Classic disc physics

III - Sources of transport

Giuseppe Lodato

University of Leicester

Lectures outline

- Lecture I - Basic equations
- Lecture II - Steady state solutions - Time dependent models - Outbursts
- Lecture III - Sources of angular momentum transport

Viscosity is essential!

- It drives the evolution of the disc

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}(\Sigma \nu R^{1/2})\right]$$

- It determines the accretion rate and the disc structure

$$\dot{M} = 3\pi\nu\Sigma$$

- Relation to the stress tensor

$$T_{\mathrm{R}\phi} = \Sigma\nu R\left|\frac{\mathrm{d}\Omega}{\mathrm{d}R}\right|$$

Classic viscosity is too low

- Molecular viscosity?

$$Re = \frac{Rv_\phi}{\nu} = \Omega t_{\rm visc} \sim \left(\frac{\Sigma\sigma_{\rm coll}}{\mu m_{\rm p}}\right)\left(\frac{H}{R}\right)^{-2} \approx 10^{10}$$

- Implies viscous time >> Hubble time!
- Reynolds number very high: turbulence?

Turbulent transport

- Separate mean flow from fluctuations
- Turbulent stresses appearing as ang. mom. sources

$$\frac{\partial}{\partial t}(\Sigma\Omega R^2) + \frac{1}{R}\frac{\partial}{\partial R}(\Sigma\Omega R^3 u) = -\frac{1}{R}\frac{\partial}{\partial R}\left(R^2\Sigma\sum_i \langle v_R^i v_\phi^i\rangle\right)$$

$$\mathbf{v}^{(1)} = \mathbf{v}$$
$$\mathbf{v}^{(2)} = \mathbf{B}/\sqrt{4\pi\rho}$$
$$\mathbf{v}^{(3)} = \mathbf{g}/\sqrt{4\pi G\rho}$$

Balbus 2005

The α-prescription

$$\nu = \alpha c_{\rm s} H = \alpha \left(\frac{H}{R}\right)^2 \Omega R^2$$

- Motivation: trying to model “turbulent viscosity”
  - Typical scales of the largest turbulent eddies
    L < H
  - Typical velocity of turbulent eddies: $v < c_s$
- Energy balance requires:

$$\alpha = \frac{4}{9\gamma(\gamma-1)} \frac{1}{\Omega t_{\rm cool}}$$

Shakura & Sunyaev 1973

The α-prescription

$$T_{\mathrm{R}\phi} = \frac{3}{2}\alpha\Sigma c_s^2 = \frac{3}{2}\alpha P$$

- Turbulent stress tensor simply proportional to pressure
- In the presence of magnetic fields, P replaced by magnetic pressure
- This prescription does not provide a model for viscosity!!!
- It only measures the stress in units of the local pressure

Shakura & Sunyaev 1973

Why should the disc be turbulent?

- High Reynolds number do not by themself imply turbulence
- Rayleigh criterion for hydrodynamic stability:

$$\kappa^2 = \frac{2\Omega}{R}\frac{\mathrm{d}(\Omega R^2)}{\mathrm{d}R} > 0$$

- Always satisfied for accretion discs
- Recent experiments (Ji et al 2006) show that quasi Keplerian discs are stable up to very large Re ~ 10^6

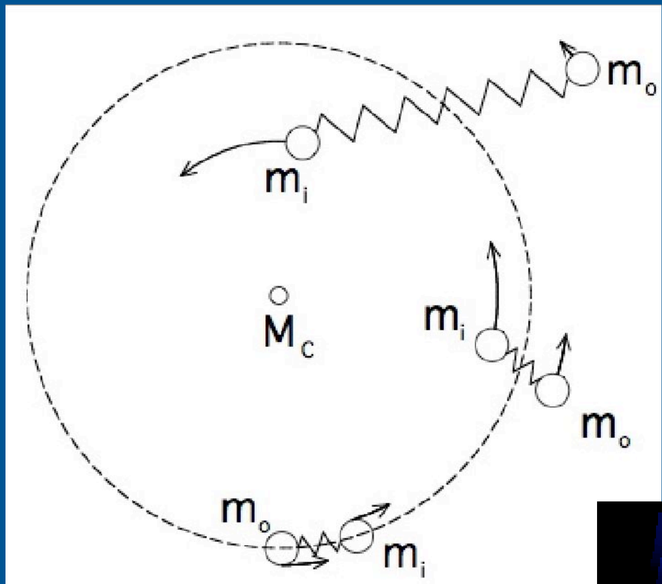
Disc instabilities

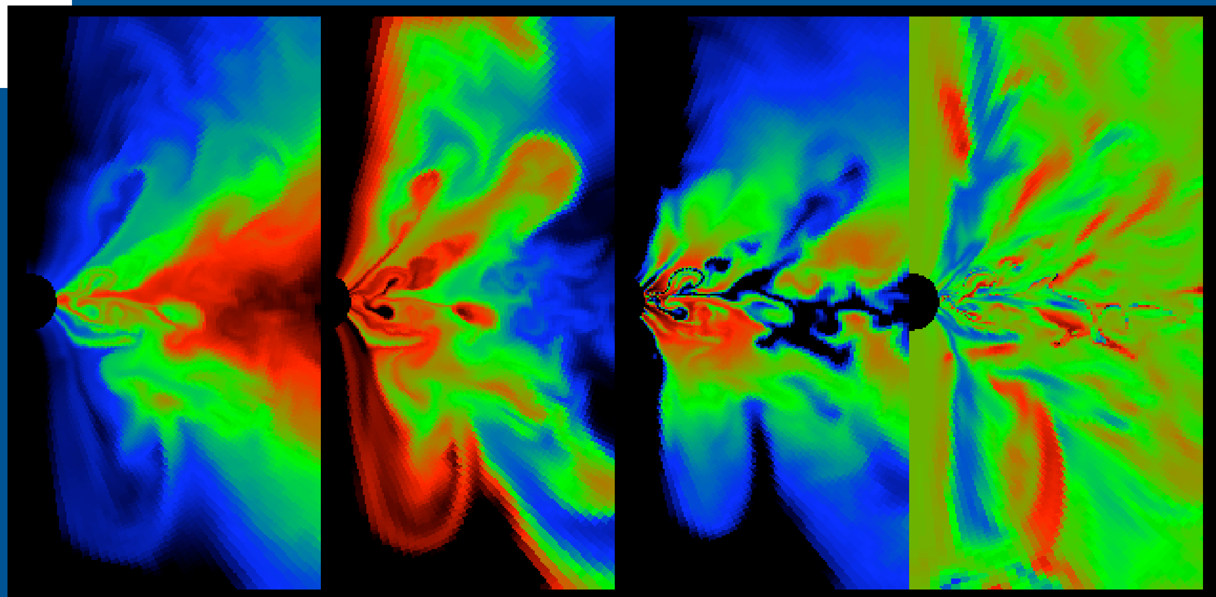
- Discs are susceptible to various kinds of instabilities:
  - MHD instabilities (MRI) -- Chandrasekhar, Balbus & Hawley, Nelson & Papaloizou
  - Gravitational instabilities (GI) -- Laughlin & Bodenheimer, Gammie, Lodato & Rice, Durisen et al.
  - Bending/warping disturbances -- Papaloizou & Pringle, Nelson, Lodato

The magneto-rotational instability

- Originally discovered by Chandrasekhar
- Applied to accretion discs by Balbus & Hawley
- The presence of a weak magnetic field radically alters stability: $$\frac{\mathrm{d}\Omega}{\mathrm{d}R} > 0$$
- Large wavelength instability: $$\frac{v_{\mathrm{A}}}{c_{\mathrm{s}}} < \frac{\lambda}{H} < 1$$
- Sub-thermal magnetic field

The magneto-rotational instability





Simulation by Jim Stone

Current challenges

- Proven to be working through numerical simulations
- Estimated $\alpha \sim 0.01$. Dependent of field geometry and resolution.
- Is it high enough? King, Pringle & Livio (2007)
- Hard to simulate very thin discs

Dead-zones

- The MRI needs a sufficient degree of ionization to work
- Protostellar discs are often not hot enough to be ionized (Fromang, Terquem, Balbus 2002)
- Dead zones: Gammie (1996)
  - Upper layers ionized by cosmic rays -- magnetically active
  - Disc midplane shielded -- dead zone
- Stone: turbulence in upper layers might extend to inner parts
- Dead zones often invoked as a way to trigger FU Orionis outbursts (Armitage, Livio & Pringle 2001, Hartmann et al. 2006)

Much more in Johnatan Ferreira's lectures

Gravitational instability

- Dispersion relation for axisymmetric GI

$$\omega^2 = c_s^2 k^2 - 2\pi G \Sigma |k| + \kappa^2$$

  - Small wavelengths stabilized by pressure
  - Large wavelengths stabilized by rotation

- There is always some unstable wavelength if

$$Q = \frac{c_s \kappa}{\pi G \Sigma} < 1$$

Toomre 1964

- Most unstable wavelength

$$k^{-1} = \frac{c_s^2}{\pi G \Sigma} = QH$$

How massive to be unstable?

- Disc are unstable if Q < 1

$$Q \sim \frac{c_{\rm s}\Omega}{\pi G\Sigma} \sim \frac{c_{\rm s}}{\Omega}\frac{GM_\star}{\pi G\Sigma R^3} \approx \frac{H}{R}\frac{M_\star}{M_{\rm disc}(R)}$$

- Instability if:

$$\frac{M_{\rm disc}(R)}{M_\star} > \frac{H}{R} \approx 0.1$$

- Required mass not exceptionally high and definitely $M_{disc} << M_*$
- Can occur at early phases (Class 0/I)

Evolution of grav. unstable discs

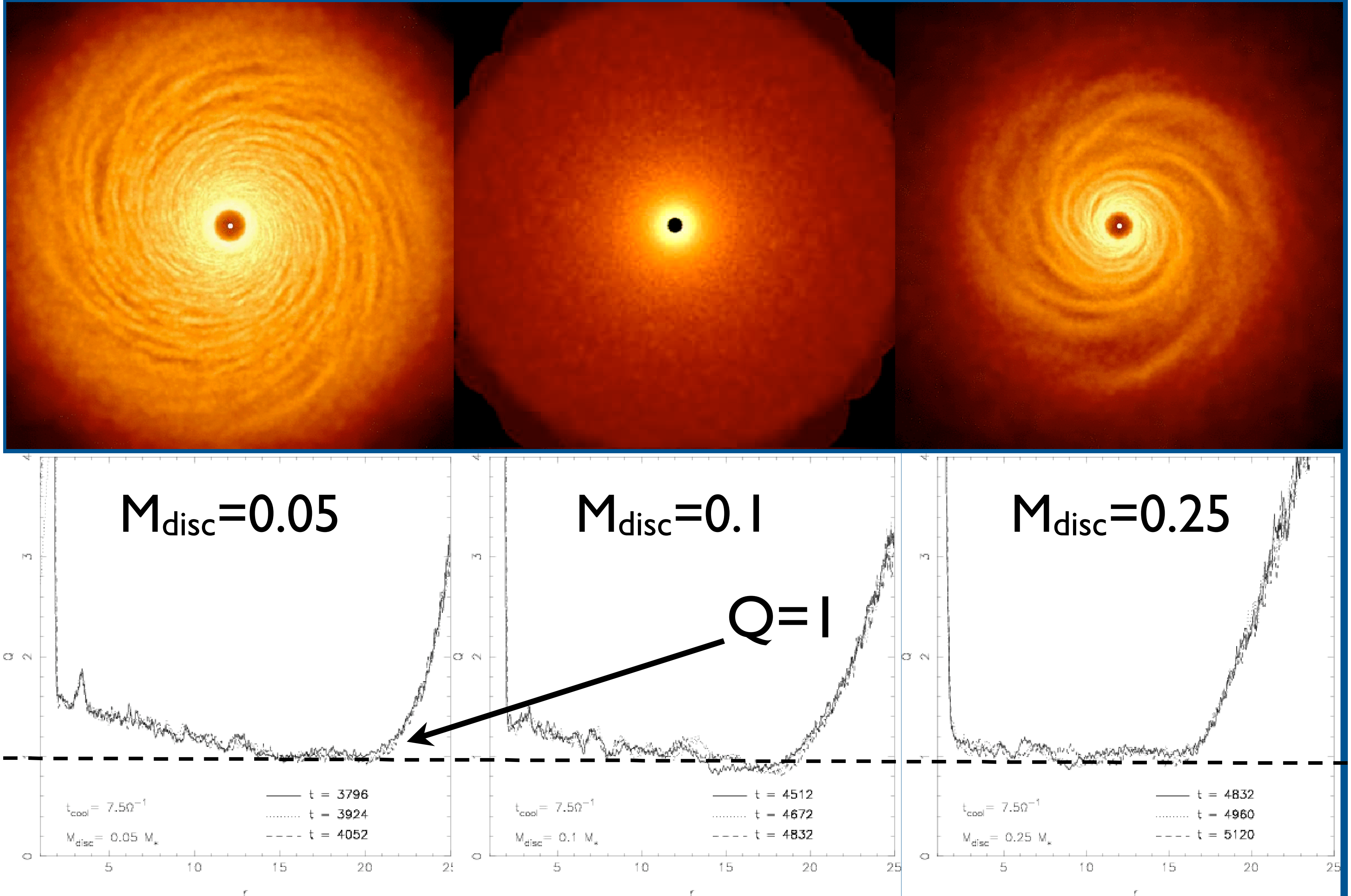
Self-regulation

- Instability acts as a 'thermostat':

$$Q = \frac{c_s \kappa}{\pi G \Sigma} < 1$$

  - Cold discs are unstable
  - Instability heats up the disc
- Equilibrium expected to have Q ~ 1

“Low”-mass discs



Local vs global transport

- The α prescription: stress determined locally
- Gravitational instabilities naturally long-range
- How much do density waves travel before being dissipated?
- Balbus & Papaloizou (1999): there can be significant non-local trasport
- Gammie (2001): local description enough

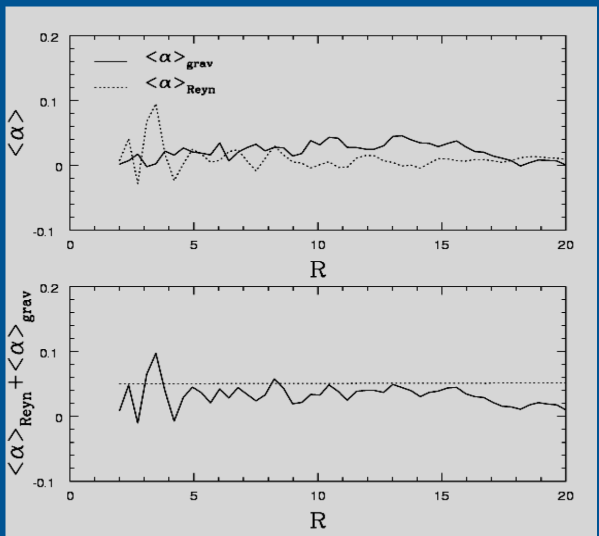
Local vs global transport

- Gammie (2001): local simulations, measured α ~ 0.02-0.07

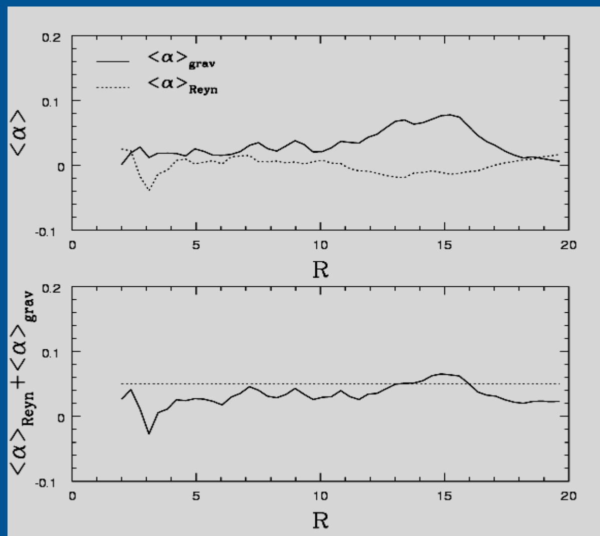
$$\alpha = \frac{4}{9\gamma(\gamma-1)}\frac{1}{\Omega t_{\rm cool}}$$

- Lodato & Rice (2004,2005): global simulations

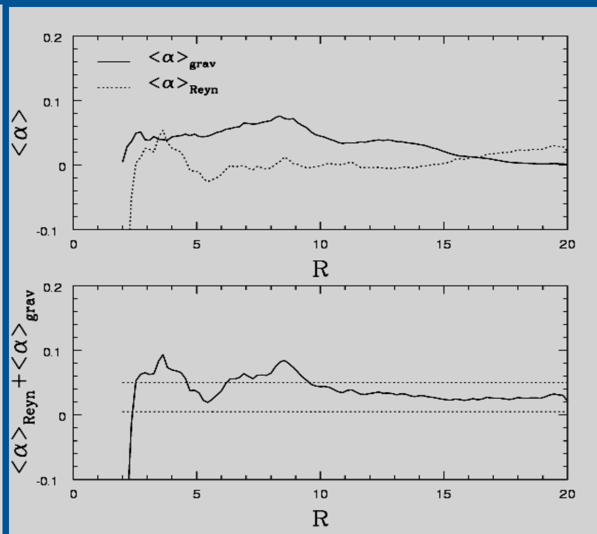
M_{disc}=0.05



M_{disc}=0.1



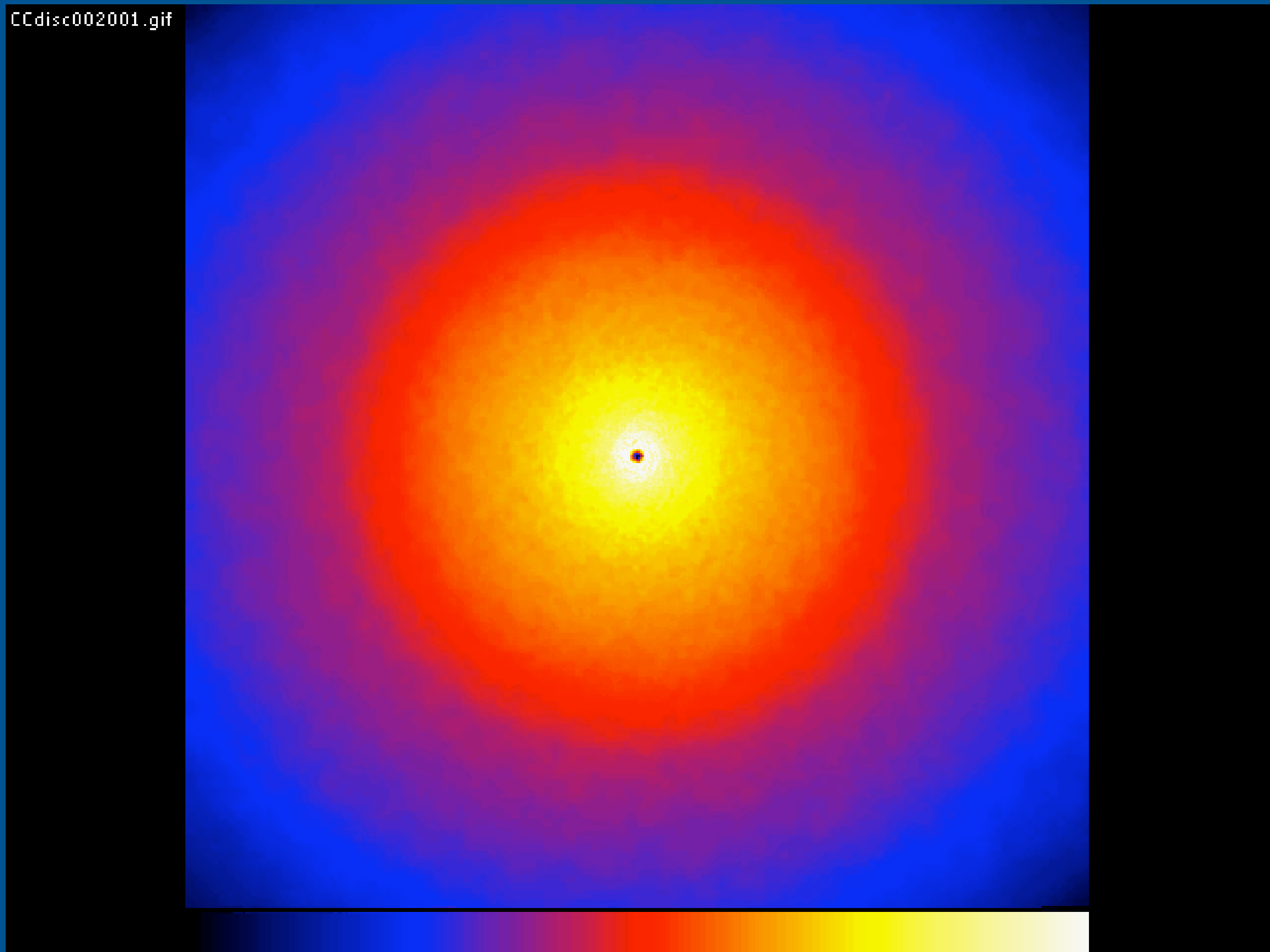
M_{disc}=0.25



Global transport?

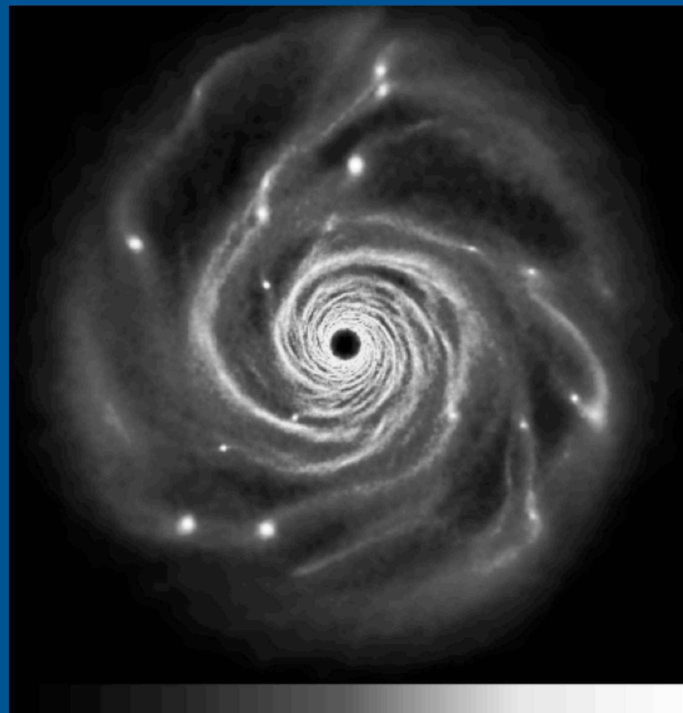
- Large disc mass ---> thicker discs
- Global properties more likely
- However, high variability observed

Global transport?



Fragmentation

- Important results: Gammie (2001), Rice, Lodato & Armitage (2005)
- Large $t_{cool} > 3\Omega^{-1}$: spiral structure, turbulence
- Small $t_{cool} < 3\Omega^{-1}$: fragmentation
- GI provides no "viscosity" larger than $\alpha \sim 0.06$
- Fragmentation might occur because of large inflow



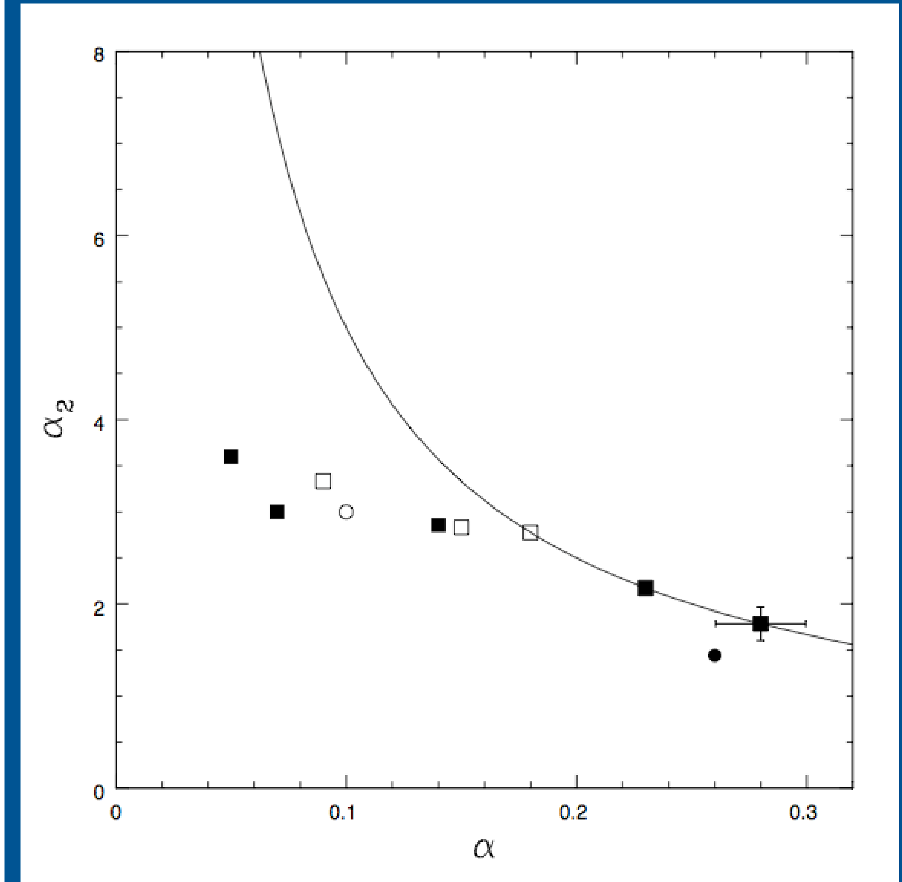
Current challenges

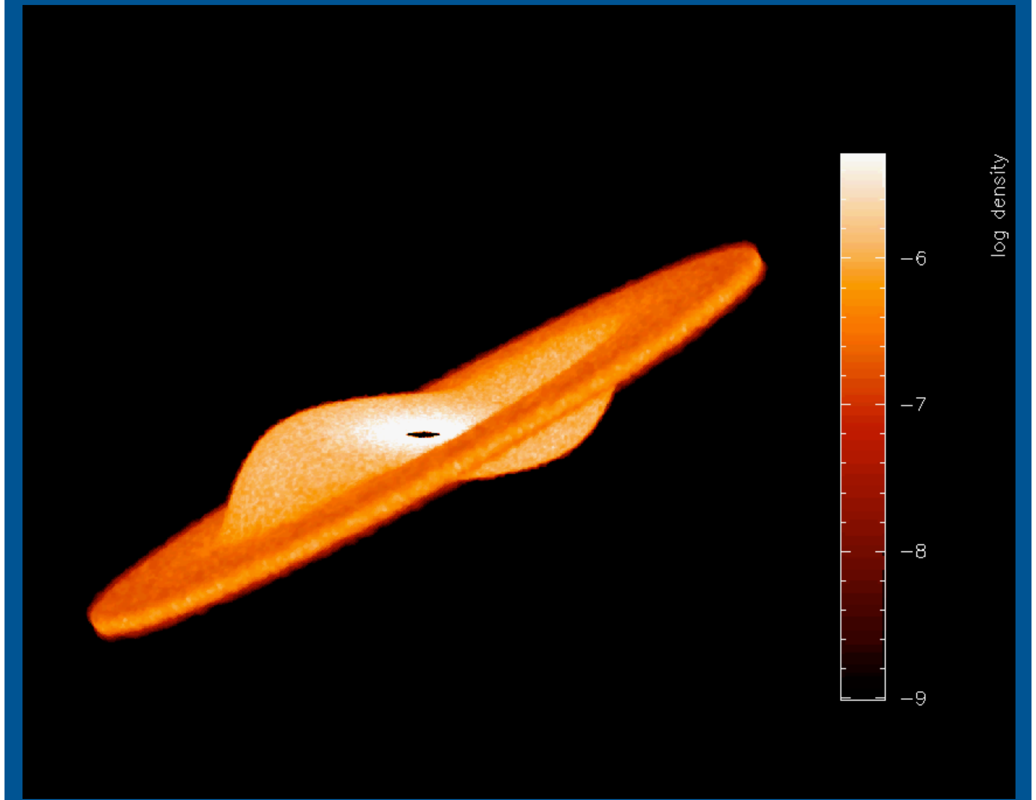
- Planet formation: can it occur by fragmentation?
  - Most likely not!
  - Maybe at large radii (100AU) can form small mass companions...
- What is the cooling time in protostellar discs?
- “Realistic” cooling simulation
- Local/global balance still unsolved

Transport due to bending waves

- Dissipation of bending waves leads to ang. mom. transport
- Discs in binaries most likely warped (KH 15D)
- Interesting property: $\alpha_2 \sim 1/\alpha$

Transport due to bending waves





Lodato & Pringle (2007)

Transport due to bending waves