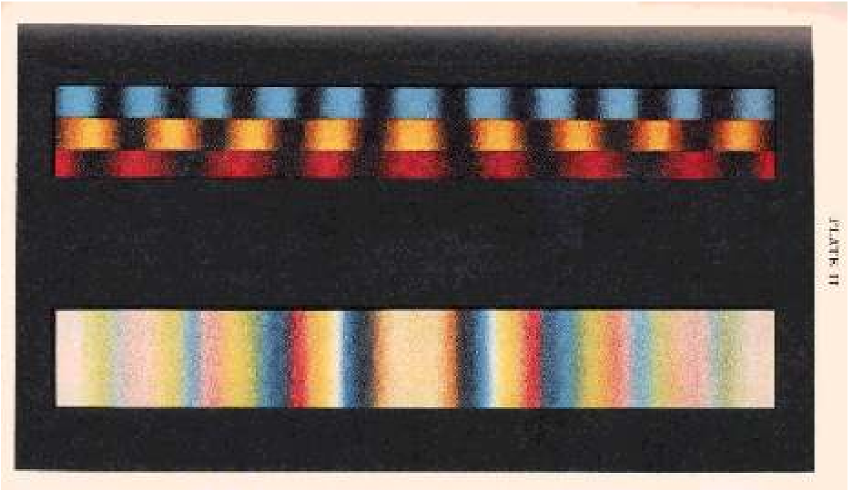
“Michelson”
(Pupil plane)



Interference patterns



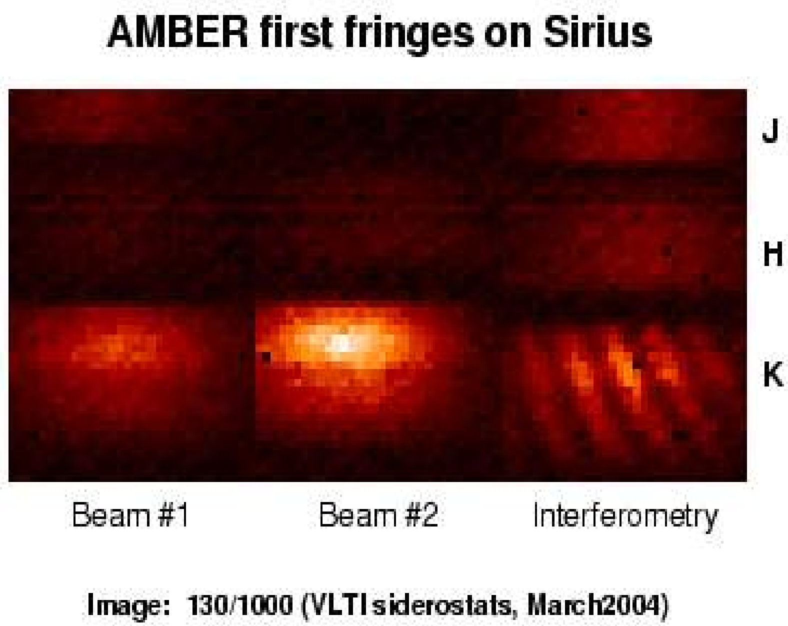
2-dimensional



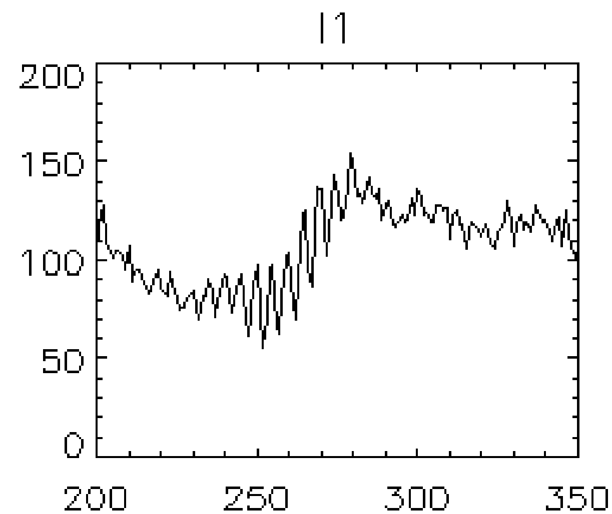
1-dim. (fringes)

Fringe detection

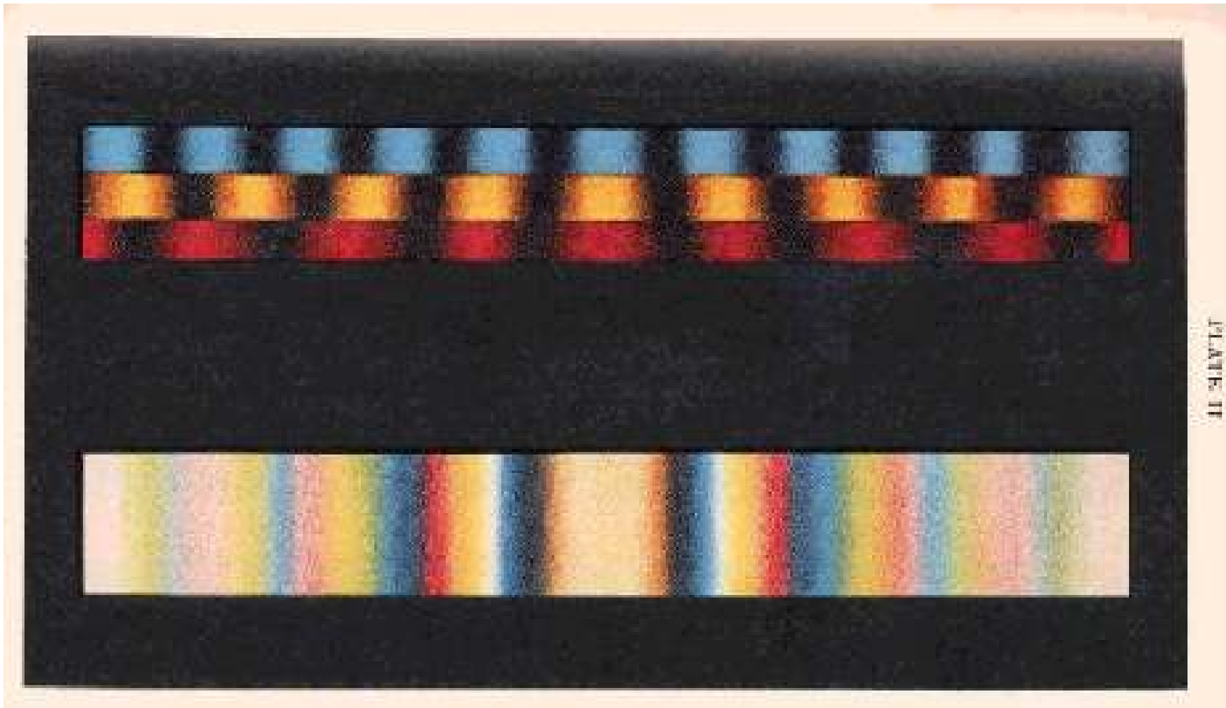
Spatial (geometric delay)

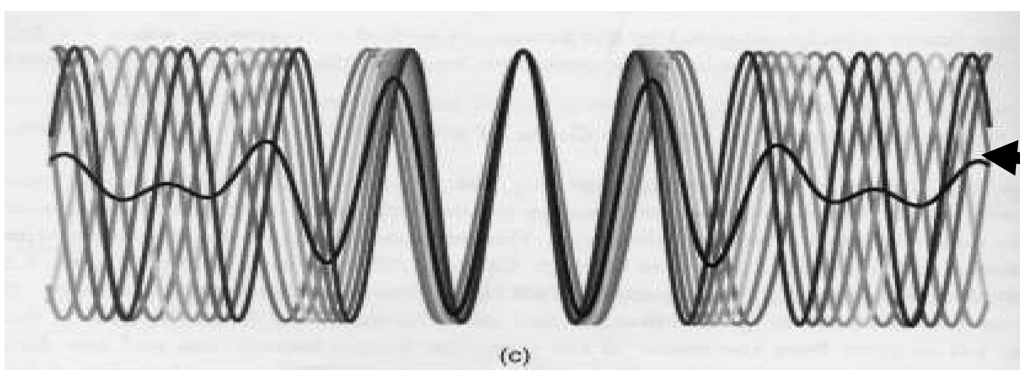


Temporal (delay modulation)



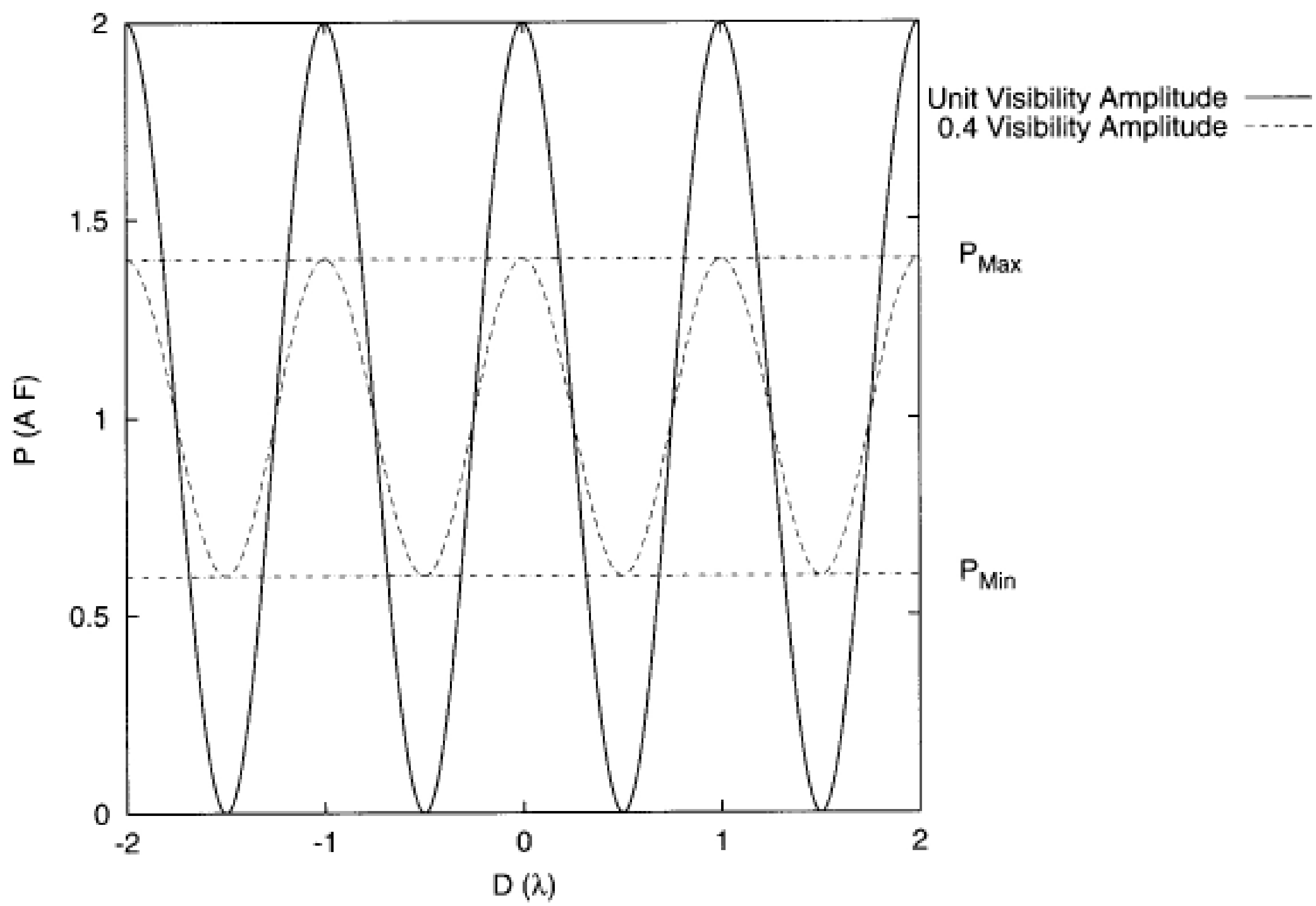
Poly-chromatic fringes





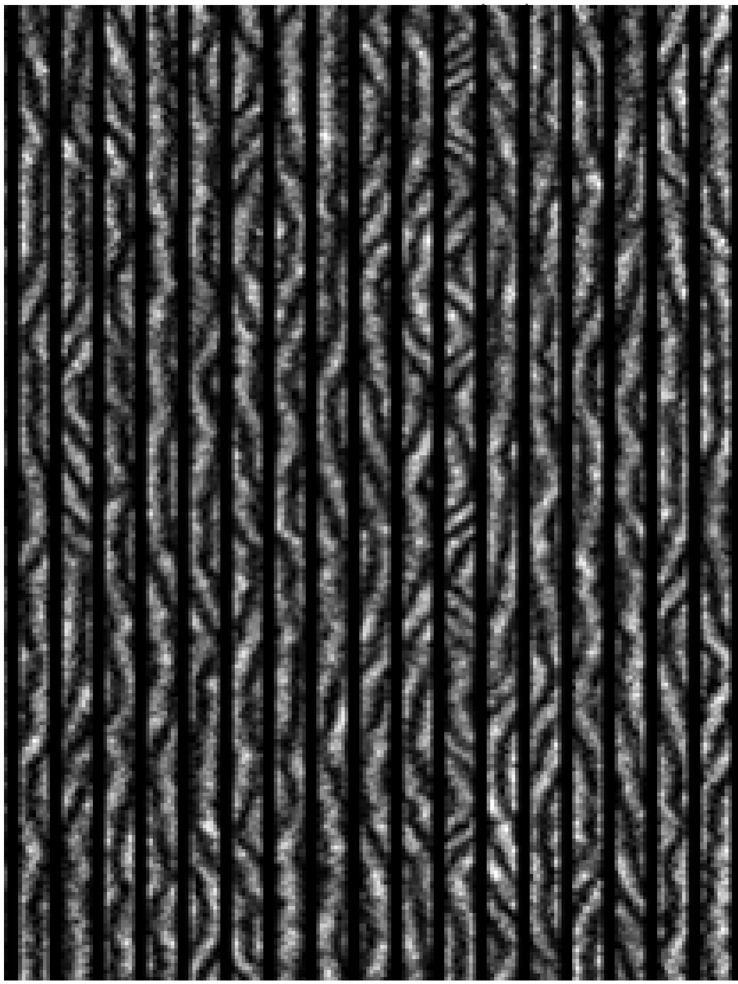
Fringe packet

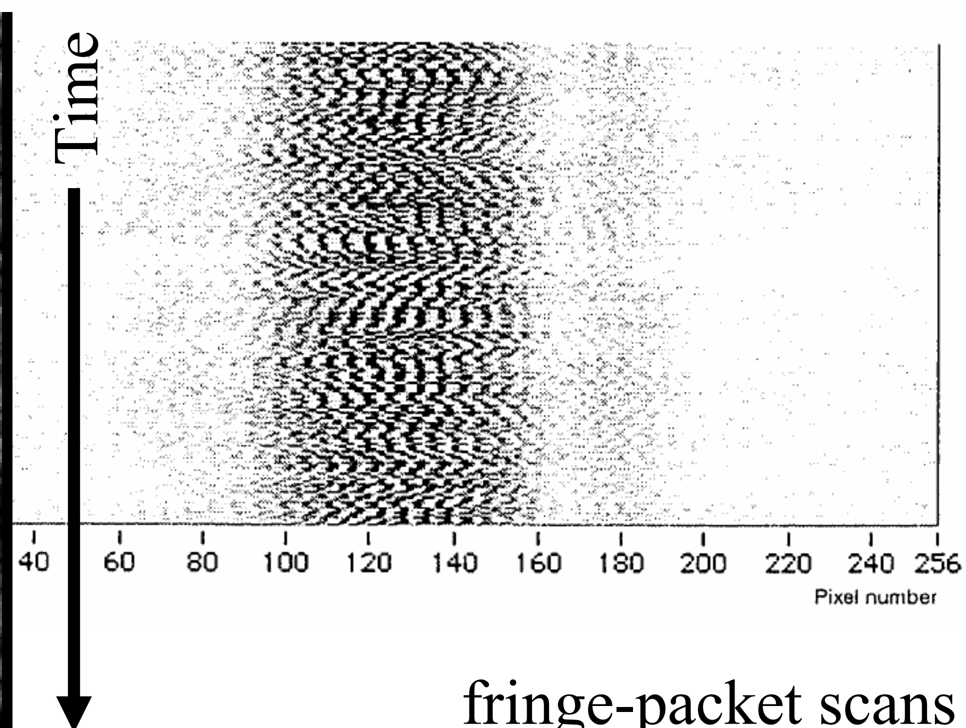
Definition of Complex Visibility



$$V = (P_{max} - P_{min}) / (P_{max} + P_{min})$$

Fringe tracking and centering



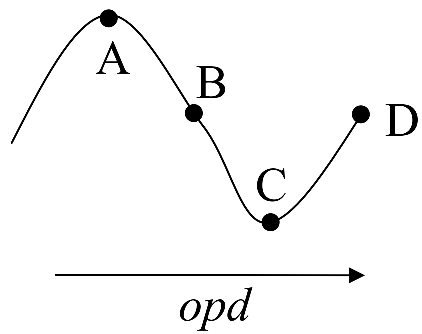


fringe-packet scans

single-fringe measurements

V^2 estimator (single-fringe)

ABCD:



$$V^2 = 4\frac{(A-C)^2 + (B-D)^2}{(A+B+C+D)^2}$$

- It is indeed a 4-point DFT !

$$V^2 = \frac{8\pi^2}{n^2\left(1-\cos\frac{2\pi}{n}\right)} \frac{< X^2 + Y^2 - N >}{< N - D >^2}$$

V or V^2 ?

$$V^2 = \frac{\pi^2}{2} \frac{X^2 + Y^2}{N^2}.$$

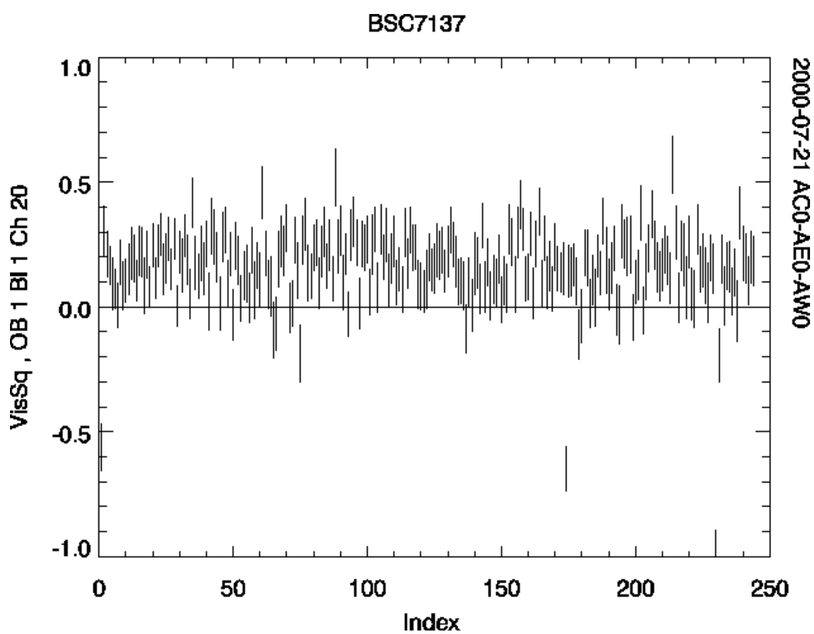
The bin counts A through D obey Poisson statistics (photon starved regime). For the Poisson process, we have $E(X) = np$ and $D(X) = E(X^2) - E(X)^2 = np$. Therefore,

$$\begin{aligned}
E(X^2 + Y^2) &= E(X^2) + E(Y^2) && (10)\\
&= E((C - A)^2) + E(Y^2) && (11)\\
&= E(C^2 - 2CA + A^2) + E(Y^2) && (12)\\
&= E(C^2) - 2E(CA) + E(A^2) + E(Y^2) && (13)\\
&= \overline{C}^2 + \overline{C} - 2\overline{C}\overline{A} + \overline{A}^2 + \overline{A} + E(Y^2) && (14)\\
&= (\overline{C} - \overline{A})^2 + (\overline{D} - \overline{B})^2 + \overline{A} + \overline{B} + \overline{C} + \overline{D} && (15)\\
&= \overline{X}^2 + \overline{Y}^2 + \overline{N}. && (16)
\end{aligned}$$

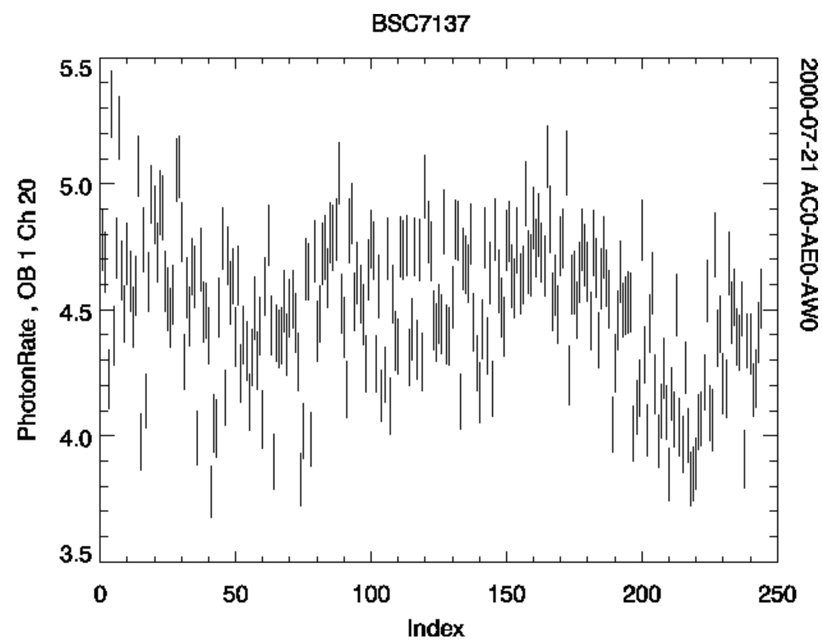
We have made use of the fact that for the variance of a Poisson process x, $var(x) \equiv E(x^2) - E(x)^2 = \overline{x}$. To obtain an unbiased estimator for the squared visibility V^2 we have to compute

$$V^2 = \frac{\pi^2}{2} \frac{< X^2 + Y^2 - N >}{< N - D >^2},$$

Some V^2 Statistics

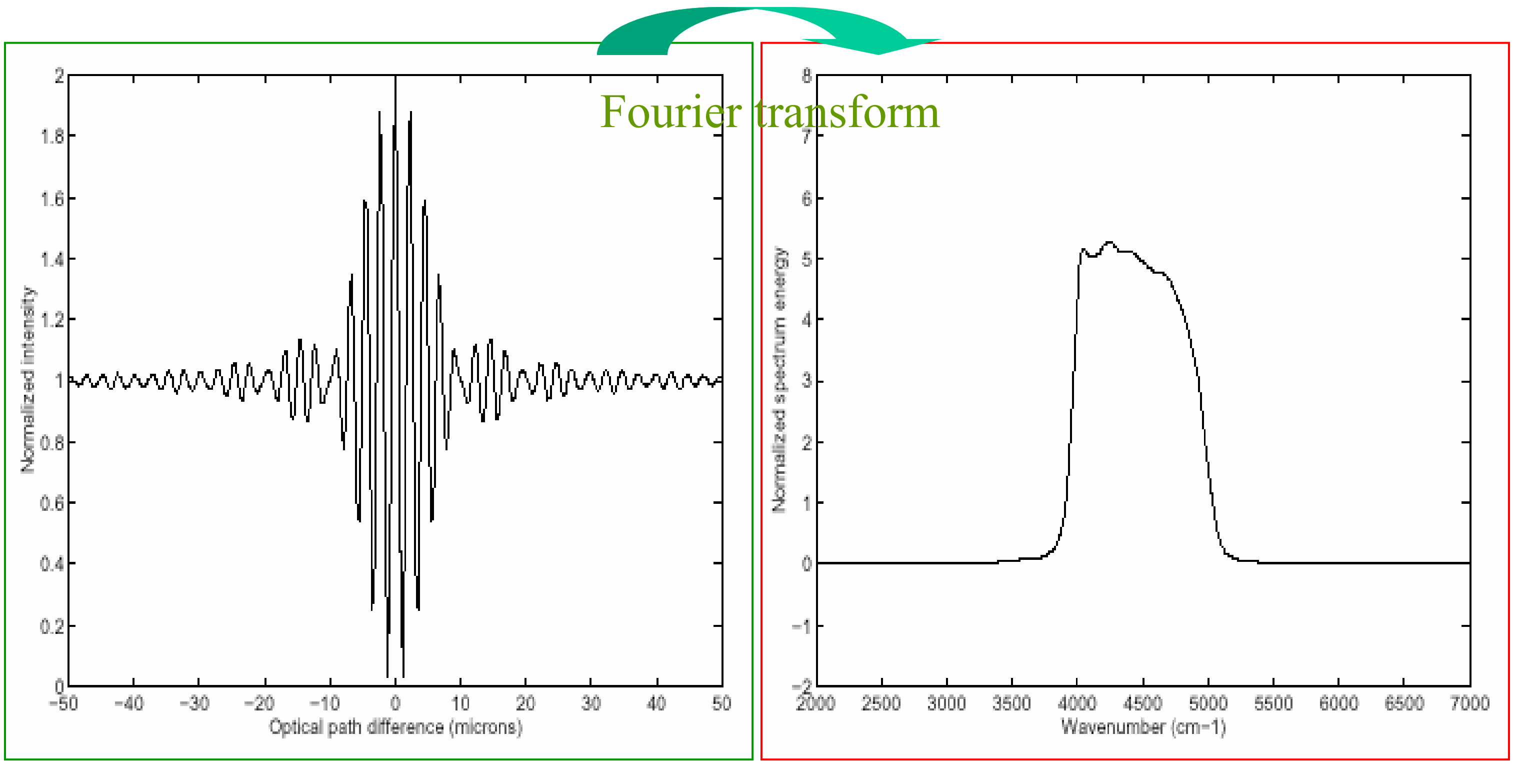


V^2



N (total of 8 bins)

Fringe scanning



Scan

Intensity vs. x opd in μm

Spectrum B(σ)

$\sigma = 1/\lambda$ = wavenumber (cm^{-1})

K band : 4000 - 5000 cm^{-1}

x and σ are conjugate variables through the Fourier Transform

Overview

- Fringe detection and estimators
- Detector bias and noise
  - Poisson statistics
  - Read noise
  - Cosmetic corrections
- Challenges of atmospheric turbulence
- Wave front filtering in the NIR
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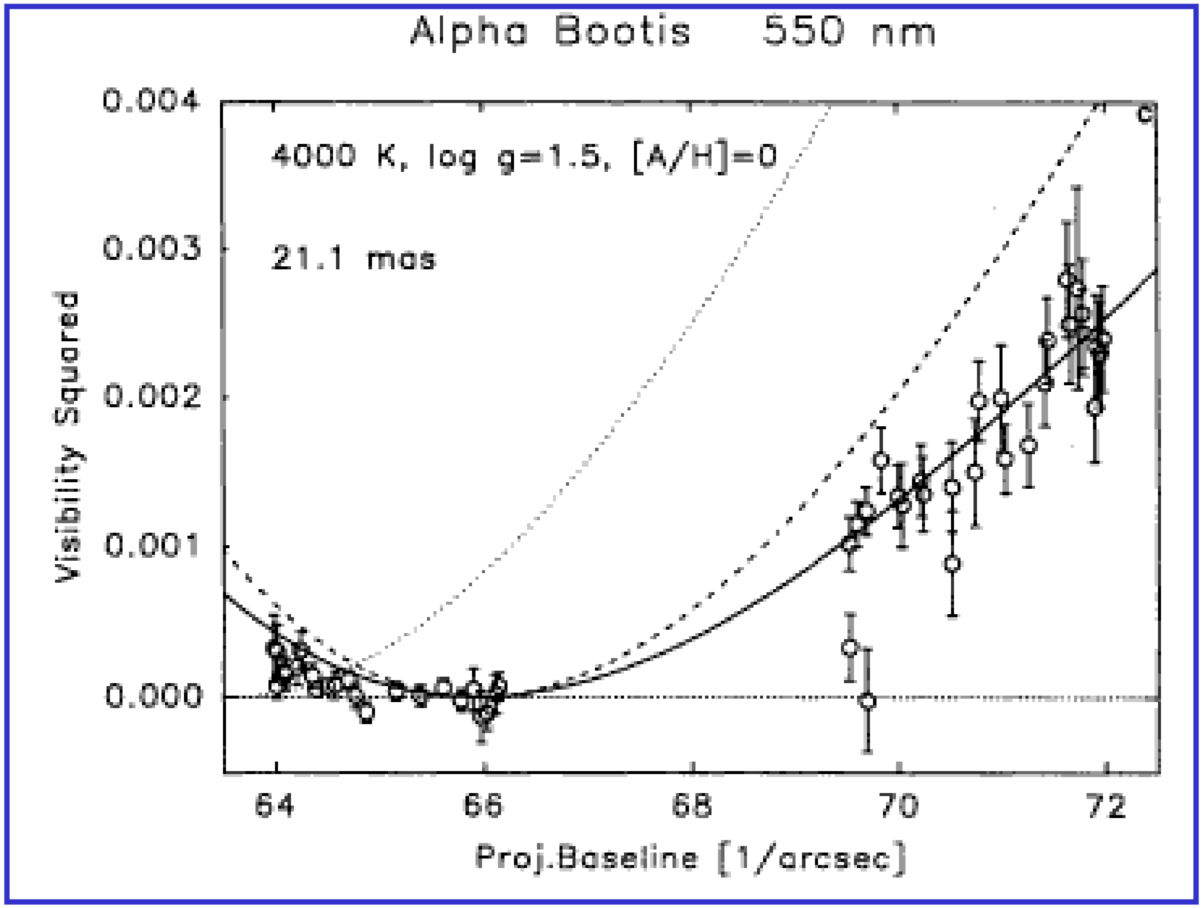
- Interferometric imaging

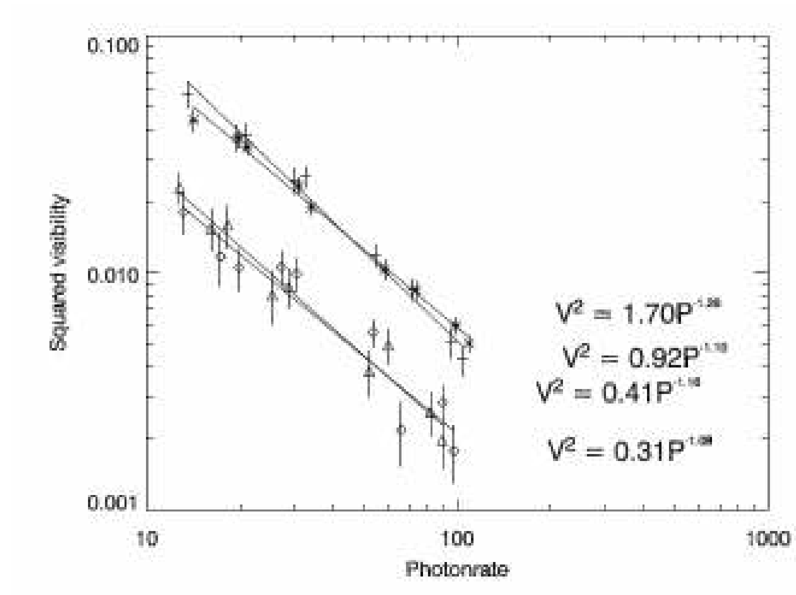
Photo Multiplier Tubes

- Low efficiency
- Delicate
- Poisson noise
- Were used on Mark III, not used anymore



Avalanche photo diodes

- High quantum efficiency
- No dark current
- After-pulsing
- Dead-time
- No array detectors



Incoherent observations, fringe signal caused by after-pulsing.

IR array detectors

- Array, or single pixel
- Cosmetic corrections may be required
  - bad pixels
  - dark current
  - flat field (i.e. relative pixel gain map)
- Read-noise issue exacerbated due to fast read-out
- Not as fast

Noise in interferograms

- ***1. Additive noise***

- Ideal interferogram: $P(s_0,B,\delta) \;=\; I_{total}\{1+\mathrm{Re}\;[V\exp[-ik\delta]]\}$

With photon and detector noise:

$$\begin{cases} P_n(s_0,B,\delta) \;=\; P(s_0,B,\delta)+n_{\det}+n_{ph} \\ \mathrm{rms}(n_{ph})=\sqrt{P(s_0,B,\delta)} \end{cases}$$

With instrument and sky background noise:

$$\begin{cases} P_{n,b}(s_0,B,\delta) \;=\; P(s_0,B,\delta)+n_{\det}+n_{ph}+Back(t)+n_{Back(t)} \\ \mathrm{rms}(n_{Back(t)})=\sqrt{Back(t)} \end{cases}$$

Noise in interferograms

- ***1. Additive noise***

$$P_{n,b}(s_0,B,\delta) \;=\; P(s_0,B,\delta) + n_{\det} + n_{ph} + Back(t) + n_{Back(t)}$$

Pure random noise
Only averages down to zero

Removable with the chopping technique (residuals will remain)

Noise in interferograms

- ***<u>2. Multiplicative noise</u>***

- Ideal interferogram: $P(s_0,B,\delta) \;=\; I_{total}\left\{1+\mathrm{Re}\;\left[V\exp[-ik\delta]\right]\right\}$

Interferogram with unbalanced beams:

$$P(s_0,B,\delta) \;=\; \mathrm{P_A}+\mathrm{P_B}+2\sqrt{\mathrm{P_A P_B}}\times\mathrm{Re}\;\left[V\exp[-ik\delta]\right]$$

Fringe contrast (phase unchanged):

$$C=\frac{2\sqrt{P_A P_B}}{P_A+P_B}\times|V|$$

$$P_A=2P_B \Rightarrow C=0.94\times V$$

Noise in interferograms

- ***<u>2. Multiplicative noise</u>***
- Interferogram with turbulence:

$$P(s_0,B,\delta) \;=\; \mathrm{P_A} + \mathrm{P_B} + 2\sqrt{\mathrm{P_A P_B}} \times e^{-\sigma_\varphi^2} \times \mathrm{Re}\left[V \exp[-ik\delta - i\varphi_p(t)]\right]$$

- $e^{-\sigma_\varphi^2}$ is the coherent energy, σ_φ^2 is the phase variance over the pupil

<u>Instantaneous</u> fringe contrast:

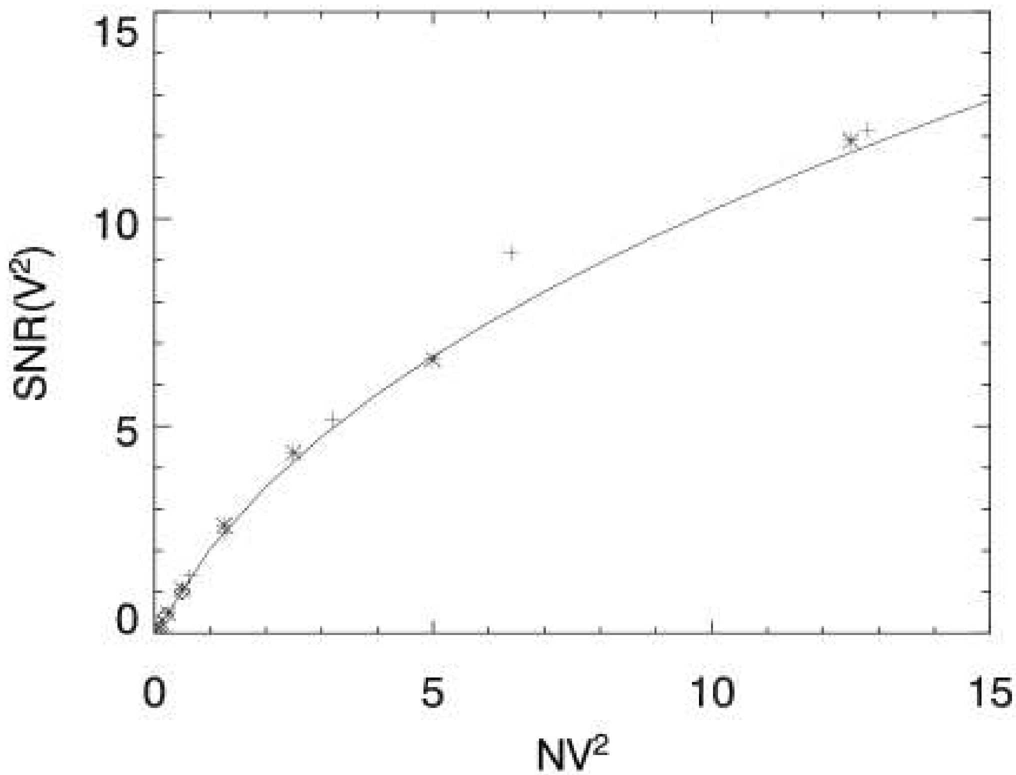
$$C = \frac{2\sqrt{P_A P_B}}{P_A + P_B} \times e^{-\sigma_\varphi^2} \times |V|$$

This is a real catastrophy when the turbulence is not stable which unfortunately is the case in real life

Overview

- Fringe detection and estimators
- Detector bias and noise
- Challenges of atmospheric turbulence
  - The need for fringe tracking and coherent integration
  - Application to baseline bootstrapping
  - Reduces visibility bias and calibrates baseline phases
- Wave front filtering in the NIR
- Back ground compensation in the MIR
- Calibration and error estimates
- Interferometric imaging

Photon-starved interferometry

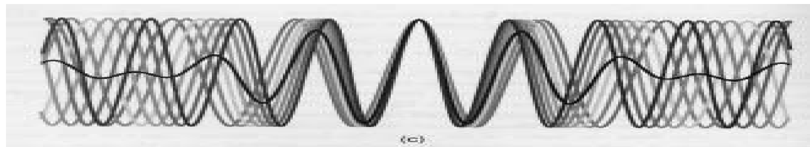


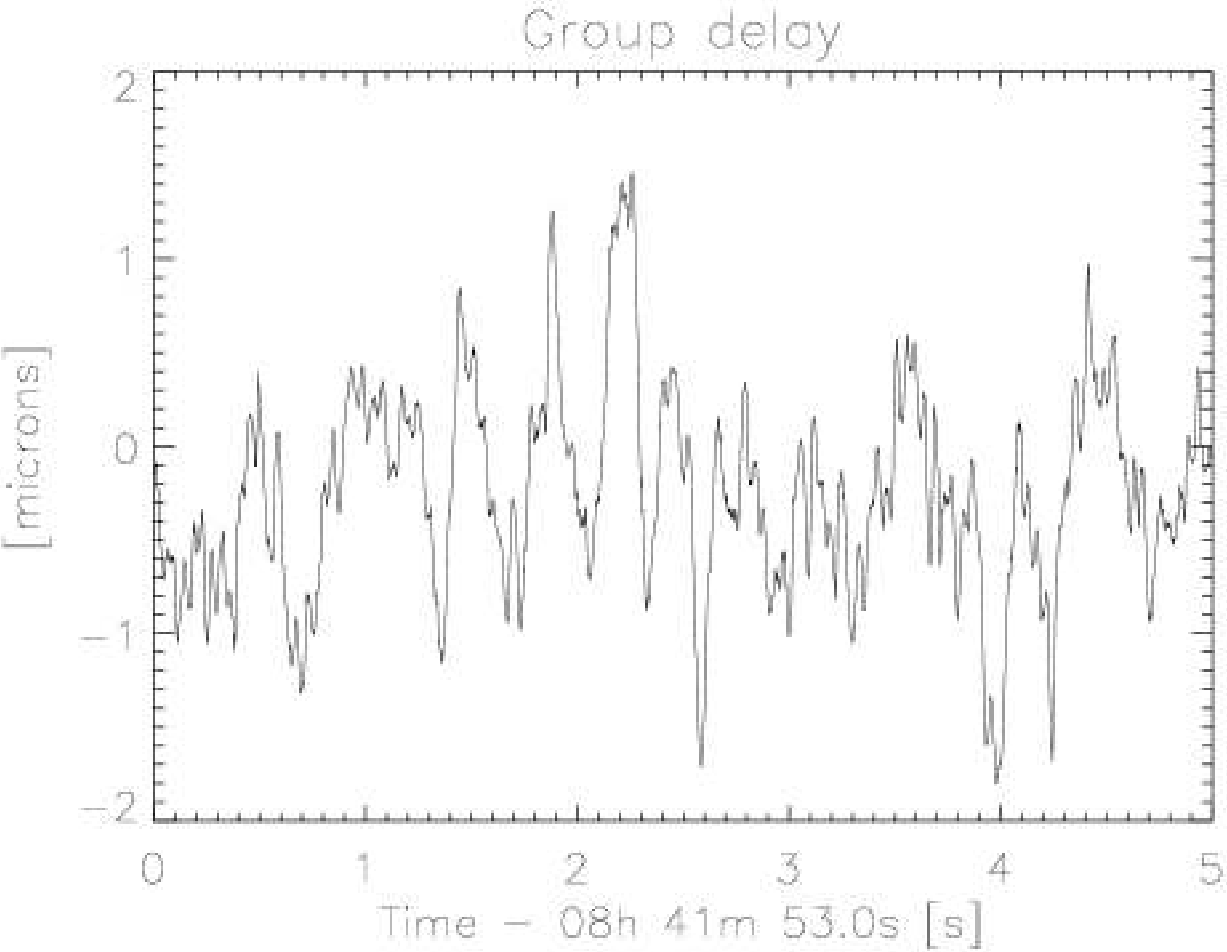
$$SNR(V^2) = \frac{1}{4} M^{1/2} N V^2 \left[1 + \frac{1}{2} N V^2 \right]^{-1/2}$$

M=number of interferograms of N photons each

Shao et al. 1988; Tango & Twiss 1980

Group delay





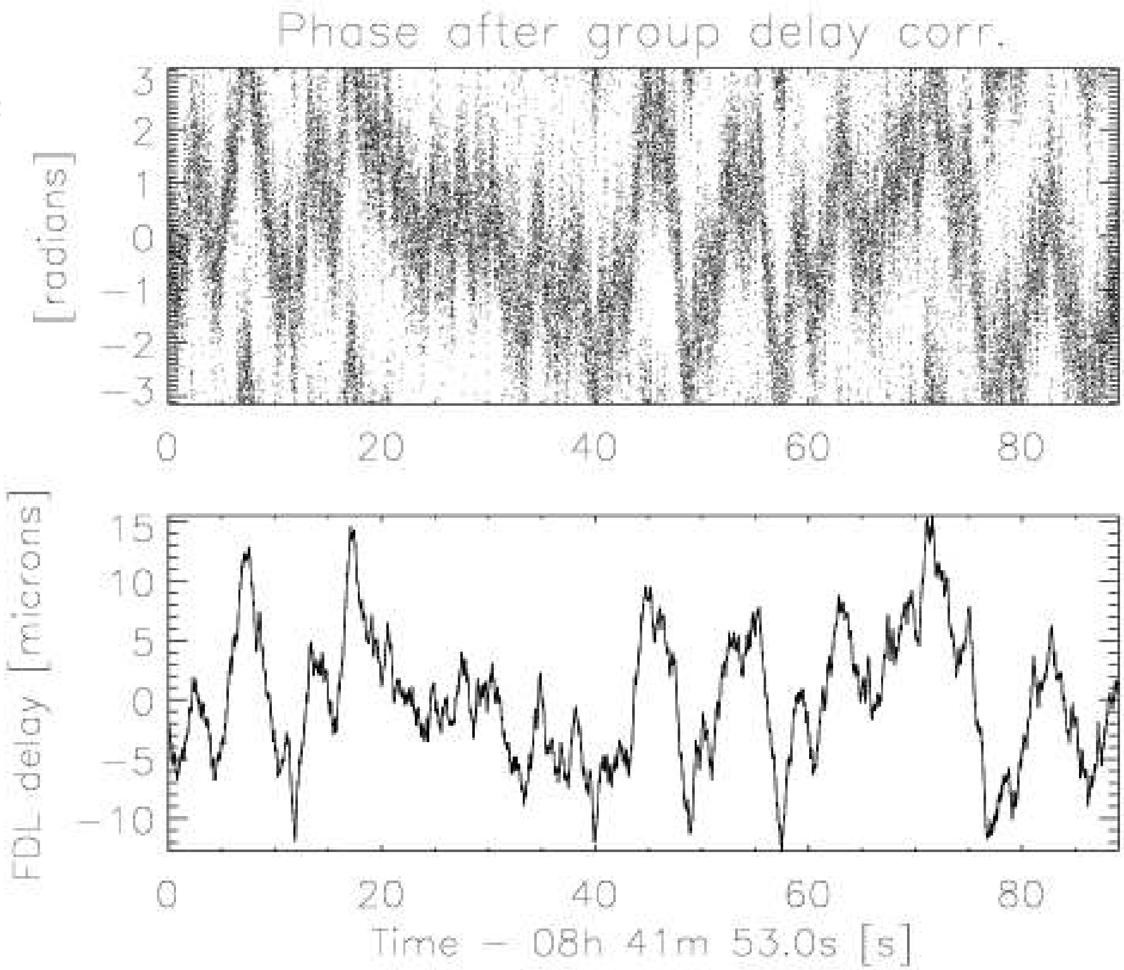
Group delay phase

$$\Phi_\mathrm{d} = \frac{2\pi}{\lambda} d A \Phi_\mathrm{d} = \frac{2\pi}{\lambda} d$$

$$n = n_0 + a/\lambda$$

$$d = A n_0$$

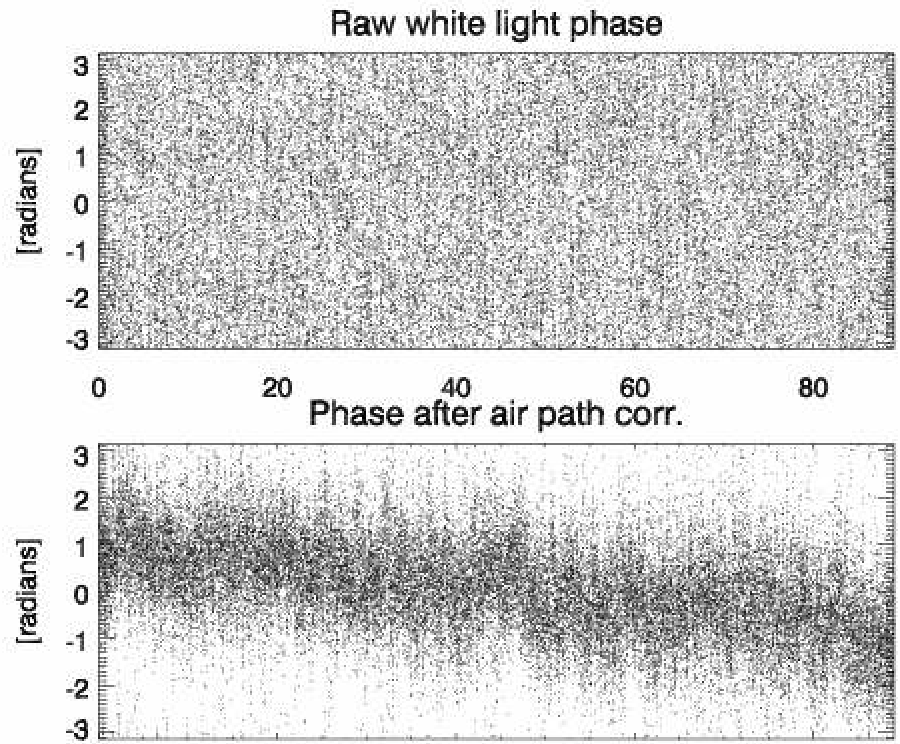
$$\Phi_\mathrm{d} = \frac{2\pi}{\lambda^2} d \frac{a}{n_0}$$



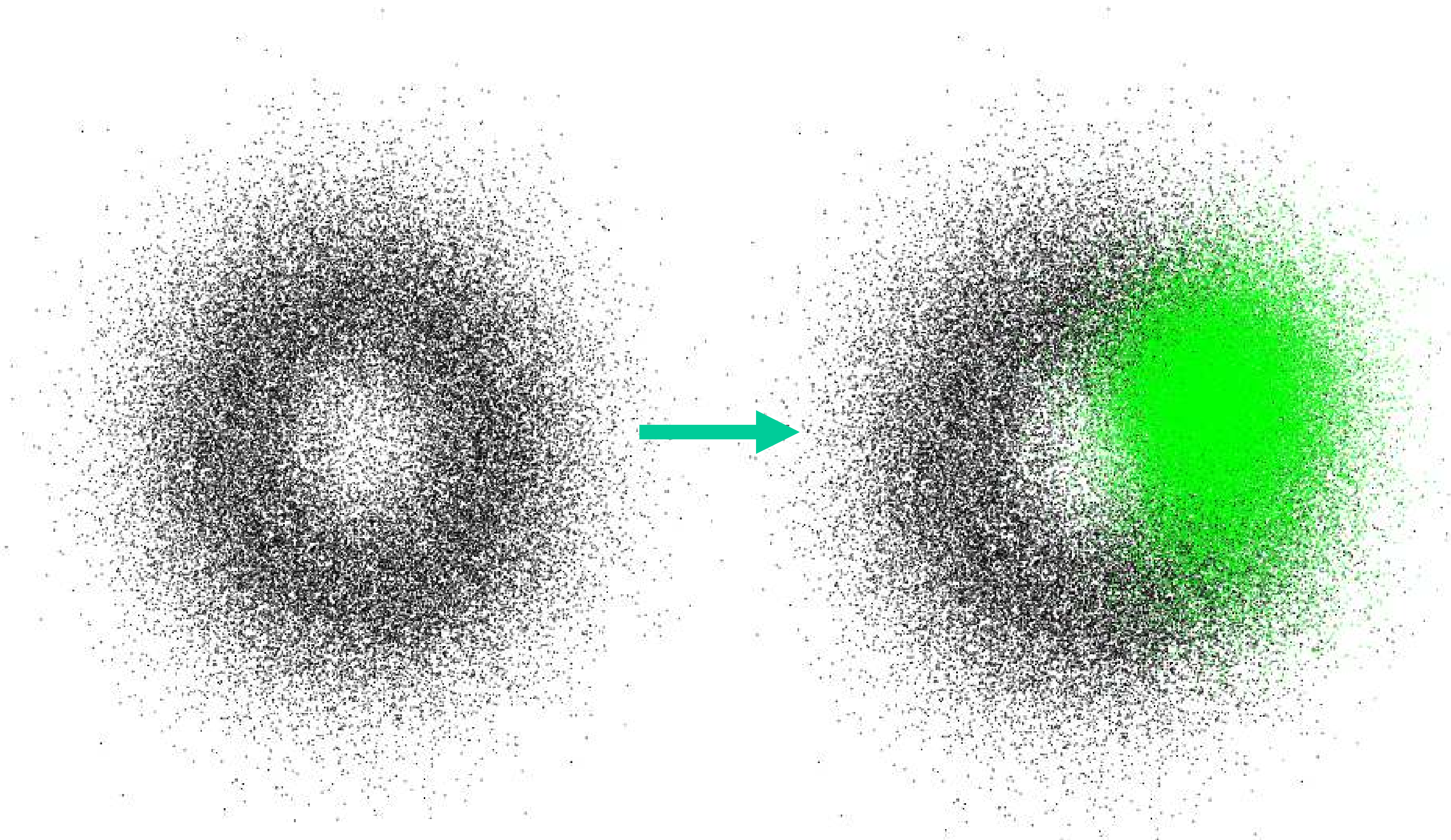
Phases for coherent integration

before

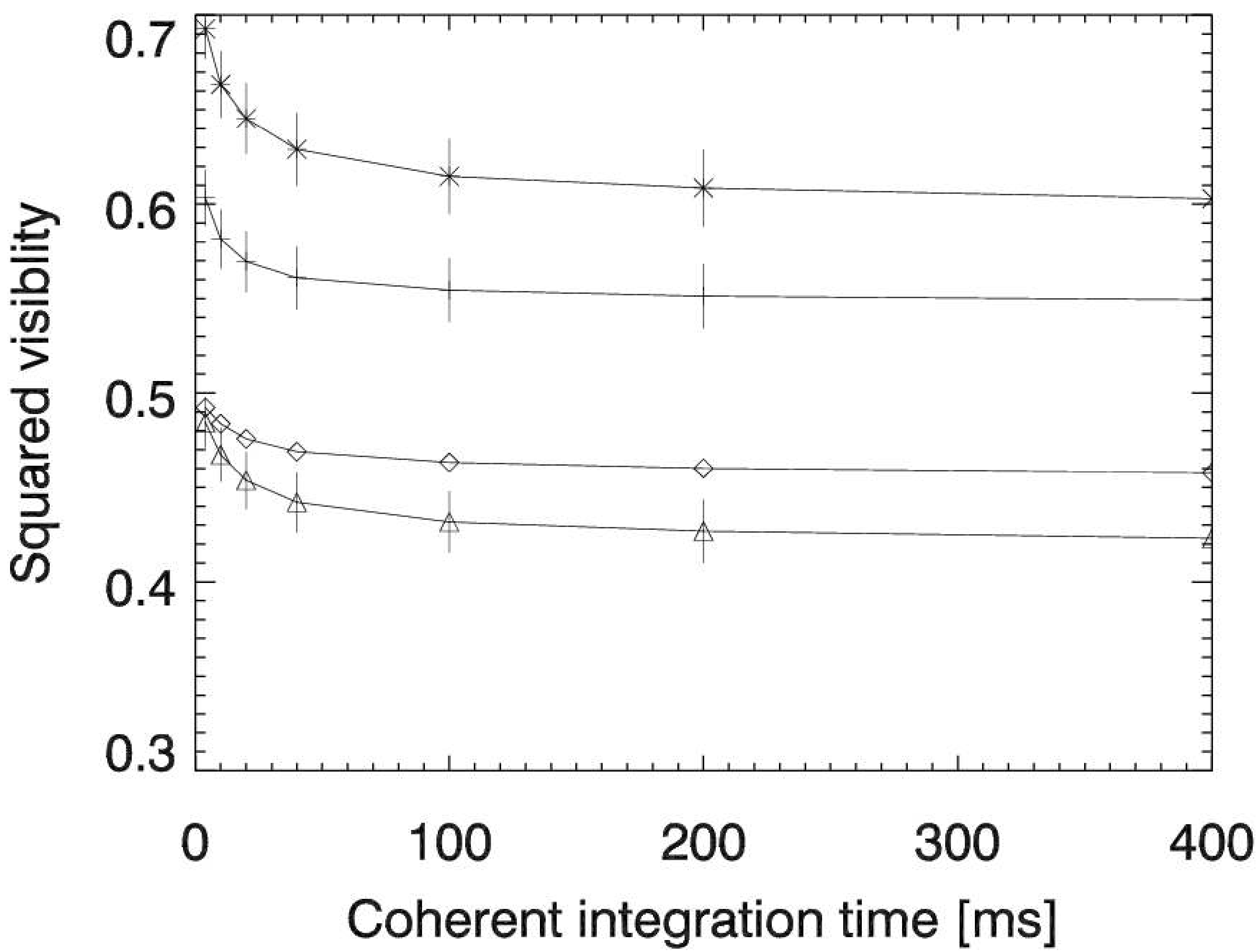
after



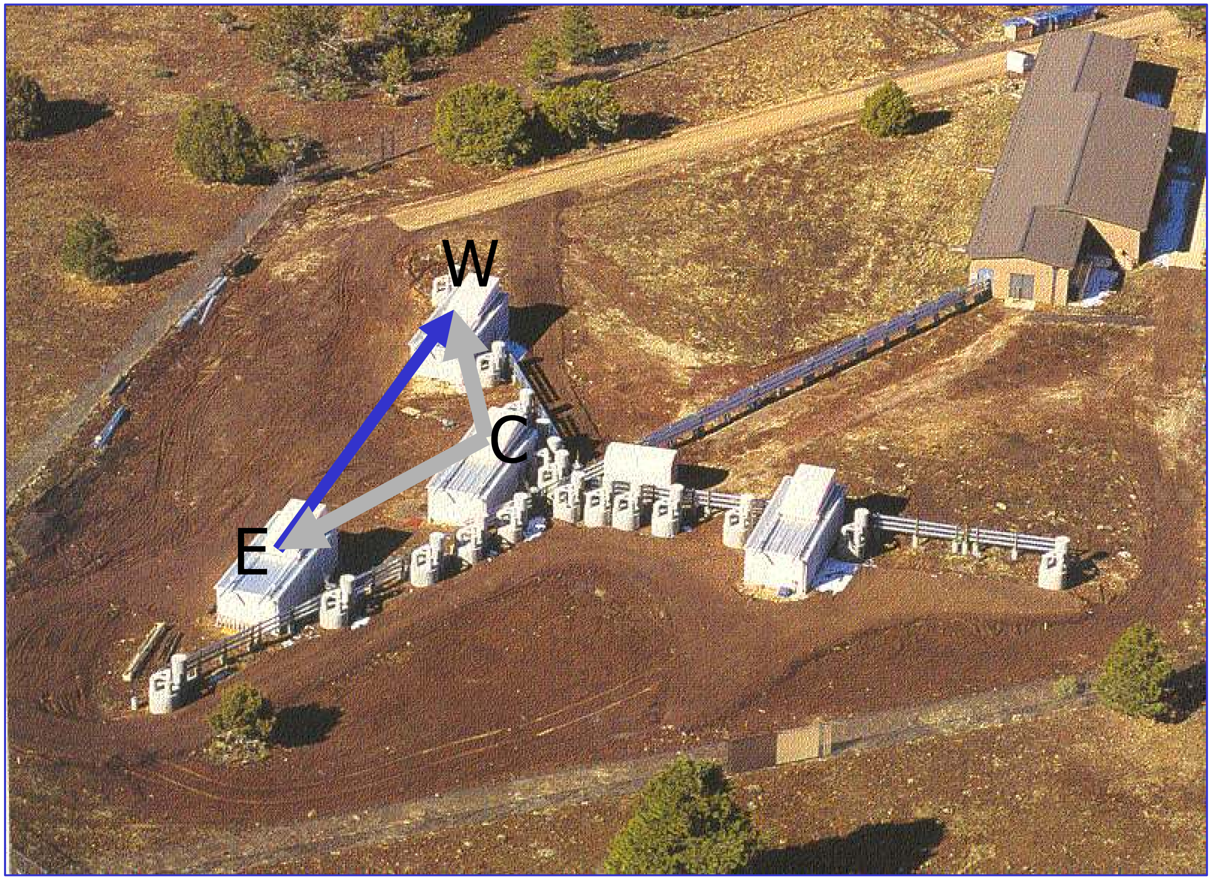
Complex visibility



Coherent integration time

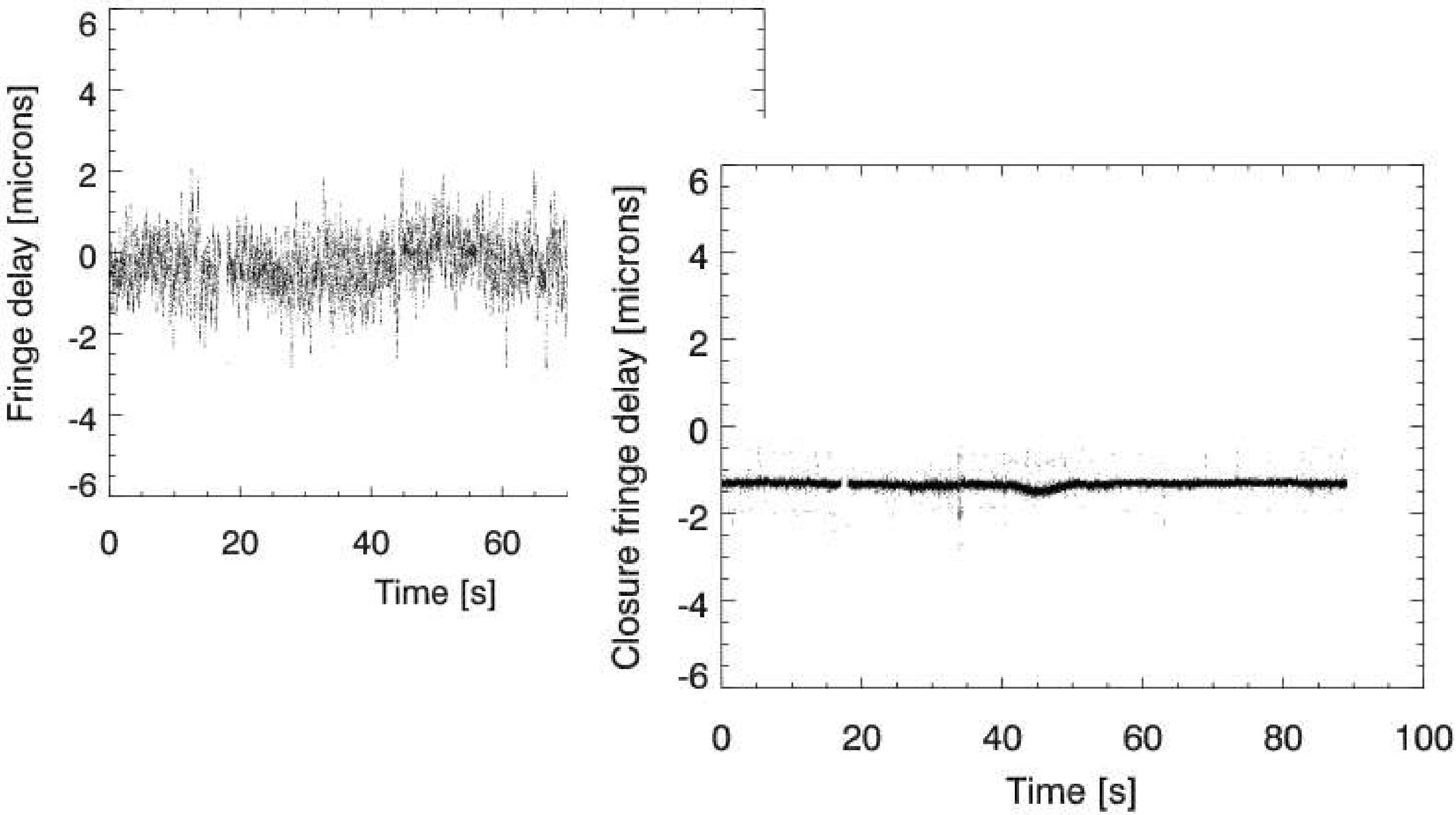


Bootstrapping

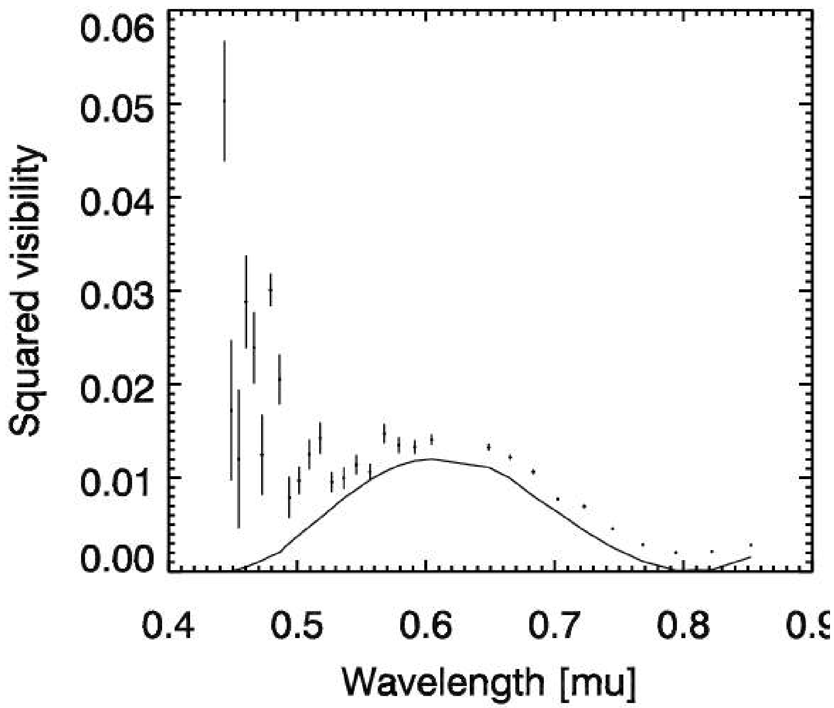


see: Armstrong et al. 1998

Group delay closure

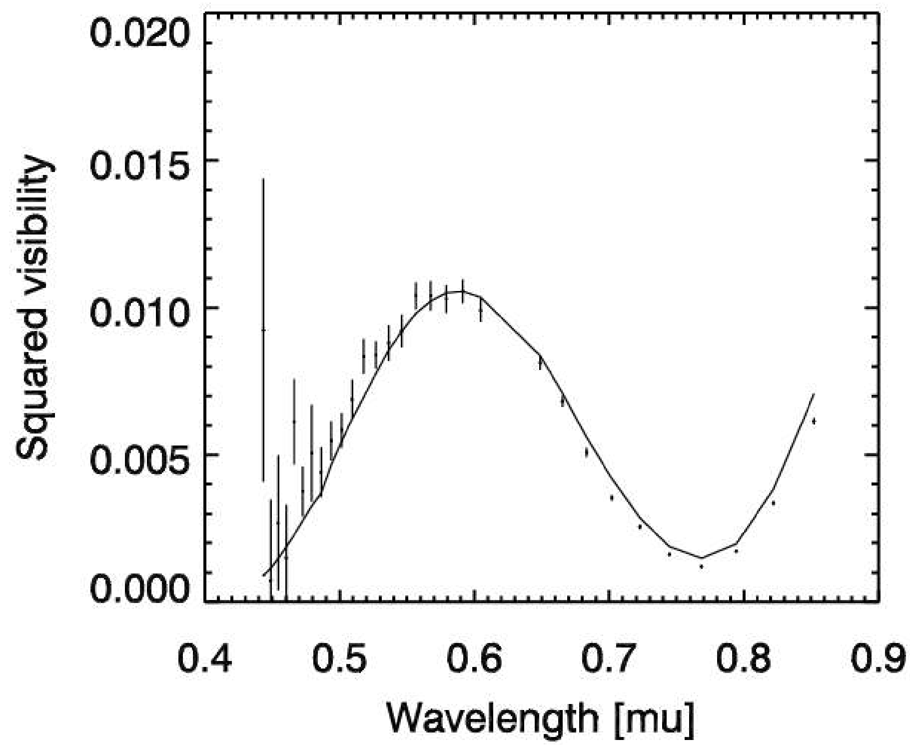


Limb darkened disk

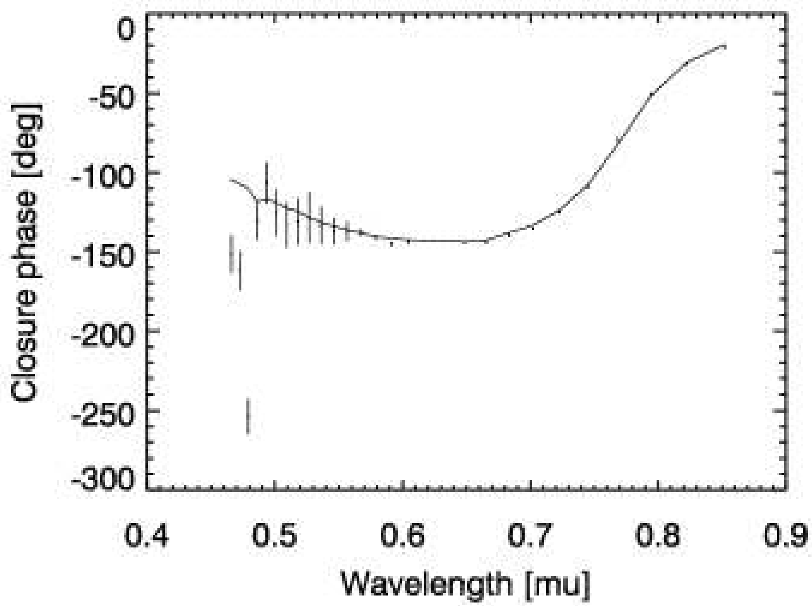


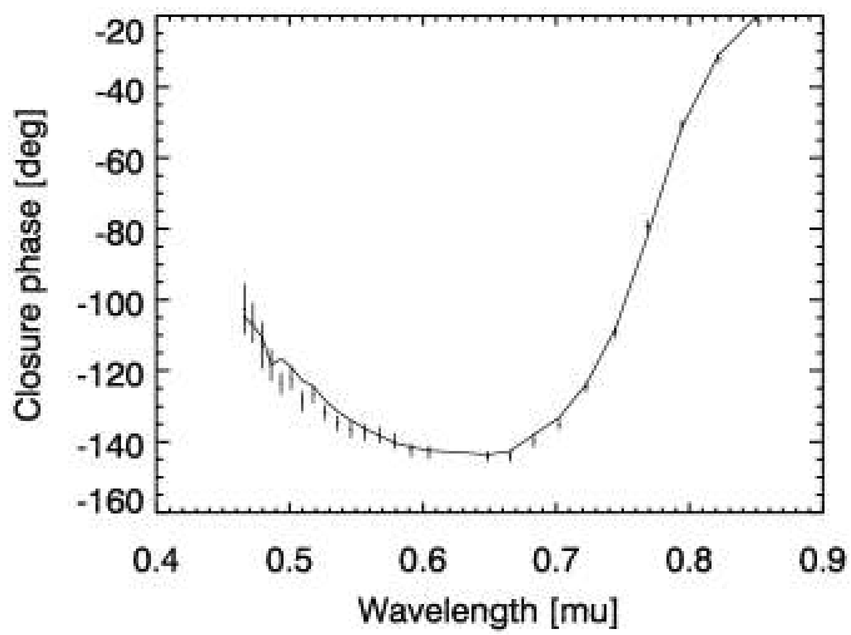
Incoherent analysis

Coherent analysis



Closure phase





Overview

- Fringe detection and estimators
- Detector bias and noise
- Challenges of atmospheric turbulence
- Wave front filtering in the NIR
  - VINCI and AMBER
- Back ground compensation in the MIR
- Calibration and error estimates
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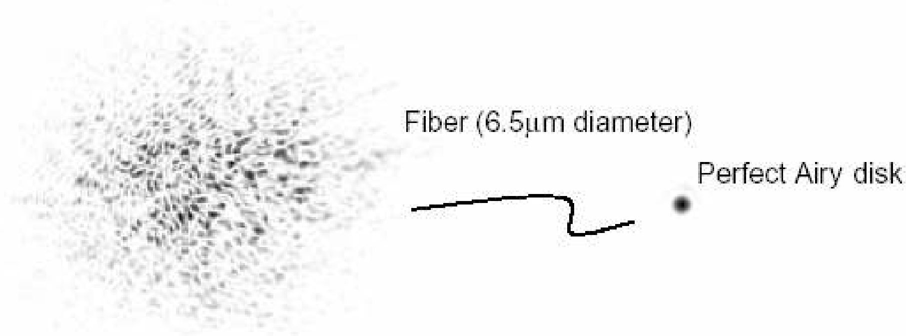
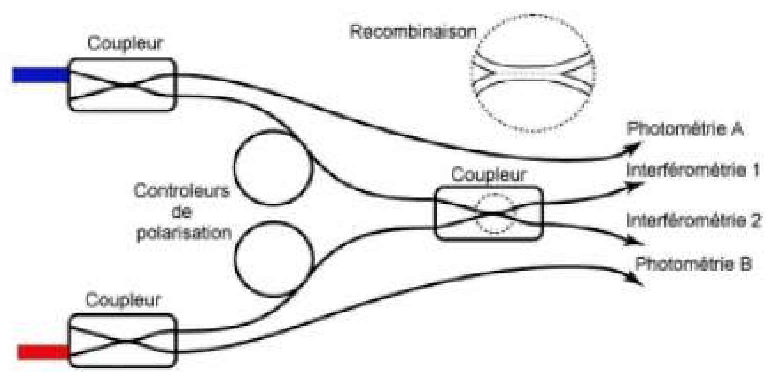
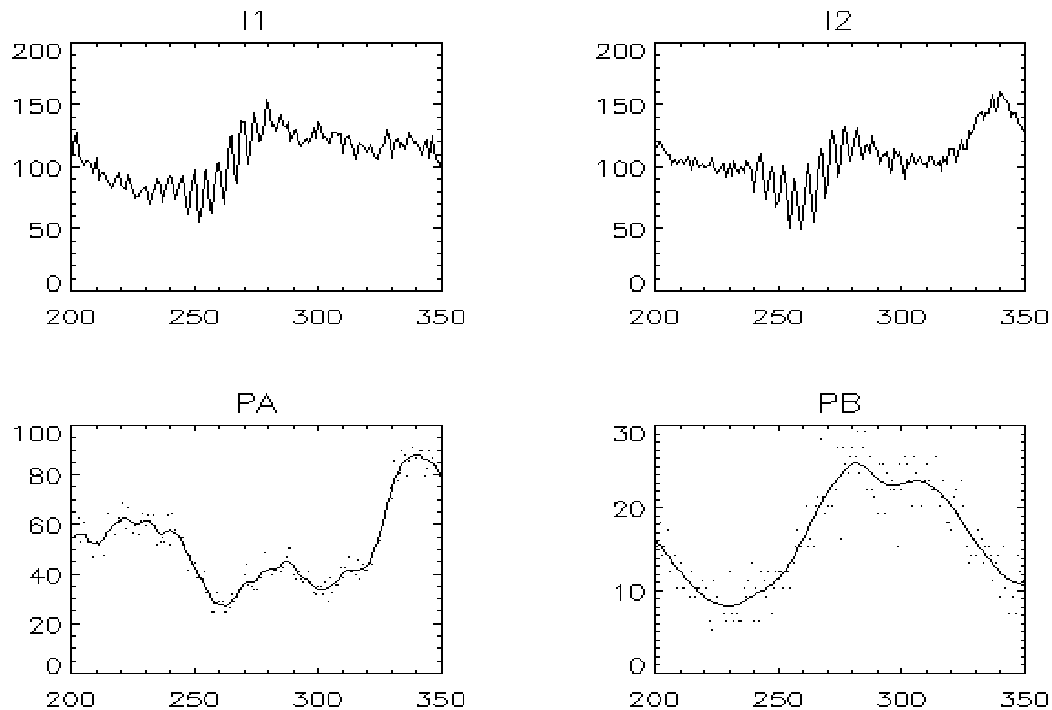
VINCI (K-band interferometry)





$$I_1 = \kappa_{1,\mathrm{A}}\, P_\mathrm{A} + \kappa_{1,\mathrm{B}}\, P_\mathrm{B}$$
$$I_2 = \kappa_{2,\mathrm{A}}\, P_\mathrm{A} + \kappa_{2,\mathrm{B}}\, P_\mathrm{B}.$$

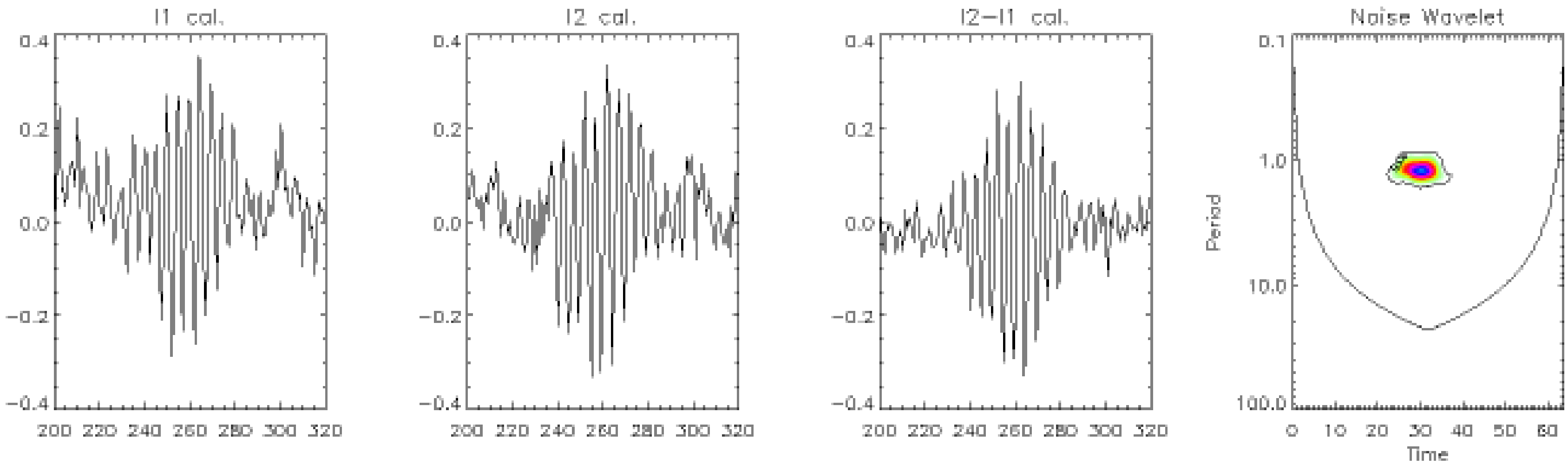
Interferometric signal (VINCI)



$$I_1 = \kappa_{1,\mathrm{A}} P_A + \kappa_{1,\mathrm{B}} P_B + \frac{1}{2}(I_{\max} - I_{\min}) \cos\left(\frac{OPD}{2\pi\lambda}\right)$$

$$\frac{1}{2}(I_{\max} - I_{\min}) = 2\sqrt{\kappa_{1,\mathrm{A}} P_A \kappa_{1,\mathrm{B}} P_B}\, V(OPD)$$

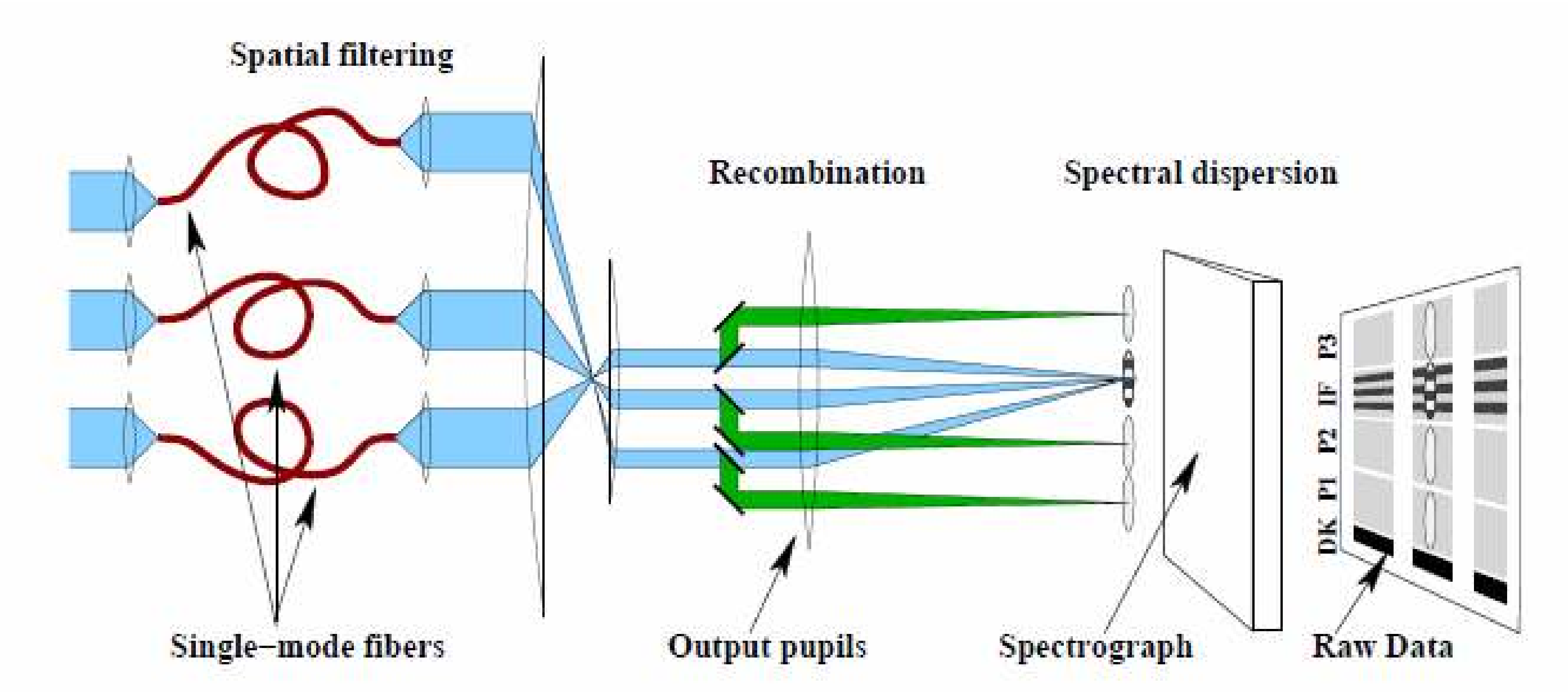
Signal calibration (VINCI)



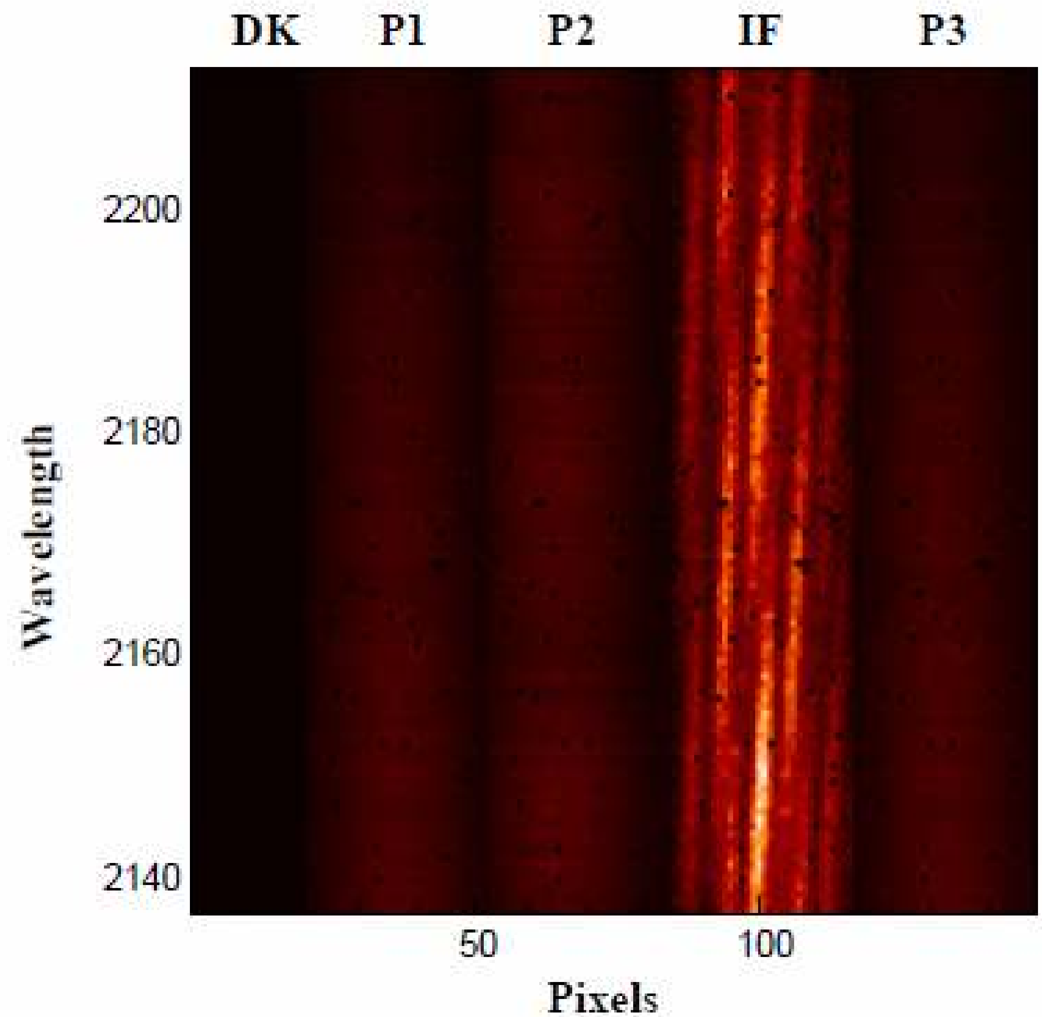
$$I_{1\,\mathrm{cal}} = \frac{1}{2\sqrt{\kappa_{1,\mathrm{A}}\ \kappa_{1,\mathrm{B}}}} \frac{I_1 - \kappa_{1,\mathrm{A}} P_\mathrm{A} - \kappa_{1,\mathrm{B}} P_\mathrm{B}}{[\sqrt{P_\mathrm{A}\ P_\mathrm{B}}]_{\mathrm{Wiener}}}$$

$$I_{2\,\mathrm{cal}} = \frac{1}{2\sqrt{\kappa_{2,\mathrm{A}}\ \kappa_{2,\mathrm{B}}}} \frac{I_2 - \kappa_{2,\mathrm{A}} P_\mathrm{A} - \kappa_{2,\mathrm{B}} P_\mathrm{B}}{[\sqrt{P_\mathrm{A}\ P_\mathrm{B}}]_{\mathrm{Wiener}}}.$$

AMBER instrument

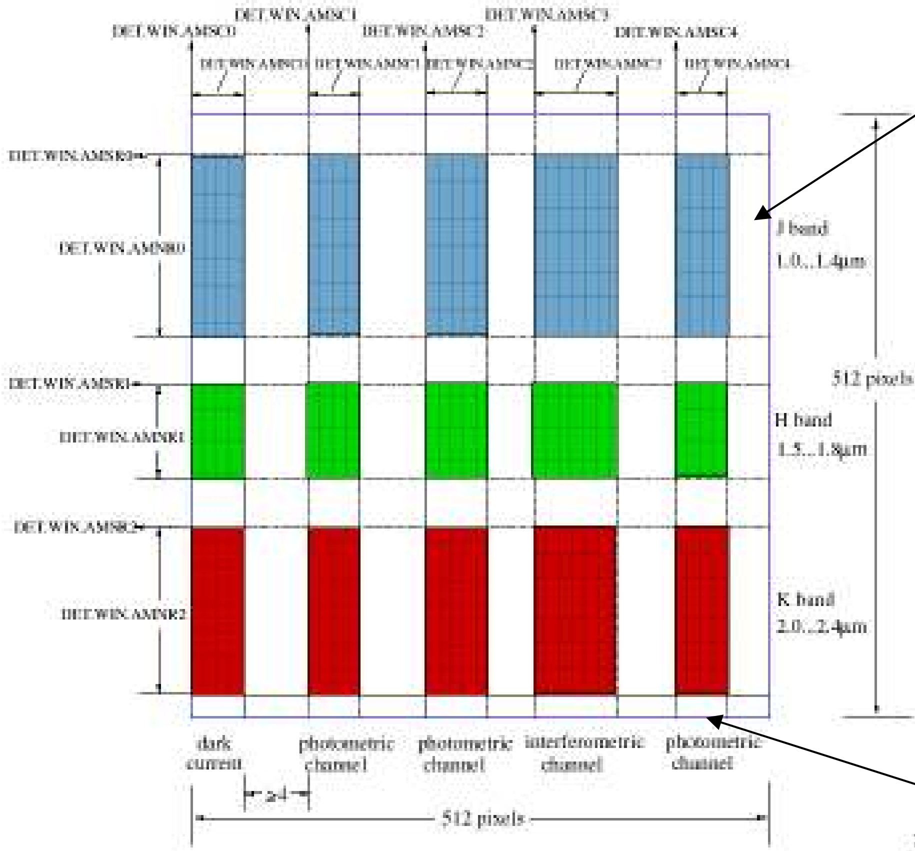


AMBER fringes



medium
resolution

• ... on an infrared Hawaii Camera:

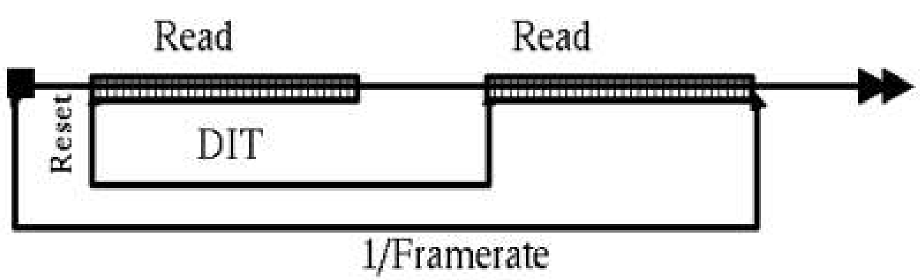


« row »

« channel »

The camera is ALWAYS illuminated (NO shutter)

The camera is divided in (max 3) ROWS of (4 or 5) , regions: Dark, P1 , P2, I [, P3]

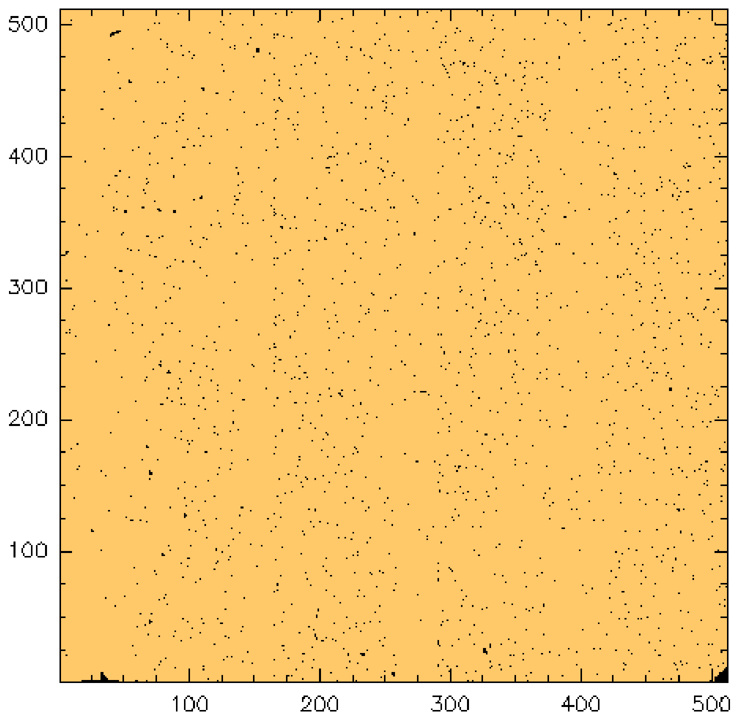


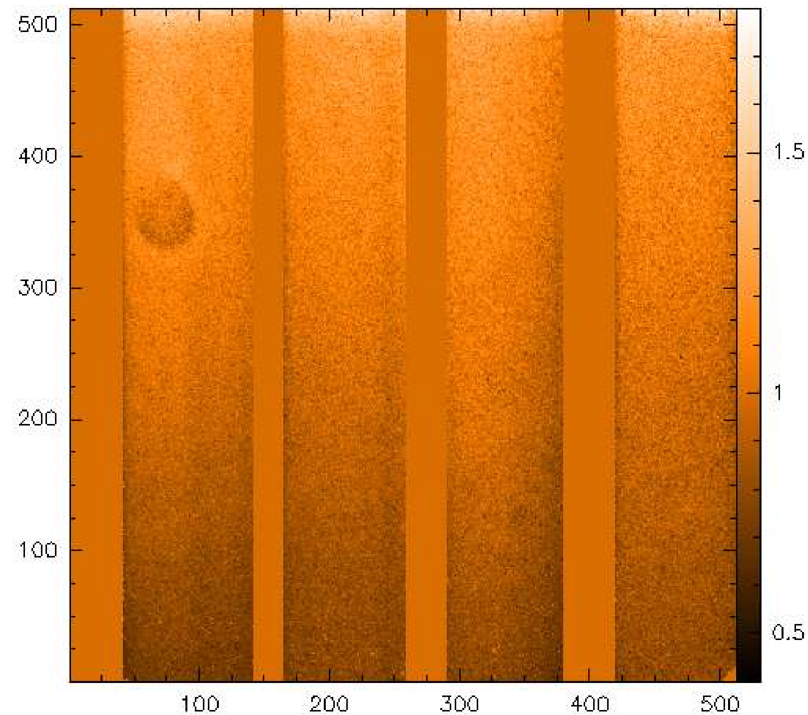
The READOUT mode used is DOUBLE-CORRELATED

Data reduction overview

- Spatially coded fringes
  - cosmetic corrections needed
  - coding calibration needed
- spectrally dispersed
  - wavelength calibration

- camera readout noise, bad pixels, flat, etc...





1. Bad Pixels -> “Bad Pixel Map” File
2. BIAS depends on the illumination of the camera and EXPOSURE TIME –> “Dark” Files
3. Relative pixel-to-pixel gain -> “Flat Field Map” File

• **spatially coded** ... the P2VM: 5 (2T) or 9 (3T) files

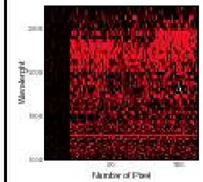
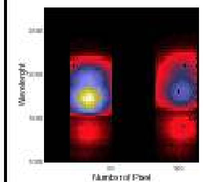
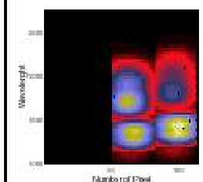
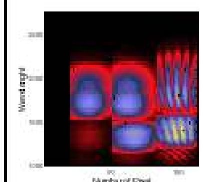
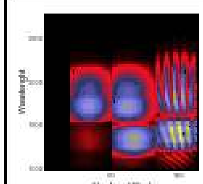
Shutter 1	Shutter 2	Shutter 3	Delaying plate	file Name	figure
Close	Close	Close	No Delay	AMBER_3TSTD_CAL_0001.fits	
Open	Close	Close	No Delay	AMBER_3TSTD_CAL_0002.fits	
Close	Open	Close	No Delay	AMBER_3TSTD_CAL_0003.fits	
Open	Open	Close	No Delay	AMBER_3TSTD_CAL_0004.fits	
Open	Open	Close	1/2 Delayed	AMBER_3TSTD_CAL_0005.fits	

Figure 3. Complete calibration sequence for 2 telescopes

Shutters and P2VM calibration files

Step	Shutter 1	Shutter 2	Shutter 3	Phase γ_0	DPR key
1	Open	Closed	Closed	NO	2P2V, 3P2V
2	Closed	Open	Closed	NO	2P2V, 3P2V
3	Open	Open	Closed	NO	2P2V, 3P2V
4	Open	Open	Closed	YES	2P2V, 3P2V
5	Closed	Closed	Open	NO	3P2V
6	Open	Closed	Open	NO	3P2V
7	Open	Closed	Open	YES	3P2V
8	Closed	Open	Open	NO	3P2V
9	Closed	Open	Open	YES	3P2V

Pixel to visibility matrix

Carrier waves

$$c_k^{ij} = C_B^{ij} \frac{\sqrt{a_k^i a_k^j}}{\sqrt{\sum_k a_k^i a_k^j}} \cos(2\pi\alpha_k f^{ij} + \phi_s^{ij} + \Phi_B^{ij})$$

$$d_k^{ij} = C_B^{ij} \frac{\sqrt{a_k^i a_k^j}}{\sqrt{\sum_k a_k^i a_k^j}} \sin(2\pi\alpha_k f^{ij} + \phi_s^{ij} + \Phi_B^{ij})$$

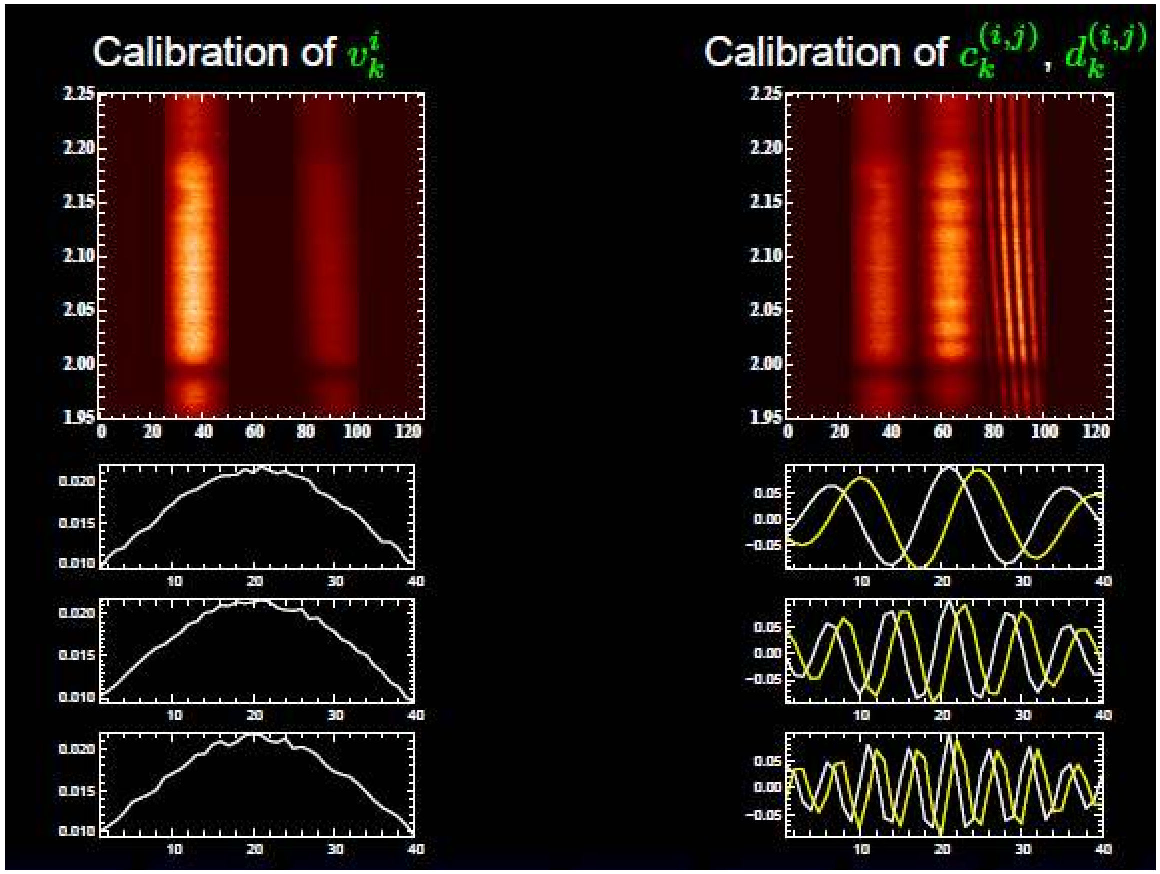
Complex correlation

$$R^{ij} = \sqrt{\sum_k a_k^i a_k^j} \mathrm{Re}\left[F_c^{ij}\right], \quad I^{ij} = \sqrt{\sum_k a_k^i a_k^j} \mathrm{Im}\left[F_c^{ij}\right]$$

DC corr. pixels

$$\begin{pmatrix} m_1 \\ | \\ m_{N_{pix}} \end{pmatrix} = \begin{pmatrix} \overbrace{\begin{matrix} .. & c_1^{ij} & .. \\ | & \vdots & | \\ .. & c_{N_{pix}}^{ij} & .. \end{matrix}}^{N_b} & \overbrace{\begin{matrix} .. & d_1^{ij} & .. \\ | & \vdots & | \\ .. & d_{N_{pix}}^{ij} & .. \end{matrix}}^{N_b} \end{pmatrix} \begin{pmatrix} \vdots \\ R^{ij} \\ \vdots \\ I^{ij} \\ \vdots \end{pmatrix} = \mathrm{V2PM} \begin{pmatrix} \vdots \\ R^{ij} \\ \vdots \\ I^{ij} \\ \vdots \end{pmatrix}$$

P2VM frames



Fringe fitting and estimation

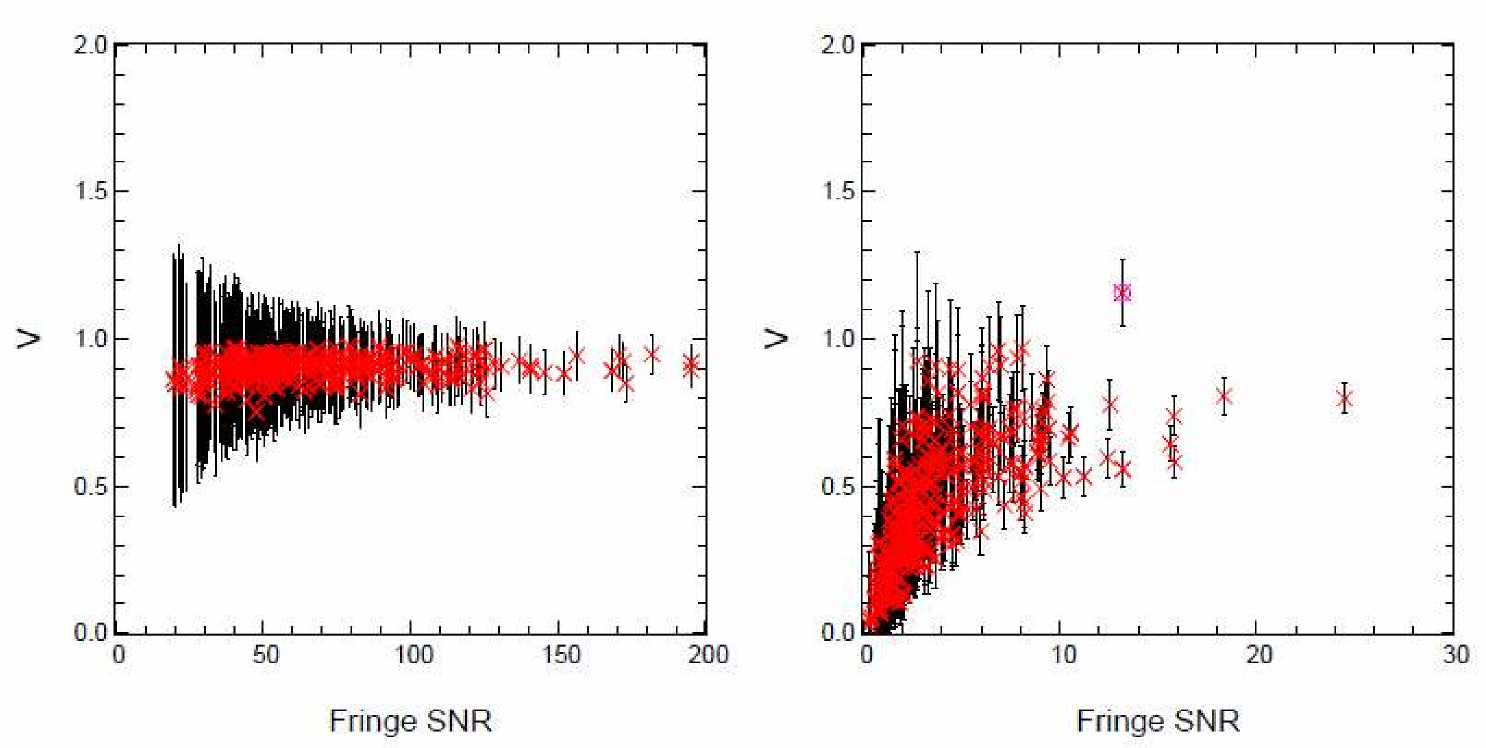
$$[\widetilde{R}^{ij}, \widetilde{I}^{ij}] = \mathrm{P2VM}[m_k]$$

where

$$\mathrm{P2VM} = [\mathrm{V2PM}^T C_M^{-1} \mathrm{V2PM}]^{-1} \mathrm{V2PM}^T C_M^{-1}$$

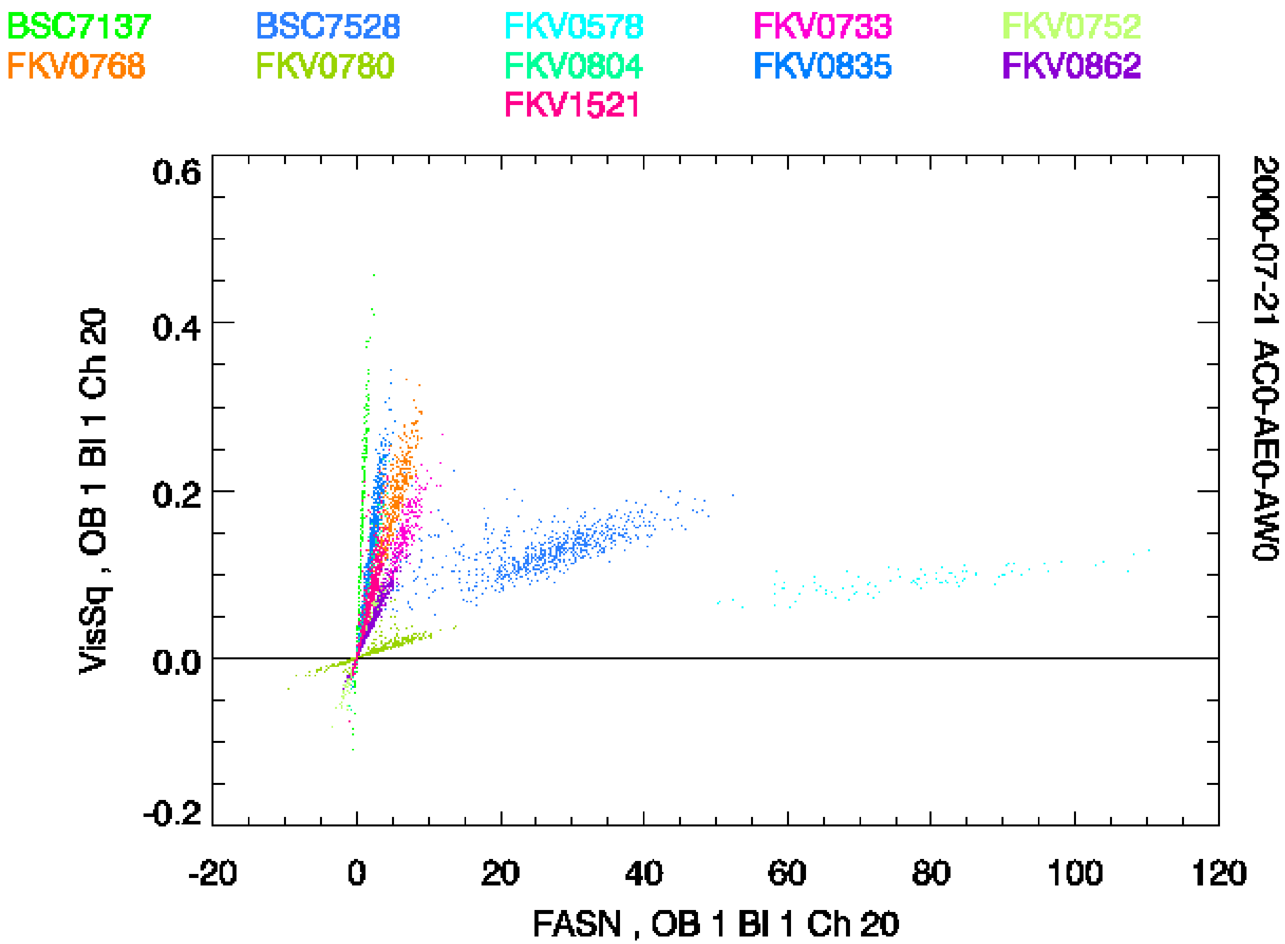
$$\frac{\widetilde{|V^{ij}|^2}}{{V_c^{ij}}^2} = \frac{\left\langle {R^{ij}}^2 + {I^{ij}}^2 \right\rangle - \ \mathrm{Bias}\{{R^{ij}}^2 + {I^{ij}}^2\}}{4 \left\langle P^i P^j \right\rangle \sum_k v_k^i v_k^j}$$

Fringe SNR



$$\mathrm{SNR}^2(t) = \frac{1}{N_b}\frac{1}{N_l}\sum_{b}^{N_b}\sum_{l}^{N_l}\left[\left(\frac{R^{b^2}(l,t)}{\sigma^2_{R^b}} - 1\right) + \left(\frac{I^{b^2}(l,t)}{\sigma^2_{I^b}} - 1\right)\right]$$

NPOI fringe SNR



Summary

- For AMBER, a lot still to be done
  - e.g. LR visibility reduction due to piston
  - multi-stage frame selection
- FINITO will stabilize AMBER visibilities
  - longer integration times, full read-out
- Not discussed: internal dispersion and differential phase issues
  - important for astrometry

Overview

- Fringe detection and estimators
- Detector bias and noise
- Challenges of atmospheric turbulence
- Wave front filtering in the NIR
- Back ground compensation in the MIR
  - MIDI data reduction
- Calibration and error estimates
- Interferometric imaging

MIDI Observation

- Fringe data (delay modulation on)
  - HIGH_SENS (no chopping)
  - SCI_PHOT (chopping)
- Photometry (chopping on)
  - shutter A open
  - shutter B open

HIGH-SENS Principles (1a)

- Observe fringes without chopping; the difference between the interferometric channels will be (almost) free of background fluctuations. This is because the background is highly correlated between the two channels, unlike VINCI where PA and PB differ due to the different fiber injection efficiencies as a function of time.

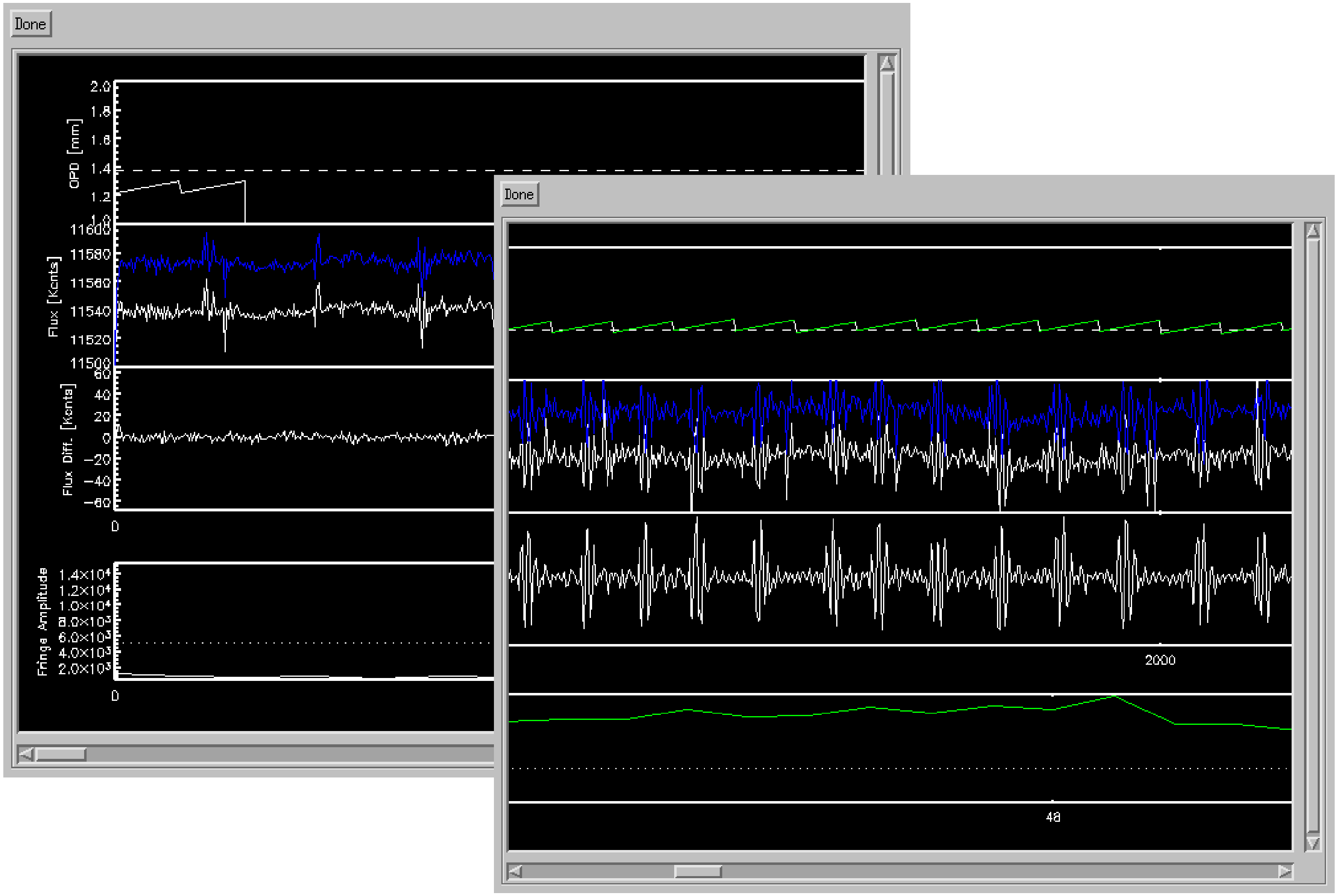
HIGH-SENS Principles (1b)

OPD

Flux

Interf.

Amp.



HIGH-SENS Principles (1c)

- The quality of the initial background cancellation depends on the splitting ratios
- A high-pass filter needs to be used to remove residual background fluctuations

$$P_B = \alpha P_A$$

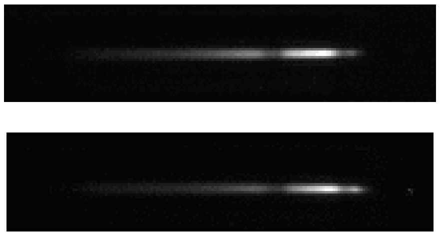
$$I_1 = \kappa_{1,\mathrm{A}} P_A + \alpha \kappa_{1,\mathrm{B}} P_A = P_A(\kappa_{1,\mathrm{A}} + \alpha \kappa_{1,\mathrm{B}})$$

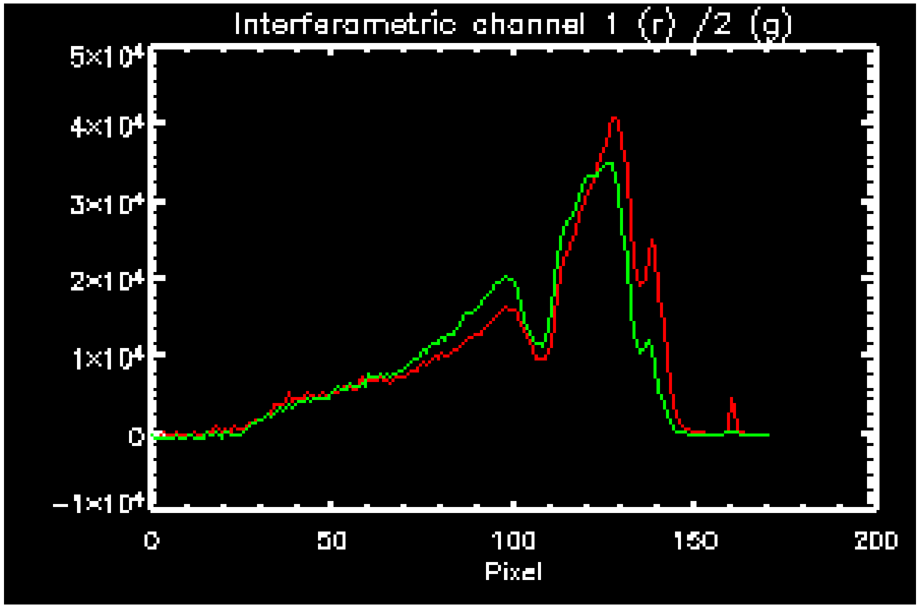
$$I_2 = P_A(\kappa_{2,\mathrm{A}} + \alpha \kappa_{2,\mathrm{B}})$$

$$I_1 - I_2 = P_A[\kappa_{1,\mathrm{A}} - \kappa_{2,\mathrm{A}} + \alpha(\kappa_{1,\mathrm{B}} - \kappa_{2,\mathrm{B}})]$$

HIGH-SENS Principles (2)

- Separate photometry with shutters A and B and chopping will measure spectra in the two interferometric channels





HIGH-SENS Principles (3)

Max. and min. field amplitudes: $A_\mathrm{A} + A_\mathrm{B}$ $A_\mathrm{A} - A_\mathrm{B}$

Max. and min. intensities:

$$I^\mathrm{max} = A_\mathrm{A}^2 + 2A_\mathrm{A}A_\mathrm{B} + A_\mathrm{B}^2 \qquad I^\mathrm{min} = A_\mathrm{A}^2 - 2A_\mathrm{A}A_\mathrm{B} + A_\mathrm{B}^2$$

Visibility amplitude: $V = (I^\mathrm{max} - I^\mathrm{min})/(I^\mathrm{max} + I^\mathrm{min})$

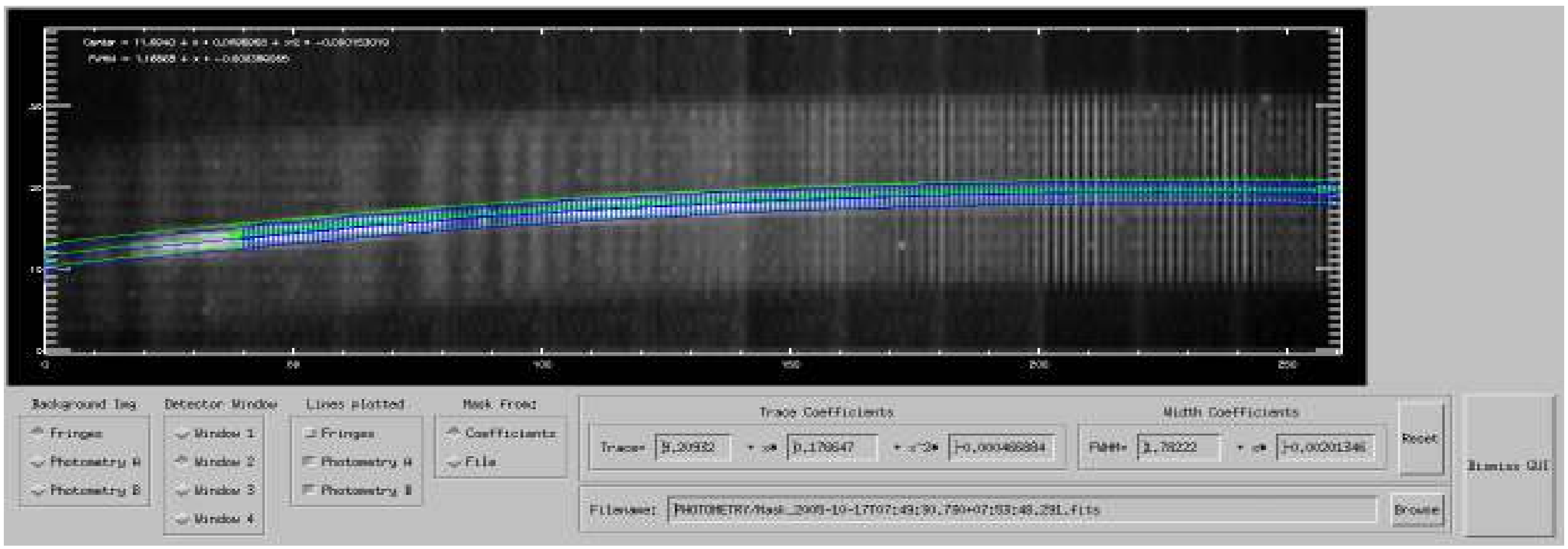
yields: $V^\mathrm{max} = 2\sqrt{I_\mathrm{A} I_\mathrm{B}}/(I_\mathrm{A} + I_\mathrm{B})$

Interferogram in one MIDI channel:

$$I_1 = I_{\mathrm{A},1} + I_{\mathrm{B},1} + (1/2)(I_1^\mathrm{max} - I_1^\mathrm{min})V \sin(2\pi OPD/\lambda)$$

Subtracting the two channels: $2V\sqrt{I_{\mathrm{A},1} I_{\mathrm{B},1}} + 2V\sqrt{I_{\mathrm{A},1} I_{\mathrm{B},1}}$

Mask definition



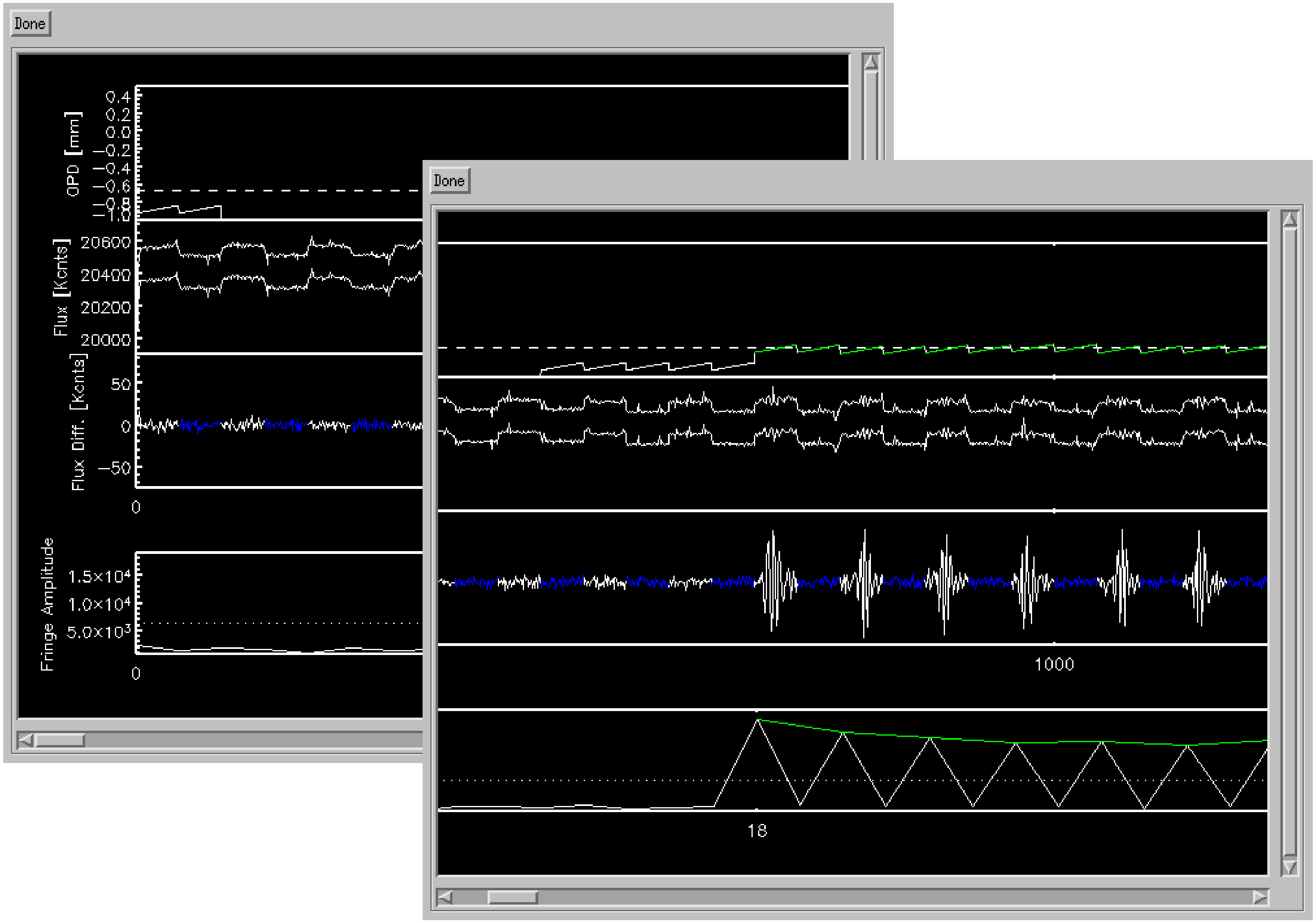
SCI_PHOT Principles (1a)

- Photometry taken simultaneously with the fringe data, but kappa matrix (i.e. splitting ratios) must be used to determine normalization
- Kappa matrix can be determined from A and B photometry (needs only to be done once per night)

$$\kappa_{1,\mathrm{A}} = I_1/(I_1 + I_2),\ \kappa_{2,\mathrm{A}} = I_2/(I_1 + I_2)$$

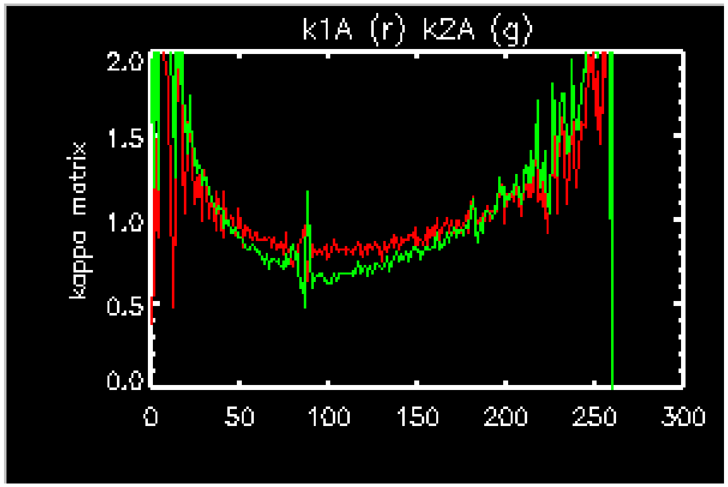
- Similar to VINCI, but background correction is different

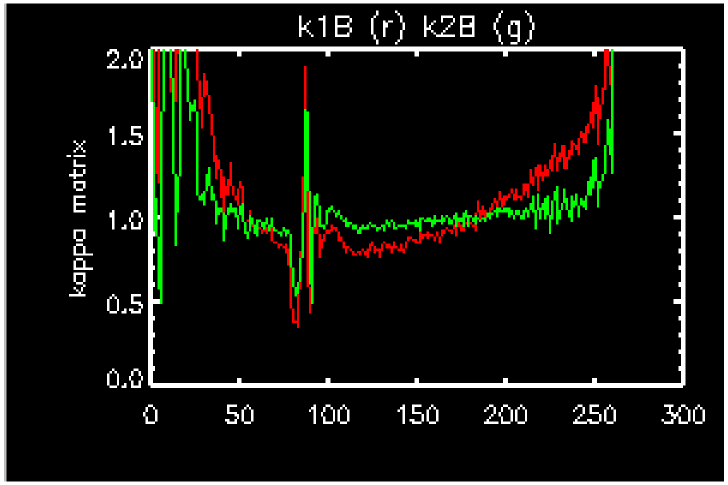
SCI_PHOT Principles (1b)



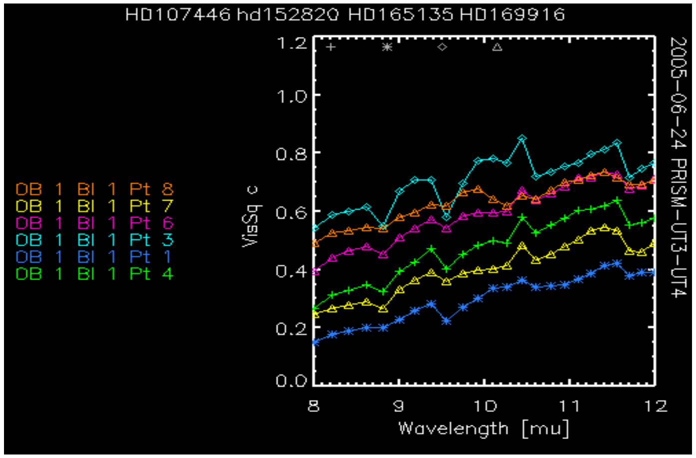
SCI_PHOT Principles (1c)

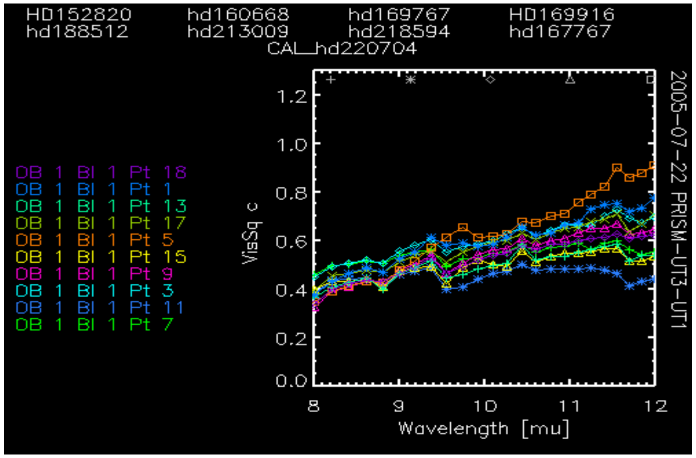
- Kappa matrix needs to be determined once per night to derive splitting ratios

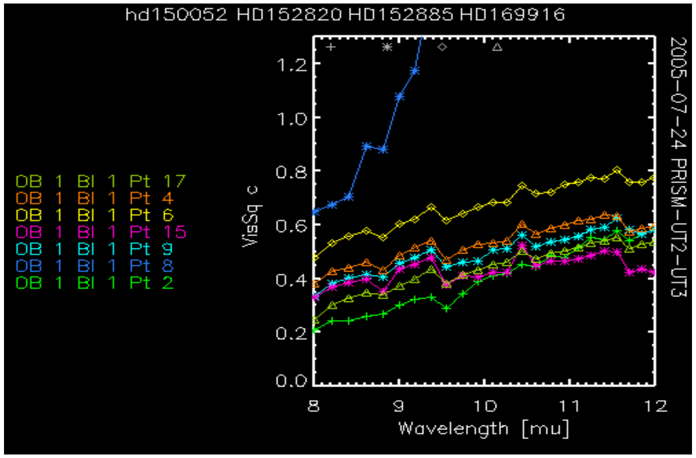


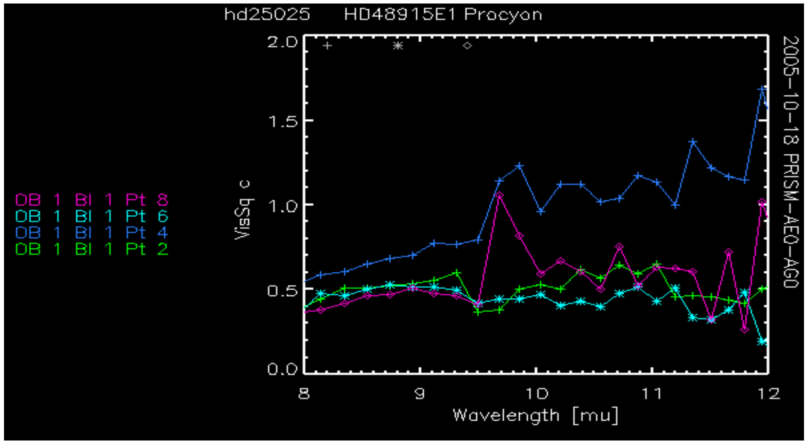


Calibrator visibility (TF)







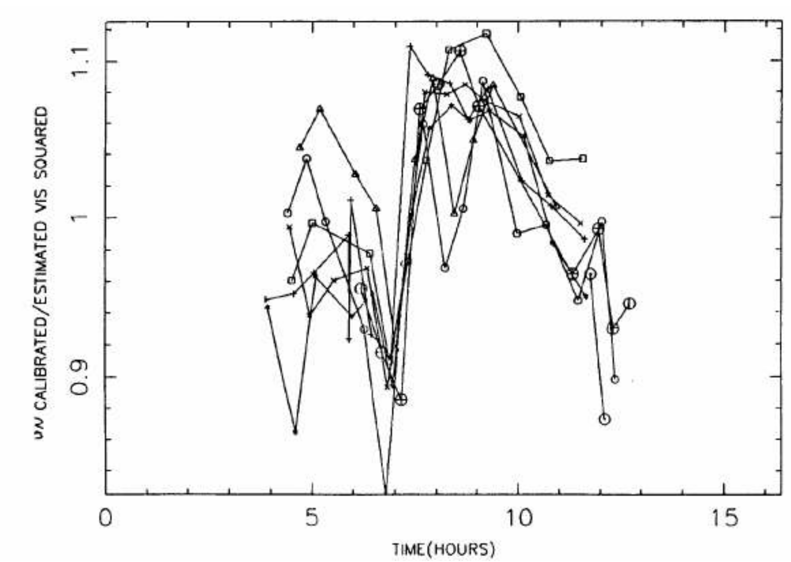


Overview

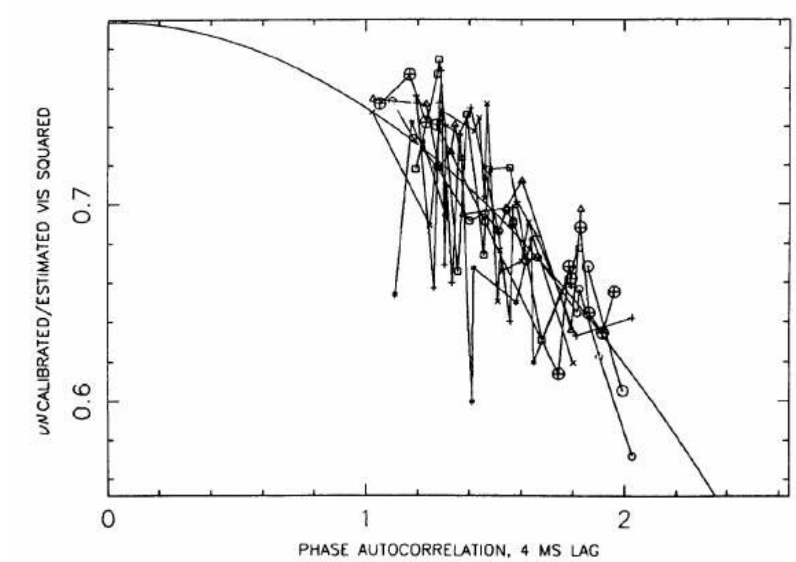
- Fringe detection and estimators
- Detector bias and noise
- Challenges of atmospheric turbulence
- Wave front filtering in the NIR
- Back ground compensation in the MIR
- Calibration and error estimates
- Interferometric imaging

Global calibration

Calibrator V^2 vs time

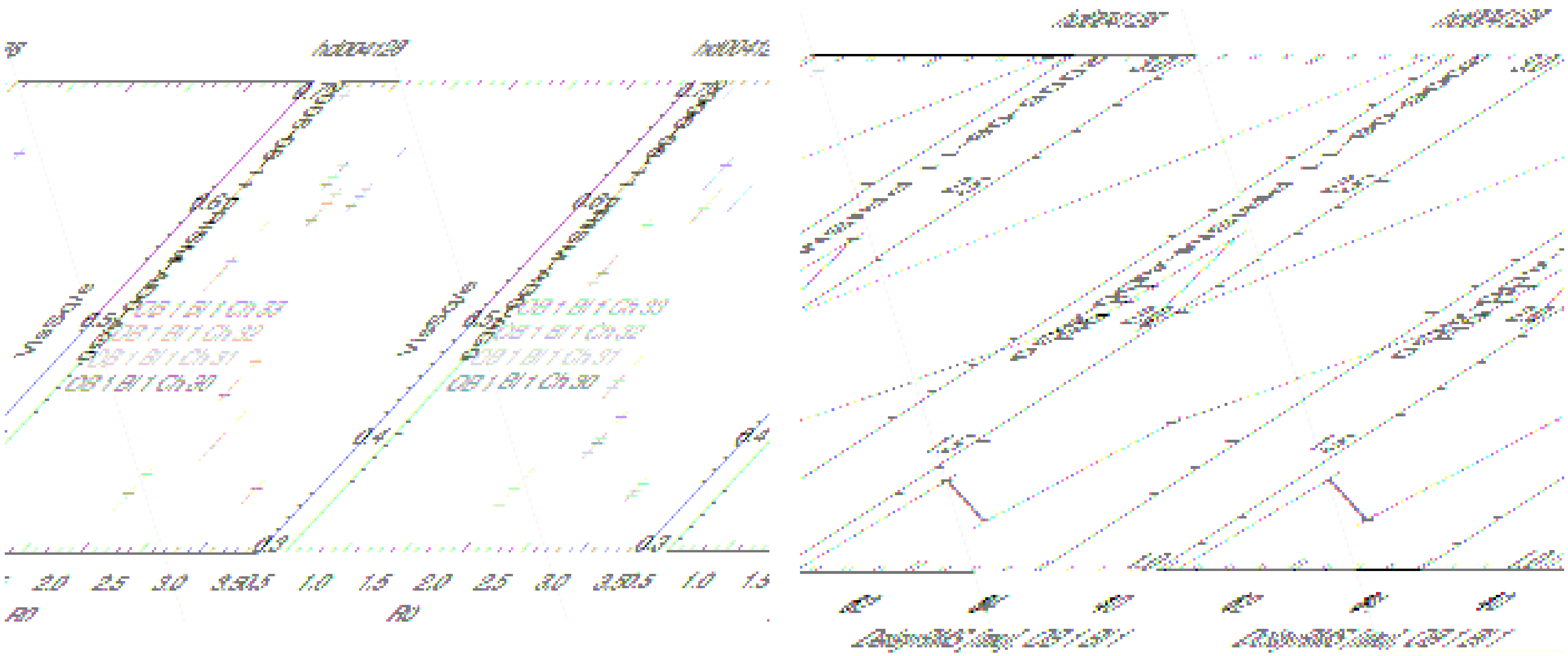


Calibrator V2 vs seeing



Mark III: Mozurkewich et al. 1991

Dependence of MIDI V^2 on seeing

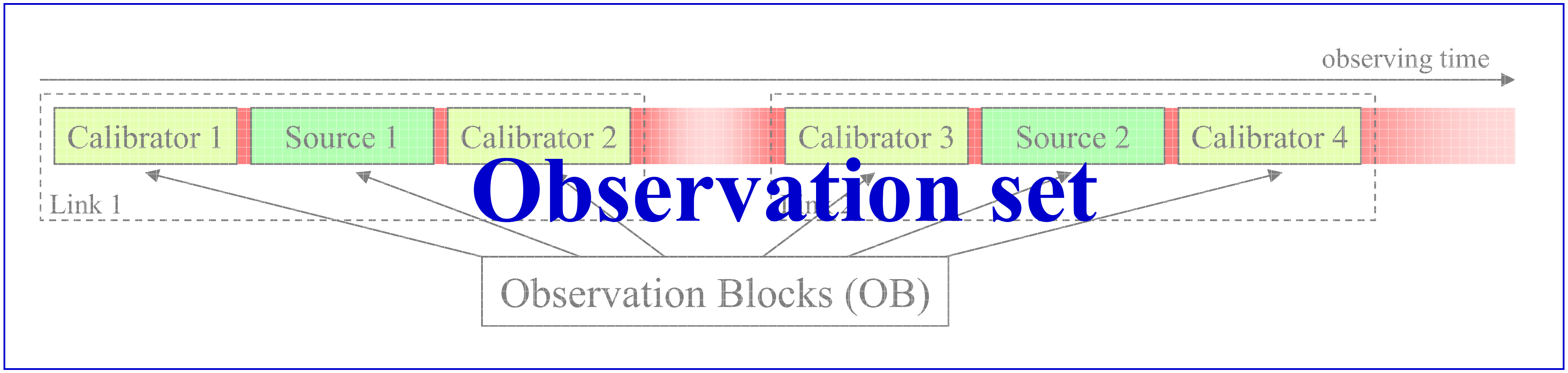


MIDI

Calibrator selection

- Close to science target
- Measured immediately before and/or after
- Small known diameter
- Similar brightness as science target

Principle of calibration



- 1 Observation set = 1 set-up
  - same night
  - same detector parameters (frame rate, number of frames, ...)
  - same filter...

Steps

- Derive the ***expected visibility of the calibrator:***

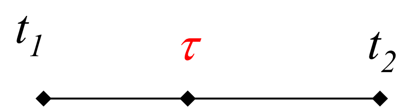
$$V_{\exp}(S) = \left| \frac{2J_1(\pi\theta_{UD}S)}{\pi\theta_{UD}S} \right|$$

- Derive the ***instantaneous transfer function*** for each channel:

$$T_i^2(t_1) = \frac{\mu_i^2}{V_{\exp}^2(S)}$$

- Estimate the transfer function ***at the time when the science target was observed:***

$$T^2(\tau) = \left(\frac{t_2 - \tau}{t_2 - t_1} \right) T^2(t_1) + \left(\frac{\tau - t_1}{t_2 - t_1} \right) T^2(t_2)$$



- ***Calibrate the visibility of the science target***

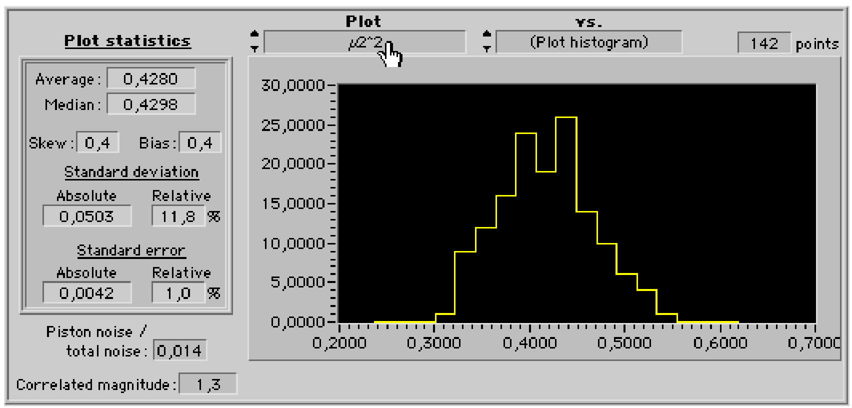
$$V^2 = \frac{\mu^2}{T^2(\tau)}$$

Uncertainty estimates

- Error bars often based on RMS of samples in an average
- Theoretical error estimates often too small
- Systematic errors due to bias or low SNR

Final μ^2 estimate and error bar

- Squared coherence factors are computed for each scan in each interferometric channel



$$\left.\begin{array}{l} \mu_+^2 \pm \sigma\left(\mu_+^2\right) \\ \mu_-^2 \pm \sigma\left(\mu_-^2\right) \end{array}\right\} \quad \Rightarrow \quad \boxed{\mu^2 \pm \sigma\left(\mu^2\right)}$$

- They define a statistics (histogramm) from which a standard deviation is derived

Propagation of errors

- <u>Sources of errors (1σ error bars):</u>
- - errors on coherence factors (detector noise, photon noise, piston noise)
- - errors on the diameter of calibrators

- <u>Propagation of errors:</u>
- - The final estimate of the squared visibility is the product and ratio of hopefully gaussian random variables.

Propagation of errors

- <u>1st method to propagate errors:</u> $V^2 = \frac{\mu^2}{\mu_c^2} \times V_c^2$

  – make an expansion of the V^2 estimator if error bars are small

$$dV^2 = \frac{V^2}{\mu^2} \times d\mu^2 + \frac{V^2}{V_c^2} \times dV_c^2 - \frac{V^2}{\mu_c^2} \times d\mu_c^2$$

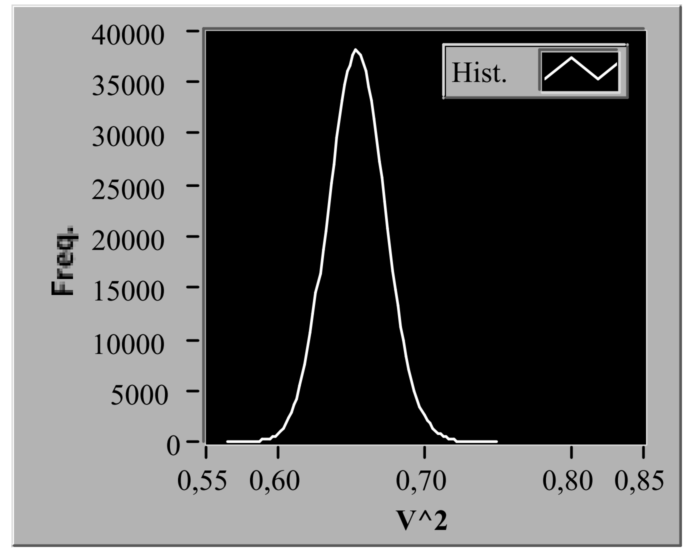
  – and sum the weighted variances of the errors

$$\sigma^2(V^2) \approx \left(\frac{V^2}{\mu^2}\right)^2 \times \sigma^2(\mu^2) + \left(\frac{V^2}{V_c^2}\right)^2 \times \sigma^2(V_c^2) + \left(\frac{V^2}{\mu_c^2}\right)^2 \times \sigma^2(\mu_c^2)$$

⚠ *only valid if errors are small*

Propagation of errors

- <u>2nd method to propagate errors:</u> $V^2 = \frac{\mu^2}{\mu_c^2} \times V_c^2$



$$\left.\begin{aligned} \mu^2 &= 0.400 \pm 0.010 \\ \mu_c^2 &= 0.600 \pm 0.010 \\ V_c^2 &= 0.980 \pm 0.001 \end{aligned}\right\} \Rightarrow \quad V^2 = 0.654 \pm 0.020$$

  - simulate the random variable and compute the variance of the simulated statistical distribution

Analytical method $\Rightarrow \quad V^2 = 0.653 \pm 0.020$

Final uncertainties

- Use designated calibrator
- Estimate (conservatively) calibration error from all calibrator measurements in same night
- Report to ESO/USD any bad calibrators

Overview

- Fringe detection and estimators
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Self-cal and CLEAN

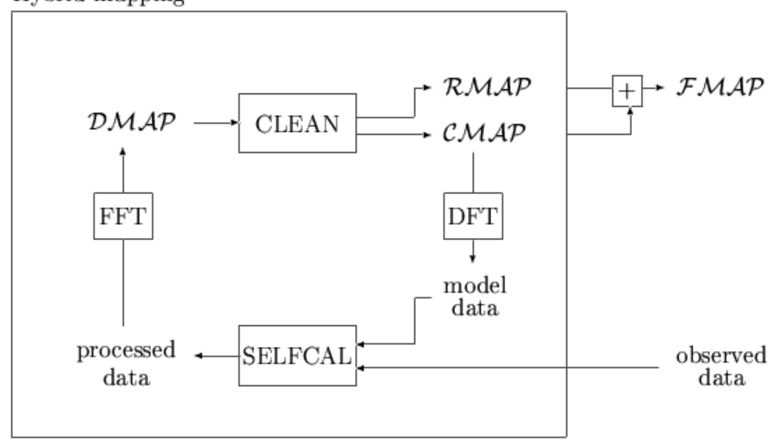
DMAP = dirty map
DMAP = FFT(V_{obs})

RMAP = residual map

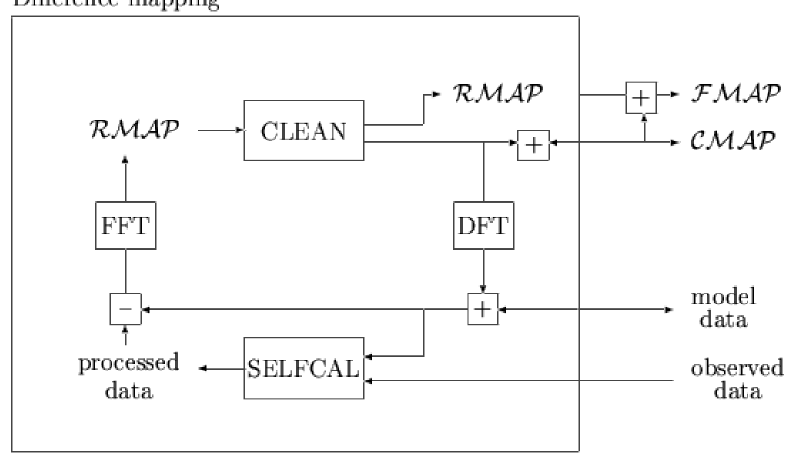
CMAP = clean map
V_{mod} = DFT(CMAP)

FMAP = final map
FMAP = RMAP + CMAP

Hybrid mapping

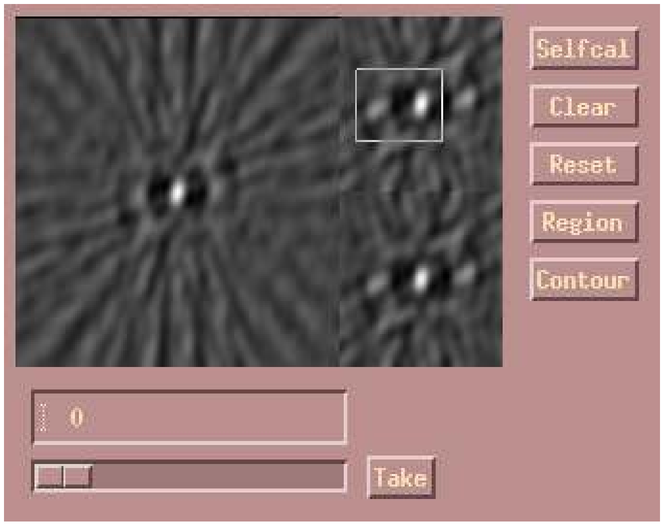


Difference mapping

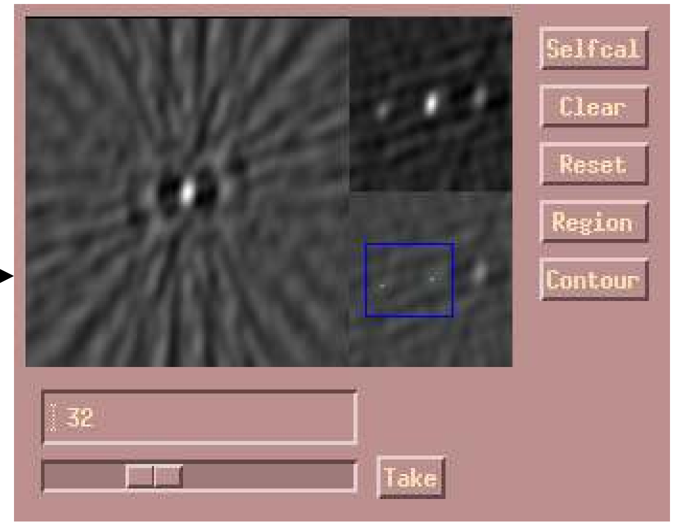


Difference mapping

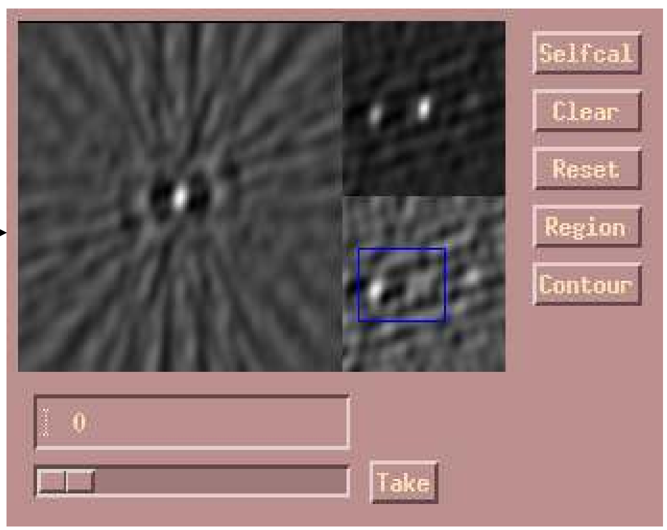
1



2



3



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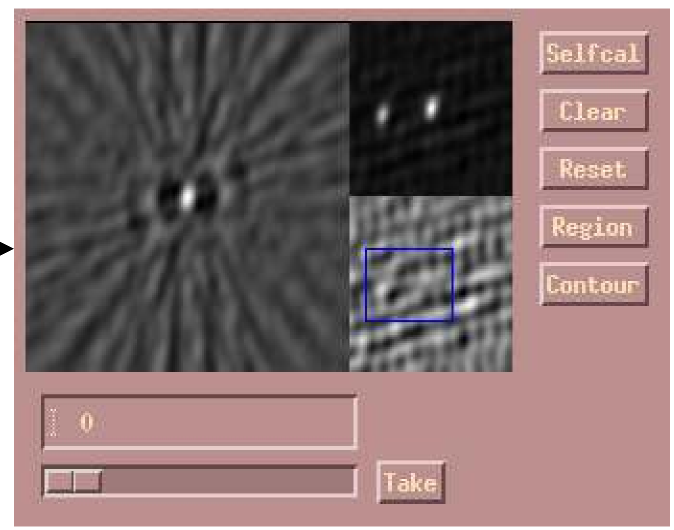
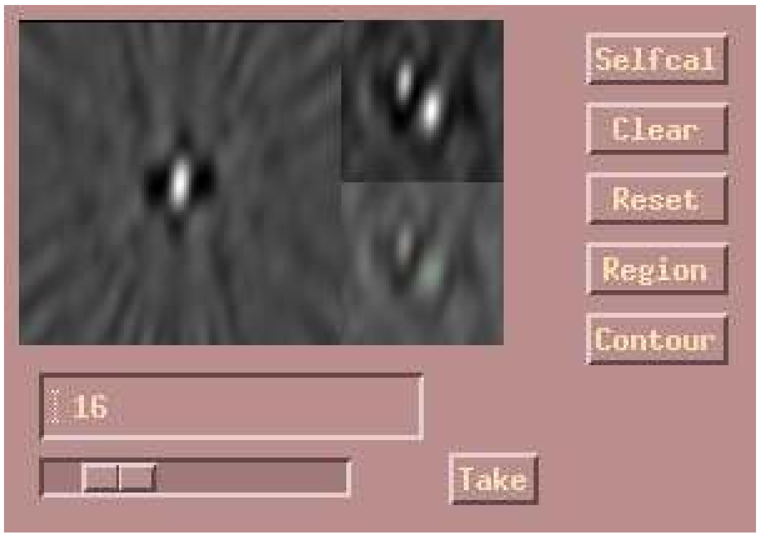
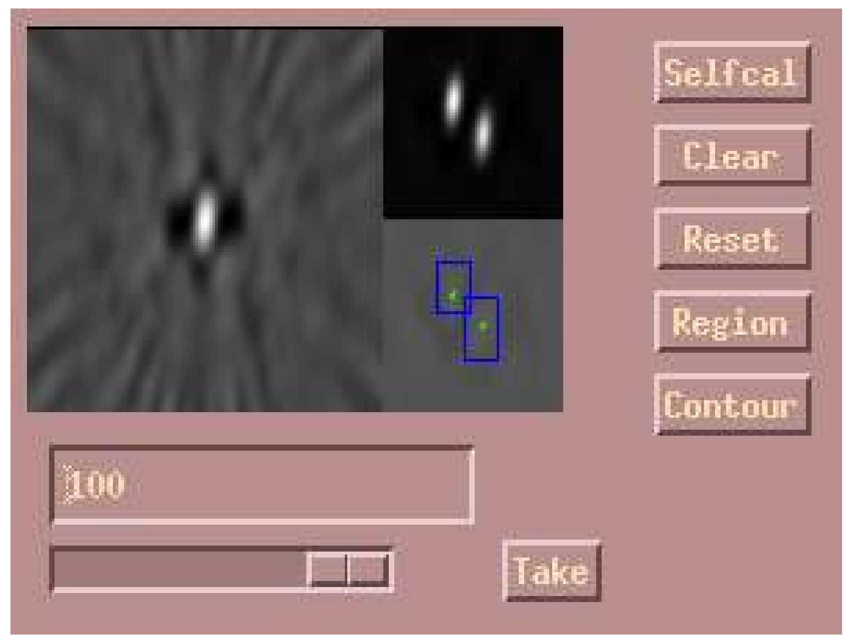


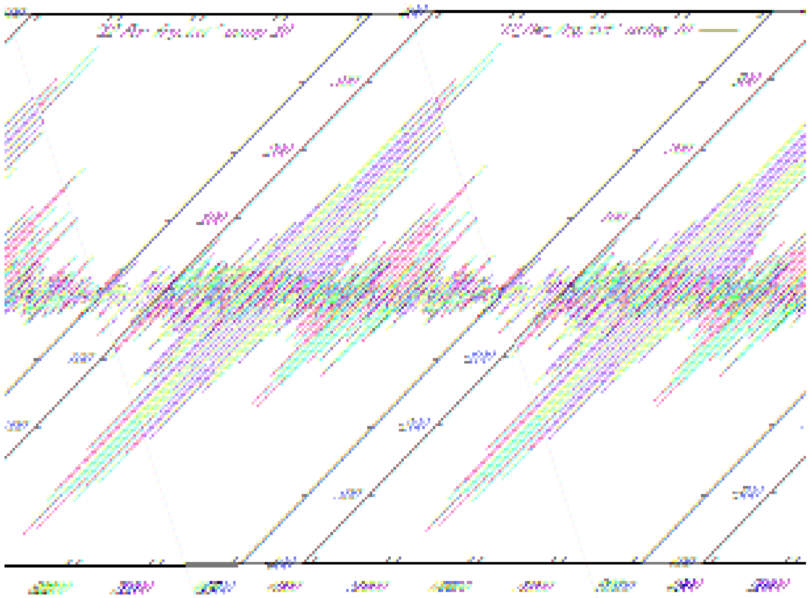
Imaging composite spectrum binaries



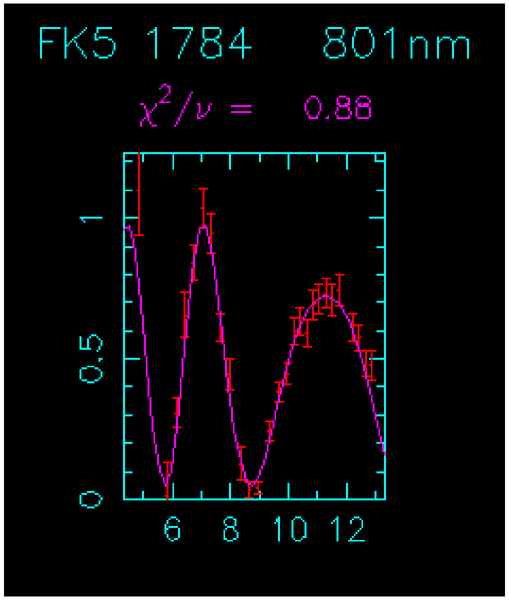


(Pearl/OYSTER)

Interferometric field of view (I)

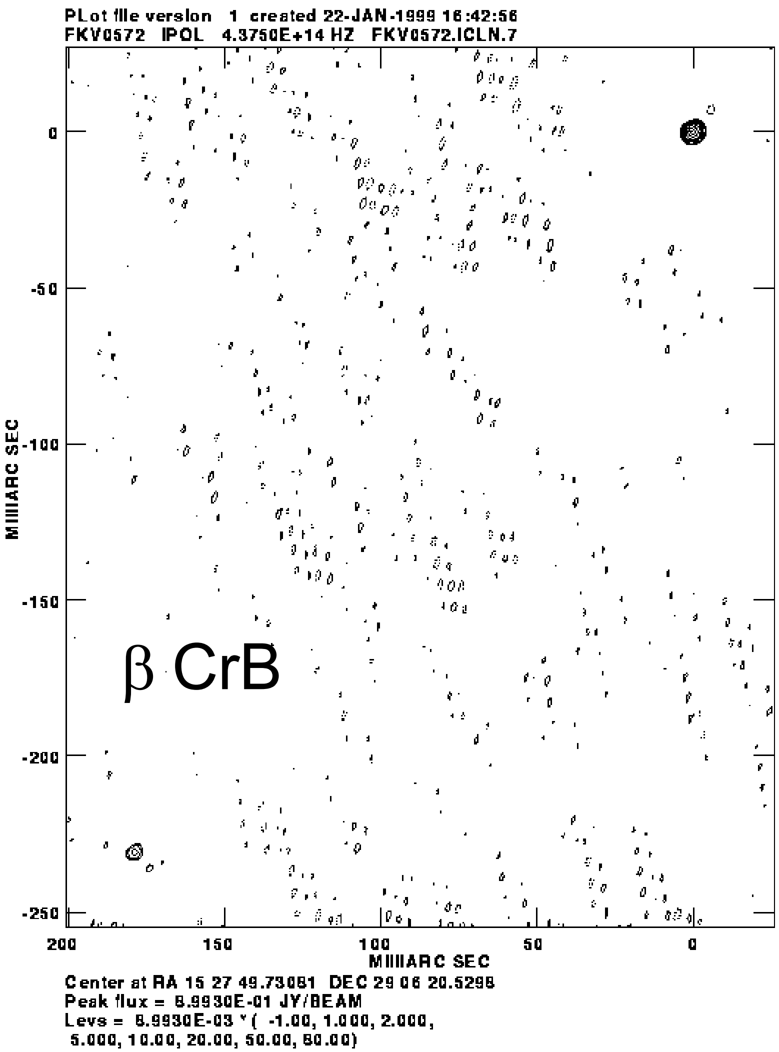


12 Persei observed on Oct 9, 2001 with the CHARA Array, K’-band, 330m baseline, separation 40 marcsec

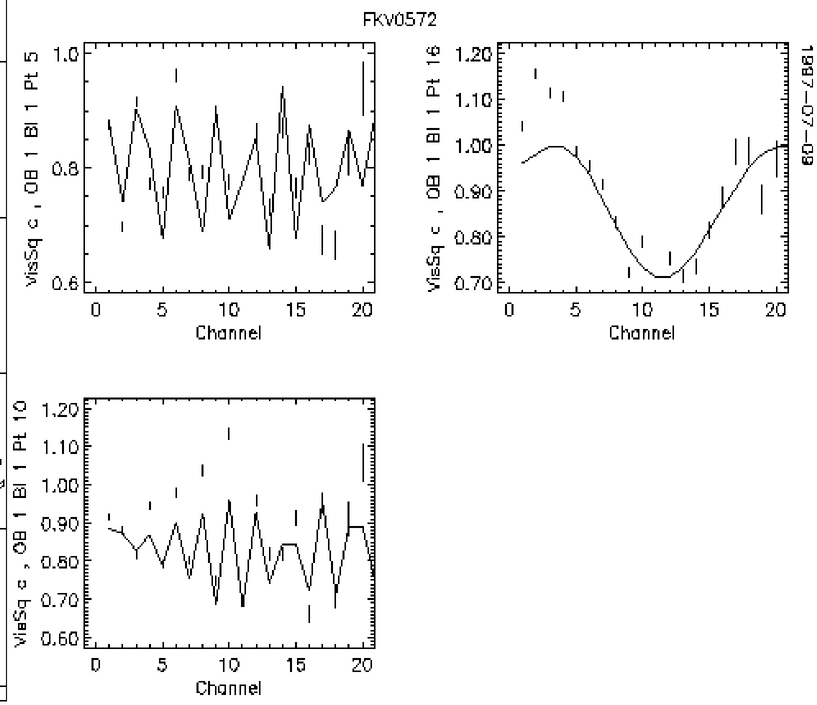


Mark 3 (Oct 8, 1992)

Interferometric field of view (II)

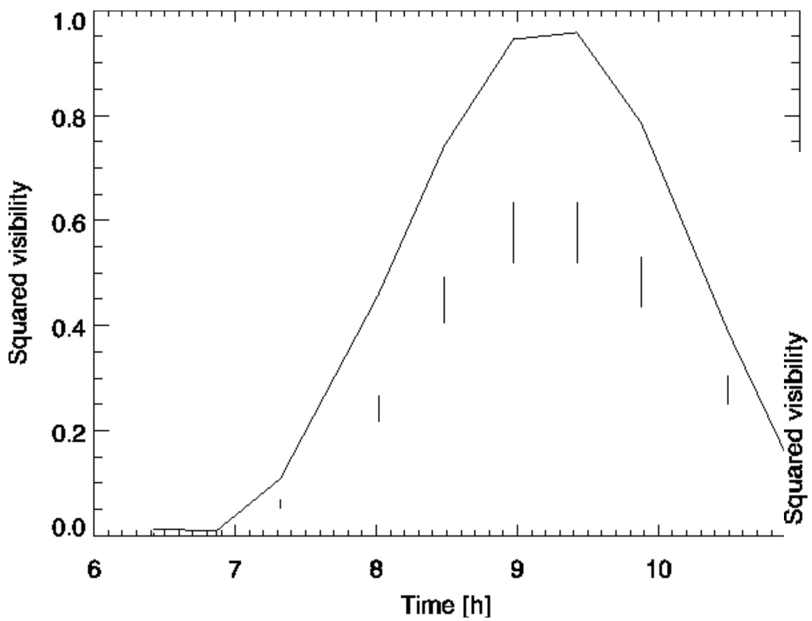


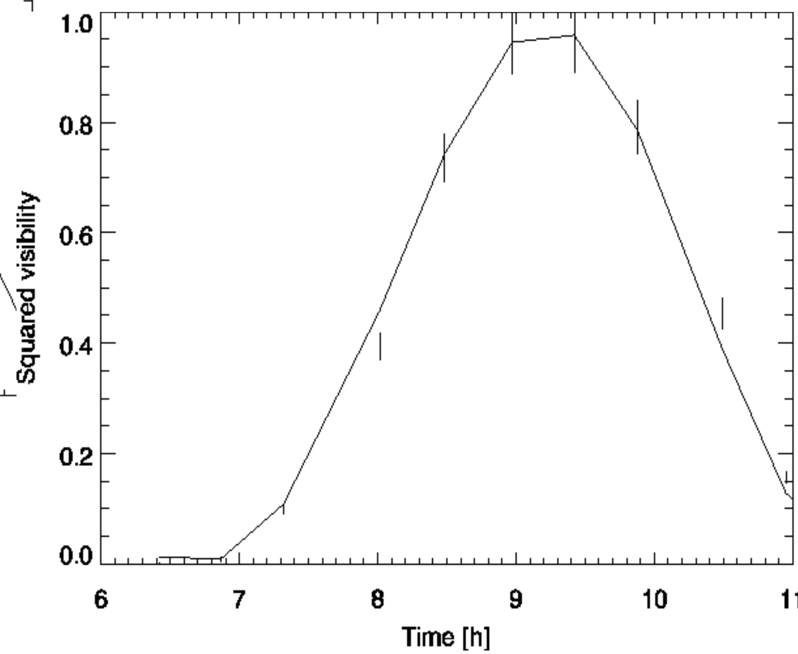
$$\Delta\alpha = R\frac{\lambda}{B}$$



(NPOI)

Photometric field of view





Mizar A (V=2.3)
with B (V=4.0) at 14″
(Mark III)

$$f = (1 + 10^{-\Delta m/2.5})^2$$