An introduction to modelling interferometric data.

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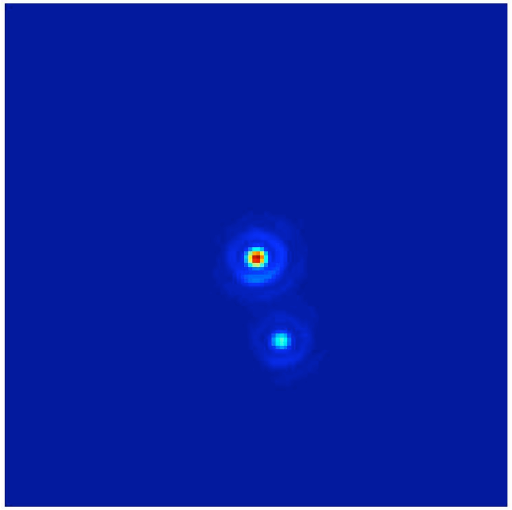
Imaging and visibilities



FT & deconvolve

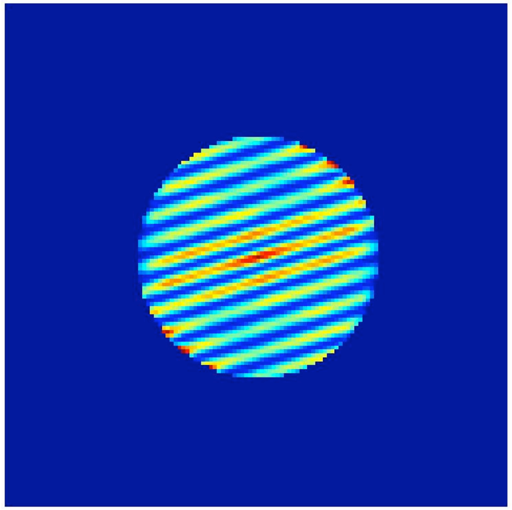


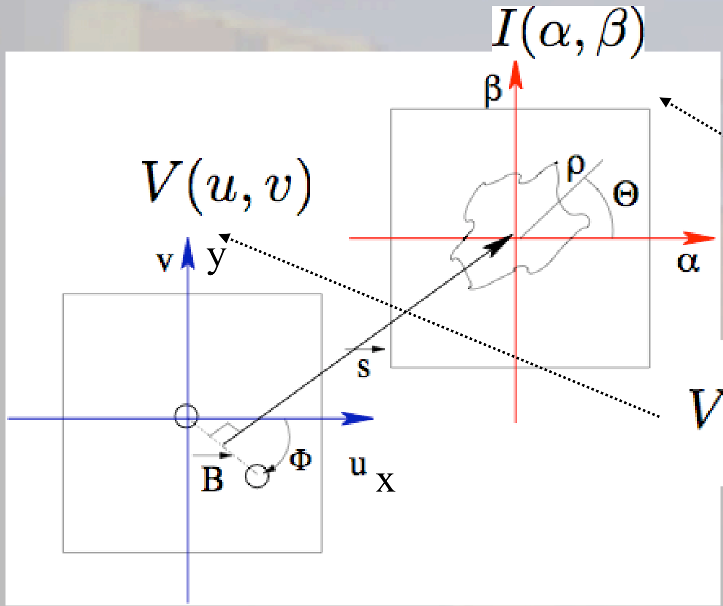
Image : I(x,y)=O*PSF

$|V(u,v)|^2$ & cut-off frequency at D/λ

Example : **resolved binary star** at Canada-France-Hawaii Telescope

Measuring visibilities with an interferometer

Practical application of the Van-Cittert Zernike theorem



$$V(u,v) = \frac{\int\int I(\alpha,\beta)\exp^{-2i\pi(\alpha u+\beta v)}\,d\alpha d\beta}{\int\int I(\alpha,\beta)d\alpha d\beta}$$

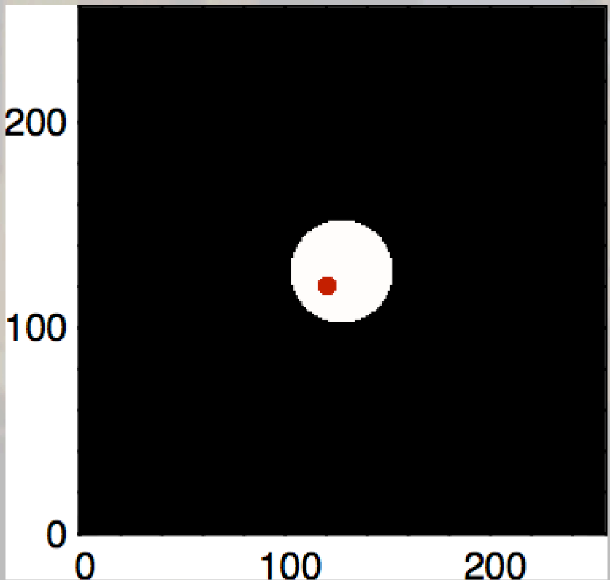
The VCZ theorem links the intensity distribution of an object in the plane of the sky (in the far field) to the complex visibility measured in the array plane.

This relation is a fourier transform. Spatial frequency coordinates $u=Bx/\lambda$, $v=By/\lambda$ where Bx and By stand for projected baselines coordinates on the x,y axes of telescopes

Measuring visibilities with an interferometer

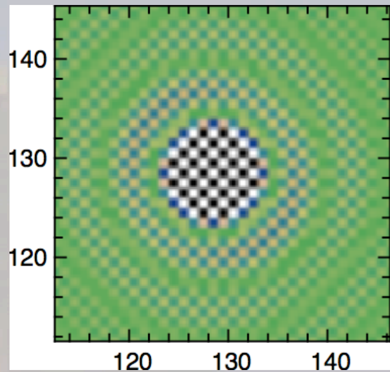
The visibility space

I(x,y)

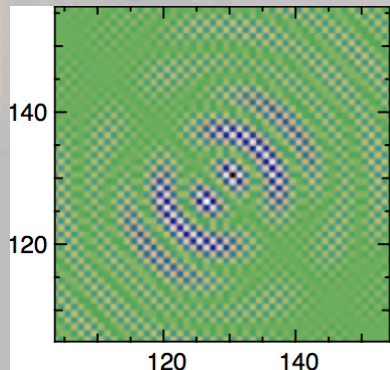


FT

Real(V(u,v))

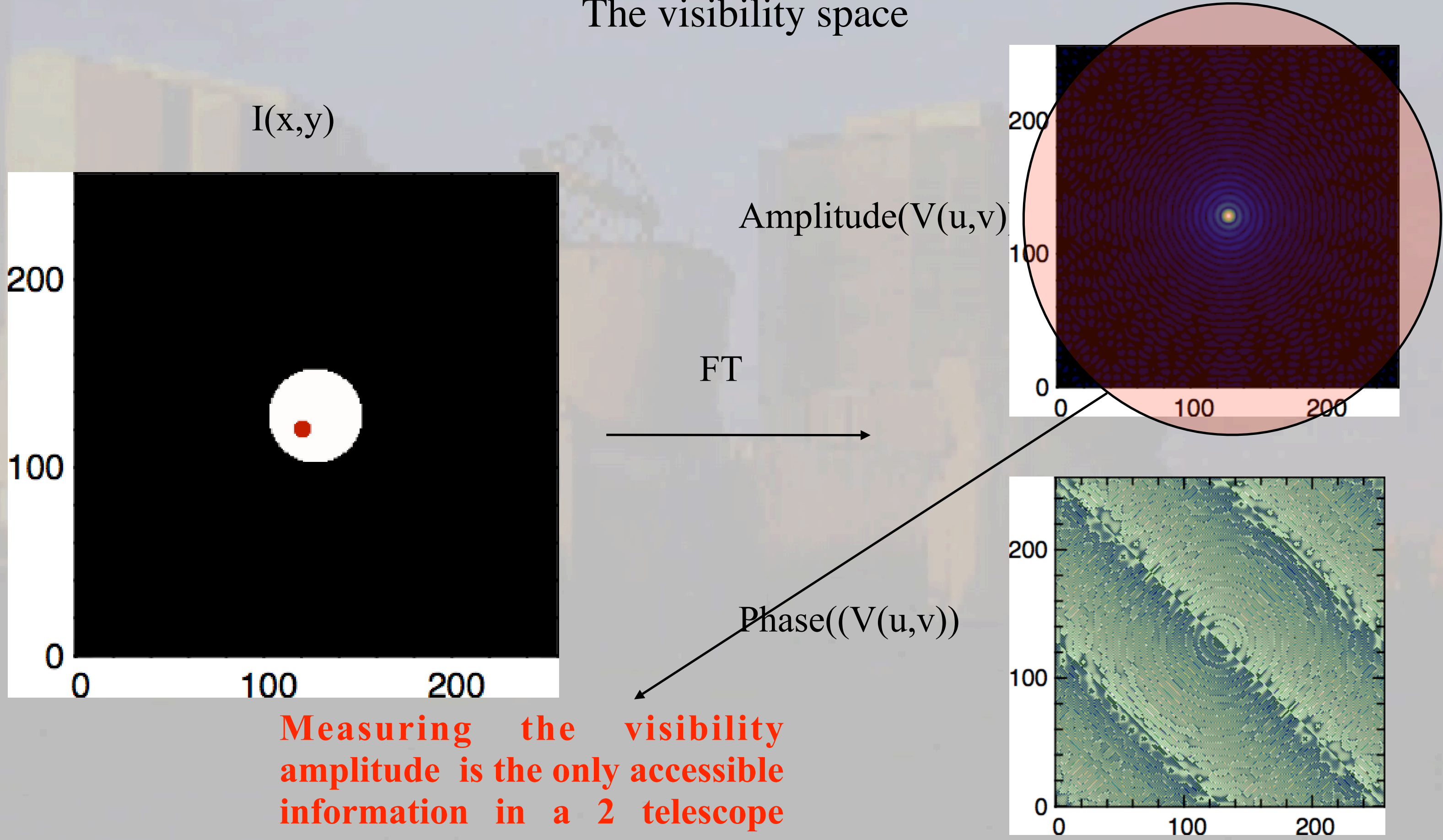


Imaginary(V(u,v))



Measuring visibilities with an interferometer

The visibility space

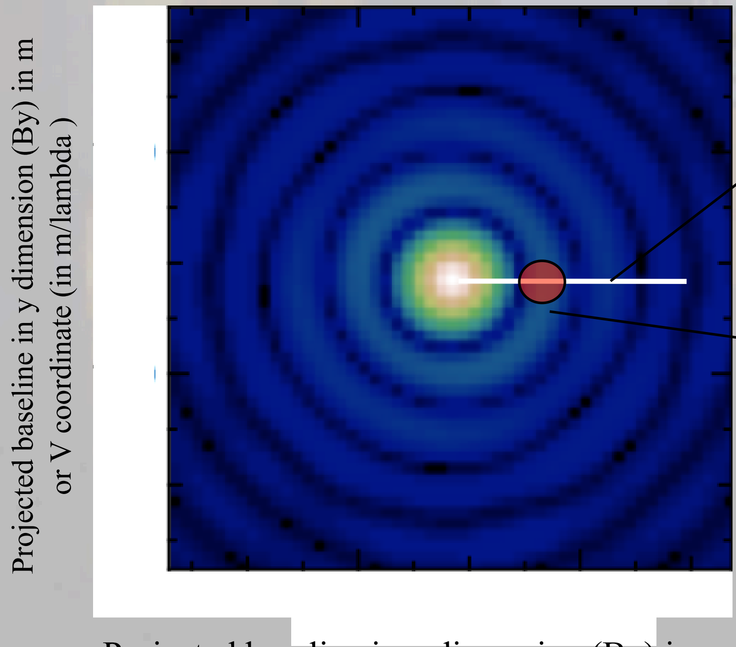


Measuring the visibility amplitude is the only accessible information in a 2 telescope interferometer (unless you have an external reference (star) or internal (continumm/line))

Measuring visibilities with an interferometer

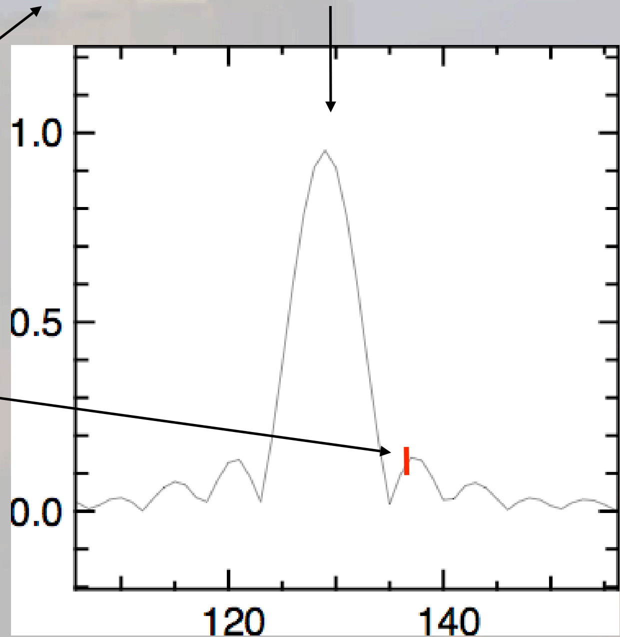
The visibility space

Amplitude V(u,v) (visibility of the fringe)



Projected baseline in x dimension (Bx) in m
or V coordinate (in m/lambda)

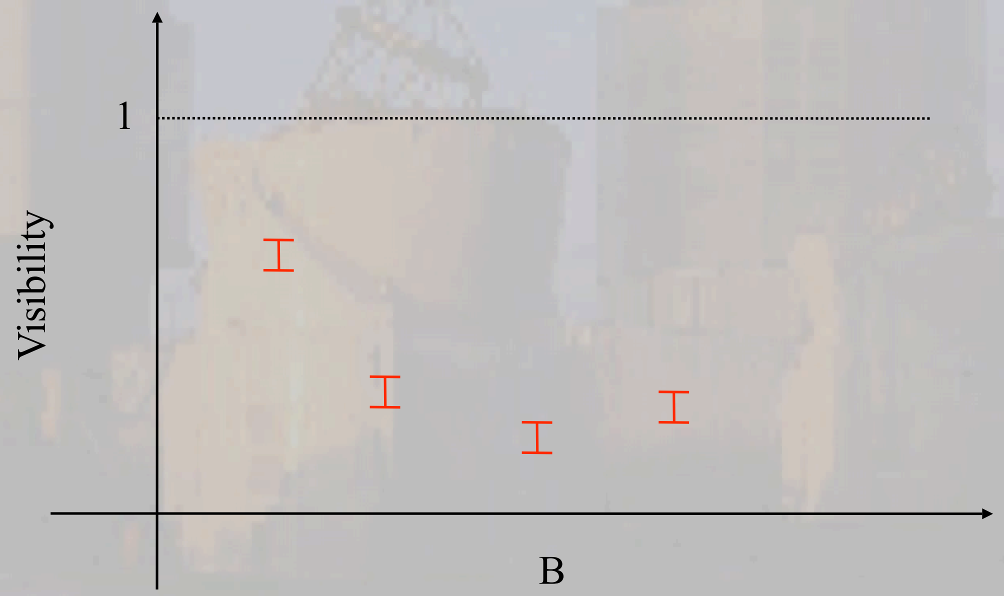
A cut in the visibility curve



In fact with one visibility measurement with one baseline you are only sampling one spatial frequency component of the visibility amplitude.

Measuring visibilities with an interferometer

This talk is about what you could do to with that ...



Simple first step: parametric analysis using basic visibility functions.

Model fitting

This talk adresses the basic issues of interpreting visibilities directly

→ **Realistic in the VLTI AMBER and MIDI contexts**

Model fitting in the visibility domain is a very attractive complement (alternative) to imaging:

- Domain where measurements are made-> errors easier to recognize
- When (u,v) plane sampling is poor
- Might be better to address some issues such as source variability

OUTLINE

1. Modeling visibilities: principles.
2. Some useful basic functions.
3. Practical issues.
4. Conclusion

Ad-hoc modeling

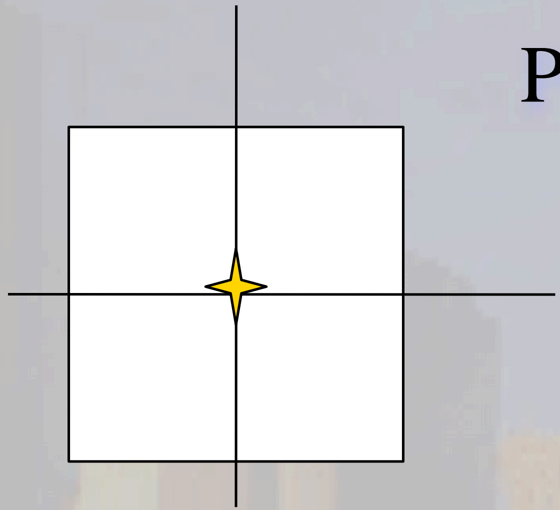
Fourier transform properties
Use of basic intensity distribution functions .

Important first step towards modeling with real physical model

Fourier transform properties:

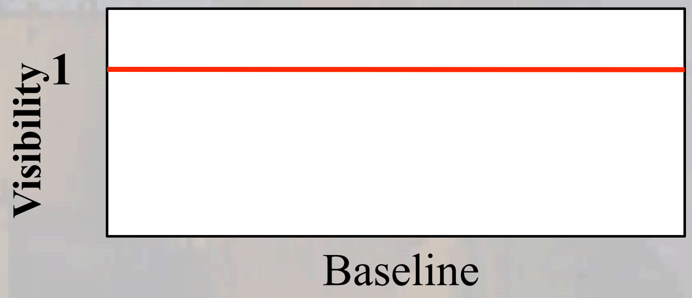
1. **Addition** $$\mathrm{FT}\{f(x,y)+g(x,y)\} = F(u,v)+G(u,v)$$
2. **Convolution** $$\mathrm{FT}\{f(x,y)\times g(x,y)\} = F(u,v).G(u,v)$$
3. **Shift theorem** $$\mathrm{FT}\{f(x-x_0,y-y_0) = F(u,v)\exp[2\pi i(ux_0+vy_0)]$$
4. **Similarity theorem** $$\mathrm{FT}\{f(ax,by)\} = \frac{1}{|ab|}F(u/a,v/a)$$

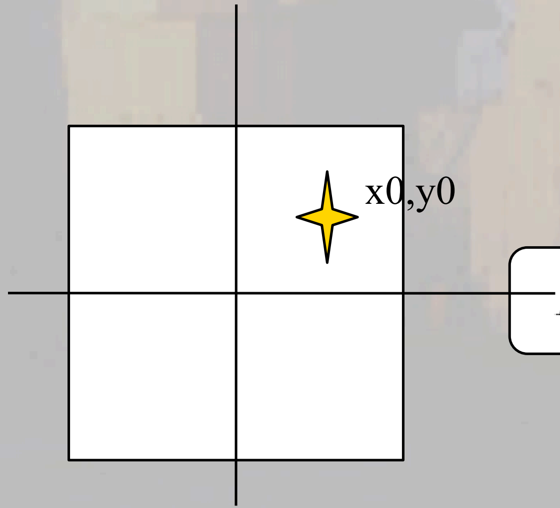
Point source function



Centered source

$$I(x,y) = \delta(x,y) \Rightarrow V(u,v) = 1$$





Off-axis source

$$I(x,y) = \delta(x-x_0)\delta(y-y_0) \Rightarrow V(u,v) = \exp[-2i\pi(x_0u+y_0v)]$$

Amplitude =1 , linear dependence for the phase

Gaussian brightness distribution.

Use: Estimate for angular sizes of envelopes-disks etc

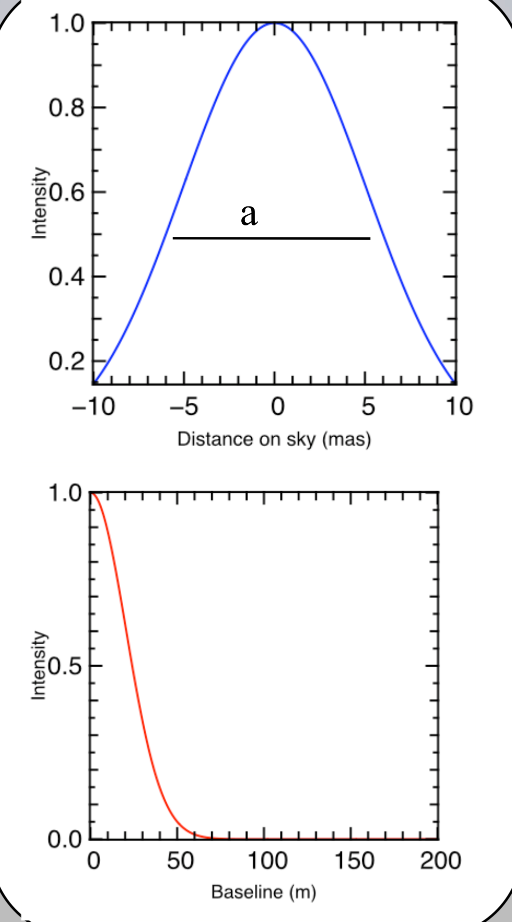
$$I(r) = \frac{I_0}{\sqrt{\pi/4\ln 2a}} \exp(-4\ln 2r^2/a^2)$$

$$V(\rho) = \exp[-(\pi a \rho)^2/(4\ln 2)]$$

Where a: FWHM intensity, Io Peak intensity

with $r = \sqrt{x^2 + y^2}$

with $\rho = \sqrt{u^2 + v^2}$



Uniform disk

Use: aproximation for brightness distribution of photospheric disk.

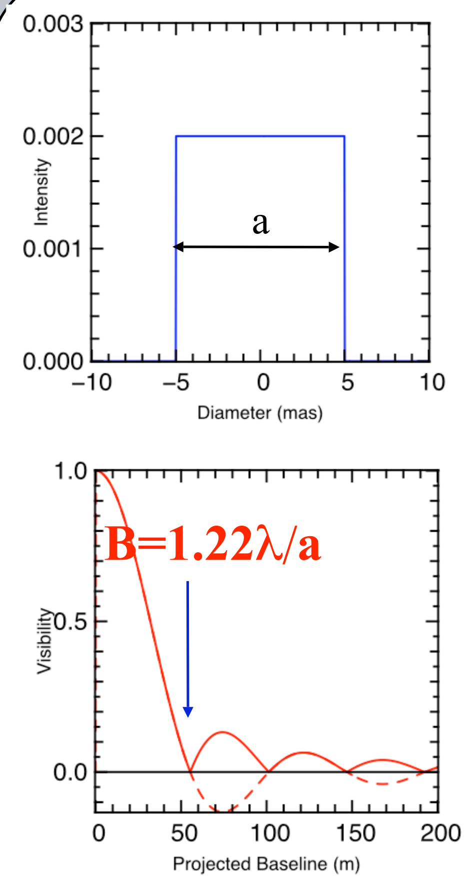
$$\begin{aligned} I(r) &= 4/(\pi a^2), \text{if } r = \sqrt{x^2 + y^2} \leq a/2 \\ I(r) &= 0 \quad \text{otherwise} \end{aligned}$$

⬇

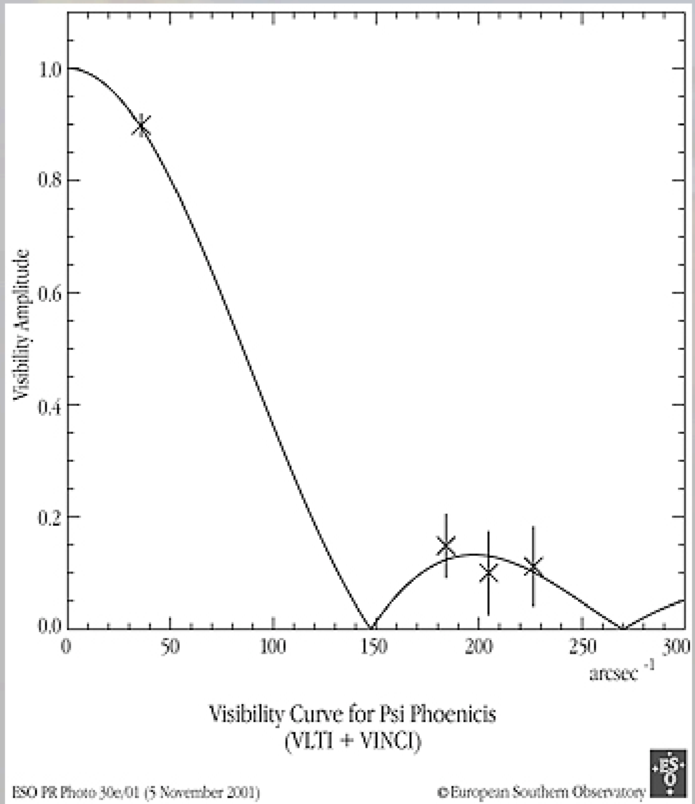
$$F(\rho) = \frac{J_1(\pi a \rho)}{\pi a \rho} \text{with } \rho = \sqrt{u^2 + v^2}$$

a: diameter

Sophistication of the model I= f(r), limb darkening
Cf Young



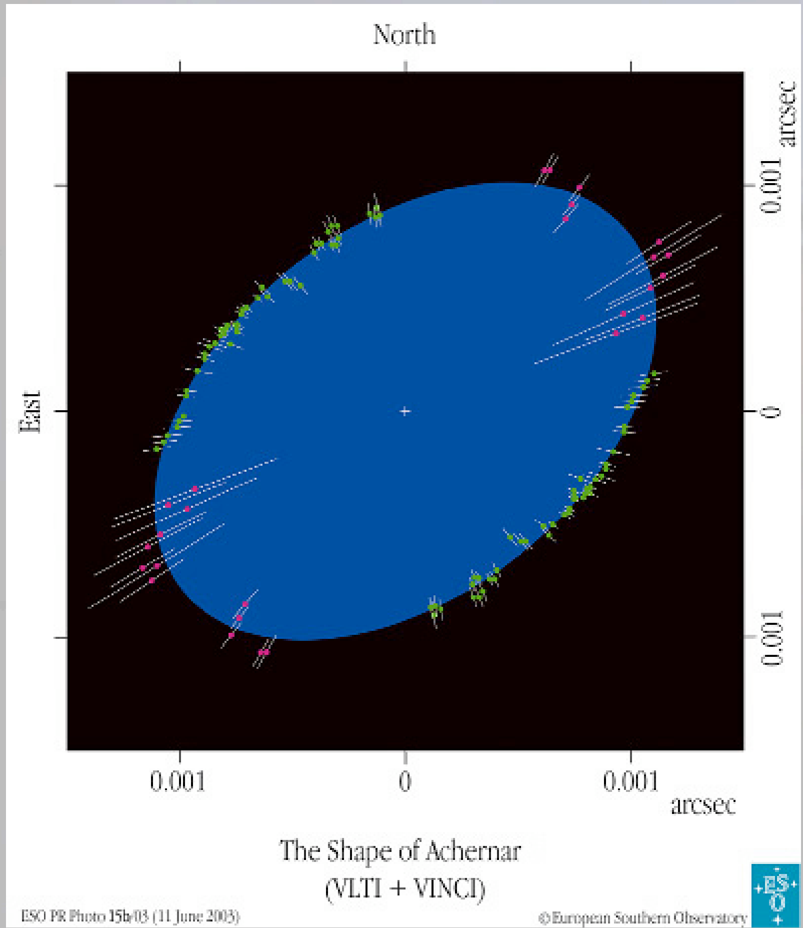
Uniform disk (example 1)



Determination of uniform diameter of Psi Phenicis with VLTI/VINCI

Second lobe points are the most constraining

Uniform disk (example 2)



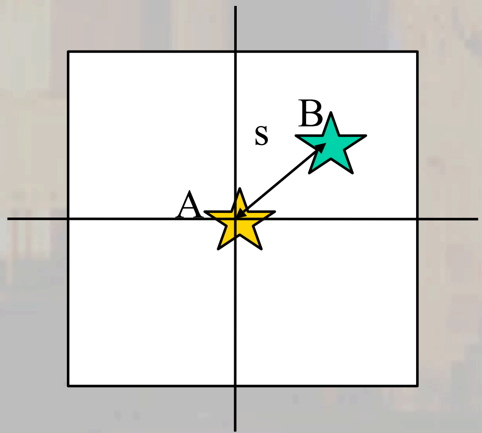
Determination of uniform diameter of Archenar (VLTI/VINCI) at different positions angles shows evidence for flattening due to to fast rotation (Dominiciano da Souza et al A&A 2003).

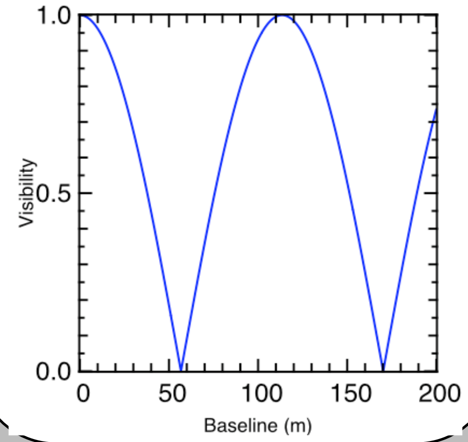
Binary

Unresolved

$$A\delta(x,y) + B\delta(x - sx, y - sy) \text{ with } s = \sqrt{sx^2 + sy^2}$$

$$\begin{aligned} V(u,v) &= \sqrt{\frac{1+r_{ab}^2+2r_{ab}\cos 2\pi \vec{L_b}\vec{s}/\lambda}{1+r_{ab}^2}} \\ \text{with } r_{ab} &= A/B \\ \text{with } \vec{L_b} &= \text{Baseline vector} \end{aligned}$$



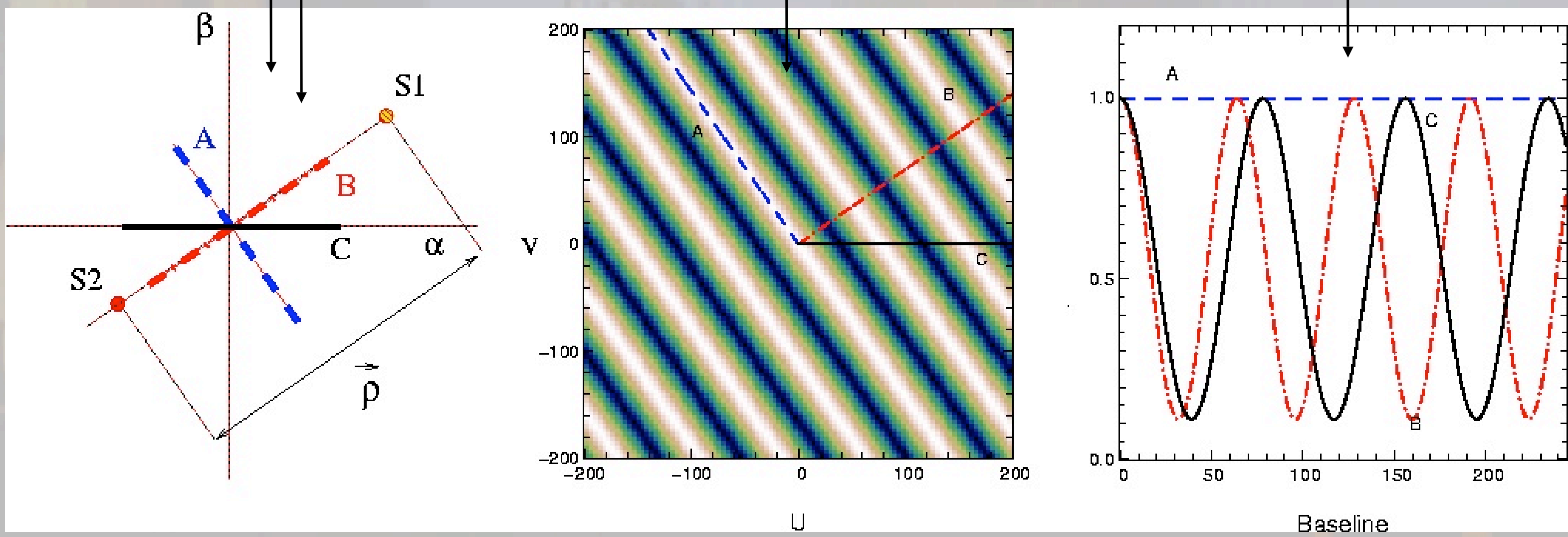


Binary (unresolved)

Projection of baseline in the plane of sky

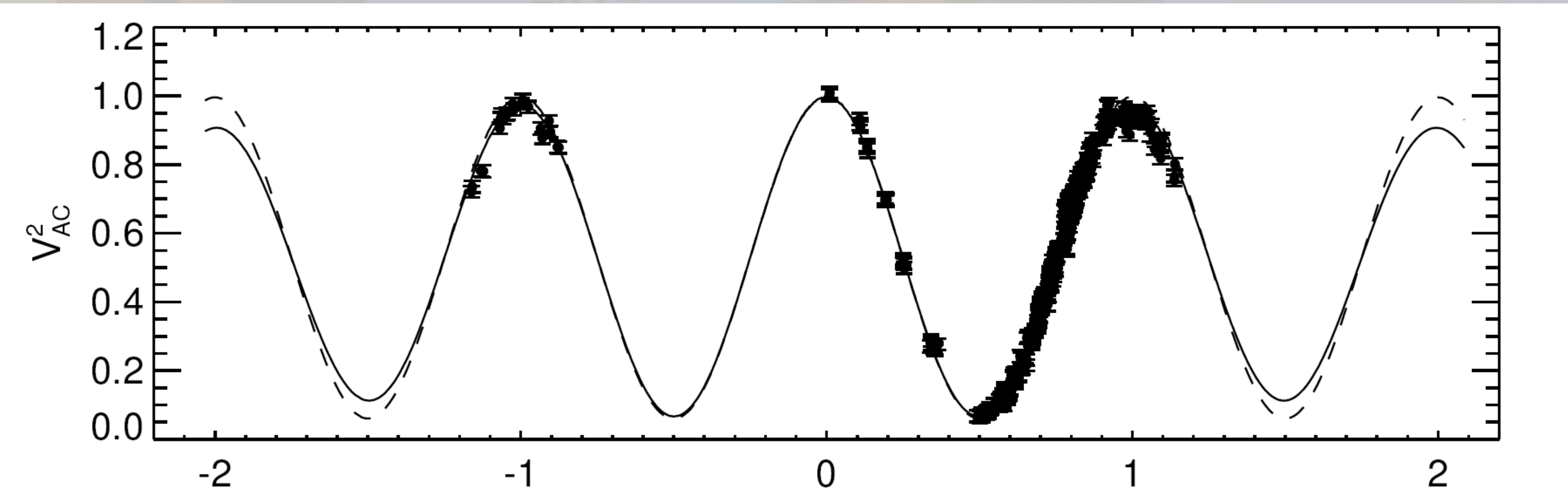
The visibility amplitude squared in (uv) plane

Squared visibility curves for three baselines as a function of baseline length



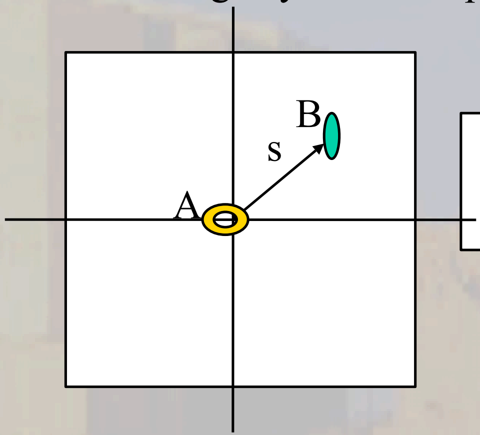
Binary (unresolved)

Binary squared visibility curve as function of hour angle (Ming et al in preparation, observed at IOTA with IONIC)



Resolved bi-structure

Use: Describing any multicomponent structure.



$$V^2(u,v) = \frac{r_{ab}^2 * V_a^2 + V_b^2 + 2r_{ab}|V_a||V_b|\cos(2\pi \vec{L_b}\vec{s}/\lambda)}{(1+r_{ab}^2)}$$

Where Va and Vb are respectively the visibility of object A and B at baseline (u,v)

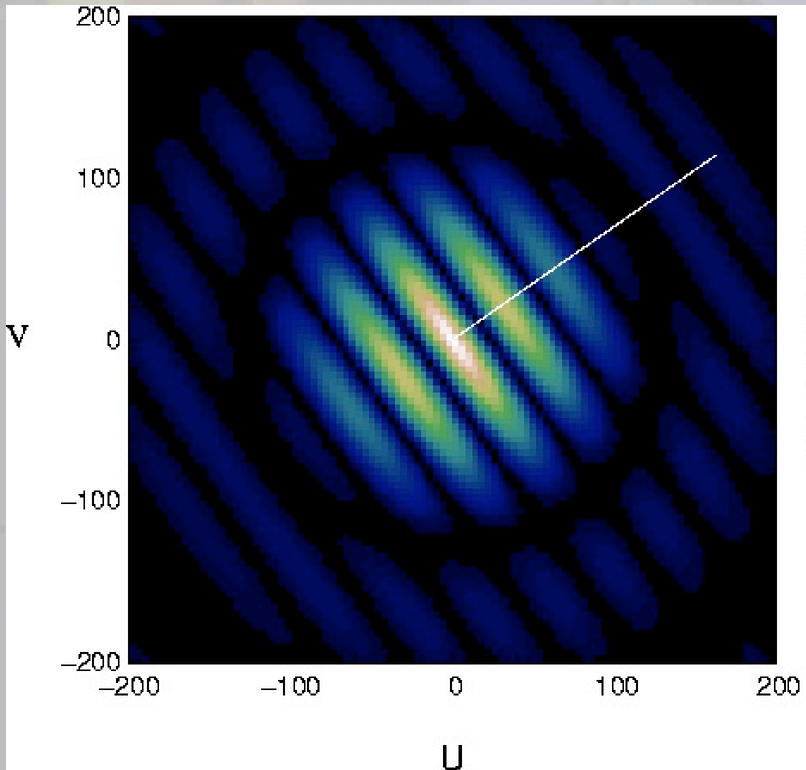
Generalization:

$$V(u,v) = \frac{\sum_{i=1}^{k} F_i V(u_i, v_i)}{\sum_{i=1}^{k} F_i}$$

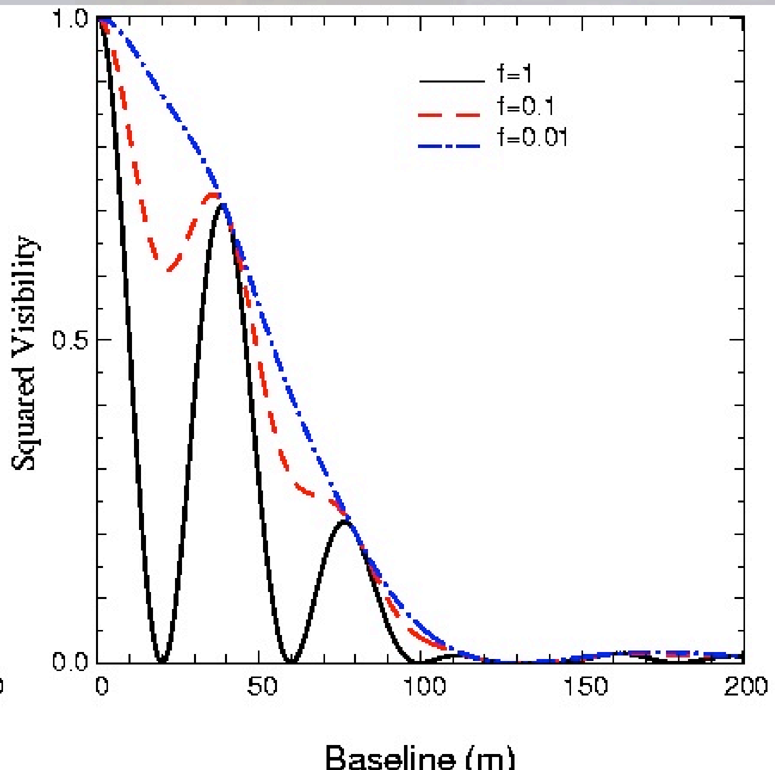
Resolved bi-structure (example)

Binary made of two resolved photometric disks: d=3mas, PA: 35deg

Squared visibility in (u,v) plane

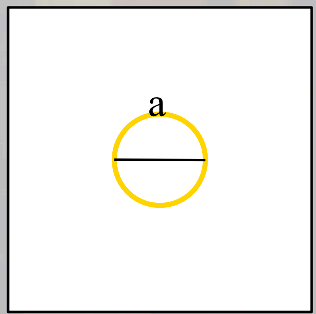


Squared visibility as a function of baseline for different flux ratios



Unresolved ring & Ellipse

Use: allowing to describe a more complex centro-symmetric structure and compute its visibility
e.g: an accretion disk made of a finite sum of annulii with different effective temperatures

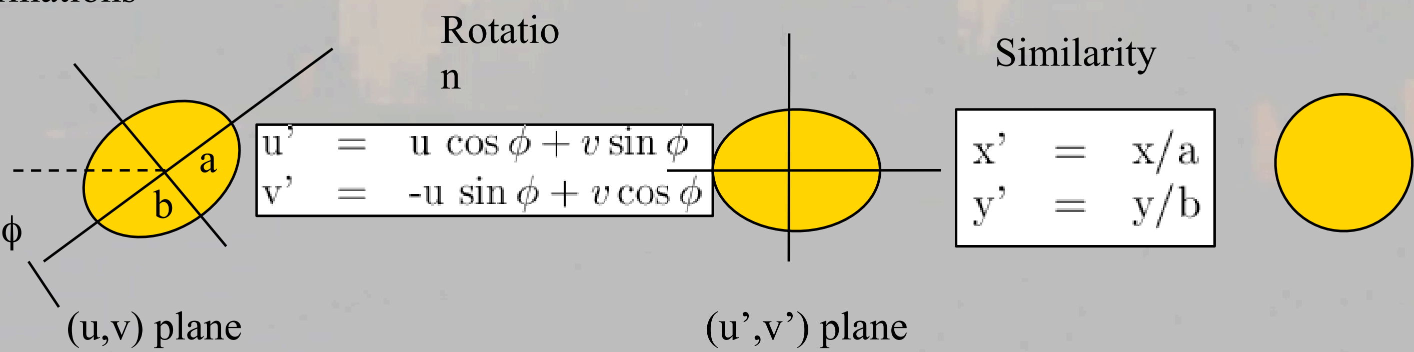


$$I(r) = 1/(\pi a)\delta(r - a/2)$$

↓

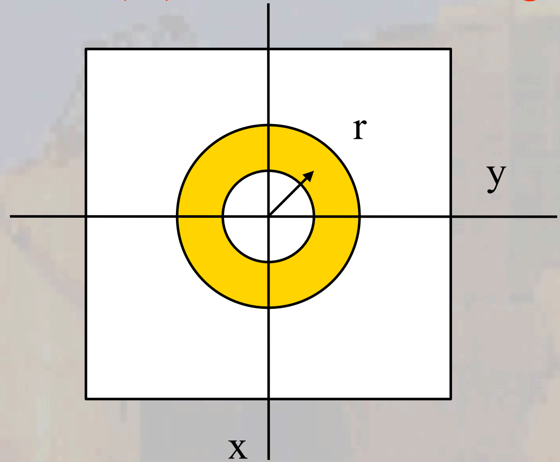
$$V(\rho) = J_0(\pi a \rho)$$

Transformations



Circularly symmetric component

Circularly symmetric component I (r) centered at the origin of the (x,y) coordinate system.



with $r = \sqrt{x^2 + y^2}$

The relationship between brightness distribution and visibility is a Hankel function

$$V(\rho) = 2\pi \int_0^\infty I(r) J_0(2\pi r \rho) r dr$$

with $\rho = \sqrt{u^2 + v^2}$

The modeling process.

Model

Parameters α, β, γ

$$I(x,y)$$

$$V(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x,y)\exp(-2\pi i(xu+yv))dxdy$$

This talk

- Perfect (u,v) sampling
- Monochromatic

Instrument

Sparse sampling $S(u,v)$

Error $\epsilon(u,v)$

$$\{..., V(u_i,v_i), ...\} i = 1..n$$

Data & Error

$$\{..., V'(u_k,v_k), ...\} k = 1..n$$

A minimization process:

$$\chi^2 = f(V(u_i,v_i), V'(u_k,v_k), \epsilon'(u,v))$$

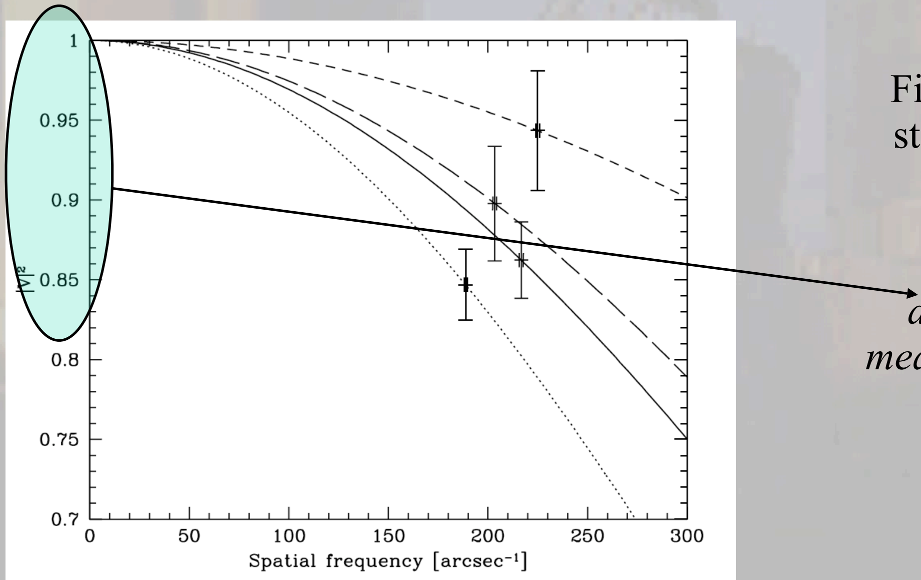
Best

$$\alpha, \beta, \gamma$$

Pushing the limits

Small diameter estimation

Model fitting is also a deconvolution process: sizes estimates or positional uncertainties can smaller than the canonical resolution (the “beam” size”): **super resolution**



First measurements of M dwarfs stars diameters (Segransan et al, 2003) .

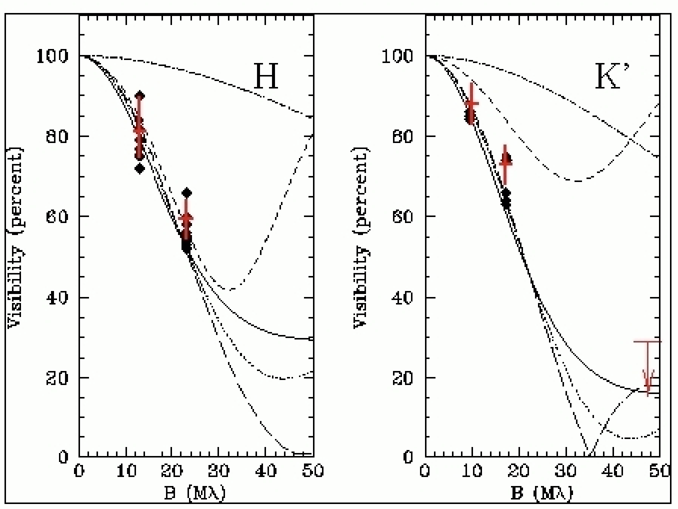
Look how small visibilities

are. No need for zero visibility measurements to retrieve diameters

Degeneracy at small baselines

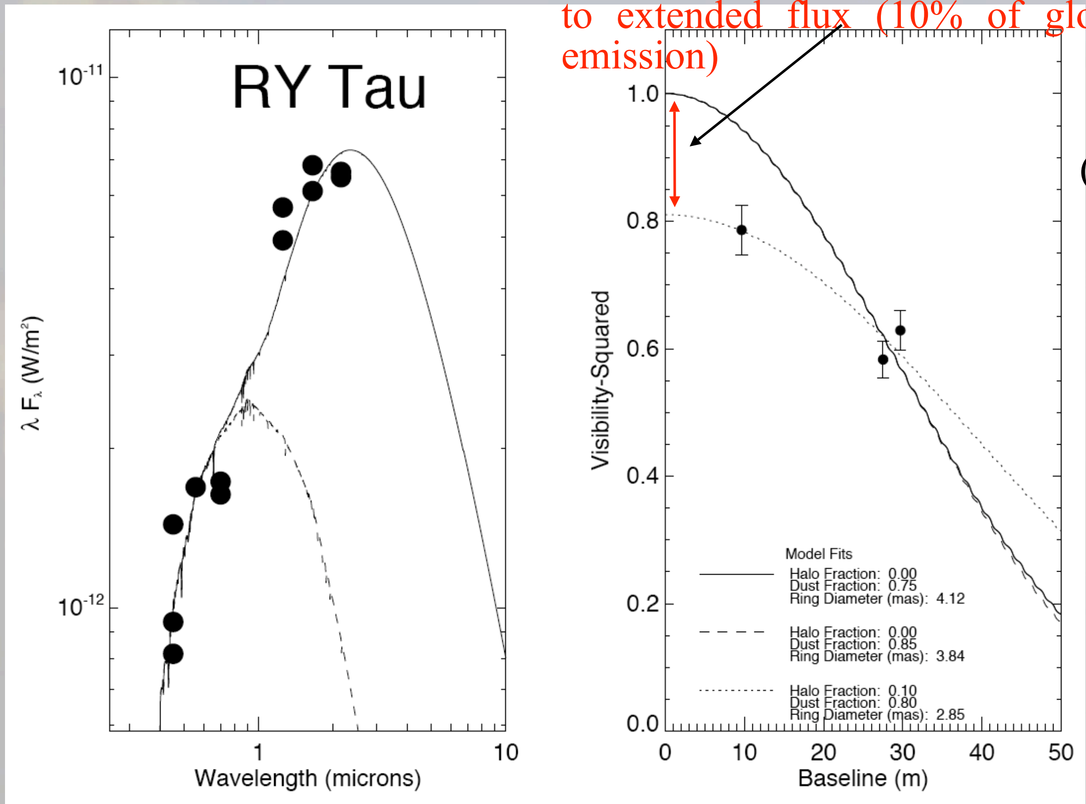
If the object is barely resolved the exact brightness distribution is not crucial - the dependance is quadratic for all the basic functions: visibility accuracy is mandatory

Modelisation of accretion disk emission around young star AB Aur. Both gaussian, uniform disk and ring fit visibilities equally well



Detecting extended emission (Monnier et al ApJ 2006)

Visibility drops rapidly: attributed to extended flux (10% of global emission)



Here a simple model of extented (totally resolved) dust emission + gaussian brings the best fit

Pushing the limits: debris disks by interferometry

(Absil et al 2006)

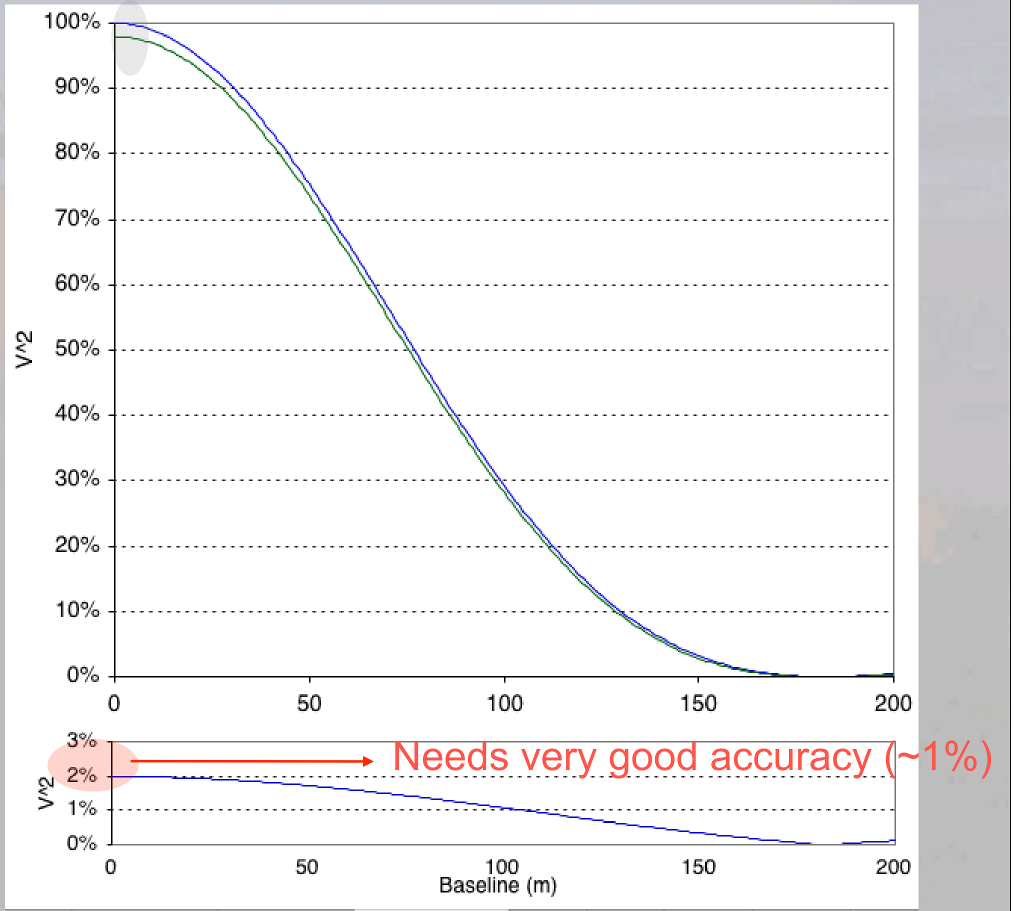
Larger than angular resolution (λ/b) → contributes as an incoherent flux

Induces a visibility deficit at all baselines

Best detected at short baselines

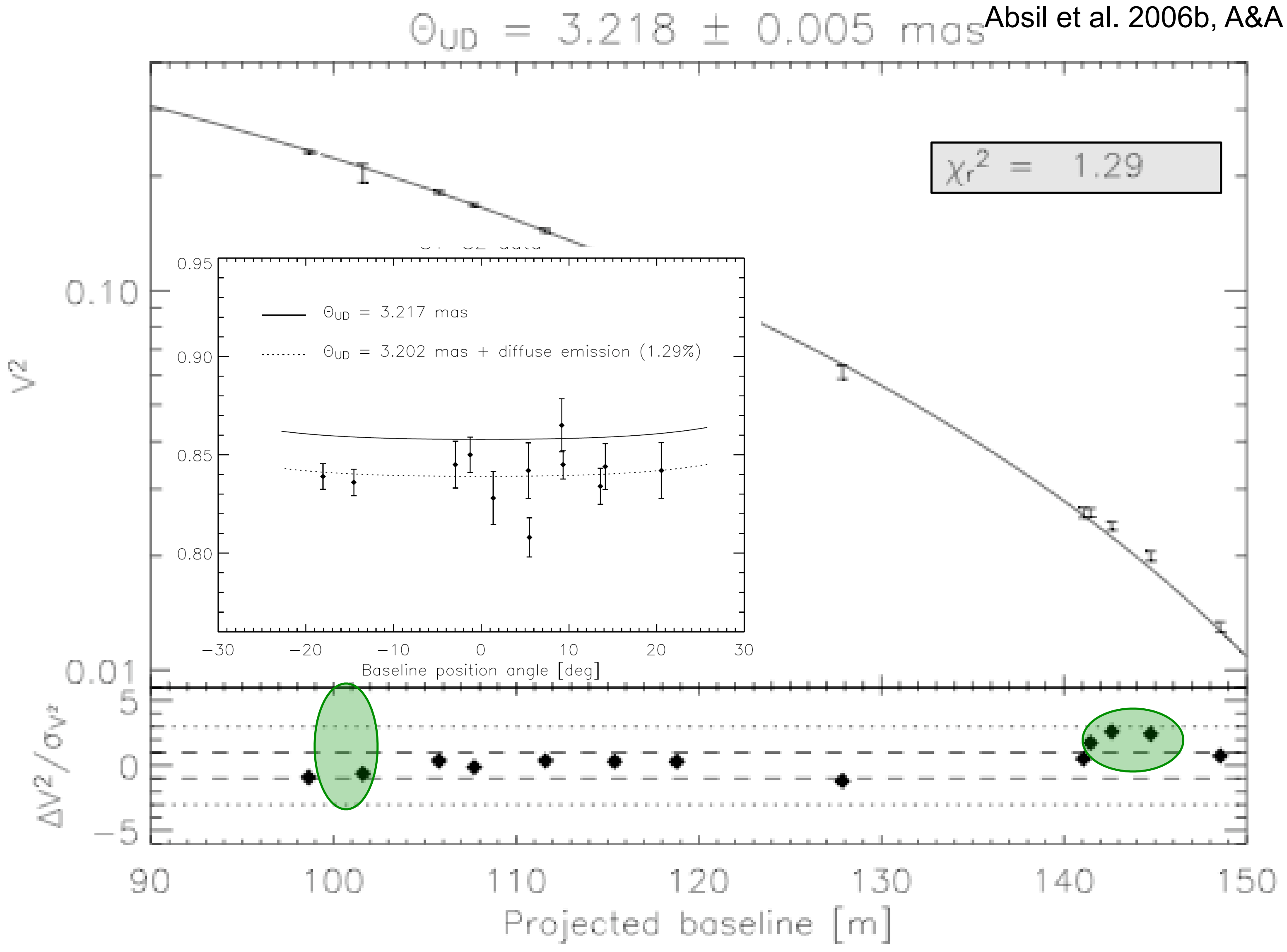
Flux ratio

$$V^2 \approx (1-2f)\left(\frac{2J_1(\pi b\theta/\lambda)}{\pi b\theta/\lambda}\right)^2$$



Pushing the limits: debris disks by interferometry

Absil et al. 2006b, A&A



Conclusion

- ✓ Visibility study without imaging can be efficient.
- ✓ The (u,v) coverage strategy is different from imaging. Limited allocated time means limited (u,v) points.
- ✓ Use basic models in order to prepare your observation and determine what is the more constraining configuration.
- ✓ Visibility space is the natural place to understand the errors of the final result.
- ✓ Always start by describing your observations in terms of basic functions. It brings quantitative information useful for further more detailed computations.