Introduction to phases and closure phases

…in the YSO and exoplanet context
part I

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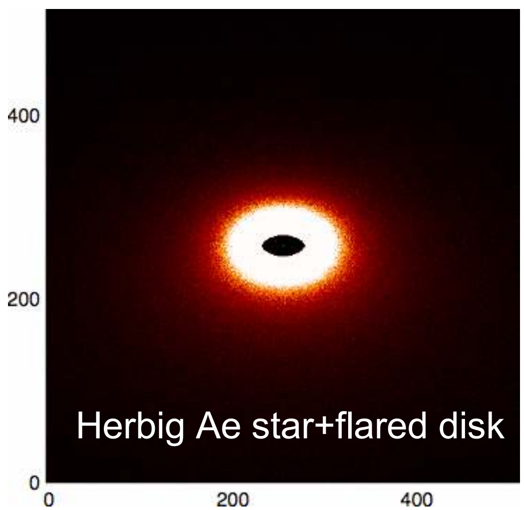
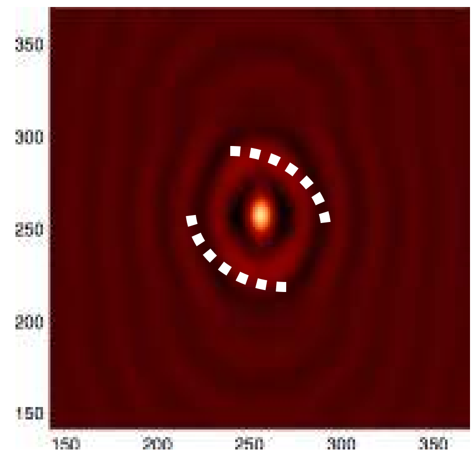
See Goutelas 2006 andMichelson 1999 Monnier's paper

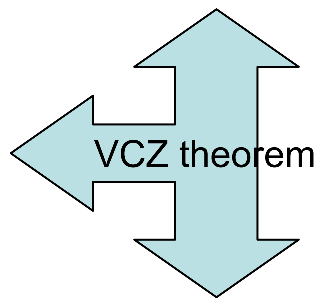
Imaging with an interferometer

The image

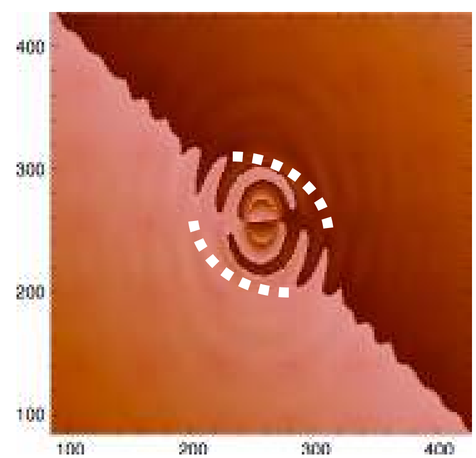


Visibilities

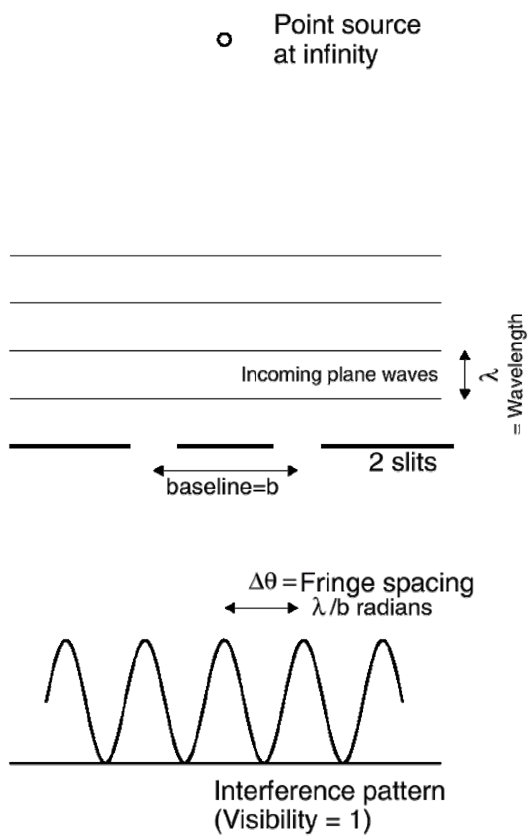


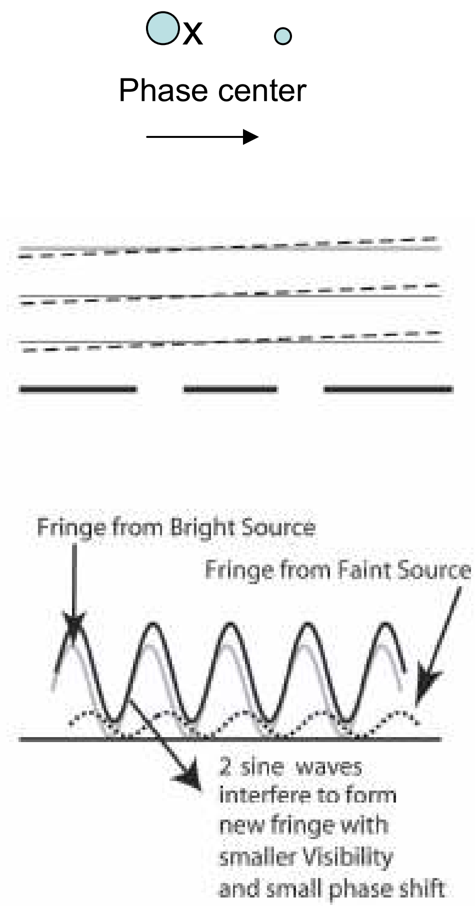


Phases

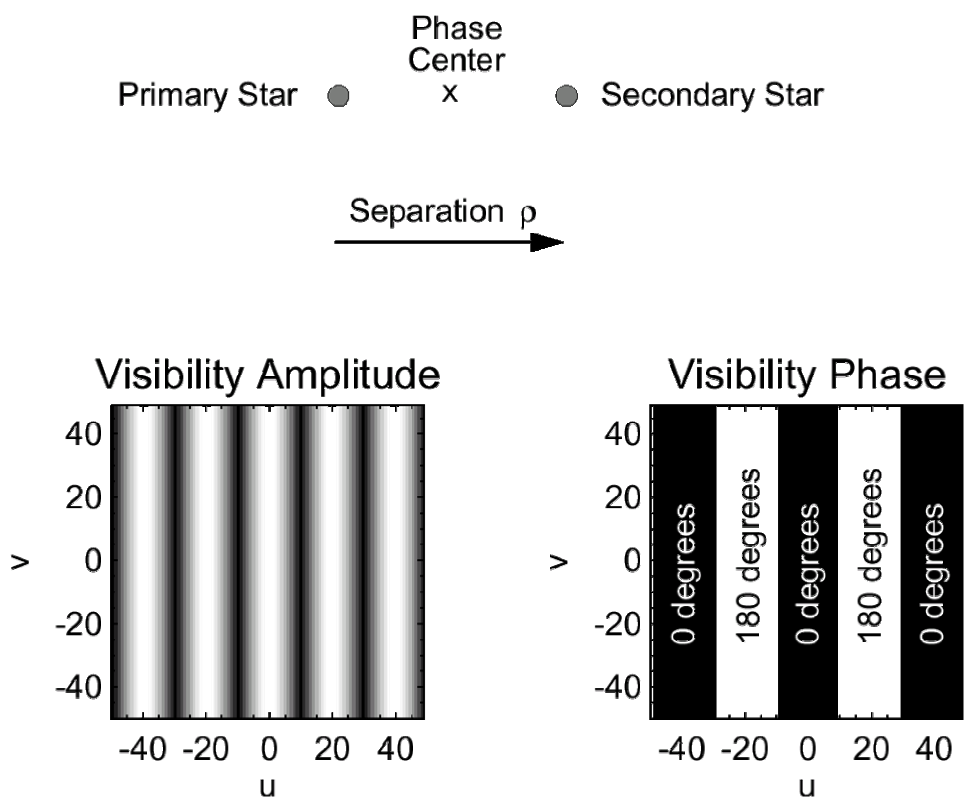


The simplest example:

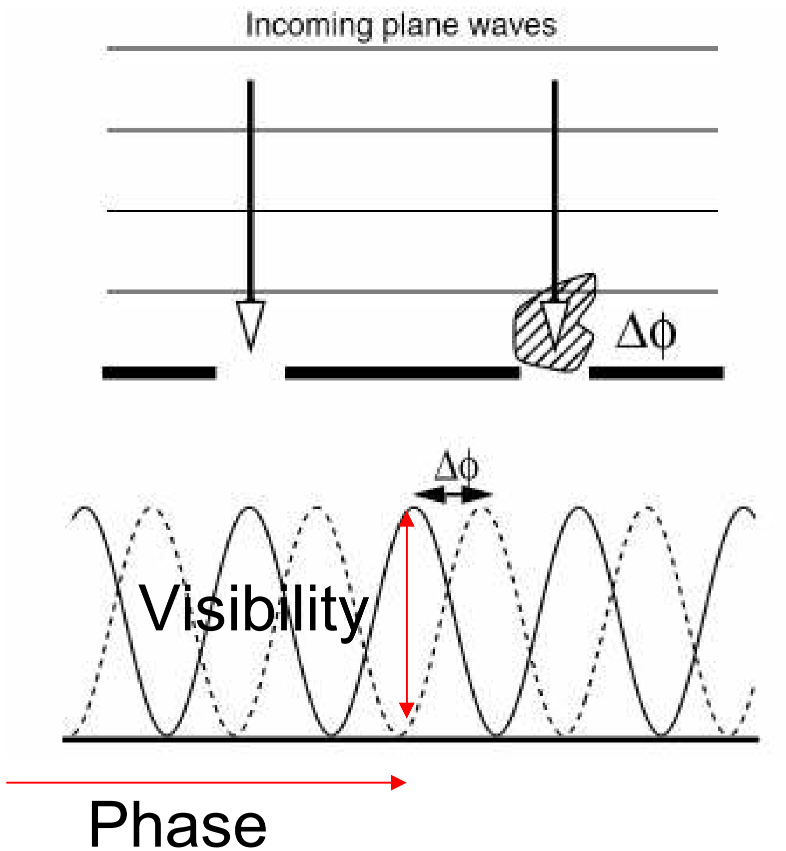




The simples example: a binary



Measuring phases (I)



- Assume a star at infinity
- A fringe pattern is centered on the spot where the path lengths are equal coming from both sides
- A phase disturbance shifts the fringe pattern
- Absolute phase is lost unless you find a way to retrieve it

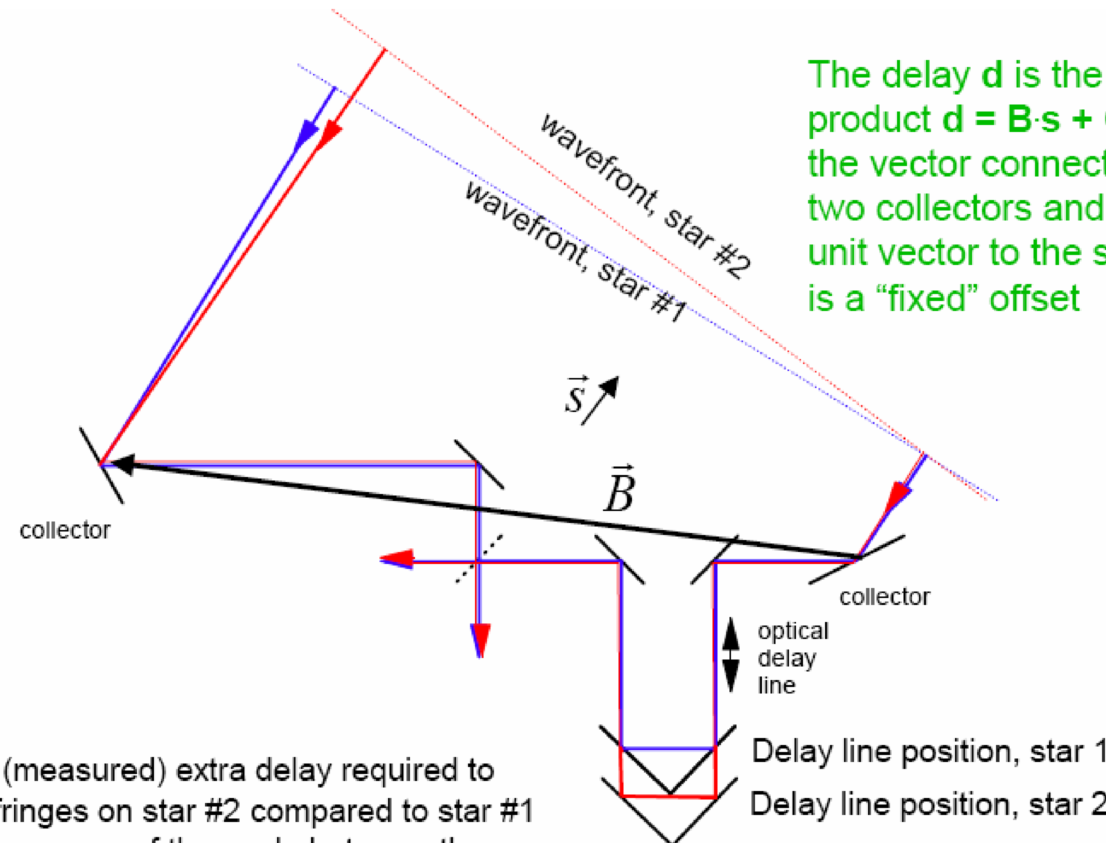
Measuring phases (II)

- Phase referencing: measuring precisely
  - the angular position of the source

  photocenter (astrometry)
- Differential phase: measuring a phase shift with respect to a reference wavelength
- Closure phase: retrieving partial but free of atmospheric biases, phase information

Phase referencing (I)

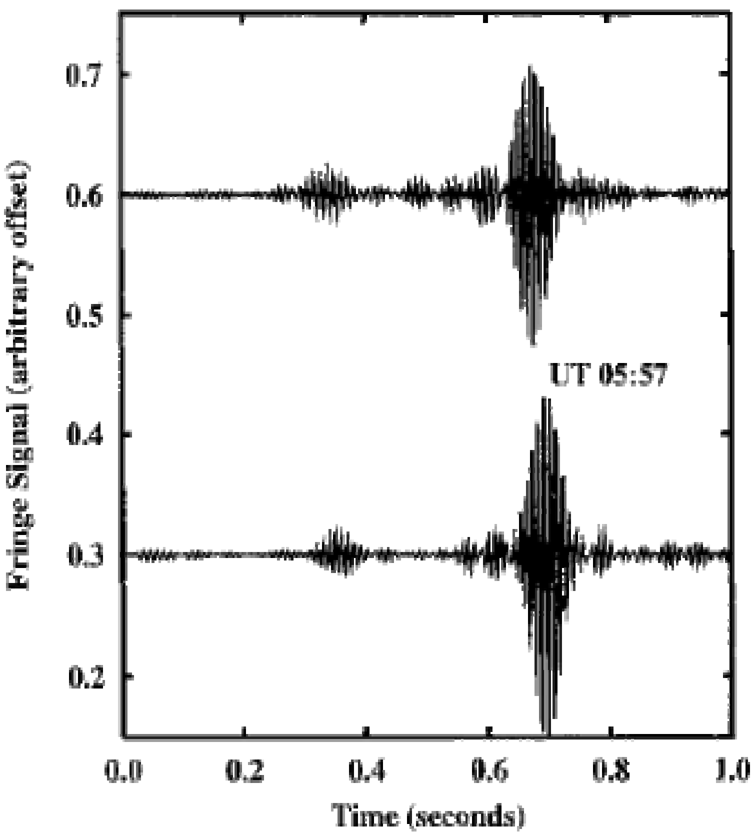
application to astrometry and imaging



The delay **d** is the dot product **d = B·s + C** of the vector connecting the two collectors and the unit vector to the star; **C** is a “fixed” offset

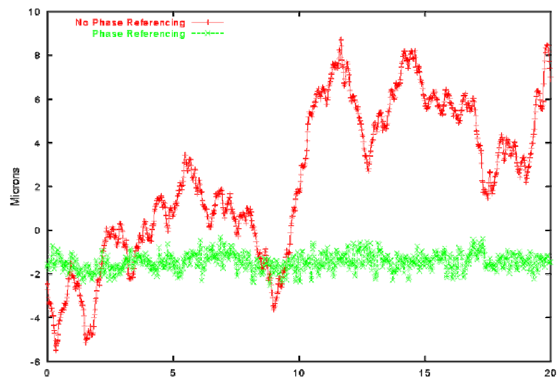
The (measured) extra delay required to get fringes on star #2 compared to star #1 is a measure of the angle between them

Phase referencing (II)

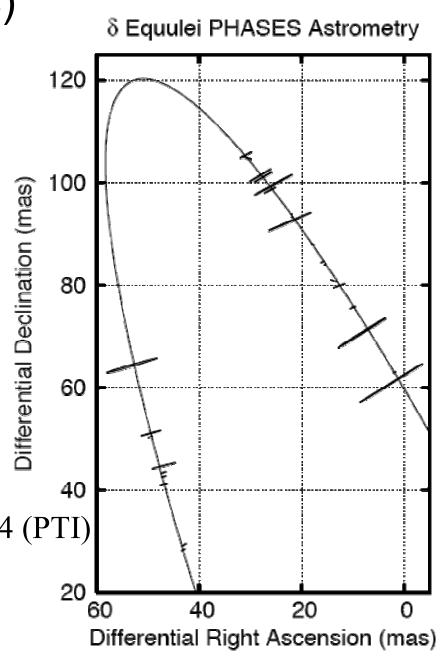


- Atmosphere is a killer
- You need a bright target at a distance smaller than the isoplanatic angle (two beams experience same atmosphere)
- Complex dual feed optics and differential metrology
- PRIMA at VLTI will offer second generations instrument with phase referencing capacity

Phase referencing (III)



Lane & Colavita (2003)



Muterspaugh et al. 2004 (PTI)

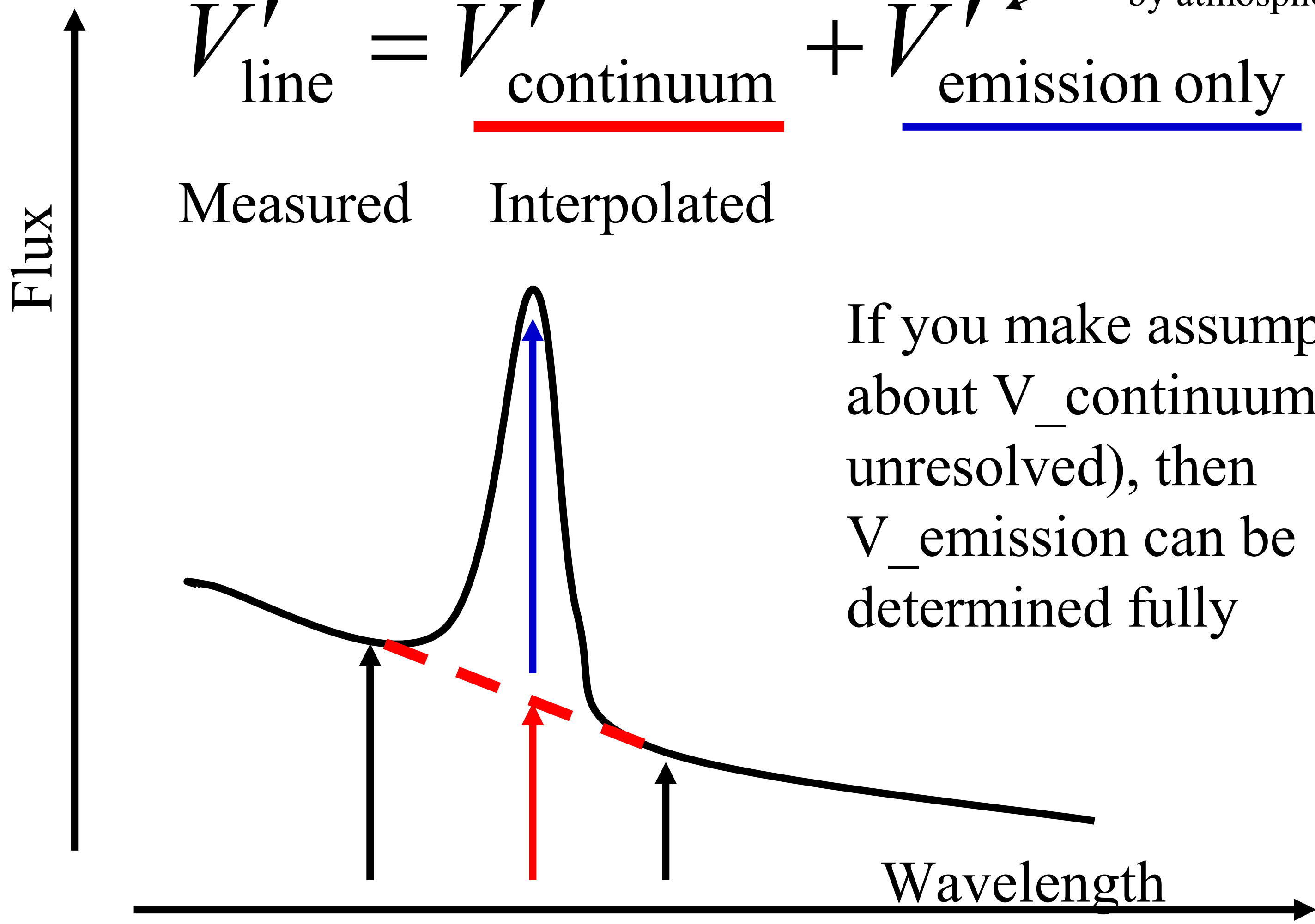
- Science case: astrometry of planet harbouring stars, massive clumps in CS disks…
- Best performance achieved:100 mas rms at PTI
- PRIMA: 50 microarcseconds should be ok, goal 10 micro arcseconds

Differential phase (I)

$$\widetilde{V}'_{\text{line}} = \widetilde{V}'_{\text{continuum}} + \widetilde{V}'_{\text{emission only}}$$

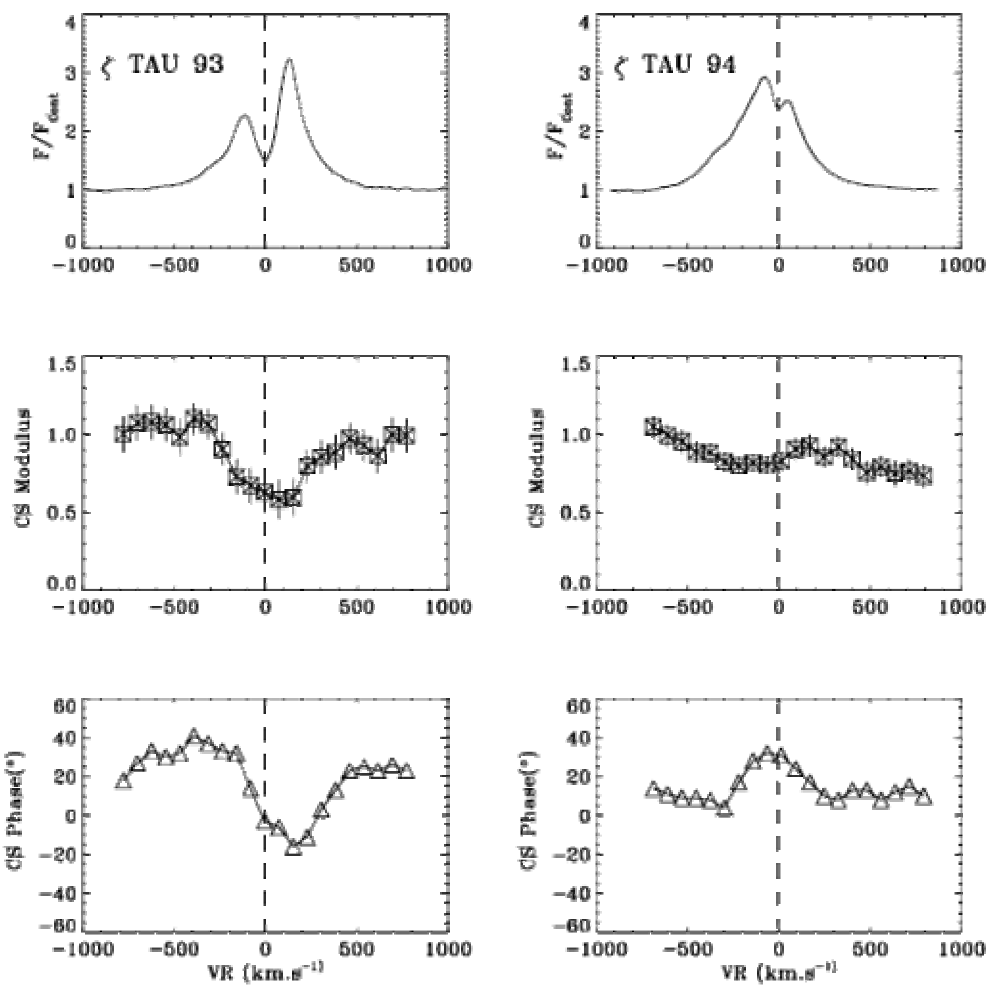
Prime indicates corrupted by atmospheric piston

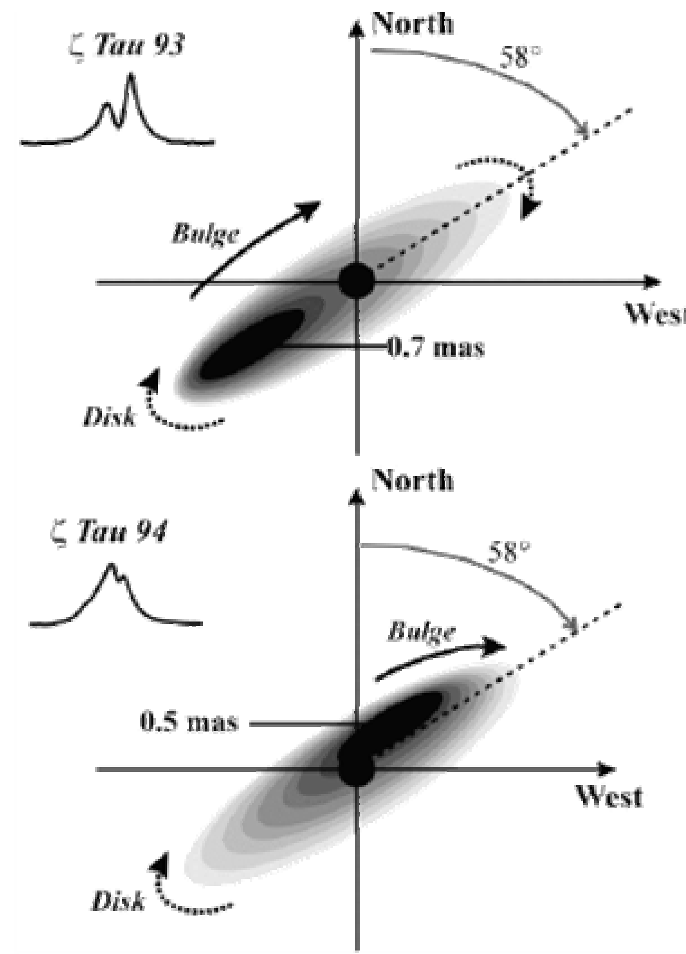
Measured Interpolated



If you make assumption about V_continuum (e.g., unresolved), then V_emission can be determined fully

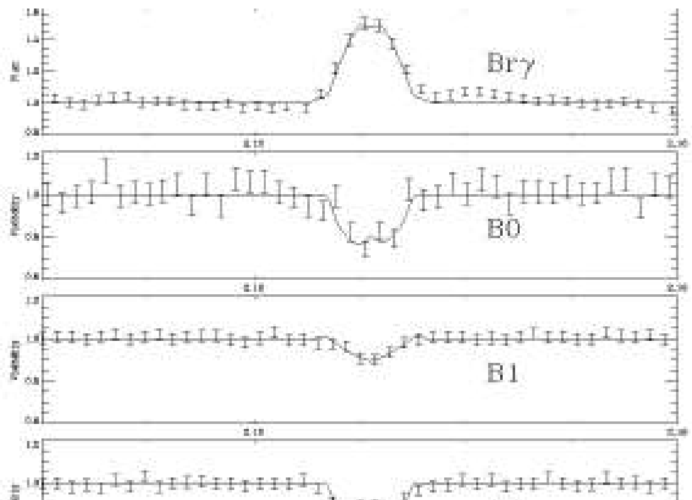
Differential phase (II)

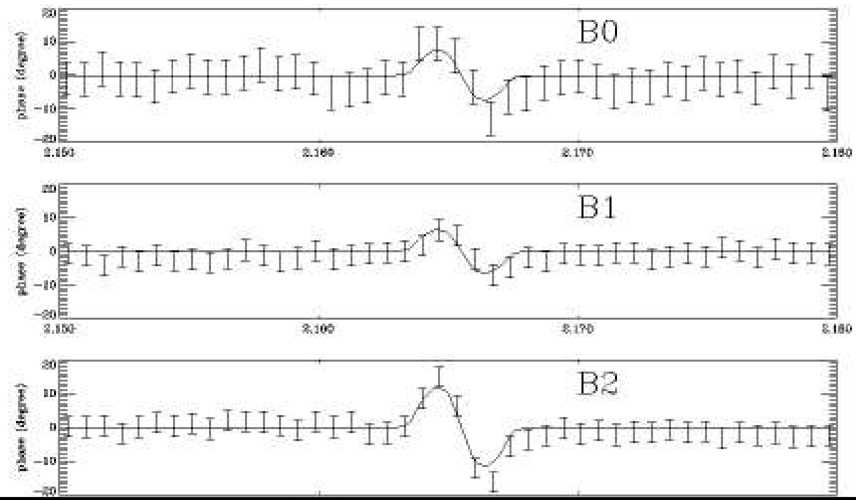


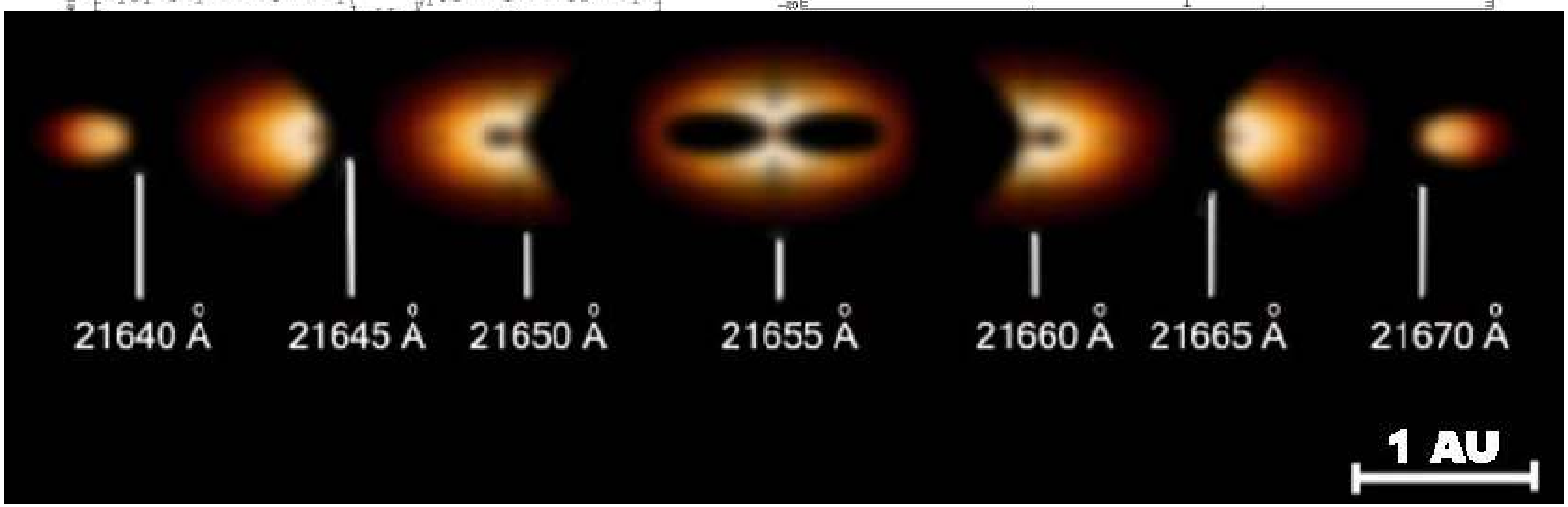


Differential phase (III)

Amber/VLTI results







Differential phase (IV)

in the YSO/exoplanet context

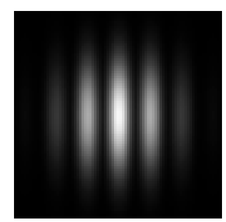
- Amber
- Rotation signatures inside bright disks (e.g CO overtone lines in Fu Ori disks)
- Wind/continuum emission localization
- Imaging but requires a good knowledge of reference wavelength distributions (continuum probably not point like in YSOS !)
- Exoplanet: very hard (atmospheric and instrumental chromatic dispersion are killers).

Closure phase (I)

$$V(u,v) = \iint A(x,y)\, I(x,y)\, e^{i2\pi(ux+vy)}$$

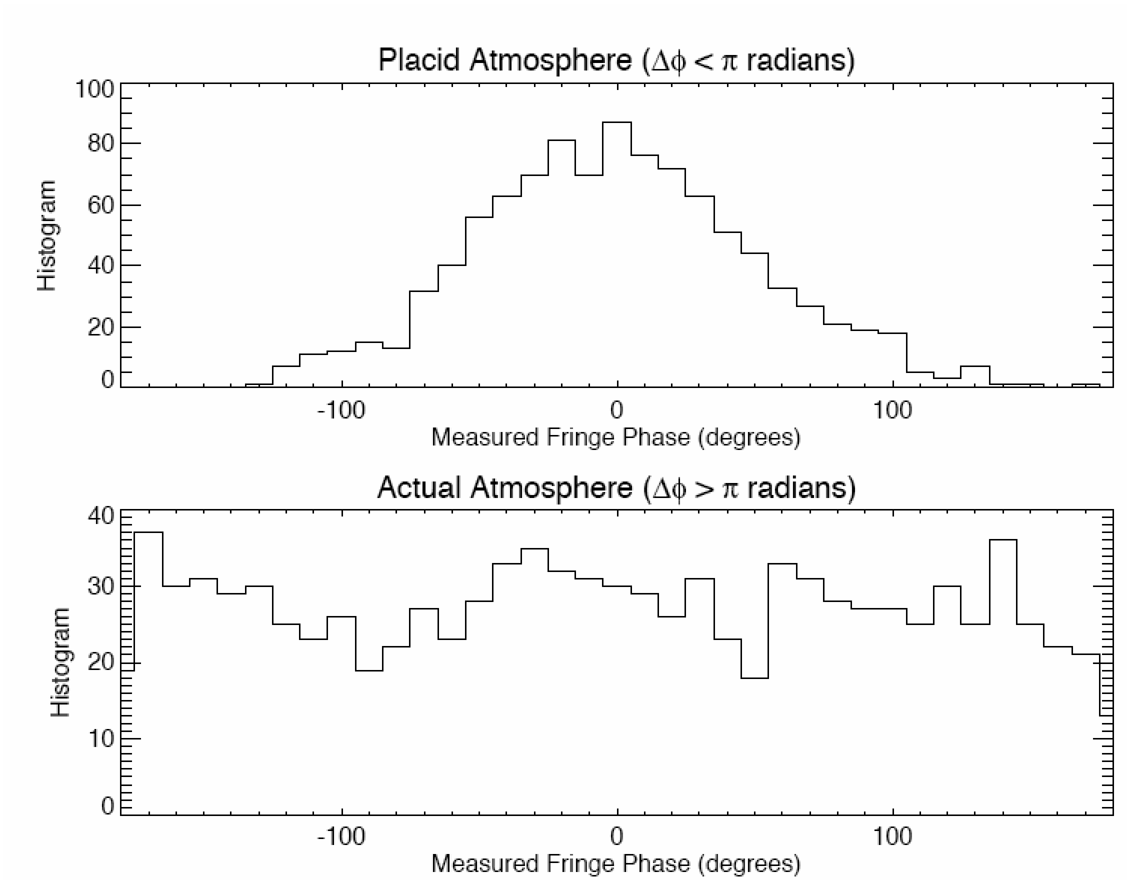
$$\begin{aligned}
\tilde{E}_i^{measured} &= \tilde{G}_i \tilde{E}_i^{true} \\
&= \left|\tilde{G}_i\right| \, e^{i\Phi_i^G} \, \tilde{E}_i^{true} \\
\tilde{V}_{i,j}^{measured} &\propto \tilde{E}_i^{measured} \, \tilde{E}_j^{*measured} \\
&\propto \left|\tilde{G}_i\right| \left|\tilde{G}_j\right| e^{i(\Phi_i^G - \Phi_j^G)} \, \tilde{E}_i^{true} \, \tilde{E}_j^{*true} \\
&= \left|\tilde{G}_i\right| \left|\tilde{G}_j\right| e^{i(\Phi_i^G - \Phi_j^G)} \, \tilde{V}_{i,j}^{true}
\end{aligned}$$

- Assume the complex gains are factorable
- Terms are due to instrumental and atmospheric effects
- Phase is lost



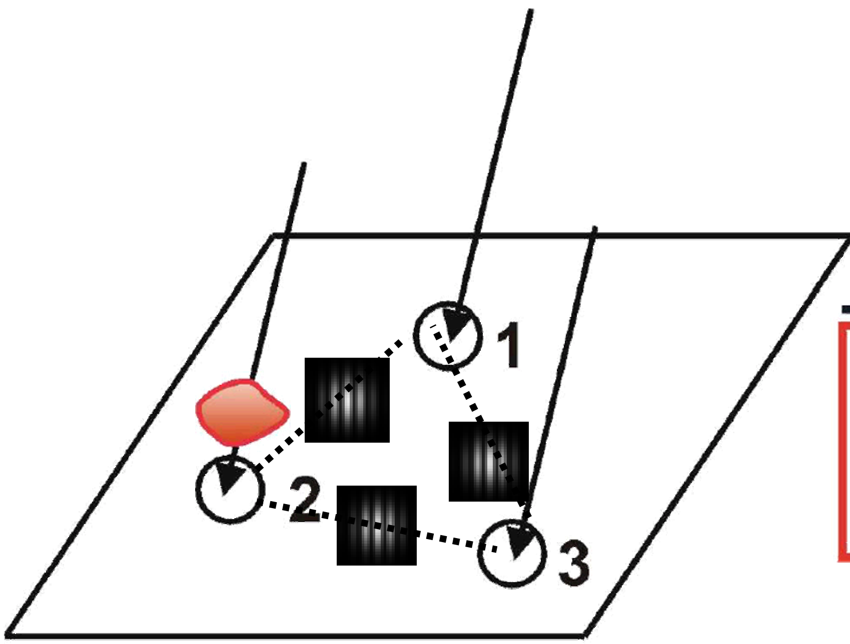
Closure phases (II)

Phases of complex visibility



Closure phase(III)

definition



Observed		Intrinsic		Atmosphere
$\Phi(1\text{-}2)$	$=$	$\Phi_o(1\text{-}2)$	$+$	$[\phi(2)-\phi(1)]$
$\Phi(2\text{-}3)$	$=$	$\Phi_o(2\text{-}3)$	$+$	$[\phi(3)-\phi(2)]$
$\Phi(3\text{-}1)$	$=$	$\Phi_o(3\text{-}1)$	$+$	$[\phi(1)-\phi(3)]$

Closure Phase (1-2-3) $= \Phi_o(1\text{-}2)+\Phi_o(2\text{-}3)+\Phi_o(3\text{-}1)$

Closure phase:

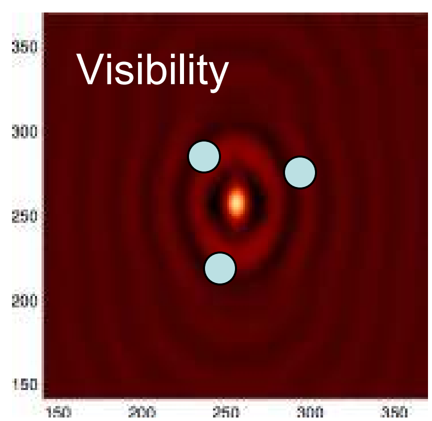
how to compute it

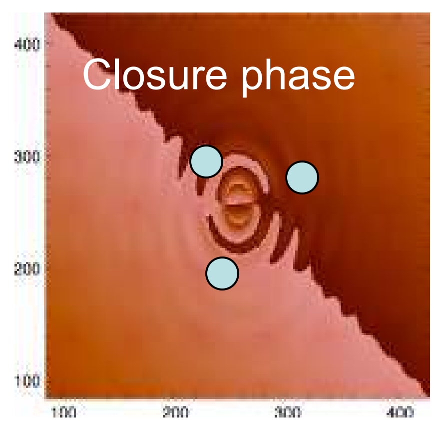
Recall:
$$\begin{aligned}\tilde{\mathcal{V}}_{ij}^{\text{measured}} &= \tilde{G}_i \tilde{G}_j^* \tilde{\mathcal{V}}_{ij}^{\text{true}} \\ &= |G_i||G_j| e^{i(\Phi_i^G - \Phi_j^G)} \tilde{\mathcal{V}}_{ij}^{\text{true}}\end{aligned}$$

Thus we define the Bispectrum:

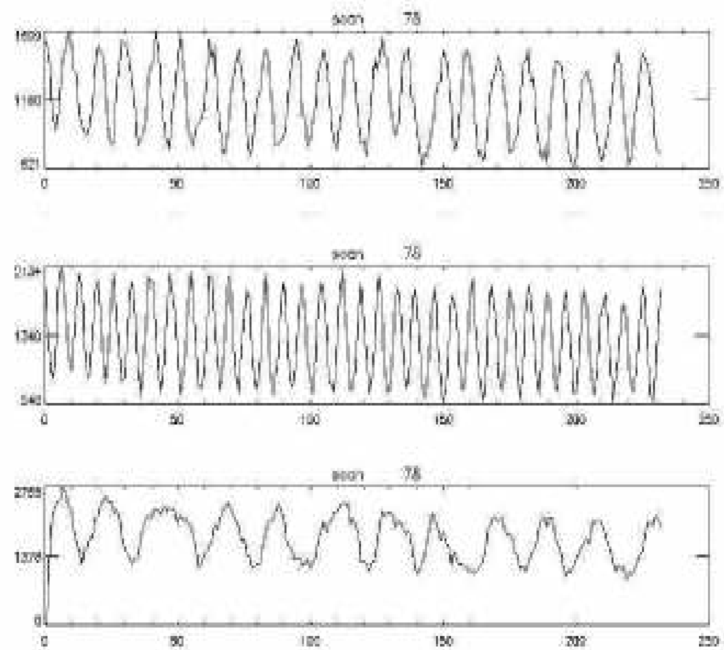
$$\begin{aligned}\tilde{B}_{ijk} &= \tilde{\mathcal{V}}_{ij}^{\text{measured}} \tilde{\mathcal{V}}_{jk}^{\text{measured}} \tilde{\mathcal{V}}_{ki}^{\text{measured}} \\ &= |G_i||G_j| e^{i(\Phi_i^G - \Phi_j^G)} \tilde{\mathcal{V}}_{ij}^{\text{true}} \cdot |G_j||G_k| e^{i(\Phi_j^G - \Phi_k^G)} \tilde{\mathcal{V}}_{jk}^{\text{true}} \cdot |G_k||G_i| e^{i(\Phi_k^G - \Phi_i^G)} \tilde{\mathcal{V}}_{ki}^{\text{true}} \\ &= |G_i|^2 |G_j|^2 |G_k|^2 \tilde{\mathcal{V}}_{ij}^{\text{true}} \cdot \tilde{\mathcal{V}}_{jk}^{\text{true}} \cdot \tilde{\mathcal{V}}_{ki}^{\text{true}}.\end{aligned}$$

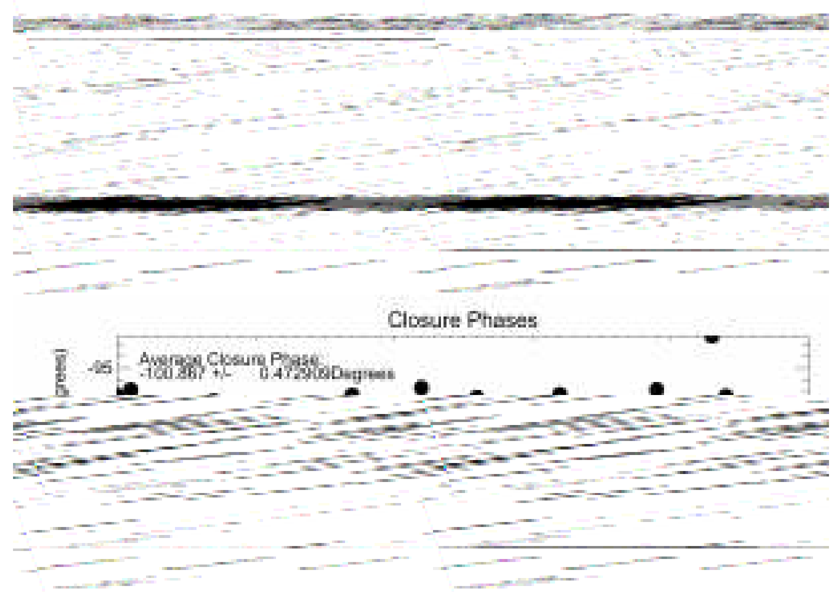
The 'argument' of this complex quantity is the Closure Phase!



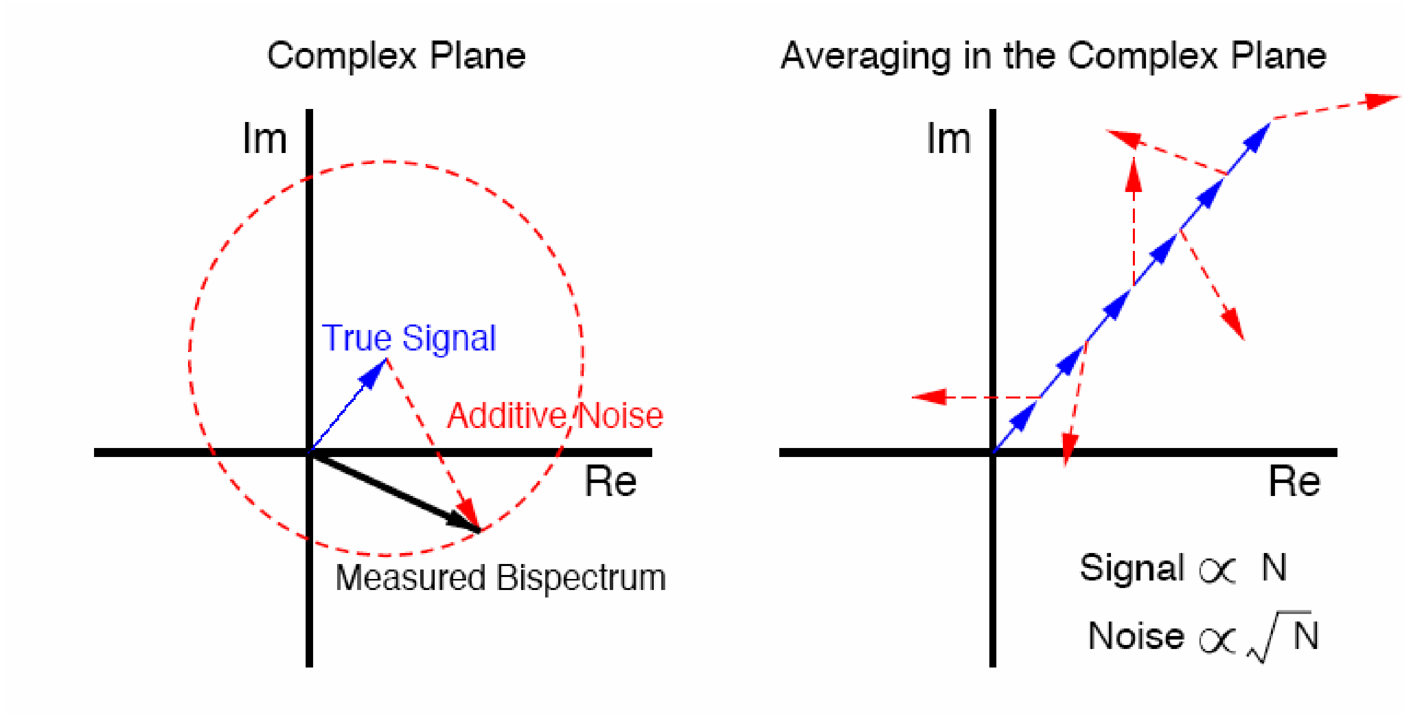


The magic of closure phases





Closure phase averaging



- Bispectrum averaging is necessary in the case of large phase noise (> 2π) per frame of data.
- For small phase noise you can directly average the closure phases.

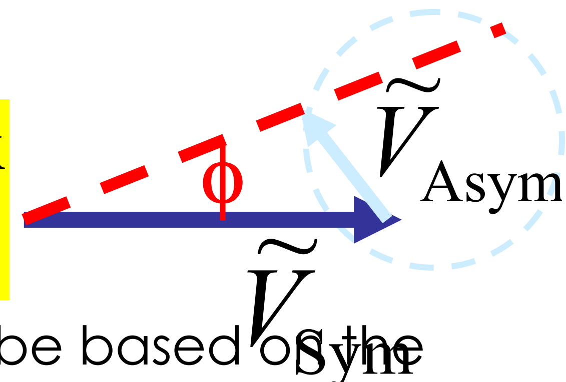
Properties of closure phases

- Aside from imaging (which requires lots of closure phases) closure phase provide a powerful mean of doing precision interferometry measurement of emission skewness.
- Closure phase are independent of all telescope-specific phase errors. Any deviation from zero observed on an unresolved sources points towards an instrumental issue (internal phase mismatchs) or non closing triangles.
- A change in the phase reference (the pointing direction) does not affect the closure phase.
- Closure phase of a point symmetric object is always 0 o 180deg.
- Unresolved objects are always symmetric. Object must be resolved in order to detect deviations from zero. Closure phase amplitude is not linear with resolution (power ~3) !

Estimate the Magnitude of Closure Phase

- It is straightforward to obtain an order-of-magnitude estimate for the "typical" closure phase for a known object distribution:

$$\left|\text{Closure Phase (radians)}\right| \approx \frac{\text{Asymmetric Flux}}{\text{Symmetric Flux}}$$



- The amount of "Asymmetric" Flux should be based on the resolution of the baselines (Nothing is asymmetric if its unresolved!)

- Example: For an unequal binary system, a closure phase measurement (radians) will typically be roughly the brightness ratio if the binary separation is resolved.

Closure phases: how many ?

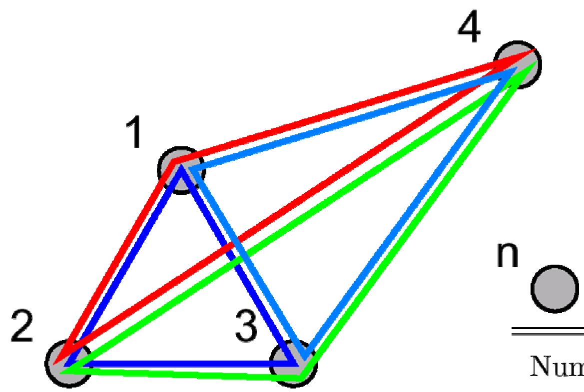
Closure Phases are not all independent from each other.

Number of Closure Phases

$$\binom{N}{3} = \frac{(N)(N-1)(N-2)}{(3)(2)},$$

Number of Fourier Phases

$$\binom{N}{2} = \frac{(N)(N-1)}{2}$$

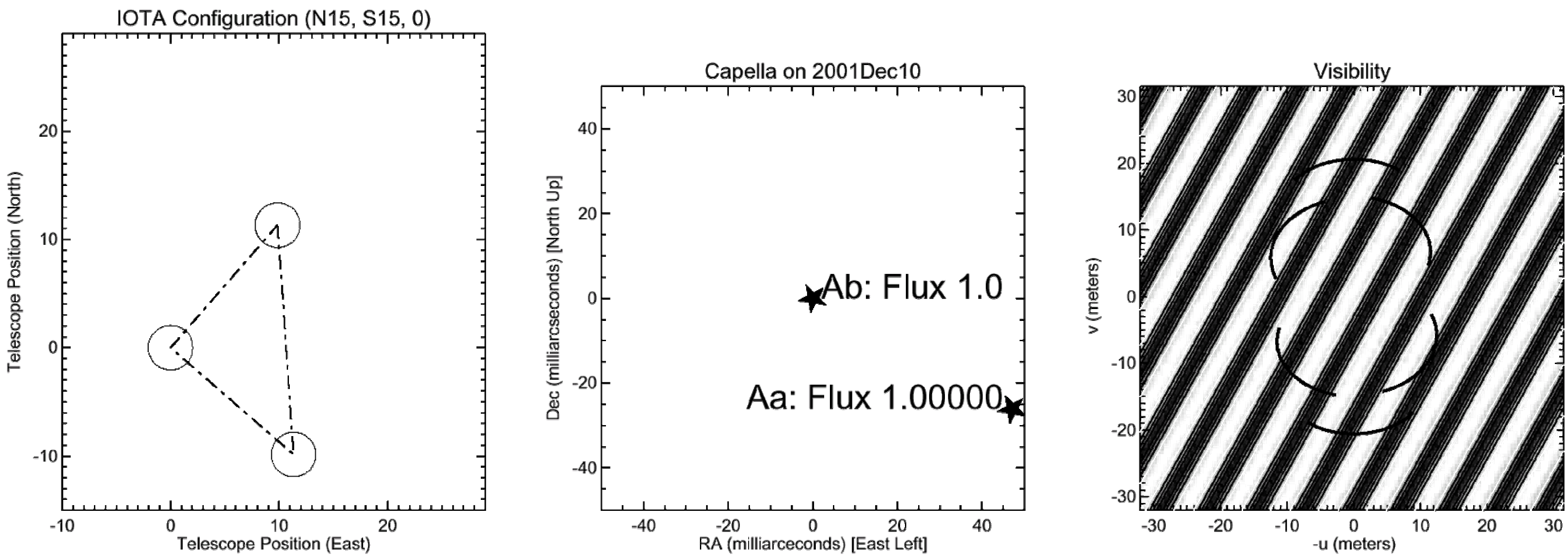


Number of Independent Closure Phases

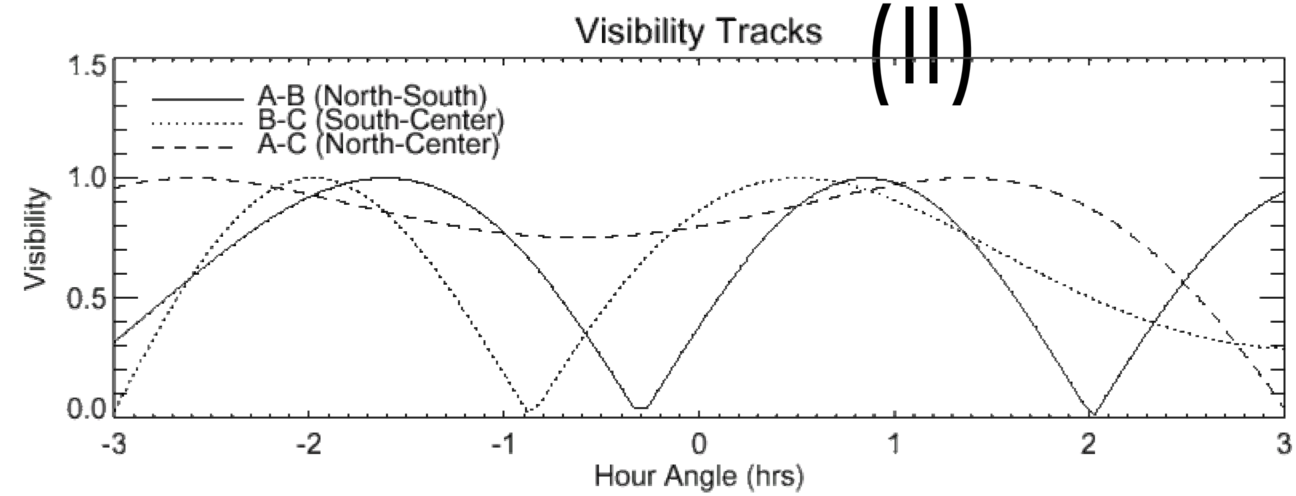
$$\binom{N-1}{2} = \frac{(N-1)(N-2)}{2}$$

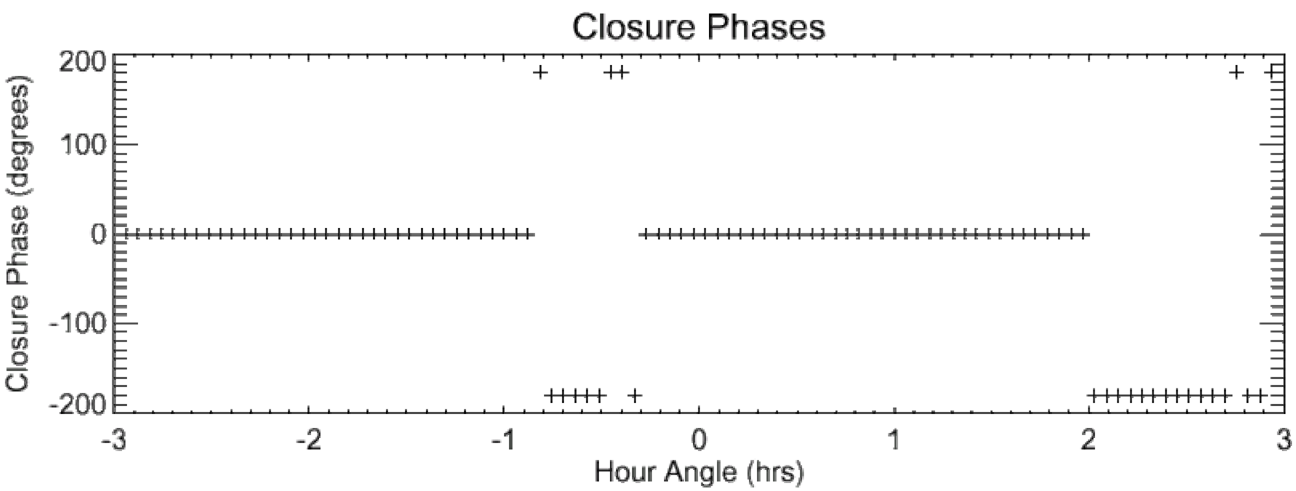
Number of Telescopes	Number of Fourier Phases	Number of Closing Triangles	Number of Independent Closure Phases	Percentage of Phase Information
3	3	1	1	33%
7	21	35	15	71%
21	210	1330	190	90%
27	351	2925	325	93%
50	1225	19600	1176	96%

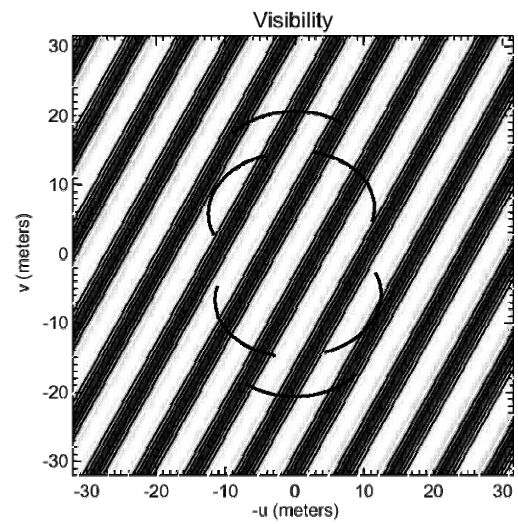
Equal Binary with 3 Telescopes



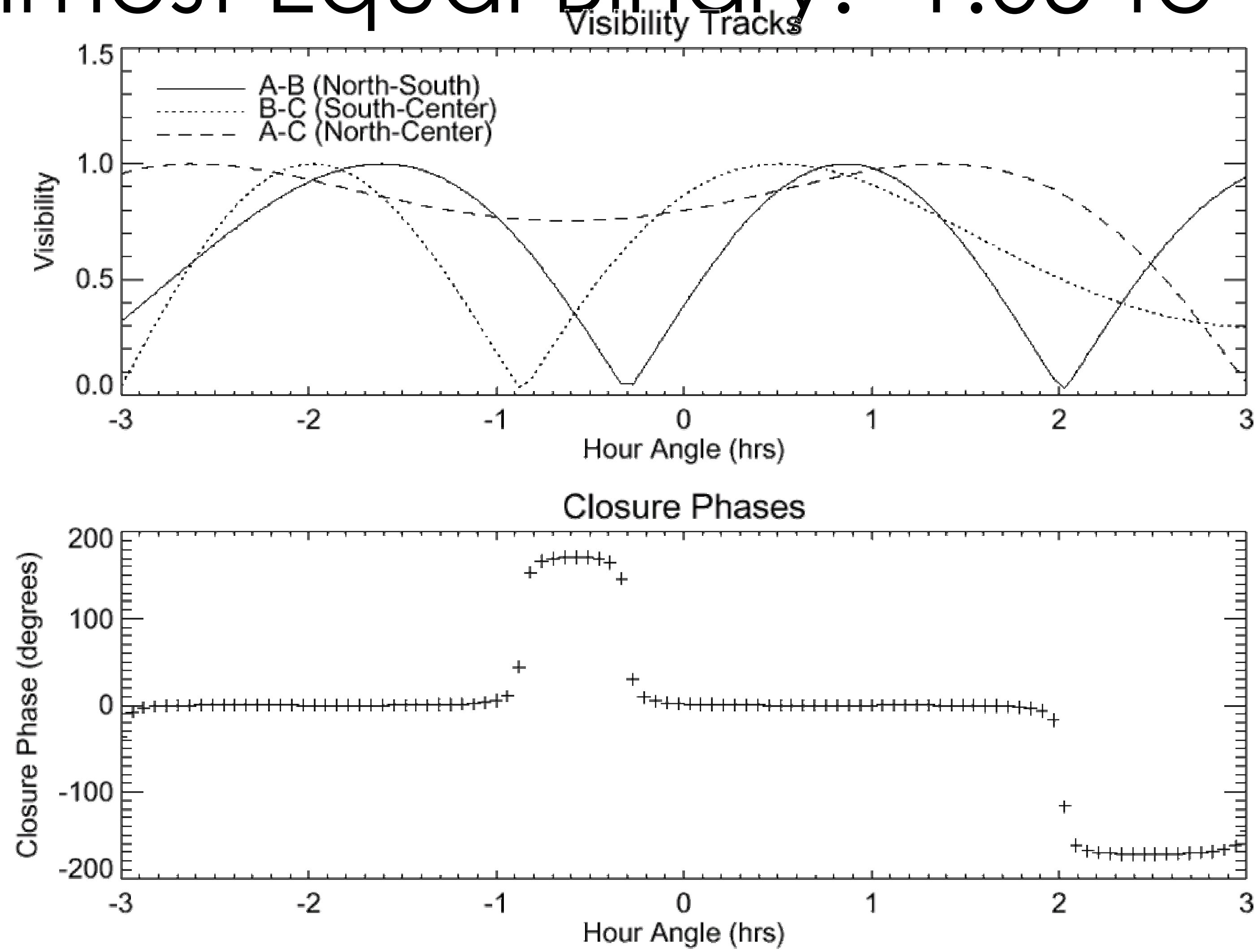
Equal Binary with 3 Telescopes (II)



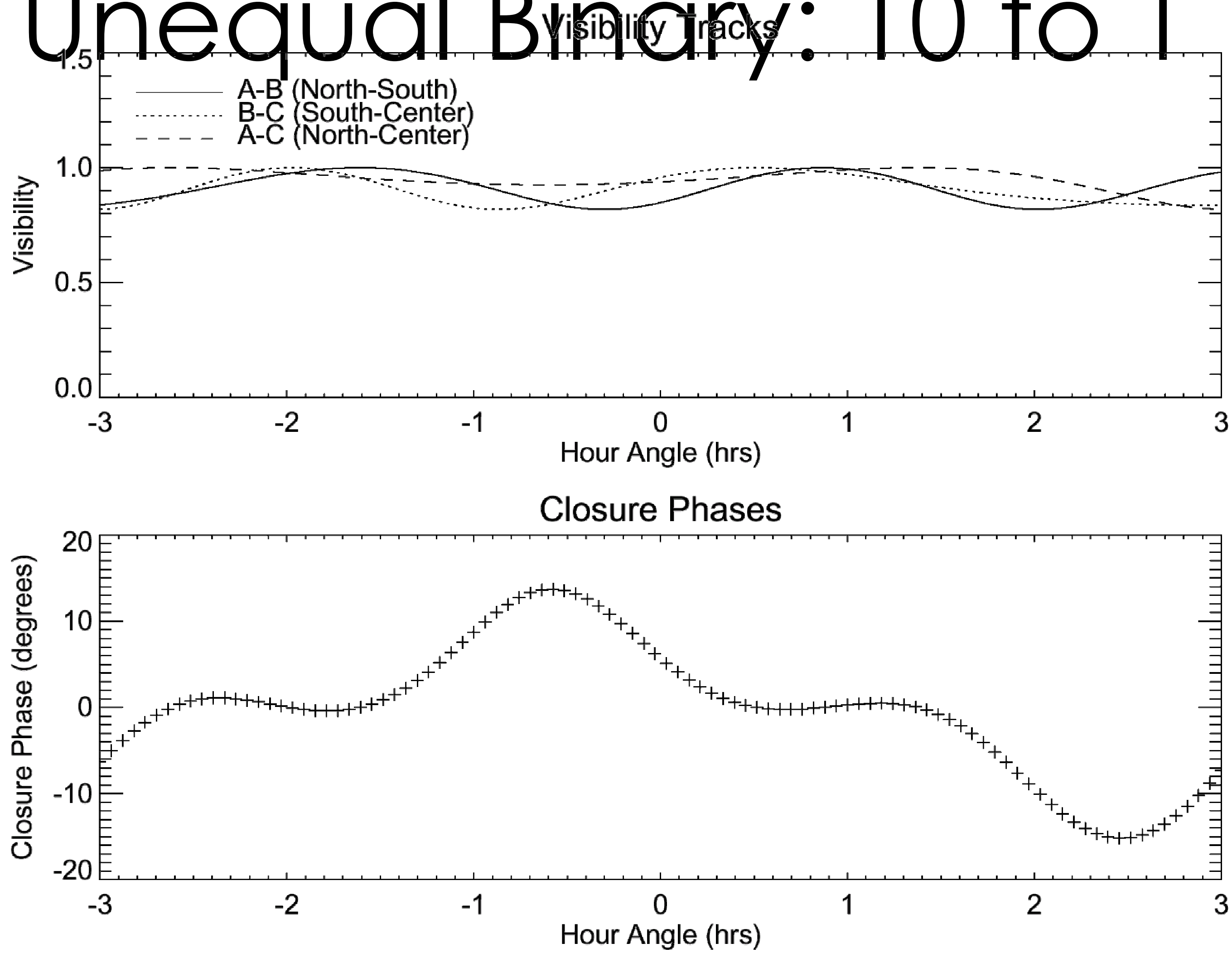




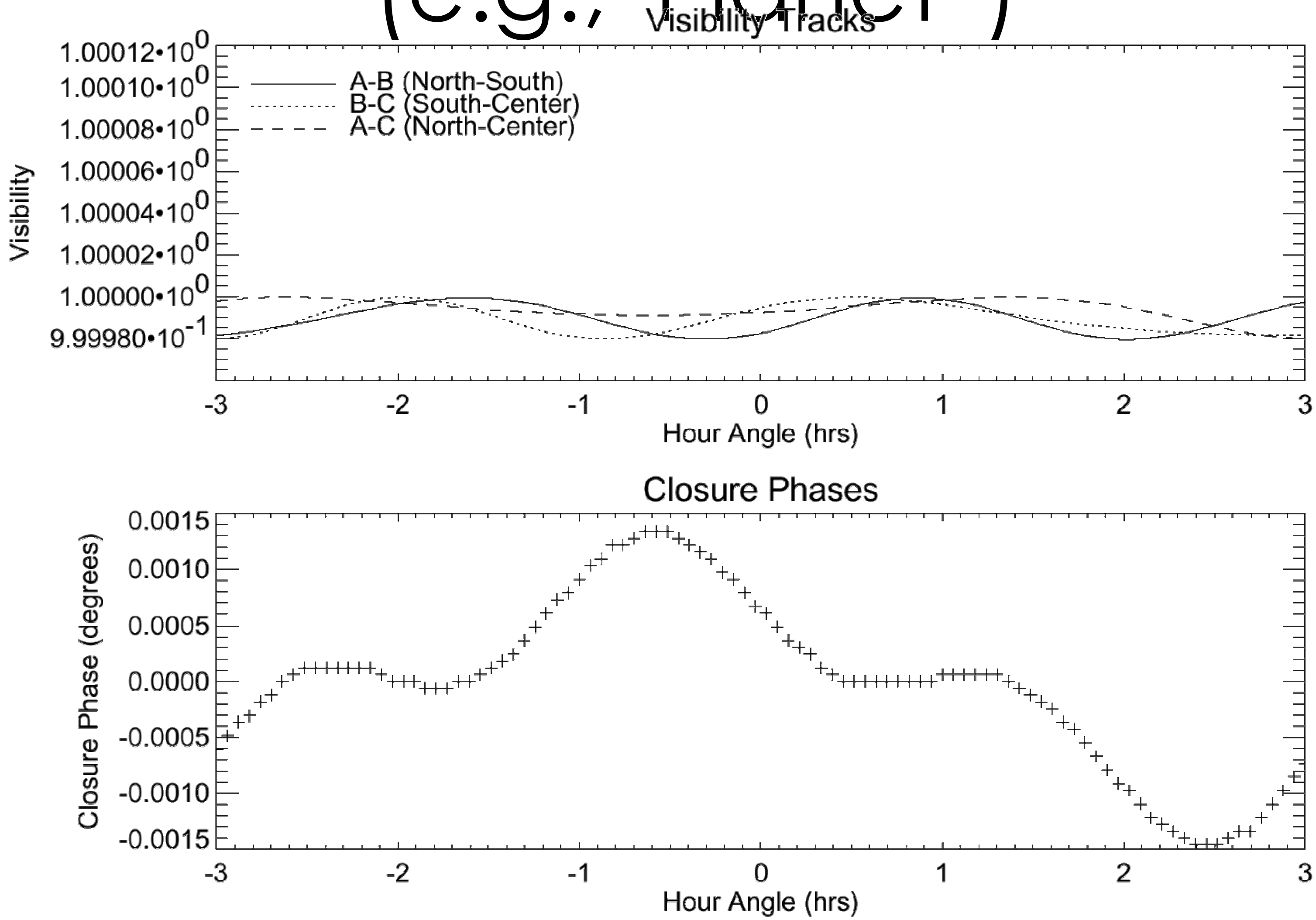
Almost Equal Binary: 1.05 to 1



Unequal Binary: 10 to 1



Very Unequal Binaries: 10^5 to 1 (e.g.,"Planet")



Application of closure phases

- Closure phases much more robust than visibilities with respect to calibration
- Essential to complement visibility diagnostics on flux distribution. It allows to quantify the flux asymmetry
- Easiest wqy to retrieve phase information and do image reconstruction