Classic disc physics
I - Basics

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Lectures outline

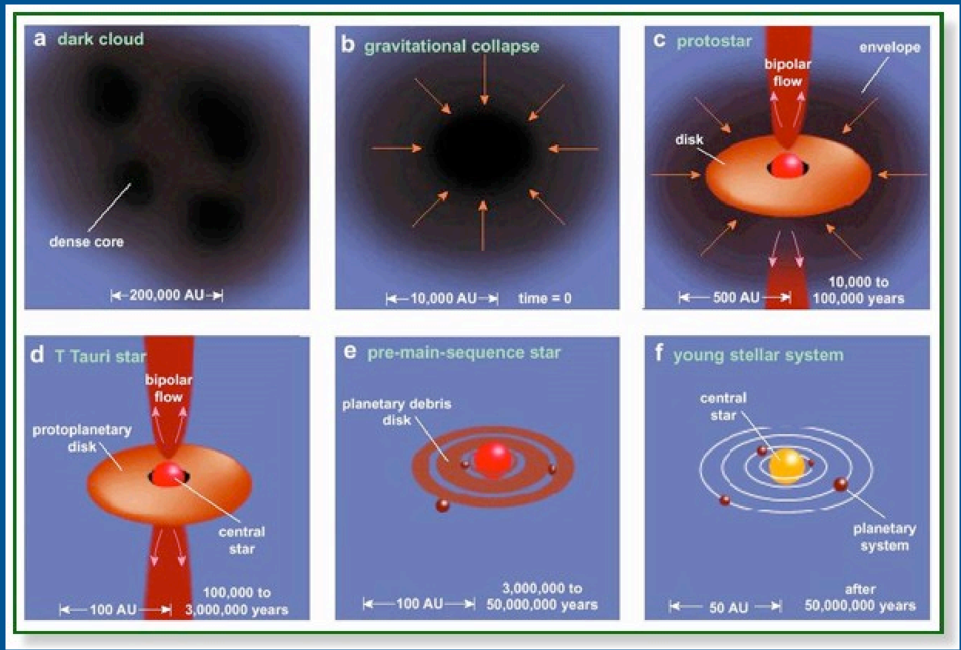
- Lecture I - Basic equations
- Lecture II - Steady state solutions - Time dependent models - Outbursts
- Lecture III - Sources of angular momentum transport

Discs in protostellar environments

- Protostars are generally surrounded by flattened structures - discs
- General picture of isolated star formation
  - (Things can be significanlty more complex in reality! see Bate, Bonnell & Bromm simulations)

Discs in protostellar environments

- General picture of isolated star formation



from Blitz

Discs in protostellar environments

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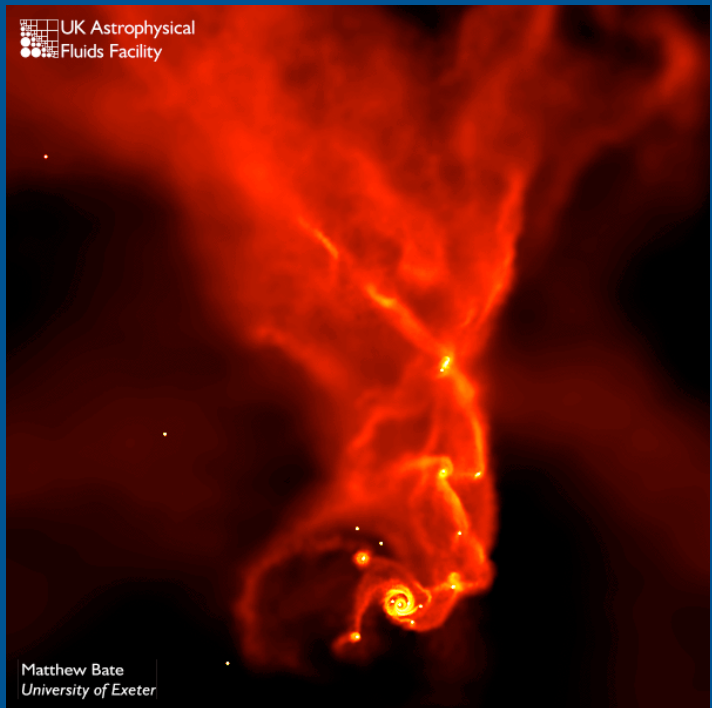

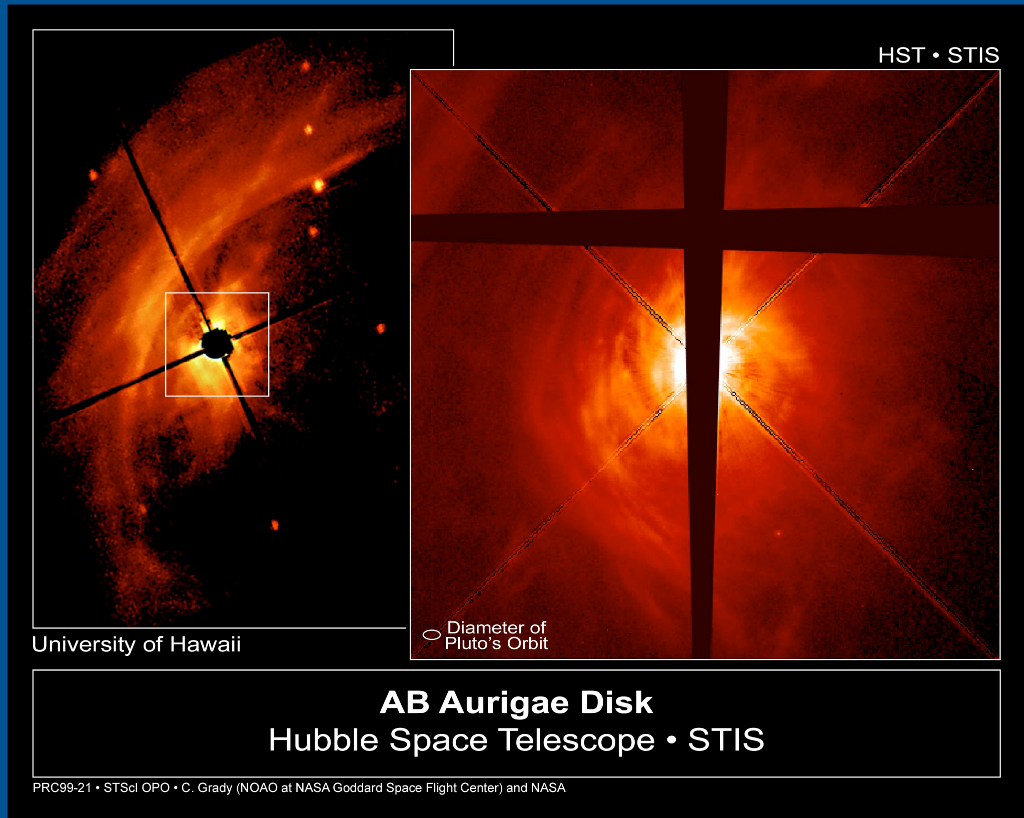

Discs in protostellar environments

- Discs found at all scales:
  - Around high-mass O-B stars
  - Around intermediate mass star (HAEBE)
  - Around solar mass stars (T Tauri, FU Orionis)
  - Around brown dwarfs
- Protostellar discs are made of:
  - Gas (dominate mass and dynamics)
  - Dust (dominate opacity and thermo-dynamics)

Main properties: mass

- Obtained from sub-mm observations (where discs are optically thin)
  - Very uncertain due to
    - Uncertainties in dust opacity -- grain growth
    - Dust to mass ratio
  - Typically: $M_{disc} \sim 0.005\text{-}0.1\ M_*$ (at least in the T Tauri regime)
  - For high-mass stars, M_{disc} can be a significant fraction of M_*
  - For brown dwarfs?

Main properties: sizes

- Theory: absolute upper limit: need to reach centrifugal equilibrium

$$t_{\rm dyn} \sim 2\pi \sqrt{\frac{R^3}{GM}} \sim 10^6 {\rm yrs}$$

$$R \sim 10^4 {\rm AU} (M/M_\odot)^{1/3}$$

  - For BD ~ a few 10^3 AU
  - For massive stars ~ a few 10^4 AU
- Observed:
  - High mass stars: pseudo-discs at 10^5 AU
  - High mass stars: discs at 10^4 AU
  - T Tauri stars: a few 100 AU (big resolved ones)
  - BD: not well determined, maybe comparable to T Tauri

Main properties

- Mass accretion rate: obtained from a number of methods:
  - strength of emission lines emitted as gas reaches the star
  - veiling of photospheric lines due to accretion shocks
  - Typical numbers:
    - 10^{-5} M_{sun}/yr for the few high-mass star discs
    - 10^{-7} -- 10^{-9} M_{sun}/yrs for T Tauri stars
    - 10^{-11} M_{sun}/yrs for brown dwarfs
    - During outbursts: 10^{-4} M_{sun}/yrs (FU Orionis)
  - *Mdot* scales as M^2?

(Natta et al. 2004, Clarke & Pringle 2006, Alexander & Armitage 2006)

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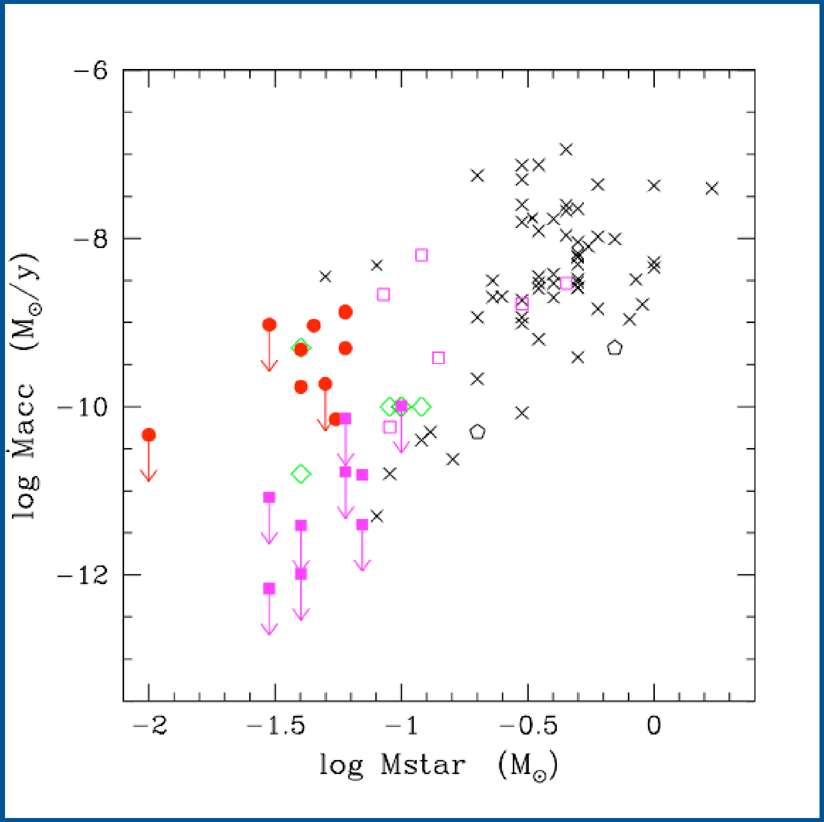
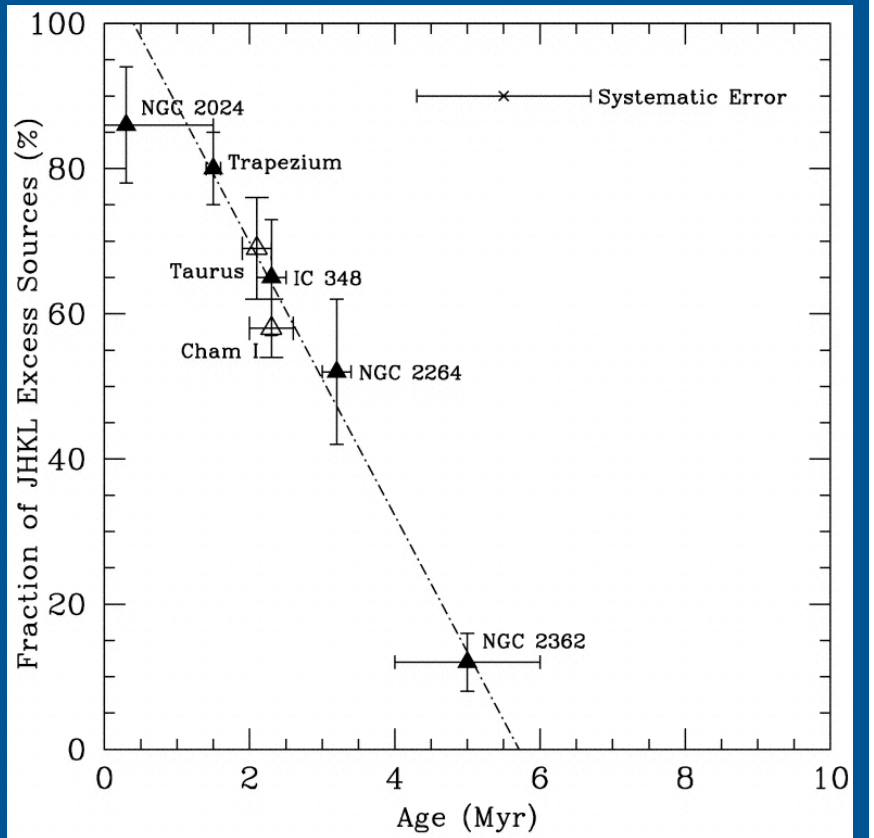


Main properties

- Lifetime: 10^6-10^7 years
- Temperature:
  - ranges from ~ 10 K in the outer parts, up to ~ 10^3 K in the hotter inner parts
  - also dependent on state: much higher (up to ~ 10^4 K) during outbursts (e. g. FU Orionis)
  - Temperature determines thickness of the disc (see later)
  - Tipically H/R ~ 0.1



Haisch et al. 2001

Why discs?

- Relatively easy for interstellar gas to get rid of energy (e.g., by radiation)
- Much more difficult to remove angular momentum
- For a given ang. mom., minimum energy orbit is a circular one
- Reach "circularization radius", where centrifugal balance holds and orbits become circular
- For a Keplerian potential (assume that accreting gas is much smaller than protostar):

$$R_{\rm circ} = \frac{L^2}{GM}$$

Viscous discs

- In a disc, the potential for angular momentum removal is larger
  - Discs are subject to a large number of instabilities that might redistribute angular momentum (Lecture 3)
  - Anisotropies, turbulence generated by instabilities lead to internal stress
  - Initial studies of discs not able to quantify such internal stress and rely on simplified viscosity prescription (α-prescription: Shakura & Sunyaev, 1973)

A brief history

- The pioneers:
  - Shakura & Sunyaev (1973): viscosity prescription and first steady-state models
  - Lynden-Bell & Pringle (1974): rigorous fluid-dynamical derivation and time-dependent models
- The ‘80s:
  - Wave propagation in discs:
    - Warps and bending waves (Papaloizou & Pringle 1983)
    - Density waves (Goldreich & Tremaine 1978)
  - Disc-satellite interaction:
    - Lin & Papaloizou (1979)
    - Goldreich & Tremaine (1980)
  - Limit-cycle instability:
    - Clarke, Lin & Papaloizou (1989), Bell & Lin (1994)
- The era of numerical simulations (from the ‘90s to present)
  - Non-linear disc dynamics, disc instabilities, source of transport
  - The magneto-rotational instability (MRI) (Balbus & Hawley 1992)
  - Gravitational instability (GI) (Laughlin & Bodenheimer 1994)

Thin discs

- Main disc property: being thin!
- At any radius R, thickness H<<R -- fundamental scaling of all quantities!
- Deal with “surface” quantities: surface density profile Σ...
- ... or with vertically averaged quantities (for example viscosity ν)
- Use cylindrical coordinates

Basic equations

- Continuity equation:

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(R\Sigma u) = 0$$

- Momentum equation:
  - There are actually three components
  - 1) Radial: centrifugal balance
  - 2) Vertical: hydrostatic balance
  - 3) Azimuthal: angular momentum conservation

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$$v_\phi^2 = \Omega^2 R^2 = \frac{GM}{R}$$

Note

- In order to obtain Keplerian rotation we have neglected several terms in Navier-Stokes eq:
  - pressure terms ~ c_s^2/R
  - Lagrangian derivative ~ u^2/R
  - Is this allowed?
- Note than specific angular momentum in Keplerian discs increases as:

$$j = \Omega R^2 = \sqrt{GMR}$$

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Vertical hydrostatic balance - Keplerian discs

$$\frac{\partial P}{\partial z} = -\frac{GMz}{R^3}\rho$$

$$\frac{\partial \rho}{\partial z} = -\frac{GMz}{c_s^2 R^3}\rho$$

$$\rho = \rho_0 \exp(-z^2/2H^2)$$

$$\frac{H}{R} = \frac{c_s}{V_\phi}$$

- Introduce sound speed (assume it is constant with z)
- Easy to solve
- Gaussian profile with thickness H
- Thin disc condition equivalent to supersonic rotation: $c_s << V_{rot}$

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Angular momentum conservation

- Navier-Stokes equation

$$\frac{\partial}{\partial t}(\Sigma\Omega R^2) + \frac{1}{R}\frac{\partial}{\partial R}(\Sigma\Omega R^3 u) = -\frac{1}{R}\frac{\partial}{\partial R}(R^2 T_{\mathrm{R}\phi})$$

- Viscous stress tensor $T_{R\phi}$

$$T_{\mathrm{R}\phi} = \Sigma\nu R\left|\frac{\mathrm{d}\Omega}{\mathrm{d}R}\right|$$

Angular momentum conservation

- Navier-Stokes equation

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Master disc evolution equation

- Obtained combining continuity equation and angular momentum conservation
- For Keplerian discs,
- .... and after some algebra, get:

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}\left(\Sigma \nu R^{1/2}\right)\right]$$

A little puzzle

- Simple kinetic arguments suggest that in a linear shearing viscous flow, momentum is transported from high to low momentum
- Analogously, one would expect ang. mom. in a disc to flow from high to low
- Ang. mom. flux at R:

$$\propto j(R-\mathrm{d}R) - j(R+\mathrm{d}R) \propto -\frac{\mathrm{d}j}{\mathrm{d}R}$$

- However, viscosity prop. to -dΩ/dR
- How come?