An introduction to the theory of interferometry

ONTHEFRINGE Summer School

Circumstellar disks and planets at very high angular resolution

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Outline

- Image formation with conventional telescopes
  - The diffraction limit
  - Incoherent imaging equation
  - Fourier decomposition
- Interferometric measurements
  - Fringe parameters
  - The van-Cittert Zernike theorem
- Imaging with interferometers
  - Rules of thumb
  - Interferometric images
  - Sensitivity

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Terminology and rationale

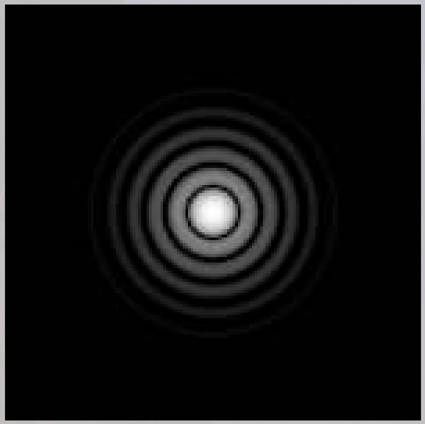
- High spatial resolution: the ability to recover information on small angular scales:
  - Positions.
  - Basic information - scale sizes, morphology, etc.
  - Detailed image structure.

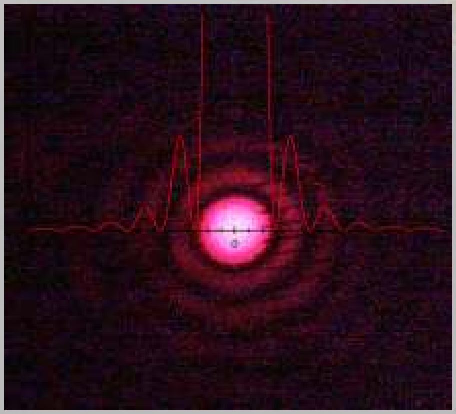
- Bandpasses:
  - Optical 0.3-1.0 μm.
  - Near-infrared 1.0-2.2 μm.
  - Thermal-infrared 3.5-20.0 μm.

- What limits our ability to investigate sources at high spatial resolution?
  - The wave nature of light.
  - The Earth's atmosphere.

What do we observe?

- Consider a perfect telescope in space observing an unresolved point source:
  - This produces an Airy pattern with a characteristic width: $\theta = 1.22\lambda/D$ in its focal plane.
  - θ is the approximate angular width of the image, called the "angular resolution".
  - λ is the wavelength at which the observation is made.
  - D is the diameter of the telescope aperture, assumed circular here.





How does this impact imaging?

- Image formation (under incoherent & isoplanatic conditions):
  - Each point in the source produces a displaced Airy pattern. The superposition of these limits the detail visible in the final image.

- But what causes the Airy pattern?
  - **Interference** between parts of the wavefront that originate from different regions of the aperture.
  - In this case, the relative amplitude and phase of the field at each part of the aperture are what matter.

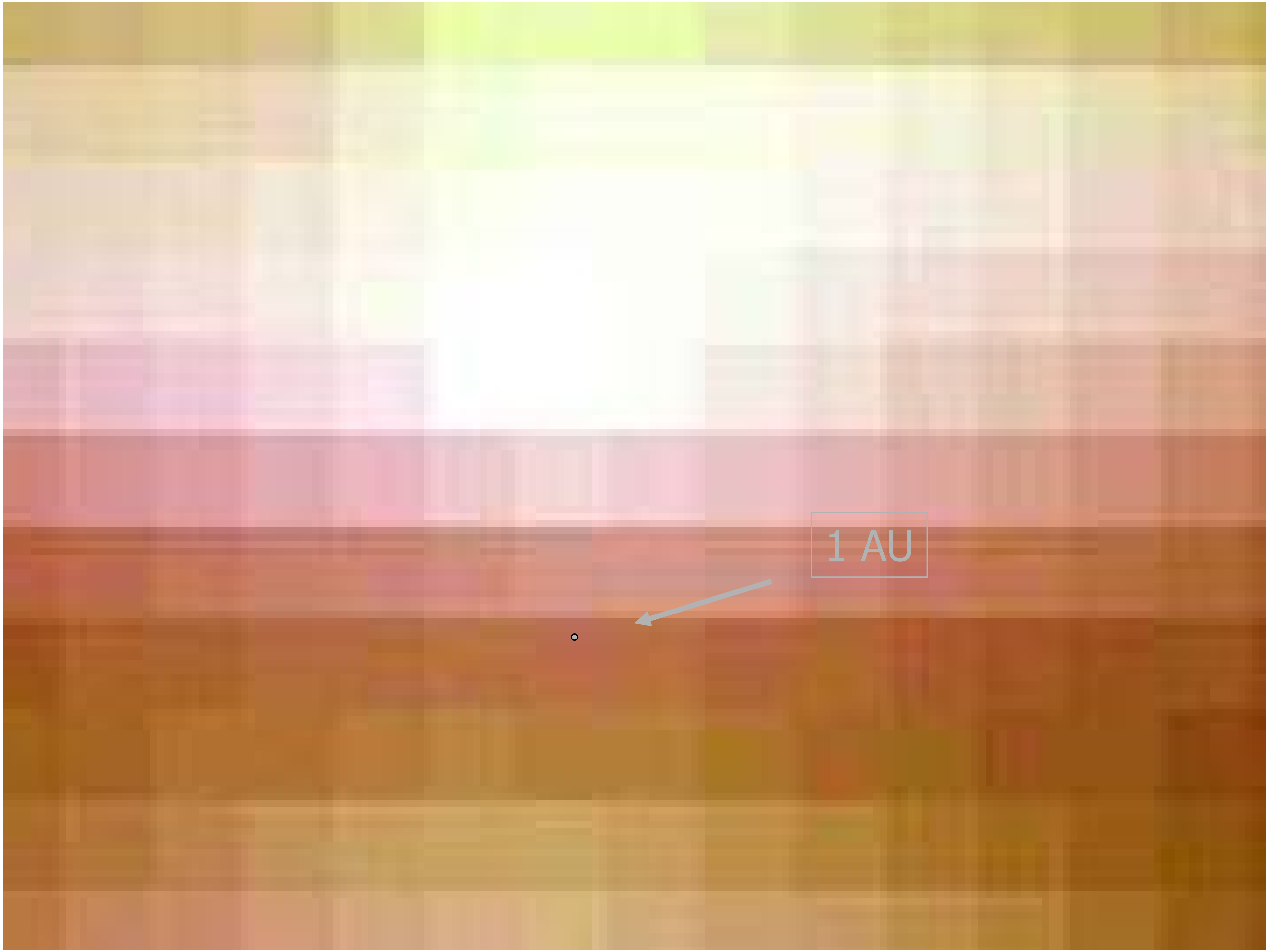

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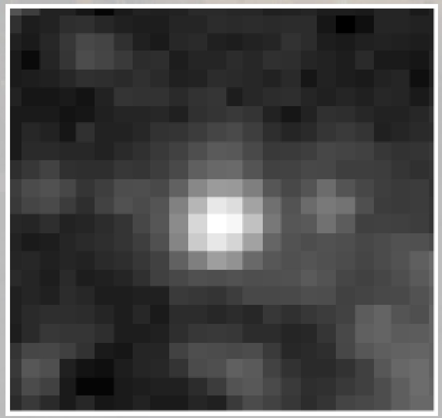
Image formation with conventional telescopes

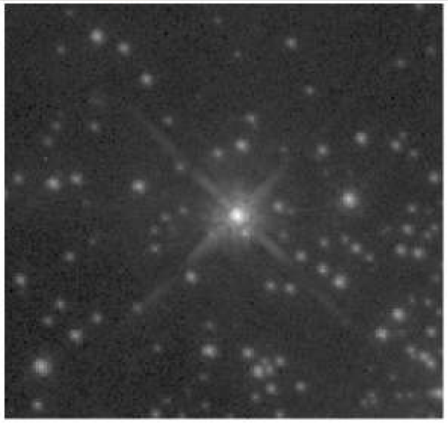
- Fundamental relationship for incoherent space-invariant (isoplanatic) imaging:

$$I(l, m) = \iint P(l-l', m-m')\, O(l', m')\, dl'\, dm' ,$$

i.e. the observed brightness distribution is the true source brightness distribution convolved with a point-spread function, P(l, m).

Note that here l and m are angular coordinates on the sky, measured in radians.





An alternative representation

$$I(u, v) = T(u, v) \times O(u, v) .$$

Here *italic* functions refer to the Fourier transforms of their roman counterparts, and *u* and *v* are now spatial frequencies measured in radians^{-1}.

- Importantly, the essential properties of the imaging system are encapsulated in a complex multiplicative transfer function, $T(u, v)$.
- Note that this is just the Fourier transform of the PSF.

What is the Transfer function?

- In general the transfer function is obtained from the auto-correlation of the complex aperture function:

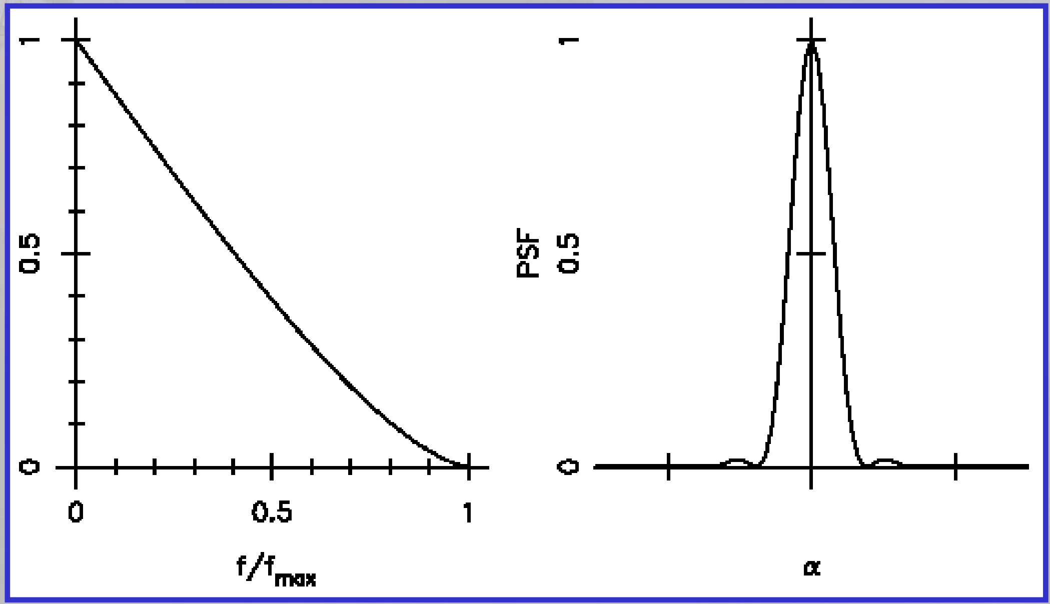
  $$T(u, v) = \iint A^*(x, y)\, A(x+u, y+v)\, dx\, dy\ ,$$

  where x and y denote co-ordinates in the aperture.

- A number of key features of this formalism are worth noting:
  - In the absence of aberrations A(x, y) is equal to 1 where the aperture is transmitting and 0 otherwise.
  - For a circularly symmetric aperture, the transfer function can be written as a function of a single co-ordinate: $T(f)$, with $f^2 = u^2 + v^2$.
  - For each spatial frequency, u, there is a physical baseline, B, in the aperture, of length λu.

The example of a circular aperture

- *T*(*f*) falls smoothly to zero at $f_{max} = D/\lambda$.
- The PSF is the familiar Airy pattern.
- The full-width at half-maximum of this is at approximately λ/D.



What should we learn from all this?

- Decomposition of an image into a series of spatially separated compact PSFs.
- The equivalence of this to a superposition of non-localized sinusoids.

- The description of an image in terms of its Fourier components.
- The action of an incoherent imaging system as a filter for the Fourier spectrum of the source.

- The association of each Fourier component (or spatial frequency) with a distinct physical baseline in the aperture that samples the light.
- The form of the point-spread function as arising from the relative weighting of the different spatial frequencies measured by the pupil of the imaging system.

- If we can measure the Fourier components of the sky-brightness distribution we can tell what the source looks like.

Quiz 1

1. Assume you have a representation of the sky in terms of a PSF and a complete list of strengths and locations at which to lay them down. What happens to the "map" if:
   a) You lose the first quarter of the list of strengths and locations? *(Assume that the list is ordered by decreasing distance from the center of the map, i.e. most distant first.)*
   b) You lose all information about the PSF between position angles 0° and 90°?
   c) You are given a smoothed version of the PSF.

2. Assume you have a representation of the sky in terms of a complete list of Fourier components and a list of strengths and phases for each of these. What happens to the "map" if:
   a) You lose the first quarter of the list of strengths and phases. *(Assume that this list is ordered by decreasing spatial frequency, i.e. highest spatial frequency first.)*
   b) The amplitude data in the list is scaled inversely with the modulus of the spatial frequency, so that the amplitudes at higher spatial frequencies are multiplied down?
   c) The phases of the last quarter of the list are randomized?

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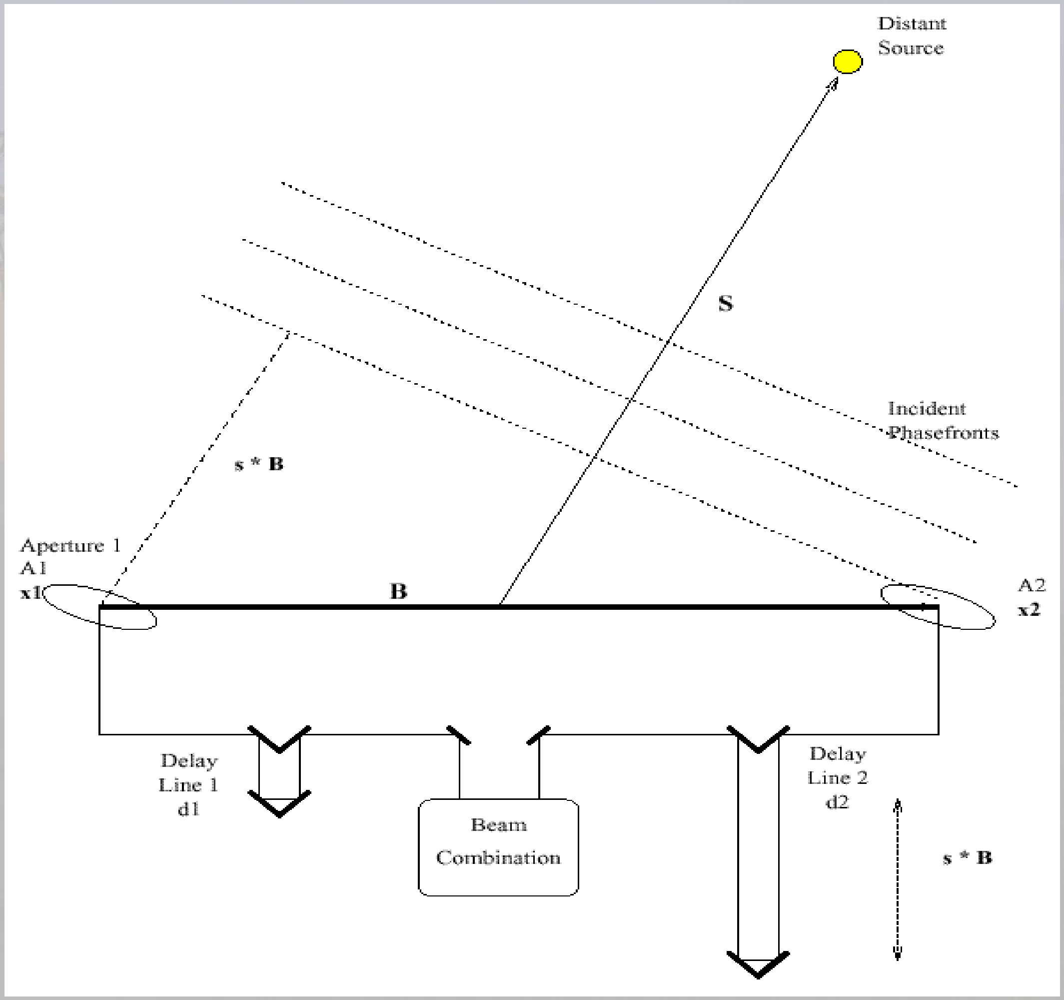
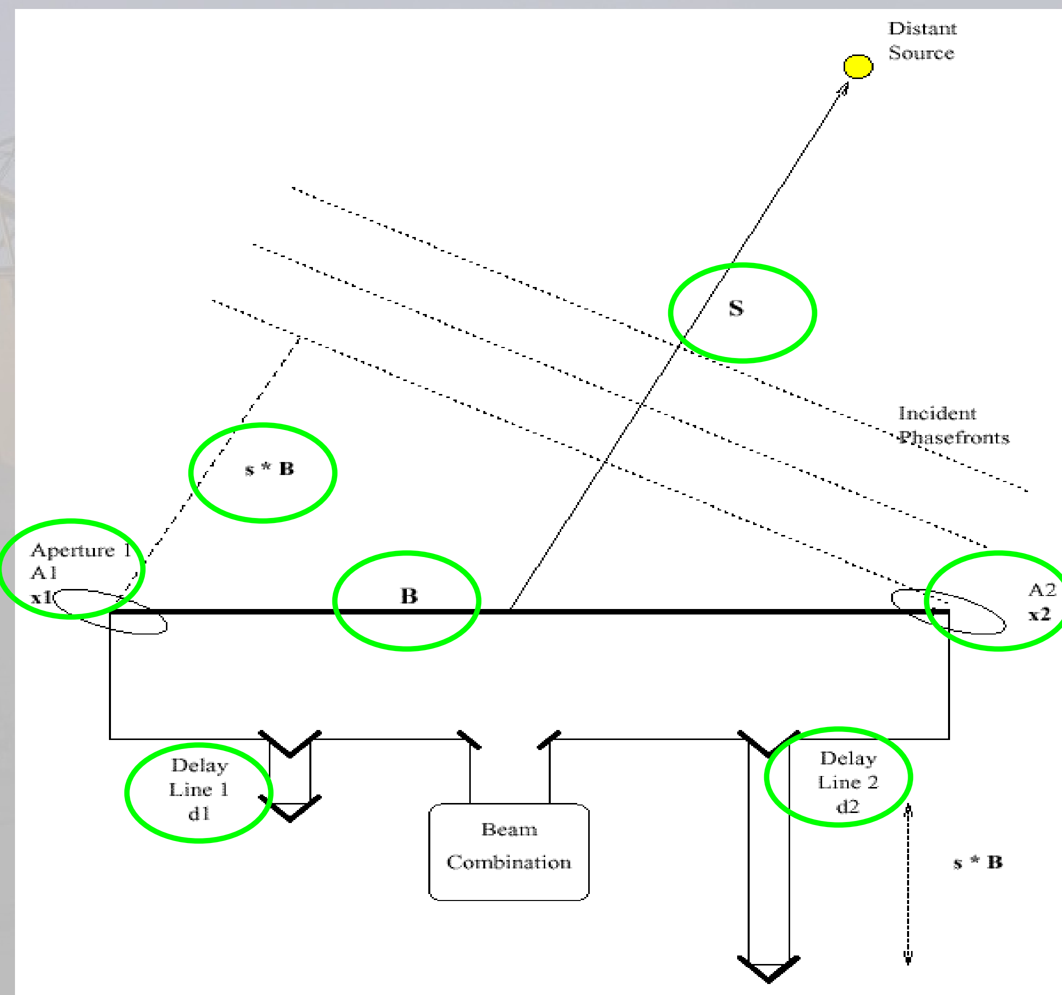
A two element interferometer - function

- Sampling of the radiation (from a distant point source).
- Transport to a common location.
- Compensation for the geometric delay.
- Combination of the beams.
- Addition and detection of the resulting output.



A two element interferometer - nomenclature

- Telescopes located at x_1 & x_2.
- Baseline $B = (x_1 - x_2)$.
- Pointing direction towards source is S.
- Geometric delay is $\hat{s}.B$, where $\hat{s} = S/|S|$.
- Optical paths along two arms are d_1 and d_2.



The output of a 2-element interferometer (i)

- At combination the E fields from the two collectors can be described as:
  - $\psi_1 = A\exp(ik[\hat{s}.B+d_1])\exp(-i\omega t)$ and $\psi_2 = A\exp(ik[d_2])\exp(-i\omega t)$.
- So, summing these at the detector we get a resultant:

$$\Psi = \psi_1 + \psi_2 = A\left[\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])\right]\exp(-i\omega t) .$$

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- Hence the time averaged intensity, $\langle\Psi\Psi^*\rangle$, will be given by:

$$\begin{aligned}\langle\Psi\Psi^*\rangle &\propto \langle\, [\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])] \times [\exp(-ik[\hat{s}.B + d_1]) + \exp(-ik[d_2])]\,\rangle \\ &\propto 2 + 2\cos(k[\hat{s}.B + d_1 - d_2]) \\ &\propto 2 + 2\cos(kD)\end{aligned}$$

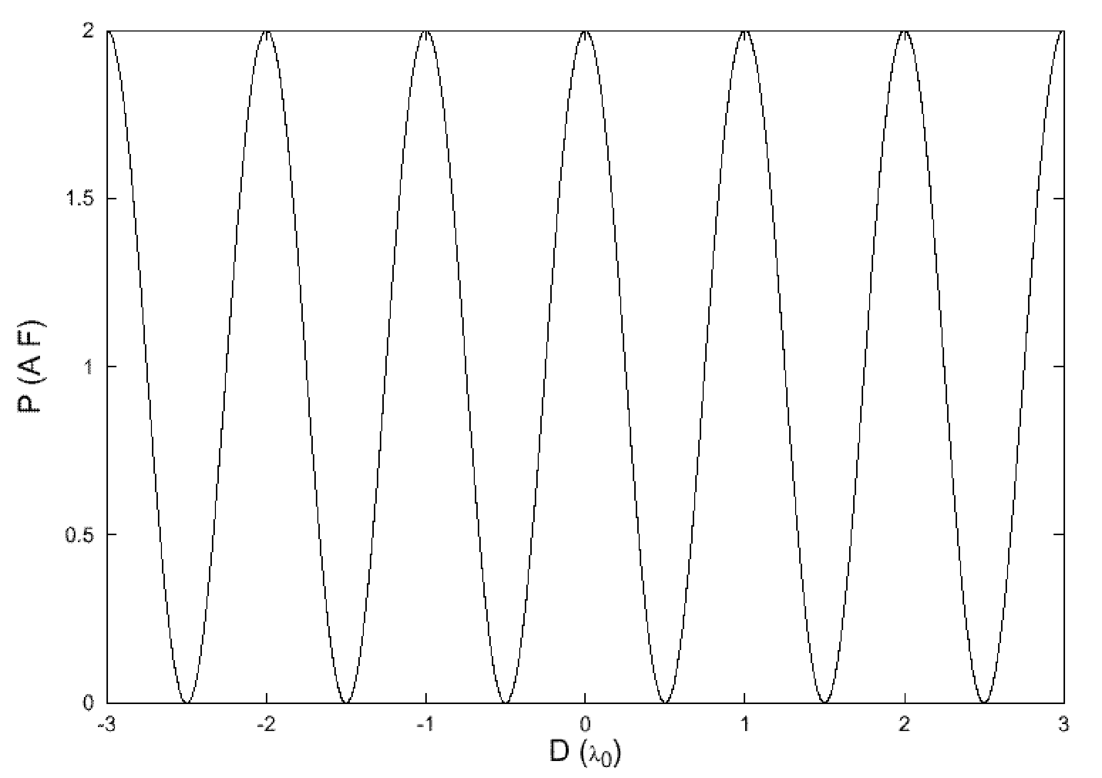
Note, here $D = [\hat{s}.B + d_1 - d_2]$. This is a function of the path lengths, d_1 and d_2, the pointing direction (i.e. where the target is) and the baseline.

The output of a 2-element interferometer (ii)

Detected power, $P = \langle \Psi\Psi^* \rangle \propto 2+2\cos(k\,[\hat{s}.B + d_1 - d_2])$

$\propto 2\times[1+\cos(kD)]$, where $D = [\hat{s}.B + d_1 - d_2]$

- The output varies co-sinusiodally with kD, with $k = 2\pi/\lambda$.
- Adjacent fringe peaks are separated by
  - $\Delta d_{1\ \text{or}\ 2} = \lambda$ or
  - $\Delta(\hat{s}.B) = \lambda$ or
  - $\Delta(1/\lambda) = 1/D$.



What properties of interference fringes matter?

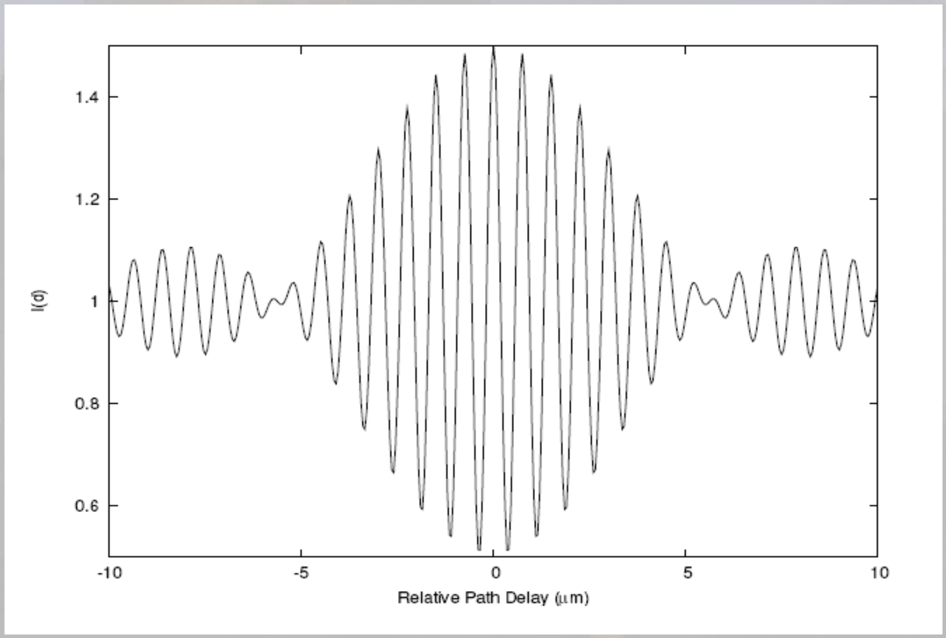
- From an interferometric point of view the key features of any interference fringe are its modulation and its location with respect to some reference point.

  – The fringe visibility:

$$V = \frac{[I_{max} - I_{min}]}{[I_{max} + I_{min}]}$$

  – The fringe phase:

    - The location of the white-light fringe as measured from some reference (radians).



We will see later that the fringe amplitude and phase actually measure the amplitude and phase of the Fourier transform of the source brightness distribution.

Quiz 2

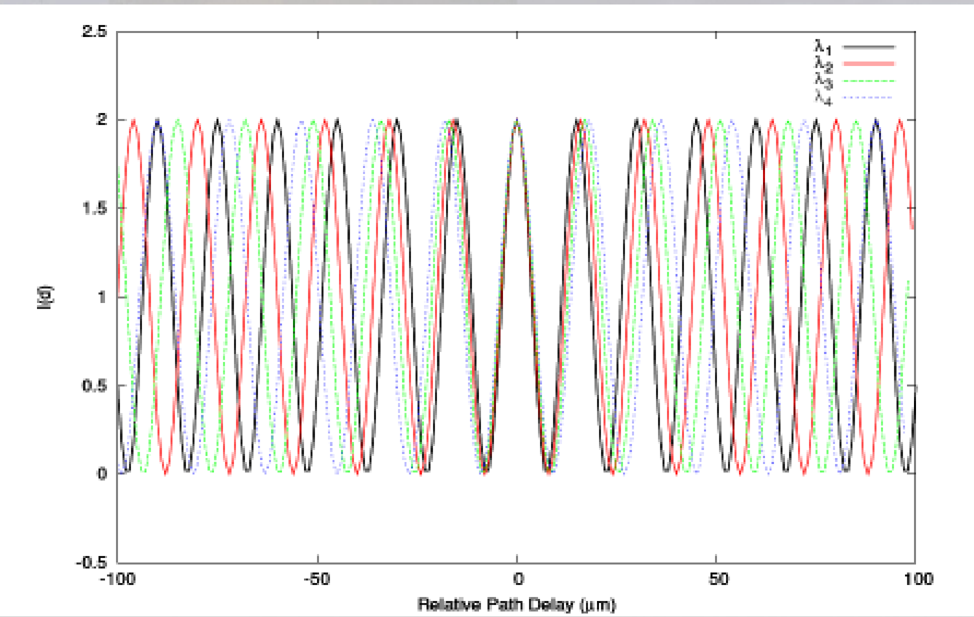
1. Check that you understand the "periodicity" of the interferometer output as a function of:
   a) The wavevector, $k = 2\pi/\lambda$.
   b) The baseline, B.
   c) The pointing direction, s.
   d) The optical path difference between the two interferometer arms.
2. How rapidly does the geometric delay change during the night? *(Only an approximate answer is needed – assume a 100m baseline interferometer observing a source at 45° elevation).*
3. Imagine an interferometer with a 100m baseline is observing a target at the zenith at a wavelength of 1 micron. If the source could be moved by some small distance in the sky, how far would you have to move it to see the interferometer response change from a maximum to a minimum and then back to a maximum again?

Extension to polychromatic light

- We can integrate the previous result over a range of wavelengths:
  - E.g for a uniform bandpass of $\lambda_0 \pm \Delta\lambda/2$ (i.e. $\nu_0 \pm \Delta\nu/2$) we obtain:

$$P \propto \int_{\lambda_0 - \Delta\lambda/2}^{\lambda_0 + \Delta\lambda/2} [2 + 2\cos(kD)]\, d\lambda$$

$$= \int_{\lambda_0 - \Delta\lambda/2}^{\lambda_0 + \Delta\lambda/2} 2\,[1 + \cos(2\pi D / \lambda)]\, d\lambda$$



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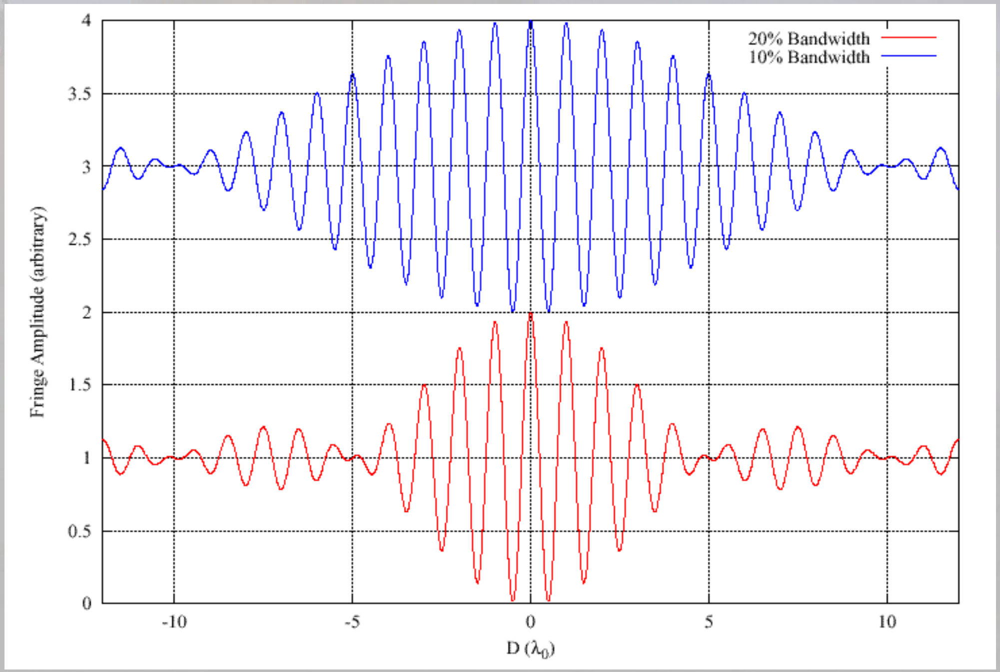
$$= \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} 2\,[1+\cos(2\pi D/\lambda)]\,d\lambda$$

$$= \Delta\lambda\left[1+\frac{\sin \pi D\Delta\lambda/\lambda^2{}_0}{\pi D\Delta\lambda/\lambda^2{}_0}\cos k_0 D\right]$$

$$= \Delta\lambda\left[1+\frac{\sin \pi D/\Lambda_{coh}}{\pi D/\Lambda_{coh}}\cos k_0 D\right]$$



So, the fringes are modulated with an envelope with a characteristic width equal to the coherence length, $\Lambda_{coh} = \lambda^2{}_0/\Delta\lambda$.

Key ideas regarding the interferometric output (i)

- The output of the interferometer is a time averaged intensity.
- The intensity has a co-sinusoidal variation – these are the “fringes”.
- The fringe varies a function of (kD), which itself can depend on:
  - The wavevector, $k = 2\pi/\lambda$.
  - The baseline, B.
  - The pointing direction, s.
  - The optical path difference between the two interferometer arms.

- If things are adjusted correctly, the interferometer output is fixed: there are no fringes. This is what most interferometers aim to achieve.

Key ideas regarding the interferometric output (ii)

- The response to a polychromatic source is given by integrating the intensity response for each color.
- This alters the interferometric response and can lead to “washing out” of the fringe modulation completely:
  - The desired response is only achieved when k [$\hat{s}$.B + d_1 - d_2]) = 0.
  - This is the so called white-light condition.
- This is the primary motivation for matching the optical paths in an interferometer and correcting for the geometric delay.

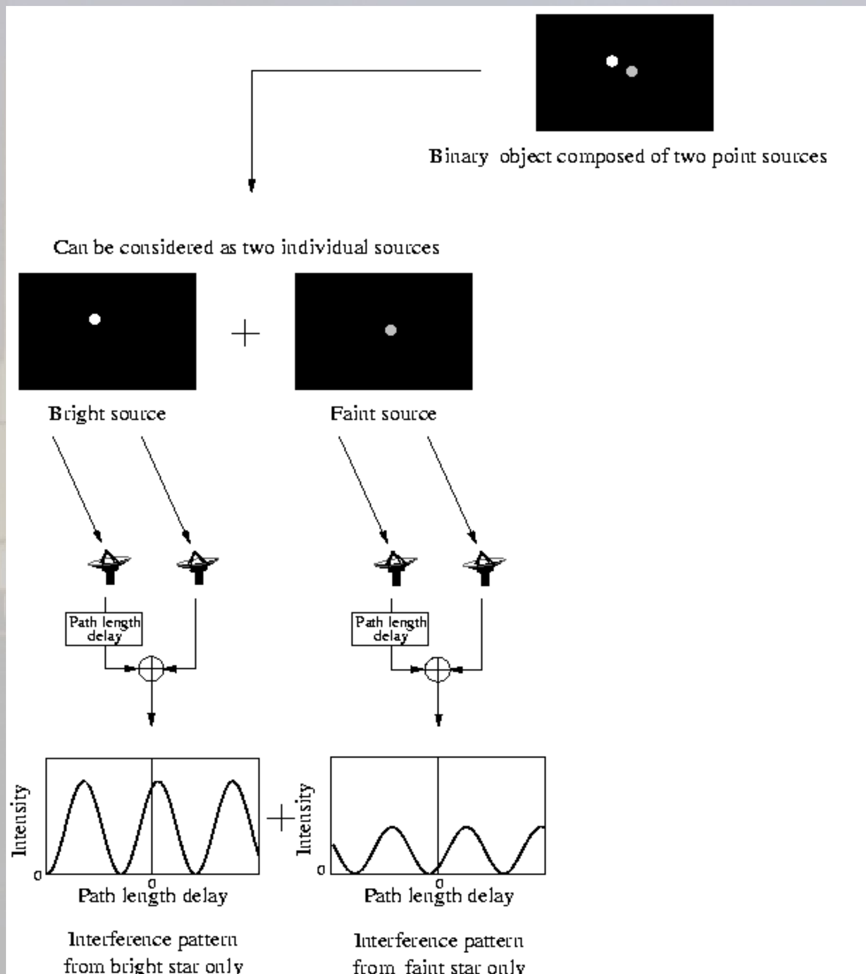
- The narrower the range of wavelengths detected, the smaller is the effect of this modulation:
  - This is usually quantified via the coherence length, $\Lambda_{coh} = \lambda^2_0/\Delta\lambda$.
  - But narrower bandpasses mean less light.

Quiz 3

1. You are observing with a 100m baseline interferometer at a wavelength of 1 micron. Due to unknown causes, the optical paths in the interferometer are unmatched by 5 microns. By how much are the maximum and minimum outputs of the interferometer altered if the fractional bandwidth of the light being collected is:
   a) 1 percent?
   b) 5 percent?
   c) 10 percent?
2. What are the coherence lengths of light passed by the standard J, H and K band near-infrared filters? *(Only approximate answers are needed.)*
3. Assume you are observing with a 100m baseline interferometer at a wavelength of 1 micron with a 10 percent fractional bandwidth. If you are looking at a target at an elevation of 45°, how far away can another target be such that the geometric delay for the secondary target is no more than a coherence length different from that of the primary target?

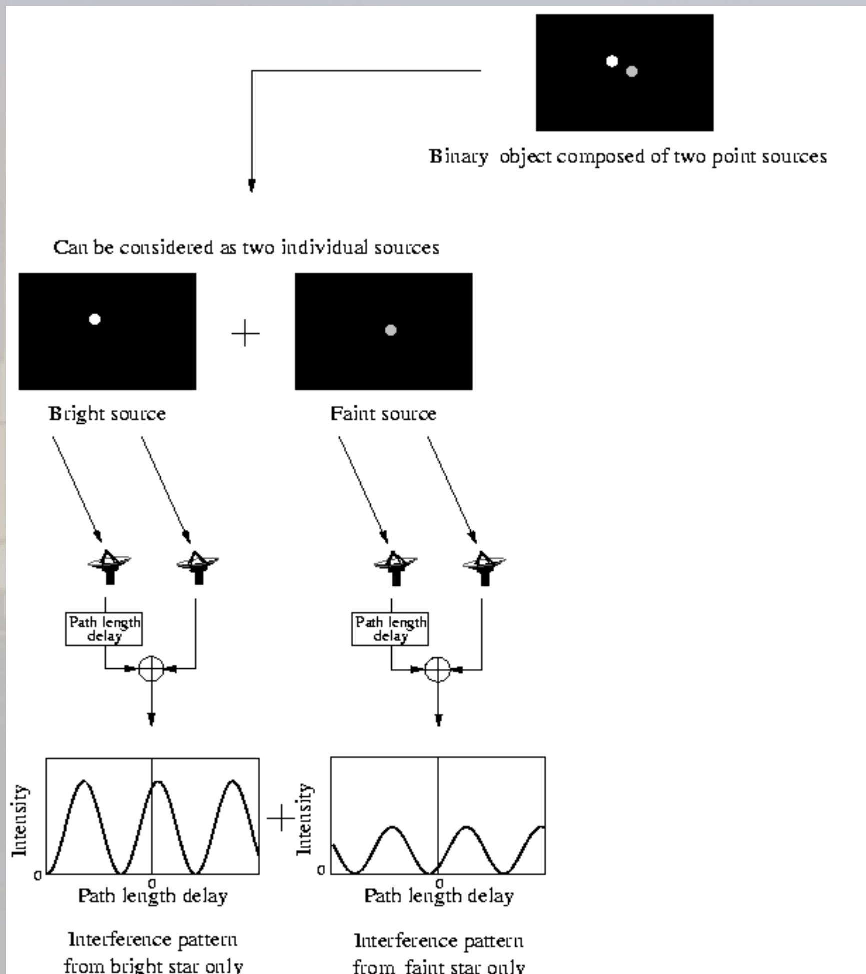
   What does this tell you about the maximum field of view of an interferometer given its baseline and its fractional bandwidth?

Heuristic operation of an interferometer



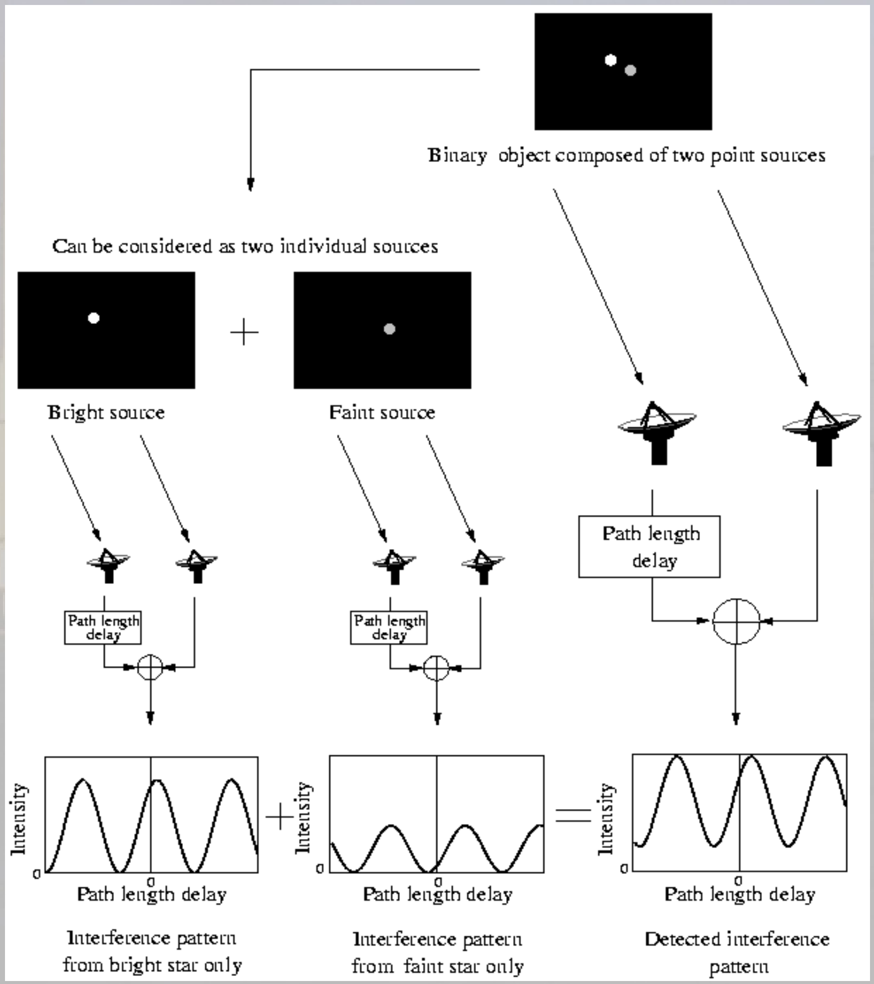
- Each unresolved element of the source produces its own fringe pattern.
- These have unit visibilities and phases that are associated with the location of that source element in the sky:
  - This is the basis for astrometric measurements with interferometers.

Heuristic operation of an interferometer



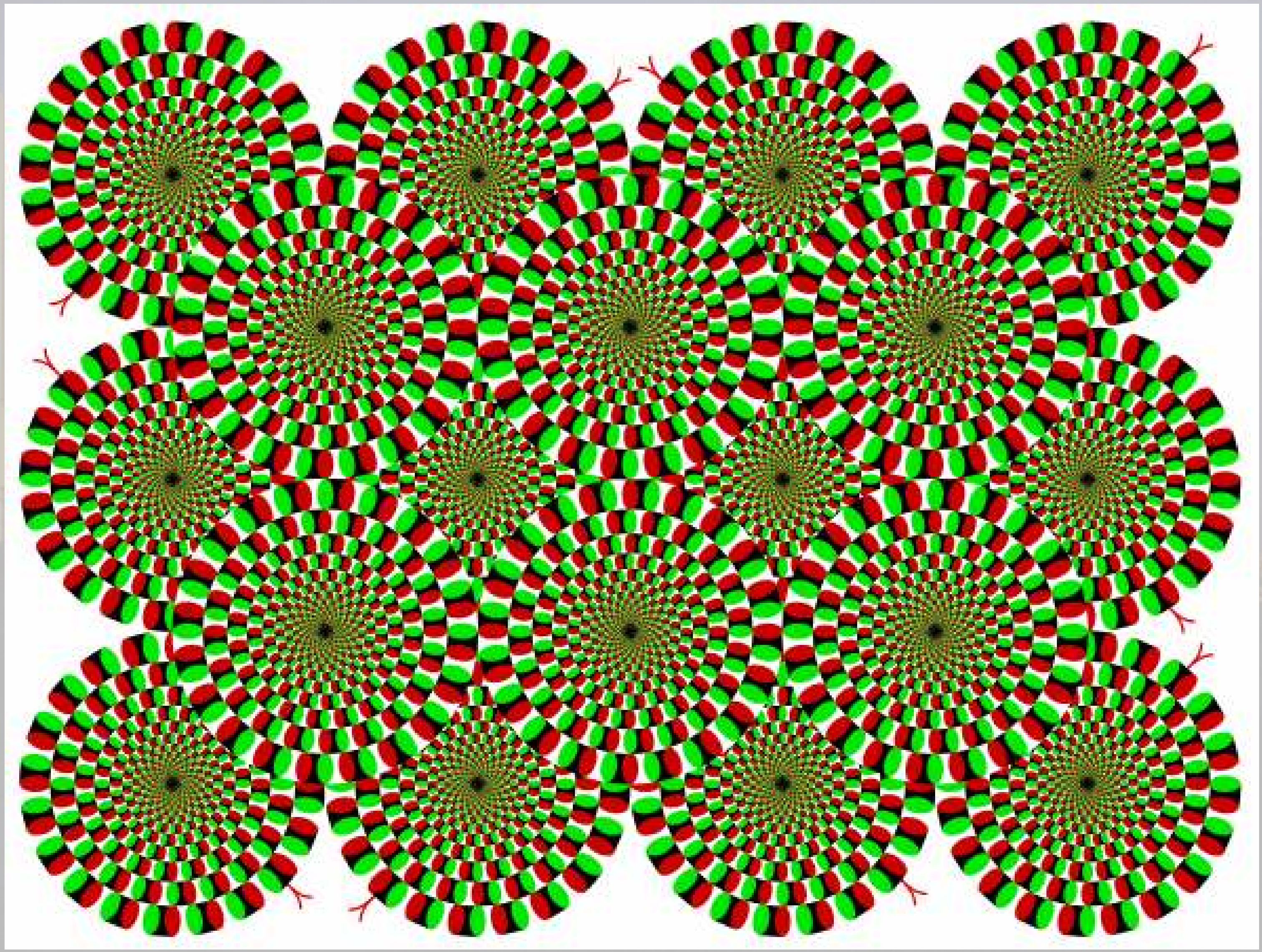
- The observed fringe pattern from a distributed source is just the intensity superposition of these individual fringe pattern.
- This relies upon the individual elements of the source being "spatially incoherent".

Heuristic operation of an interferometer



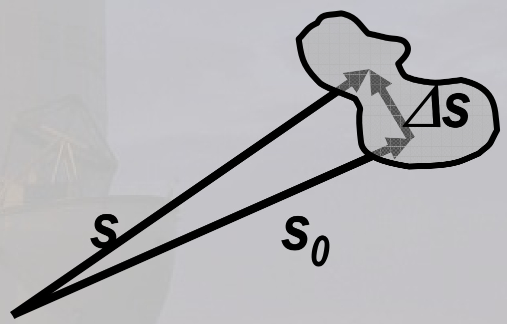
- The resulting fringe pattern has a contrast that is reduced with respect to that from each source individually.
- The positions of the sources are encoded (in a scrambled manner) in the resulting fringe phase.

Some visual refreshment



A more mathematical treatment

- Consider looking at an incoherent source whose brightness on the sky is described by I(ŝ). This can be written as $I(\hat{s}_0 + \Delta s)$, where $\hat{s}_0$ is a vector in the pointing direction, and Δs is a vector perpendicular to this.



- The detected power will be given by:

$$\begin{aligned} P(s_0, B) &\propto \int I(s)\left[1+\cos kD\right]d\Omega \\ &\propto \int I(s)\left[1+\cos k(s.B + d_1 - d_2)\right]d\Omega \\ &\propto \int I(s)\left[1+\cos k([s_0 + \Delta s].B + d_1 - d_2)\right]d\Omega \\ &\propto \int I(s)\left[1+\cos k(s_0.B + \Delta s.B + d_1 - d_2)\right]d\Omega \\ &\propto \int I(\Delta s)\left[1+\cos k(\Delta s.B)\right]d\Omega' \end{aligned}$$

Heading towards the van Cittert-Zernike theorem

- Consider now adding a small path delay, δ, to one arm of the interferometer. The detected power will become:

$$P(s_0, B, \delta) \propto \int I(\Delta s)\left[1 + \cos k(\Delta s.B + \delta)\right] d\Omega'$$

$$\propto \int I(\Delta s)\, d\Omega' + \cos k\delta \cdot \int I(\Delta s) \cos k(\Delta s.B)\, d\Omega'$$

$$- \sin k\delta \cdot \int I(\Delta s) \sin k(\Delta s.B)\, d\Omega'$$

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- We now define something called the complex visibility V(*k*,*B*):

$$V(k, B) = \int I(\Delta s) \exp[-ik\Delta s.B]\, d\Omega',$$

so that we can simplify the equation above to:

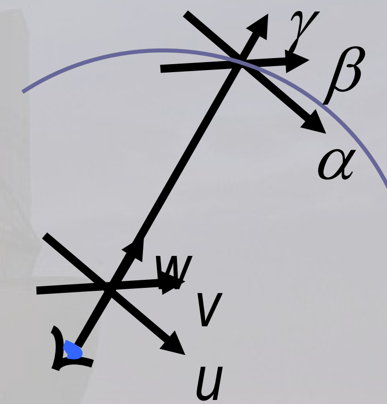
$$P(s_0, B, \delta) \propto \int I(\Delta s)\ \ d\Omega' + \cos k\delta\, \mathrm{Re}[V] + \sin k\delta\, \mathrm{Im}[V]$$

$$P(s_0, B, \delta)\ =\ I_{total} + \mathrm{Re}\left[V \exp[-ik\delta]\right]$$

What is this V that we have introduced?

- Lets assume $\hat{s}_0 = (0,0,1)$ and Δs is $\approx (\alpha,\beta,0)$, with α and β small angles measured in radians.

$$
\begin{aligned}
V(k,B) &= \int I(\Delta s)\exp[-ik\Delta s.B]\,d\Omega' \\
&= \int I(\alpha,\beta)\exp[-ik(\alpha B_x + \beta B_y)]\,d\alpha\,d\beta \\
&= \int I(\alpha,\beta)\exp[-i2\pi(\alpha u + \beta v)]\,d\alpha\,d\beta
\end{aligned}
$$



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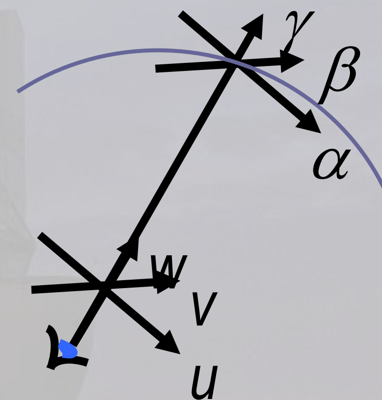


  - Here, u (= B_x/λ) and v (= B_y/λ) are the projections of the baseline onto a plane perpendicular to the pointing direction.
  - These co-ordinates have units of rad^{-1} and are the spatial frequencies associated with the physical baselines.
  - These are the same u and v we saw earlier when we introduced the Fourier representation of the incoherent imaging equation.

So, the complex quantity V we introduced is the Fourier Transform of the source brightness distribution.

How this all fits together

- We can put this all together as follows:
  - Our interferometer measures: $P(s_0, B, \delta) = I_{total} + \mathrm{Re}\left[V \exp[-ik\delta]\right]$
  - So, if we make measurements with, say, two value of δ (= 0 and λ/4), this recovers the real and imaginary parts of the complex visibility, V.
  - And, since the complex visibility is nothing more than the Fourier transform of the brightness distribution, we have our final results:

The output of an interferometer "measures" the Fourier transform of the source brightness distribution.

This amplitude and phase of the interferometer fringes are the amplitude and phase of the FT of the brightness distribution.

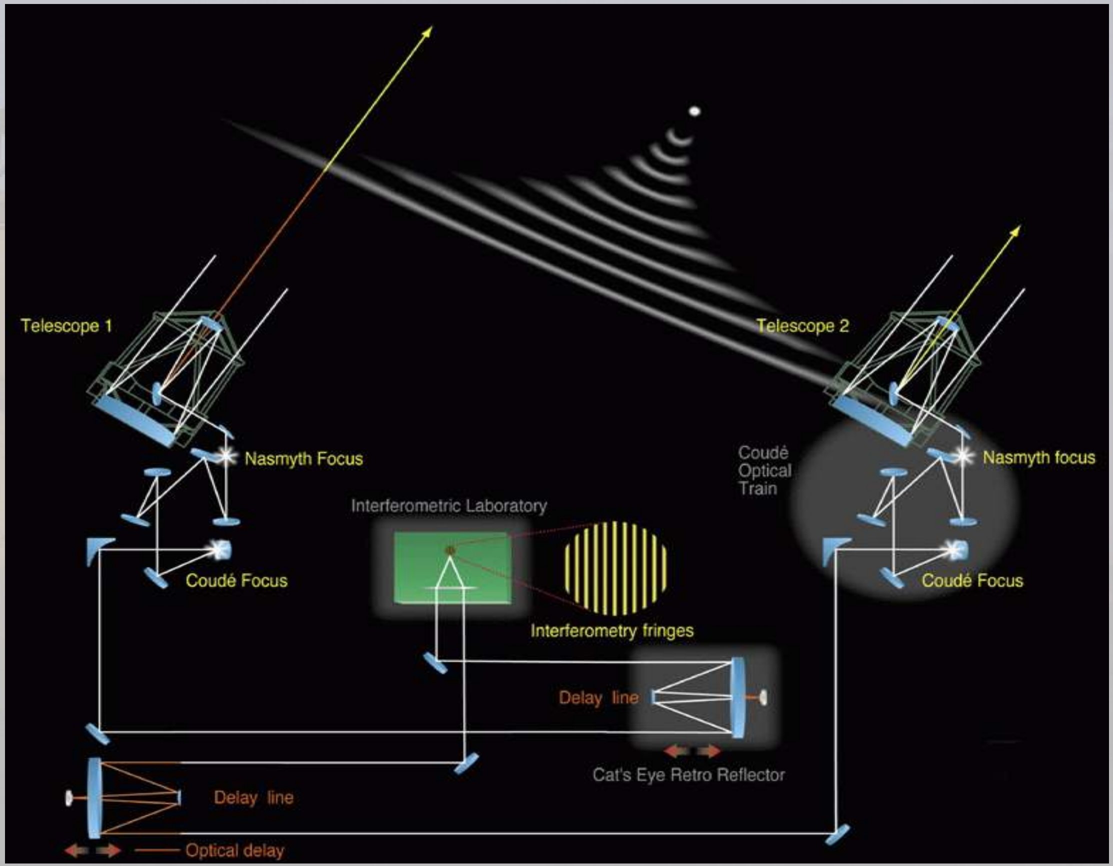
Long interferometer baselines probe small structures, shorter baselines probe larger structures.

Quiz 4

1. Check that you understand how introducing two values of δ (= 0 and $\lambda/4$), allows one to recover the Real and Imaginary parts of the complex visibility.
2. What happens to the interferometer output, $P(s_0, B, \delta)$ if δ is changed smoothly from zero to 2λ? Sketch it. *(You will need to assume values for I_{total} and V – let I_{total} equal 1.0 and let V (don't forget it is complex) have an amplitude of 0.5 and a phase of 45°.)*
3. Assume you are using an interferometer with a baseline of 100m, oriented in an E-W direction at a latitude of 45°, and observing a target at declination 15°. If the source is observed at ±3 hours around transit, what values do *u* and *v* (the projected baseline components) take? Sketch them on a plot of the *uv* plane.

A reality check

- How is all this related to the VLTI?
- Telescopes sample the fields at r_1 and r_2.
- Optical train delivers the radiation to a laboratory.
- Delay lines assure equal optical paths.
- The instruments mix the beams and detect the fringes.



"Imaging" with interferometers

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What have we learnt so far?

- Physical basis of interferometry is the van Cittert-Zernike theorem:
  - The amplitude and phase of the interferometric fringes measure the Fourier transform of the brightness distribution.
  - This function, the visibility (or coherence) function, $V(u, v) = V(B_x/\lambda, B_y/\lambda)$, is measured at locations determined by the projected interferometer baselines.

- Some questions we might wish to ask ourselves are:
  - How easy is it to measure $V(u, v)$?
  - How easy is it to interpret measurements of $V(u, v)$?
  - What do we do with measurements of $V(u, v)$ if we wish to make a map of the sky?

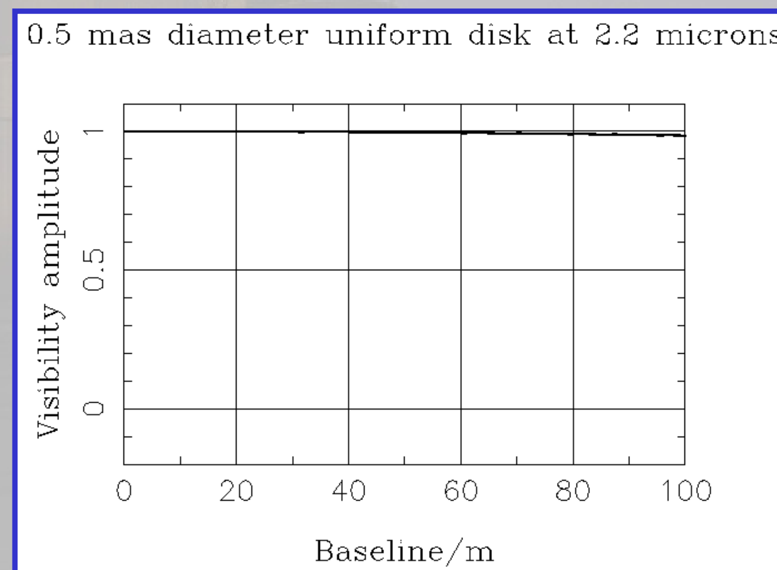
Visibility functions of simple 1-d sources (i)

$$V(u) \propto \int I(l)\, e^{-i2\pi(ul)}\, dl.$$

Point source of strength A_1 and located at angle l_1 relative to the optical axis.

$$V(u) = \int A_1 \delta(l-l_1)\, e^{-i2\pi(ul)}\, dl \div \text{total flux}$$
$$= e^{-i2\pi(ul_1)}\,.$$

- The visibility amplitude is unity $\forall\, u$.
- The visibility phase varies linearly with u (= B/λ).
- Sources such as this are easy to observe since the interferometer output gives fringes with high contrast.

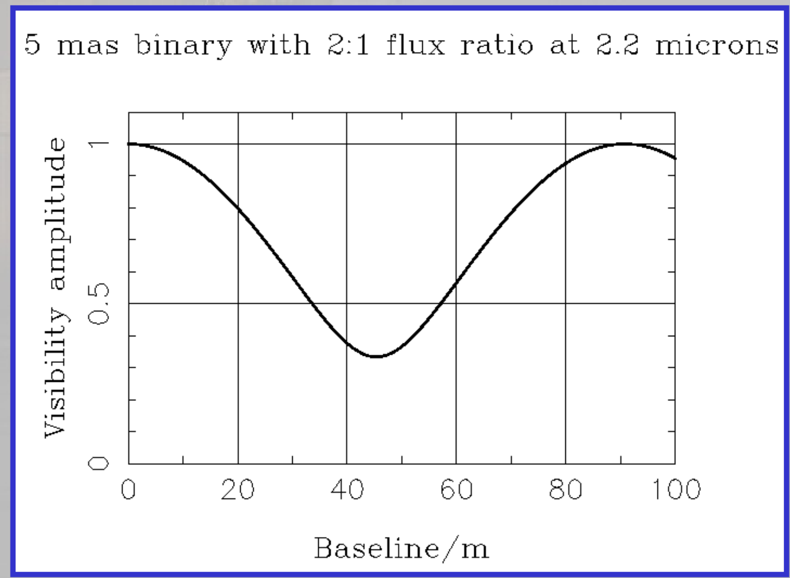


Visibility functions of simple sources (ii)

A double source comprising point sources of strength A_1 and A_2 located at angles 0 and l_2 relative to the optical axis.

$$V(u) = \int [A_1\delta(l) + A_2\delta(l-l_2)]\, e^{-i2\pi(ul)}\, dl \div \text{total flux}$$
$$= \propto [A_1 + A_2 e^{-i2\pi(ul_2)}] \,.$$

- The visibility amplitude and phase oscillate as functions of u.
- To identify this as a binary, baselines from $0 \rightarrow \lambda/l_2$ are required.
- If the ratio of component fluxes is large the modulation of the visibility becomes increasingly difficult to measure, i.e. the interferometer output looks the same for all baselines.



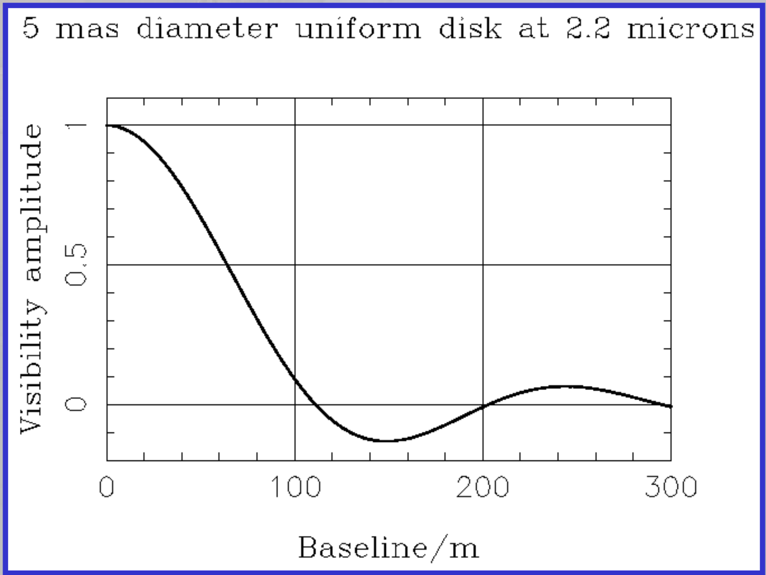
Visibility functions of simple sources (iii)

A uniform on-axis disc source of diameter θ.

$$V(u_r) \propto \int^{\theta/2} \rho \, J_0(2\pi\rho u_r) \, d\rho$$

$$= 2J_1(\pi\theta u_r) \div (\pi\theta u_r) \, .$$

- The visibility amplitude falls rapidly as u_r increases.
- To identify this as a disc requires base-lines from $0 \rightarrow \lambda/\theta$ at least.
- Information on scales smaller than the disc diameter correspond to values of u_r where $V \ll 1$, and is hence difficult to measure. This is because the interfe-rometer output gives fringes with very small modulation.



What can we learn from this?

- Unresolved, sources have visibility functions that remain high whatever the baseline, and produce high contrast fringes all the time.
- Resolved sources have visibility functions that fall to low values at long baselines, giving fringes with very low contrast.
  - ⇒ Fringe parameters for resolved sources will be difficult to measure.
- To usefully constrain a source, the visibility function must be measured adequately. Measurements on a single, or small number of, baselines are normally not enough for unambiguous interpretation.
- Imaging – which necessarily requires information on both small and large scale features in a target, will generally need measurements where the fringe contrast is both high and low.

Quiz 5

1. Plot the visibility amplitude and phase for a 5mas separation binary (see slide 42) as a function of projected baseline length up to 100m assuming you are taking observations at 1 micron and that the binary has:
   a) a flux ratio of 2:1,
   b) a flux ratio of 100:1,
   c) a flux ratio of 1000:1.

   (Remember, $V = V(u)$ and $u = B/\lambda$).
2. Assuming observations at 1 micron, what baseline lengths are needed to resolve main sequence stars of spectral types O, A, G, and M, if their apparent magnitudes are:
   a) 0,
   b) 6,
   c) 12.

   What do these results tell you about observing faint targets with an optical interferometer?

   (You will need to look up the absolute magnitudes and physical radii of these stars in a textbook.)

Making maps with interferometers

- How the properties of the FT allow for inversion of the interferometer data.
- How the image recovered in this way is related to the true sky brightness distribution.
- How the recovered image can be used to infer the true sky brightness distribution.
- Rules of thumb that you should be aware of.

Note that, for simplicity here, we will assume perfect measurements of the visibility function.

- You will learn later how the effects of noise and the atmosphere are dealt with.

How maps are recovered

- We start with the fundamental relationship between the visibility function and the normalized sky brightness:

$$\iint V(u, v)\, e^{+i2\pi(ul + vm)}\, du\, dv = I_{norm}(l, m).$$

- In practice what we measure is a sampled version of $V(u, v)$, so the image we have access to is to the so-called "dirty map":

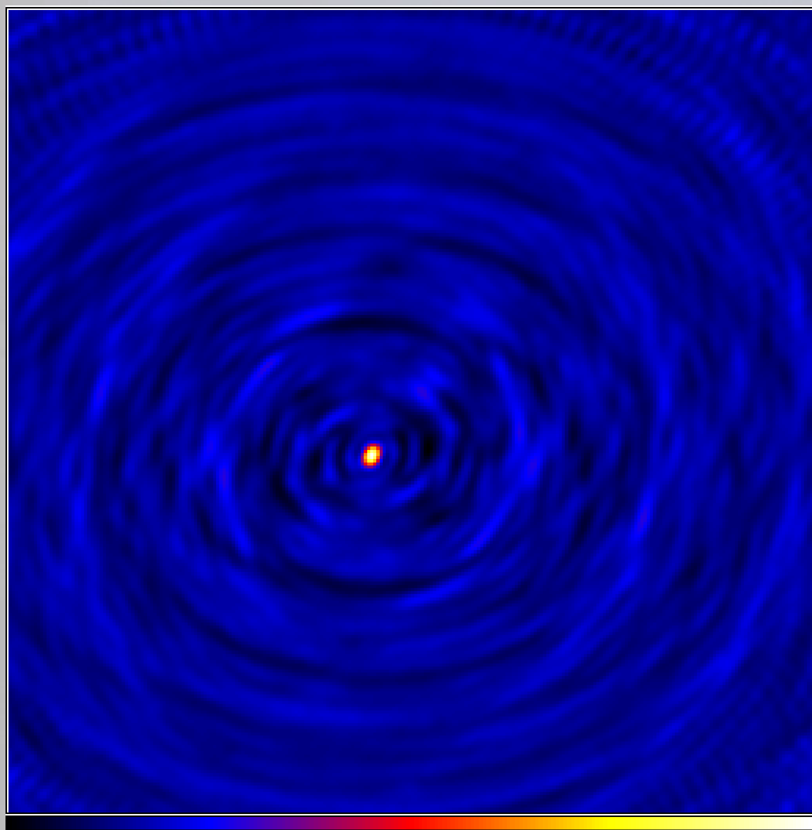
$$\iint S(u, v)\, V(u, v)\, e^{+i2\pi(ul + vm)}\, du\, dv = I_{dirty}(l, m)$$

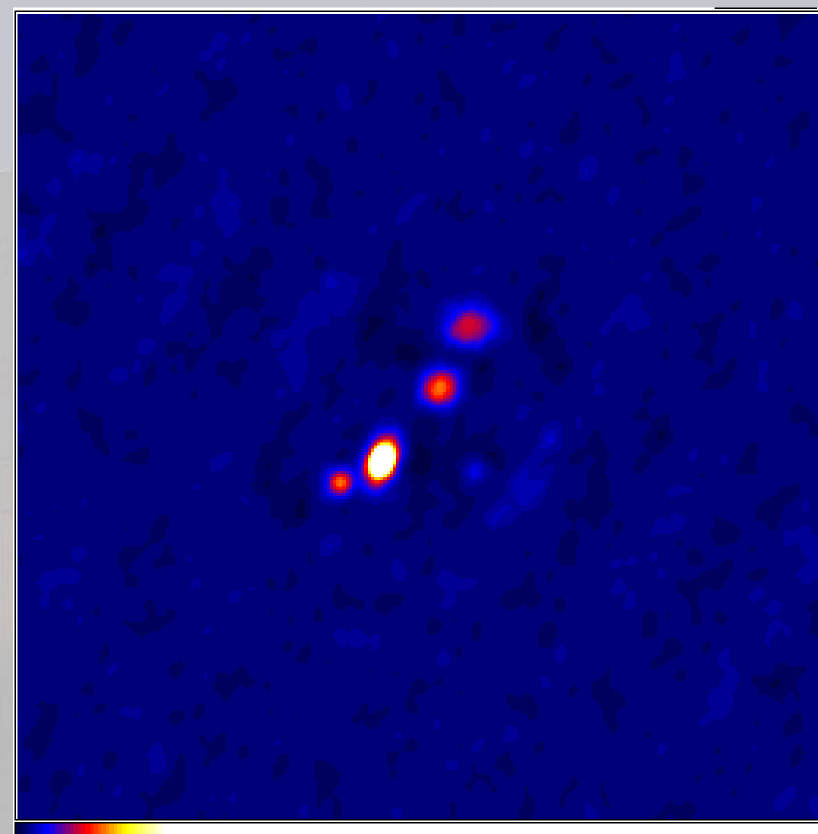
$$= I_{norm}(l, m) * B_{dirty}(l, m),$$

  where $B_{dirty}(l,m)$ is the Fourier transform of the sampling distribution, or the so-called dirty-beam.

- The dirty-beam is the interferometer PSF. While it is generally far less attractive than an Airy pattern, it's shape is completely determined by the samples of the visibility function that are measured.

The effect of the sampling distribution





- Interferometric PSF’s can often be horrible. Correcting an interferometric map for the Fourier plane sampling function is known as deconvolution and is broadly speaking straightforward.

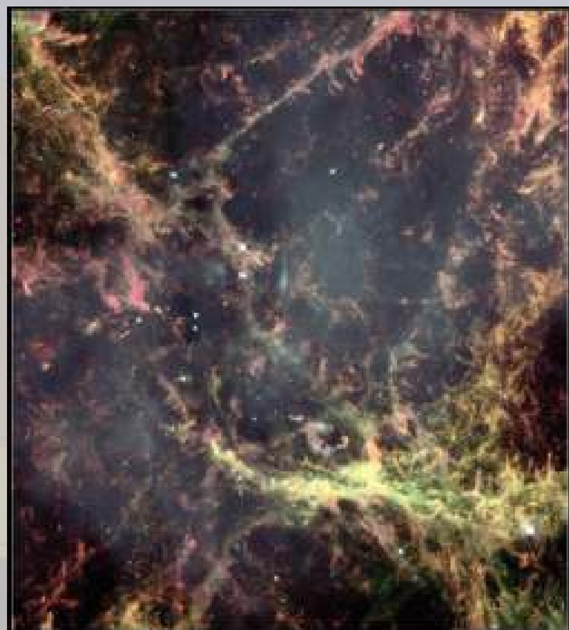
Important rules of thumb

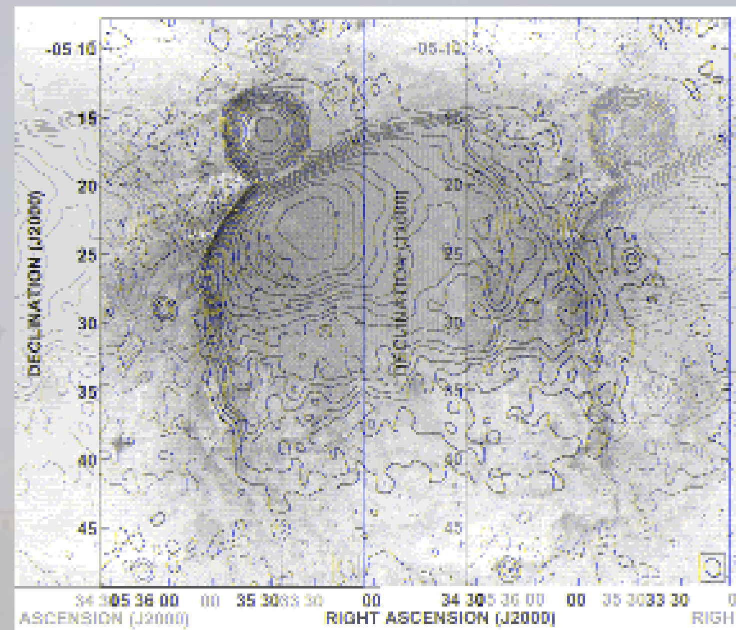
- The number of visibility data ≥ number of filled pixels in the recovered image:
  - N(N-1)/2 × number of reconfigurations ≥ number of filled pixels.
- The distribution of samples should be as uniform as possible:
  - To aid the deconvolution process.
- The range of interferometer baselines, i.e. B_{max}/B_{min}, will govern the range of spatial scales in the map.
- There is no need to sample the visibility function too finely:
  - For a source of maximum extent θ_{max}, sampling very much finer than $\Delta u \sim 1/\theta_{max}$ is unnecessary.

Other useful rules of thumb

- The FOV will depend upon:
  - The field of view of the individual collectors. This is often referred to as the primary beam.
  - The FOV seen by the detectors. This is limited by vignetting along the optical train.
  - The spectral resolution. The interference condition OPD < $\lambda 2/\Delta\lambda$ must be satisfied for all field angles. Generally ⇒ FOV $\leq [\lambda/B][\lambda/\Delta\lambda]$.
- Dynamic range:
  - The ratio of maximum intensity to the weakest believable intensity in the image.
  - $> 10^5$:1 is achievable in the very best radio images, but of order several × 100:1 is more usual.
  - $DR \sim [S/N]_{per\text{-}datum} \times [N_{data}]^{1/2}$
- Fidelity:
  - Difficult to quantify, but clearly dependent on the completeness of the Fourier plane sampling.

Conventional vs. interferometric imaging





- Optical HST (left) and 330Mhz VLA (right) images of the Crab Nebula and the Orion nebula. Note the differences in the:
  - Range of spatial scales in each image.
  - The range of intensities in each image.
  - The field of view as measured in resolution elements.

What does sensitivity mean for an interferometer?

- The "source" has to be bright enough to:
  - Allow stabilisation of the interferometer against any atmospheric fluctuations.
  - Allow a reasonable signal-to-noise for the fringe parameters to be build up over some total convenient integration time. This will be measured in minutes.
- Once this achieved, the faintest features one will be able to interpret reliably will be governed by S/N ratio and number of visibility data measured.
- In most cases the sensitivity of an interferometer will scale like some power of the measured fringe contrast × another power of the number of photons detected while the fringe is being measured.

This highlights fringe contrast <u>and</u> throughput as both being critical.

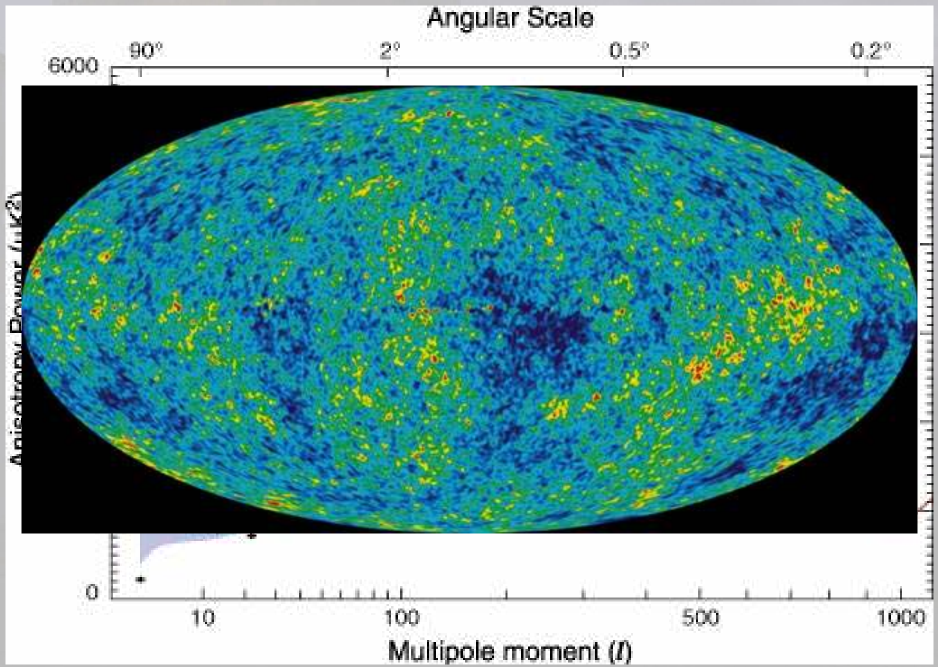
This also highlights the difficulty of measuring resolved targets where V is low.

Quiz 6

1. Why might the total integration time for an measurement of the visibility function with an interferometer be limited to a few minutes? *(You may wish to consider the rotation of the Earth and the how much u and v have to change so that you are measuring an independent value of the Fourier transform of the target.)*
2. What are the typical spatial and temporal scales associated with the atmosphere for observations made at 1 micron? How do these compare to the equivalent scales at centimetric radio wavelengths (e.g. at the VLA site in New Mexico) and at sub-mm wavelengths (e.g. at the ALMA site in Chile). *(You will need to bug the lecturers for this information or do some literature sleuthing to find the answers.)*
3. If the S/N for some interferometric observation of an unresolved target (V=1) is X, how much brighter (in magnitudes) must a target be to be measured with the same S/N if it is sufficiently resolved that its visibility function, V, is equal to 0.1? *(Assume that you are working in the regime where the S/N scales like $V \times \sqrt{(flux)}$.)*

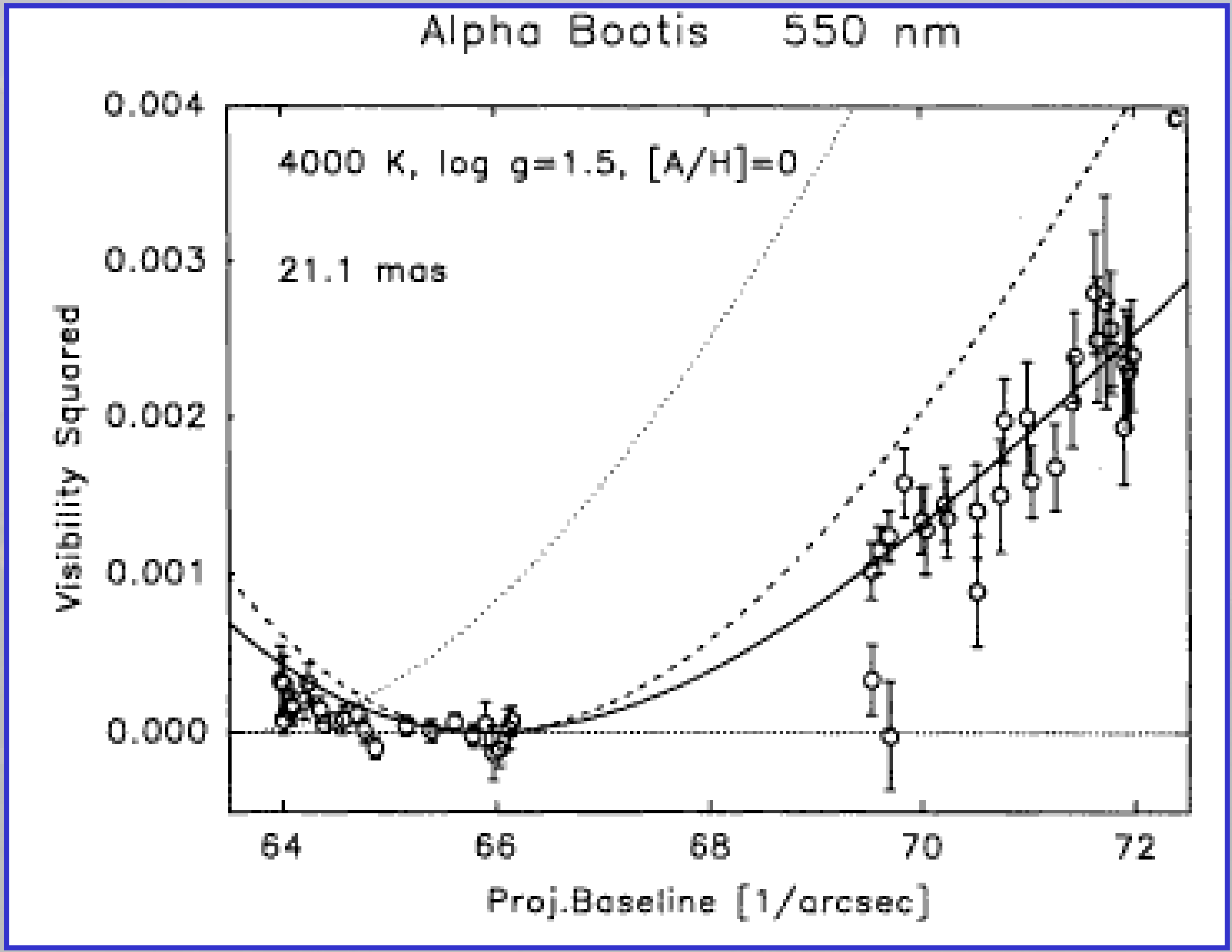
Closing thoughts

- Once we have measured the visibility function of the source, we have to interpret these data. This can take many forms:
  - Small numbers of telescopes:
    - Model-fitting.
  - Moderate numbers of telescopes:
    - Model-fitting.
    - Rudimentary imaging.
  - Large numbers of telescopes:
    - Model-fitting.
    - Model-independent imaging.

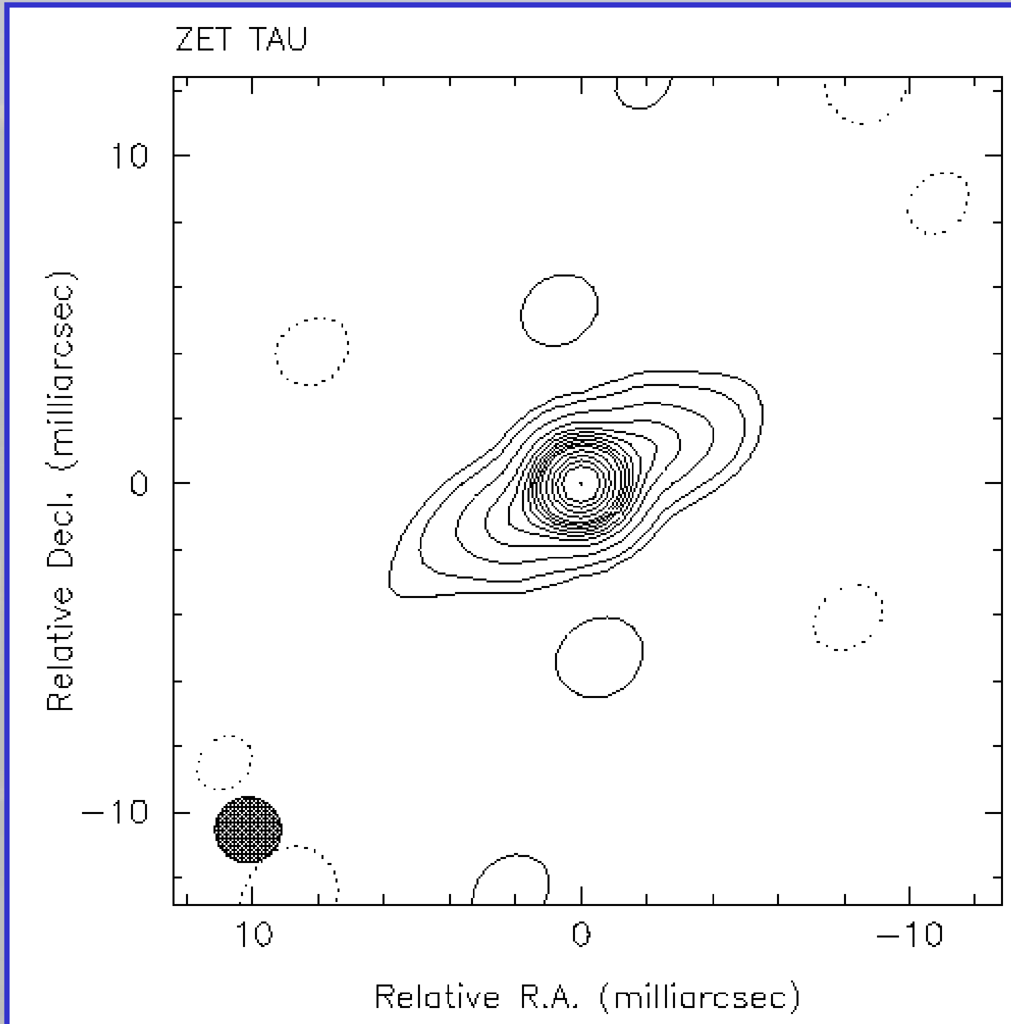


- Don’t forget that making an image is not a requirement for doing good science:

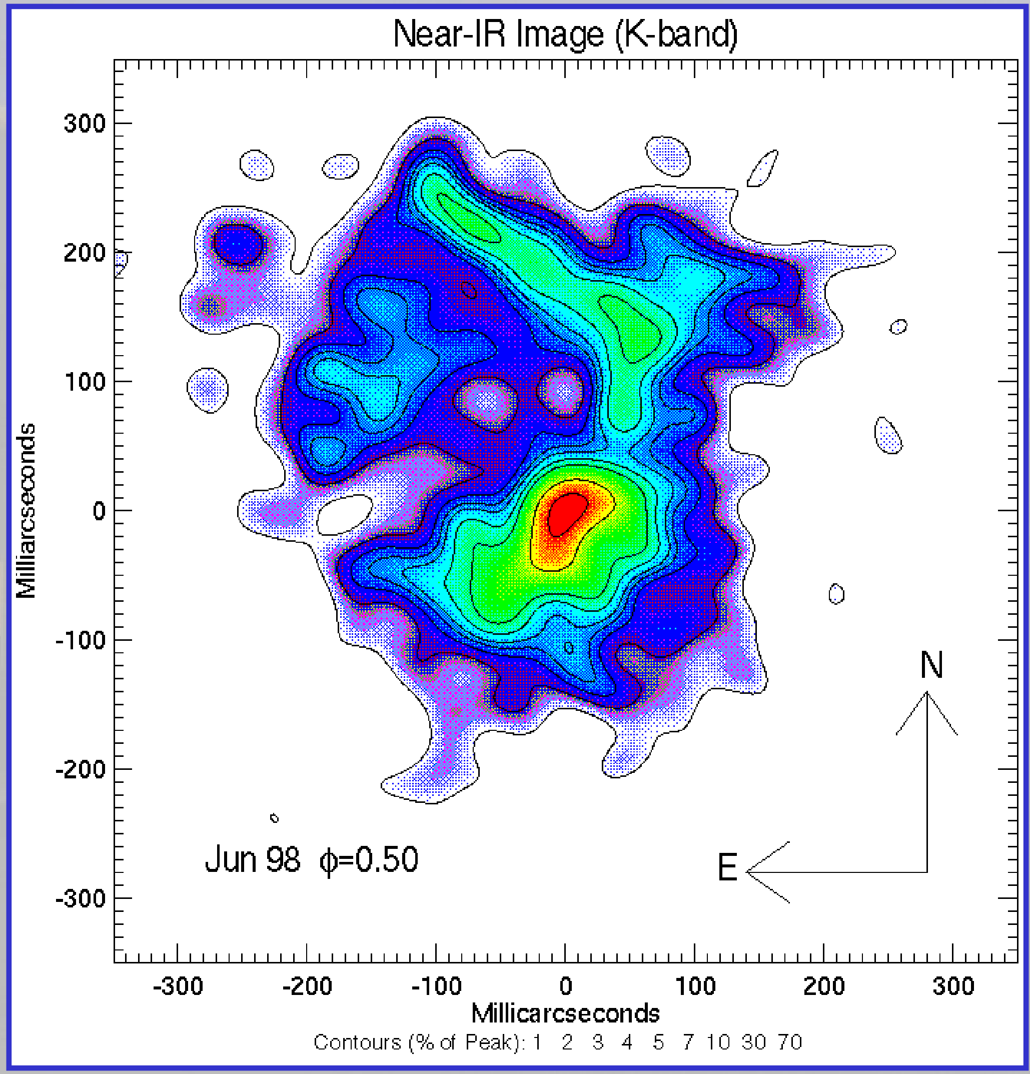
Interferometric science – 2 telescopes



Interferometric science – 5 telescopes



Interferometric science – 21 telescopes



Key lessons to take away

- Alternative methods for describing images:
  - Fourier decomposition, spatial frequencies, physical baselines.

- Interferometric measurements:
  - Interferometer make fringes.
  - The fringe amplitude and phase are what is important.
  - More precisely, these measure the FT of the sky brightness distribution.
  - A measurement with a given interferometer baseline measures a single Fourier component.

- Science with interferometers:
  - Multiple baselines are obligatory for studying a source reliably.
  - Resolved targets produce fringes with low contrast – these are difficult to measure well.
  - Once a number of visibilities have been measured, reliable interpretation can take multiple forms – making an image is only required if the source is complex.