Biases and *Systematics*

EuroSummer School

Observation and data reduction with the Very Large Telescope Interferometer

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Goal of this lecture

The goal of this lecture is to present some difficulties with the calibration of interferometric data and with the use of interferometric data.

It is also to show that methods exist to overcome these issues.

The list of biases and systematics is not exhaustive.

Outline

1. Definitions
2. Sources of biases
3. Miscalibrations and biases
4. Model fitting and biases

Outline

- ✓ Definitions

2. Sources of biases

3. Miscalibrations and biases

4. Model fitting and biases

Definitions

Biased estimator

- estimator whose average is different from the expected value
- example: modulus of the visibility estimator:

$$\langle |V+n| \rangle \neq |V|$$

In a more general sense, any source of misinterpretation of the data

Systematic error

- error (realization of) which is common to different realizations of an estimator or to different estimators
- example: $X_1 \ldots X_n$ are random variables affected by the noises $N_1 \ldots N_n$ and S

$$\tilde{X}_i = X_i + N_i + S$$

S is a systematic noise or error. It does not average down to zero in $\frac{1}{n}\sum_i \tilde{X}_i$

Definitions

Different types of biases

- those common to the source and the calibrator which disappear after calibration
- those with different magnitudes on the source and the calibrator
- those that arise from the use of a wrong model
- certainly some others …

Outline

- ✓ Definitions
- ✓ Sources of biases

3. Miscalibrations and biases
4. Model fitting and biases

Sources of visibillity biases (some)

polarization

- loss of coherence
- differential polarization changes (reflexion angles)

dispersion of refraction of index (also called longitudinal dispersion or dispersion)

- loss of coherence
- differential dispersion changes (aerial pathlength in the visible)

vibrations

- fringes get blurred

atmospheric turbulence

- loss of coherent energy over each telescope pupil
- differential piston

Sources of visibillity biases (some)

calibrator visibility

- misknowledge of the source geometry (uniform, limb-darkened disk)
- misknowledge of the source size

Instrument field of view

- single-mode instrument with object lightwave projected on the lobe of the waveguide
- limited *interferometric* field of view
- problem for extended objects

Sources of phase biases (some)

dispersion of refraction of index (also called longitudinal dispersion or dispersion)

- bias of the differential phase
- case of non-evacuated delay lines in the blue and in the mid-IR (water vapor, …
- case of long lengths of single-mode fibers
- bias of the closure phase in wide-band (?)

closure phase

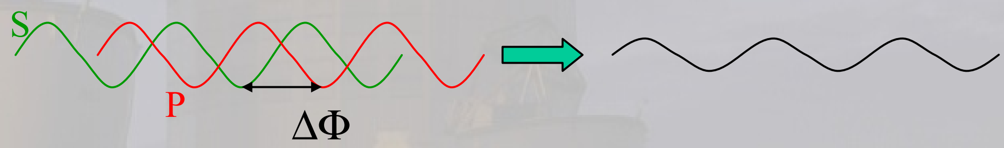
- time delays in the measurement of the central fringe of each baseline
- fluctuations of delay in non-common paths after beam splitting
- extended objects with limited field of view (case of a binary system)

Polarization

Differential polarization effects

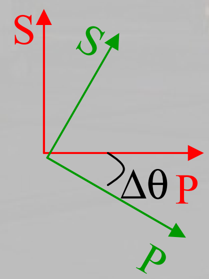
Differential birefringence: phase delay between the two polarization axes S and P

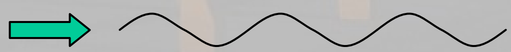
$$C = V \times \left| \cos\left(\frac{\Delta\Phi}{2}\right) \right|$$



Differential rotation: differential rotation of polarization planes between the two interferometer arms.

$$C = V \times \frac{2\left|\cos(\Delta\Theta)\right|}{1+\cos^2(\Delta\Theta)}$$





(polarizer on S or P)

Sources of polarization issues

- <u>Sources:</u>
  - Optical coatings -> phase shifts between S and P
  - Reflections -> polarization rotations

- <u>Solutions:</u>
  - Matched coatings
  - Same sequences of reflections

These traditional solutions are enough to make sure the contrast is large.

Calibration however remains necessary because the fringe contrast on a point source is never 100%.

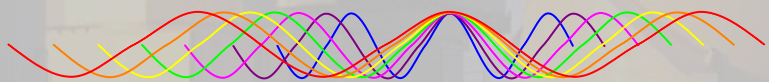
Phase shifts and rotations depend upon the source direction: calibrators need to be chosen near the science target

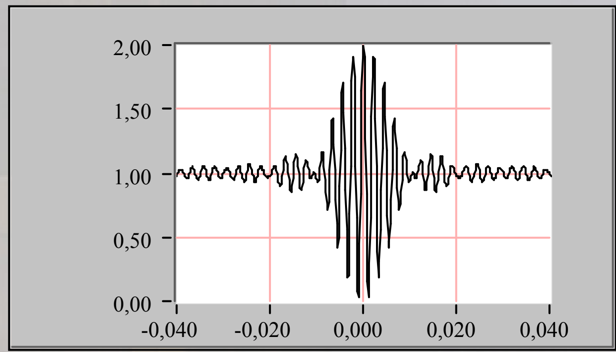
Dispersion

Differential chromatic dispersion

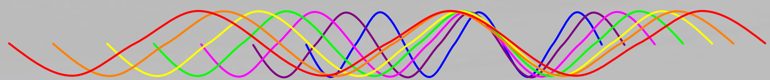
The zero optical path difference may be wavelength dependent: $\delta(\lambda) = n_1(\lambda)L_1 - n_2(\lambda)L_2$

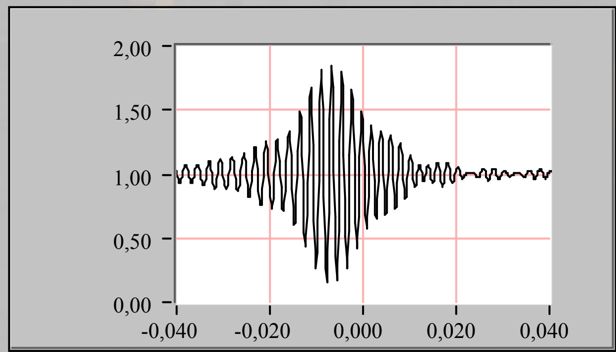
No differential dispersion



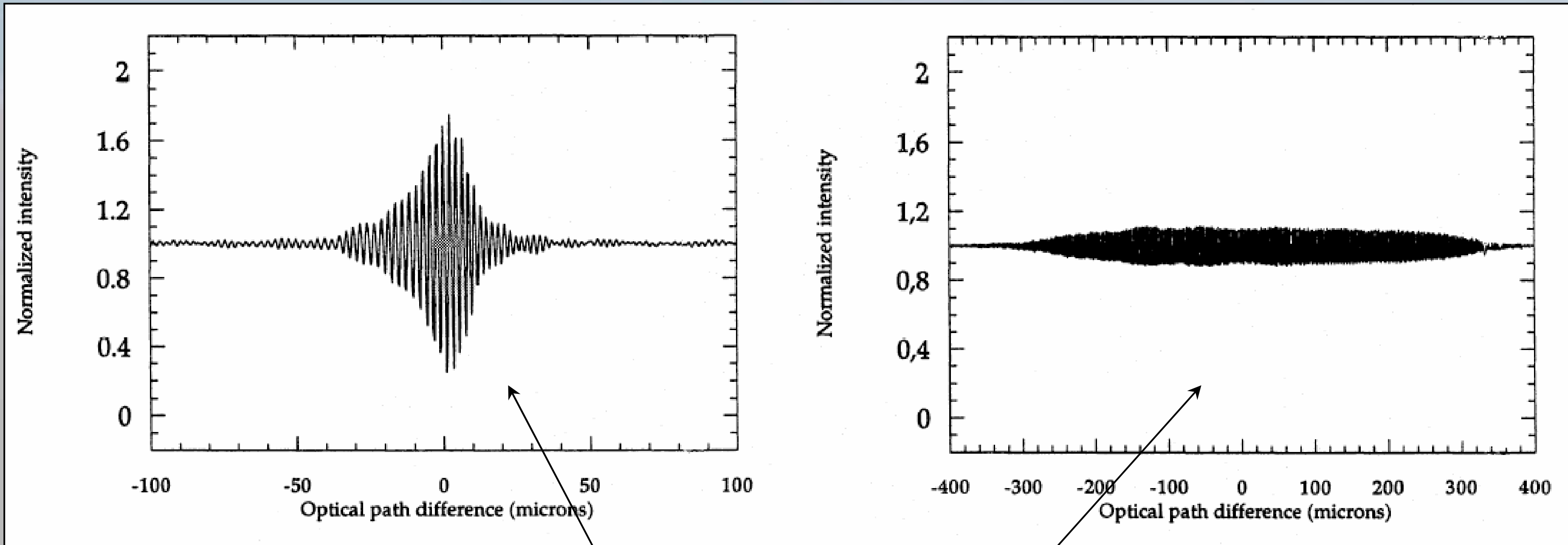


With differential dispesion

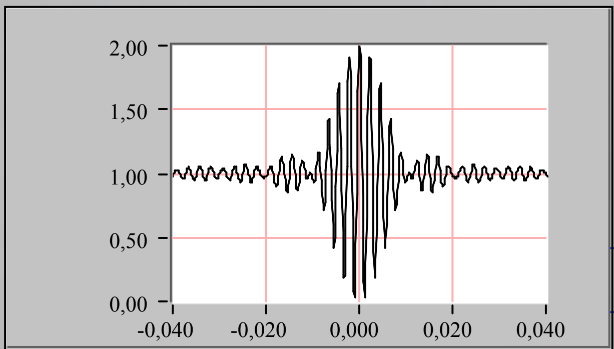




Examples of dispersion in long single-mode fibers



Coudé du Foresto, Perrin & Boccas (1995)



Coupler

77 m fiber

Dispersion and phase

Expansion of the phase to the third order:

$$\phi(\sigma) = a_0 + a_1(\sigma - \sigma_0) + a_2(\sigma - \sigma_0)^2 + a_3(\sigma - \sigma_0)^3 + \cdots$$

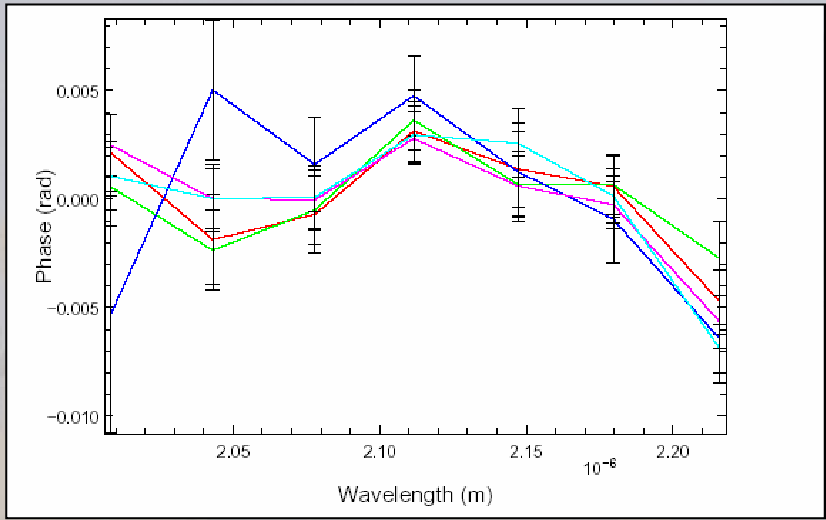
interferogram opd shift

second order dispersion main contributor

Differential phase:

$$\phi(\sigma) - \phi(\sigma_0) = a_1(\sigma - \sigma_0) + a_2(\sigma - \sigma_0)^2 + a_3(\sigma - \sigma_0)^3 + \cdots$$

Differential phase measurement with AMBER



Amplitude of the effect = 0.01 rad in this particular case

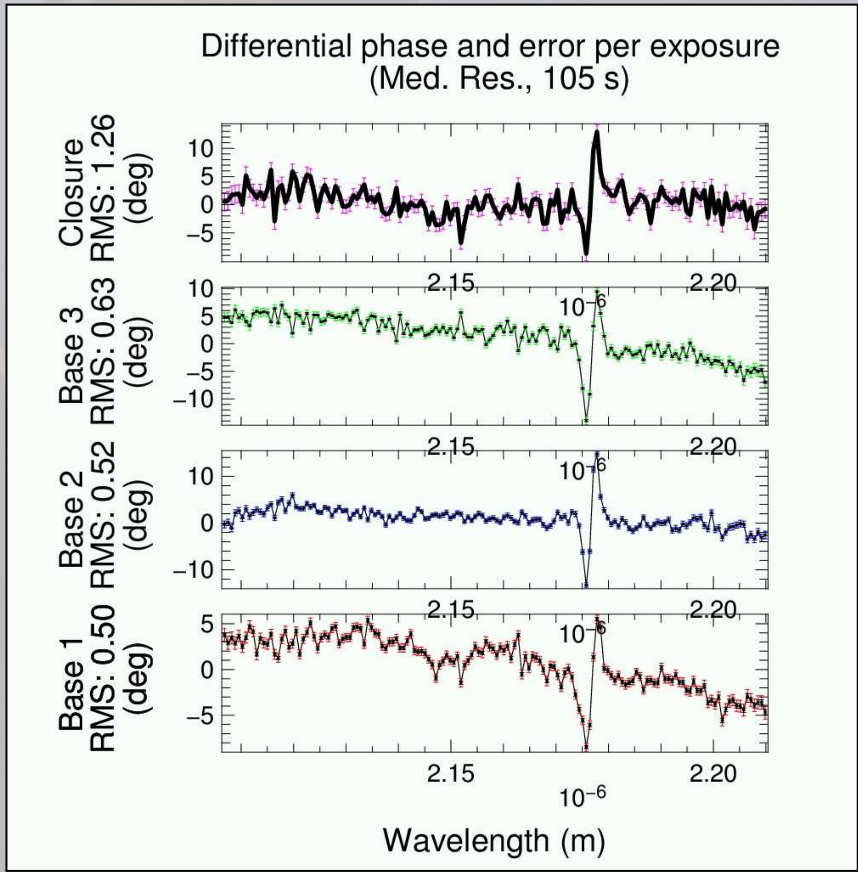
But may vary with source position and therefore with time

Potential problem for exoplanet search without evacuated delay lines

Closure phase measurement with AMBER

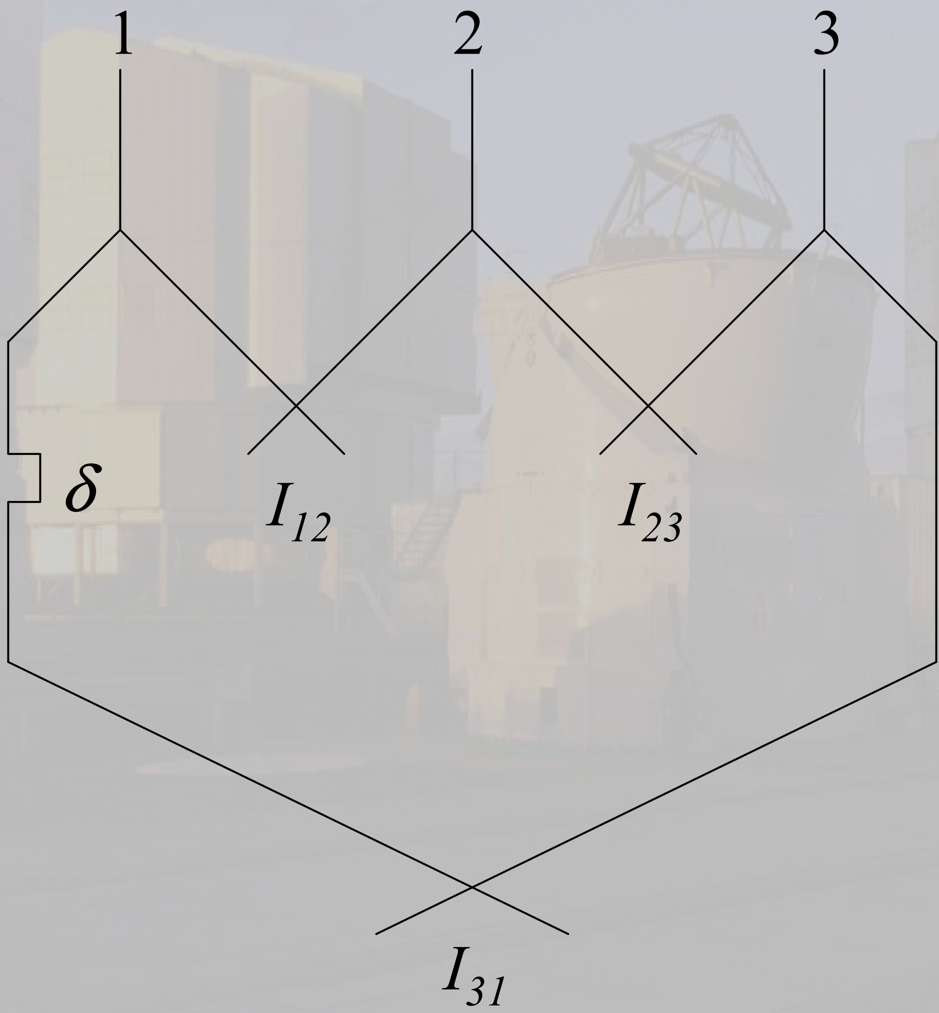
Closure phase: residual = dispersion or single-mode fiber issue ?

Differential phase: larger error (0.15 rad)



Time delays and closure phase

Beam combiner time delays



$$\varphi^{obs}{}_{12} = \varphi_{12} + \varepsilon_1 - \varepsilon_2$$
$$\varphi^{obs}{}_{23} = \varphi_{23} + \varepsilon_2 - \varepsilon_3$$
$$\varphi^{obs}{}_{31} = \varphi_{31} + \varepsilon_3 - \varepsilon_1 - 2\pi\delta\sigma$$

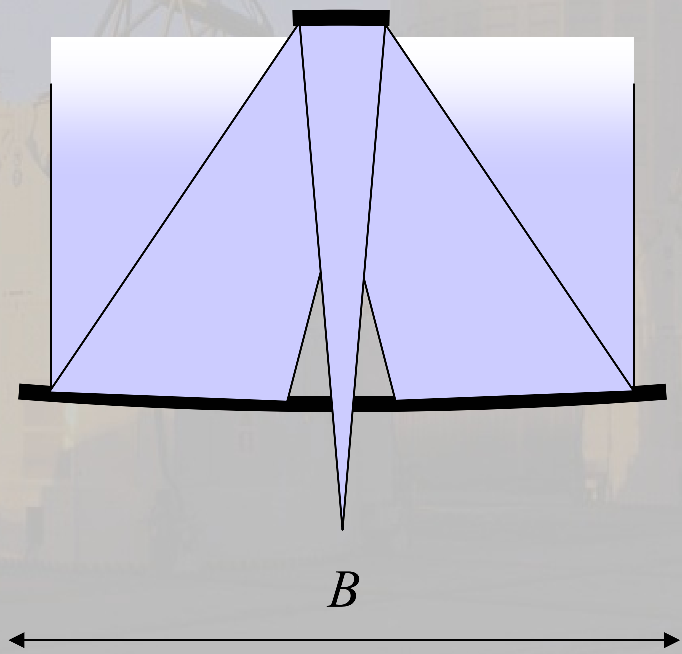
$$\Sigma\, \varphi^{obs}{}_{ij} = \Sigma \varphi_{ij} - 2\pi\delta\sigma$$

The asymmetry of the beam combiner introduces a bias in the closure phase

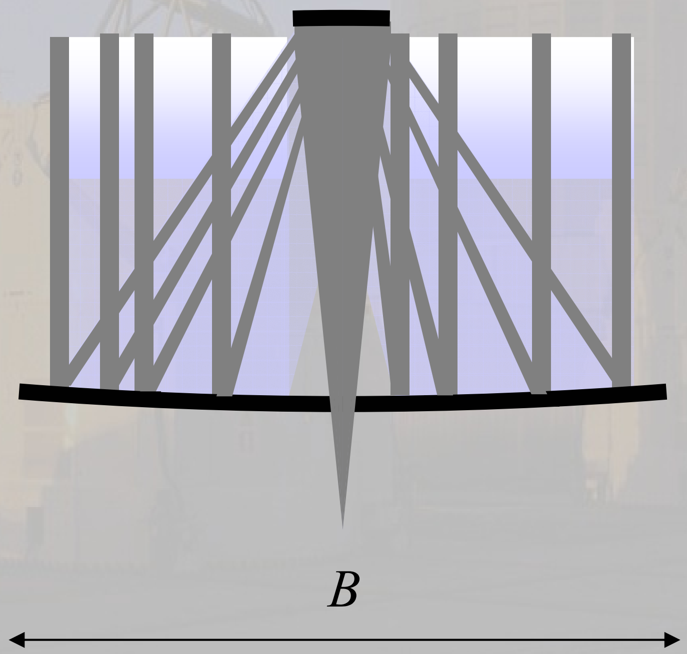
An unstable beam combiner will introduce a bias difficult to calibrate

The field of view issue

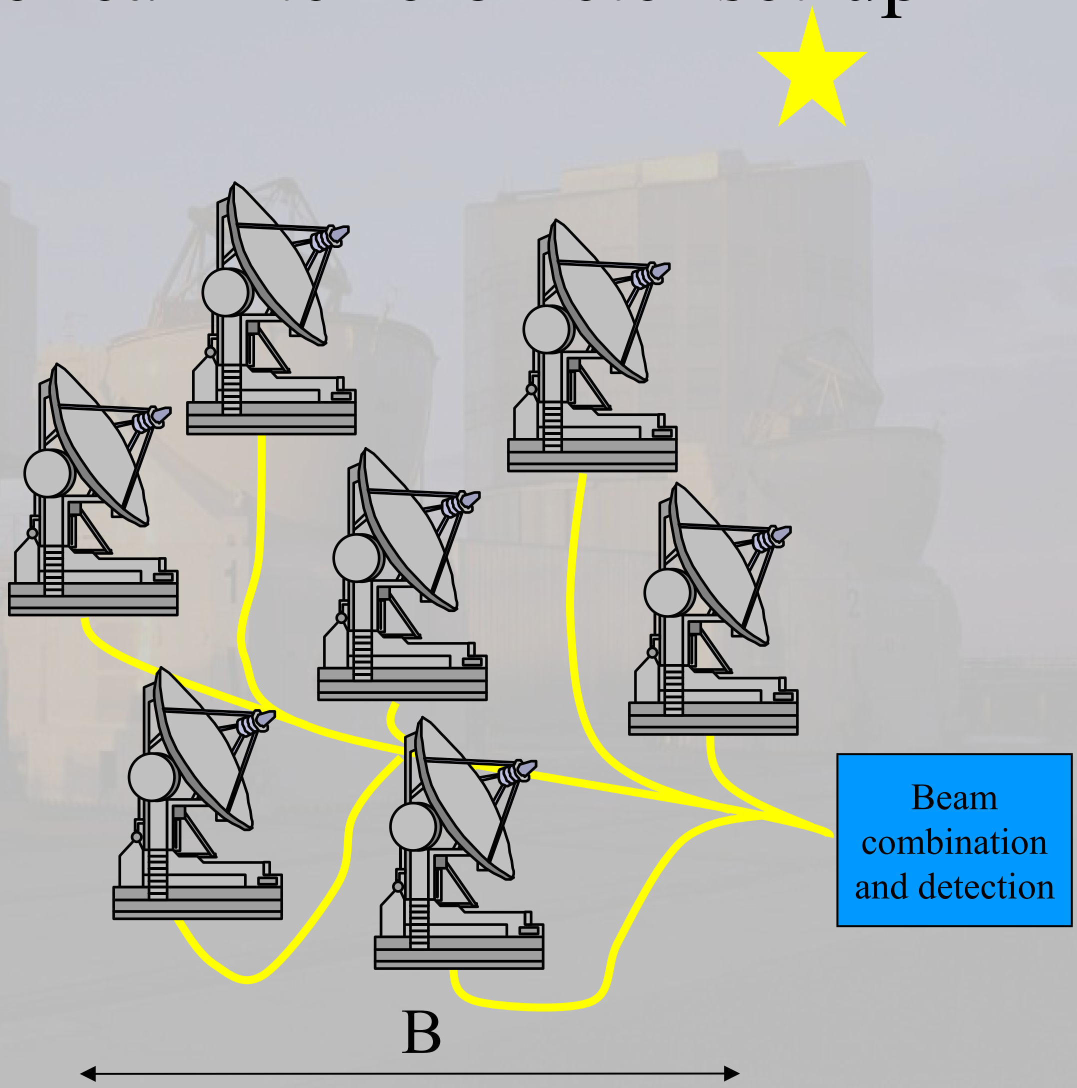
The Fizeau type interferometer



The Fizeau type interferometer

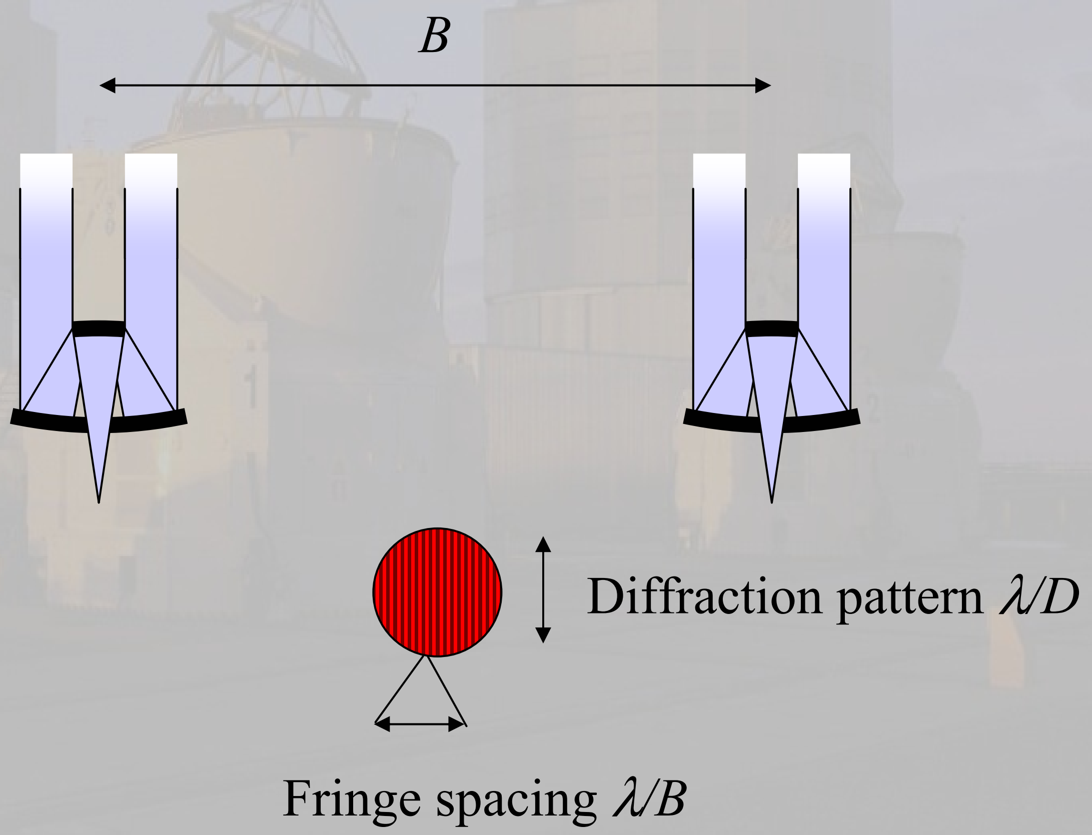


The real interferometer set-up

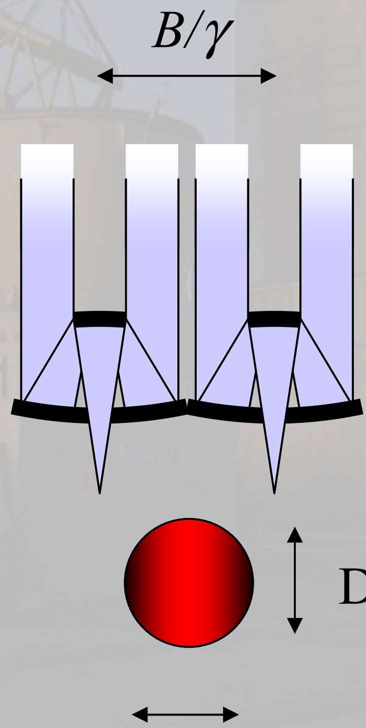


The field of view issue
(multi-axial beam combiner)

Entrance pupil



Exit pupil

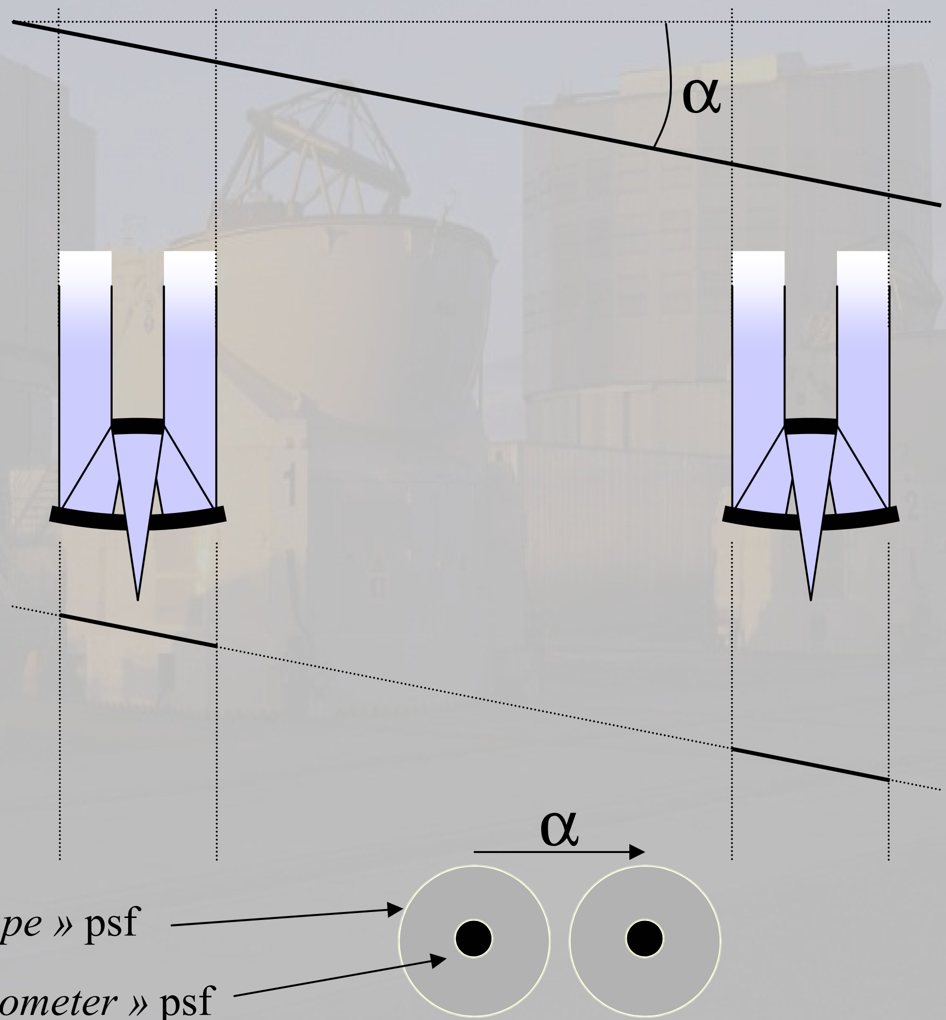


Diffraction pattern λ/D

Fringe spacing $\gamma\lambda/B$ (= λ/D)

Impact on wavefront

Entrance wavefront for an off-axis object



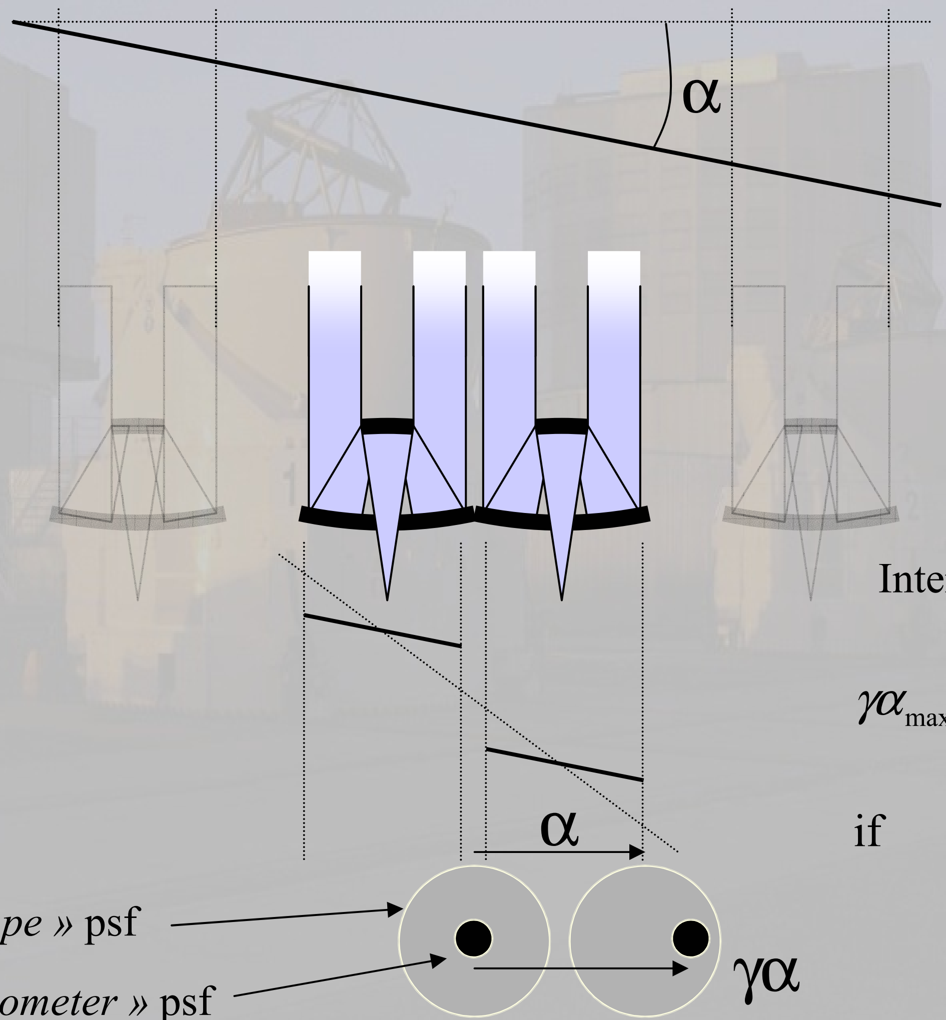
Exit wavefront

« *telescope* » psf

« *interferometer* » psf

Impact on wavefront

Entrance wavefront for an off-axis object



Exit wavefront

Interferometric field of view :

$$\gamma\alpha_{\max} = \frac{\lambda}{2D} \Rightarrow \alpha_{\max} = \frac{1}{\gamma} \times \frac{\lambda}{2D}$$

$$\text{if} \quad \gamma = \frac{B}{D} \quad , \quad \alpha_{\max} = \frac{\lambda}{2B}$$

« *telescope* » psf

« *interferometer* » psf

Interferometric field of view

The golden rule of interferometry, W. Traub, 1986:

The field of view is maximum when the interferometer entrance and exit pupils are homothetic.

Optical solution = Fizeau type beam combiner

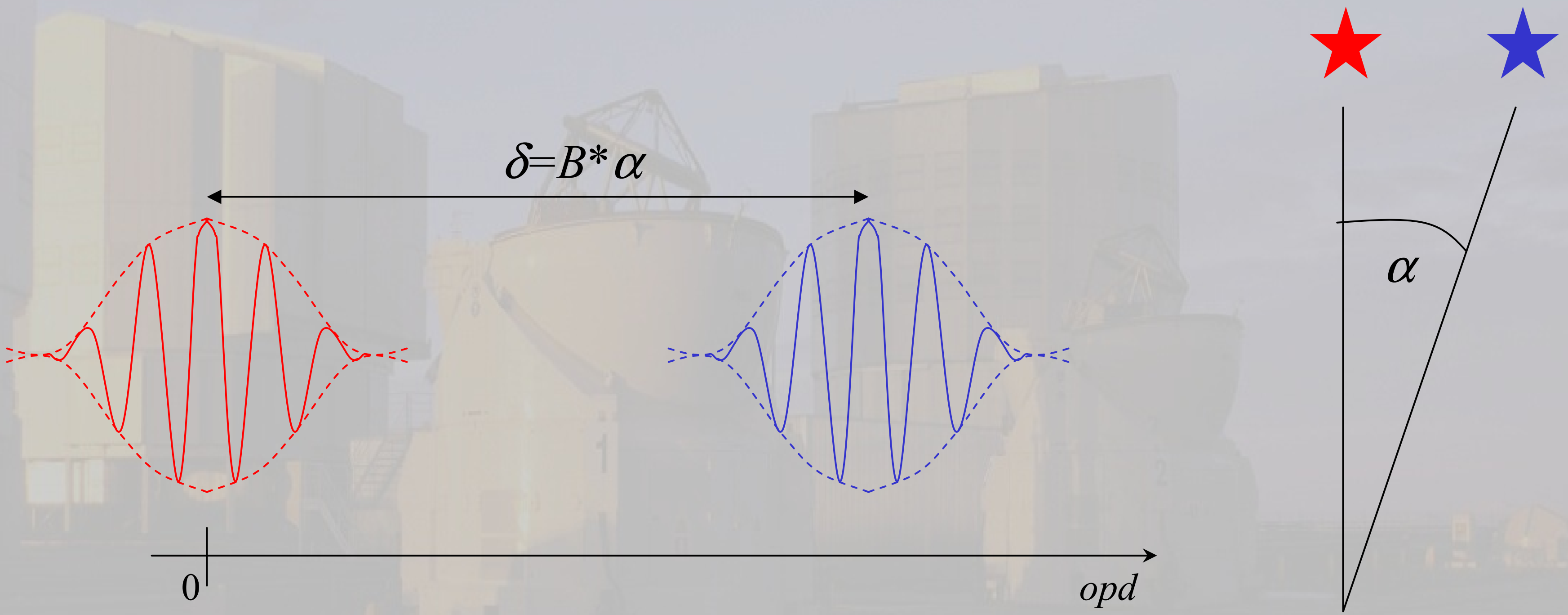
In this case only, the image is the convolution of the object by the psf.

Otherwise the convolution relation is lost.

Major drawback with diluted apertures = the *psf* is diluted over a large number of peaks which is not favorable for sensitivity (trade-off between sensitivity and field of view)

The field of view issue (co-axial beam combiner)

Field of view and co-axial combination



Condition for the off-axis object to contribute to the fringe pattern at zopd: $B \times \alpha \leq \frac{\lambda^2}{\Delta\lambda}$

Hence the field of view: $\alpha_{\max} = \frac{\lambda}{B} \times \frac{\lambda}{\Delta\lambda}$

The field of view is the product of the spectral and spatial resolutions

The field of view issue

- The field of view is limited either by the interferometer configuration, the spectral resolution (interferometric field of view) and/or the lobe of the single-mode fiber
- This is not an issue for point-like sources like calibrators
- However it is an issue for sources with an extent larger than the interferometric field of view -> the visibility of the source is overestimated
- The effect needs to be taken into account for the modeling.
- A good modeling of the effect needs to be done if the source is observed with both UTs and ATs or with different baselines at the same spectral resolution.

Calibrators

Selection of calibrators

Calibrator stars must provide very predictible visibilities

1st solution: calibrator star diameter tends to 0 (*V* tends to 1 with 100% confidence)

➲Not possible in practice (sensitivity)

2nd solution: a calibrator is a simple star (spherical compact and featurless atmosphere) with a well known diameter

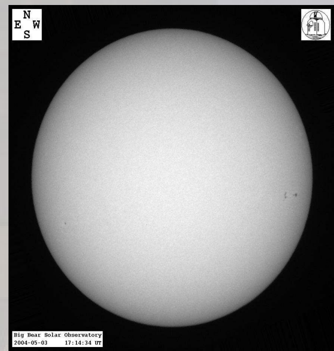
Limb darkening

Stellar photospheres are not uniform but darker on the limb

The limb darkening makes the star appear smaller than it is actually

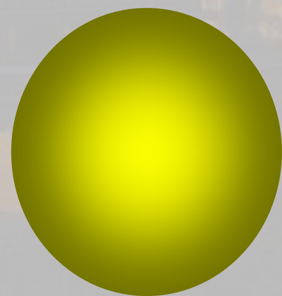
A correction has to be taken into account to produce an equivalent uniform disk (UD) diameter

UD Visibilities are an excellent approximation at high visibility



Limb darkening of the solar photosphere in the visible

Uniform disk



Limb darkened disk

Precision on diameters: direct methods

Demonstrated accuracy ~ 0.5% in K

e.g. Kervella et al. (2003) with 60m baseline on α Cen A (θ_{LD}=8.5mas) and α Cen B (θ_{LD}=6.0mas)

Extrapolated to a 200m baseline and in the J band this means that VLTI should be able to measure all stellar diameters larger than 1mas with an accuracy better than 0.5%

But calibrators for VLTI should rather be 0.1 mas sources

Precision on diameters: indirect methods

All indirect methods aim at predicting the zero-magnitude diameter (θ_{zm}) as a function of a color (or spectral type) indicator

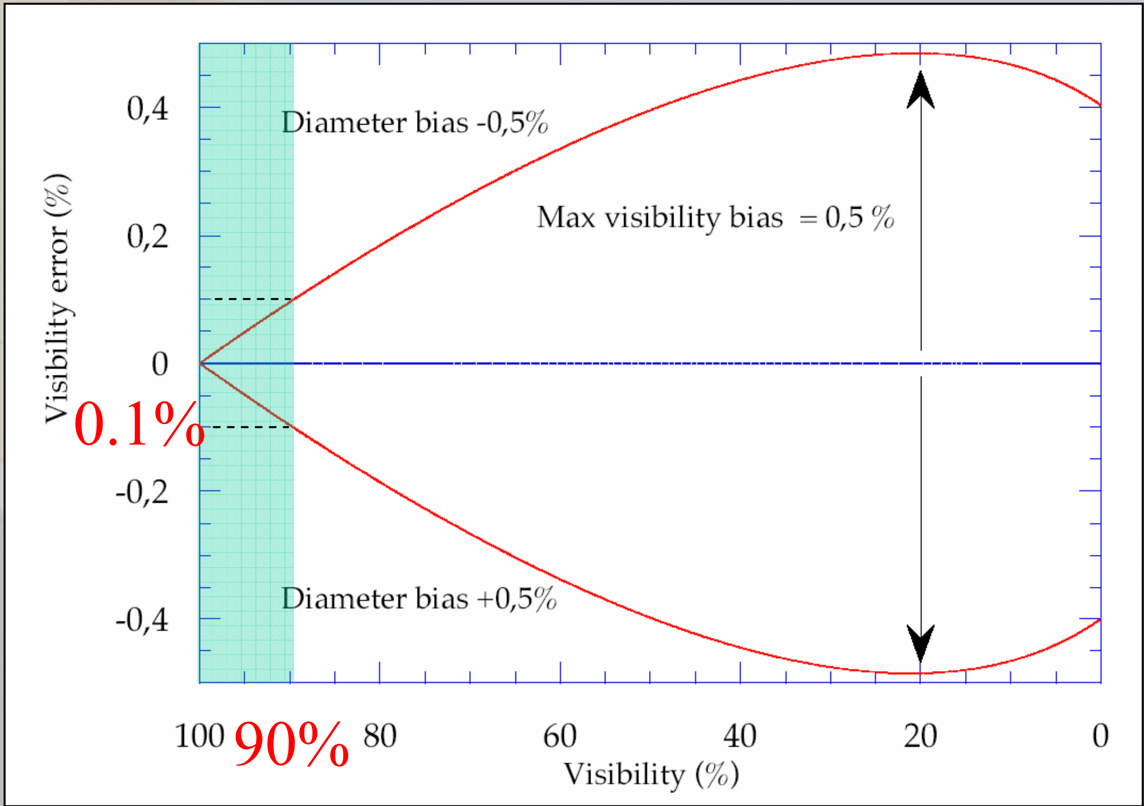
- Stellar diameter follows from $\theta_* = \theta_{zm} \times 10^{-m/5}$

Typical error is ~ 5% if all types of stars are taken into account

The prediction error can be reduced to ~1.2% for carefully selected A0 through M0 giants, using accurate photometry and atmosphere modeling (e.g. Bordé et al. 2002)

Empirical surface brightness relationships for selected dwarfs (Kervella et al. 2005) : best correlation for dereddened (B-L) colors: residual error better than 1%, can be as low as 0.5%

Propagation of the 0.5% calibrator diameter error on the estimated visibility



$$V(B,\Theta,\lambda) = \left| \frac{2J_1(\pi\Theta B/\lambda)}{\pi\Theta B/\lambda} \right|$$

B and λ need to be known with a better than 0.5% accuracy

Outline

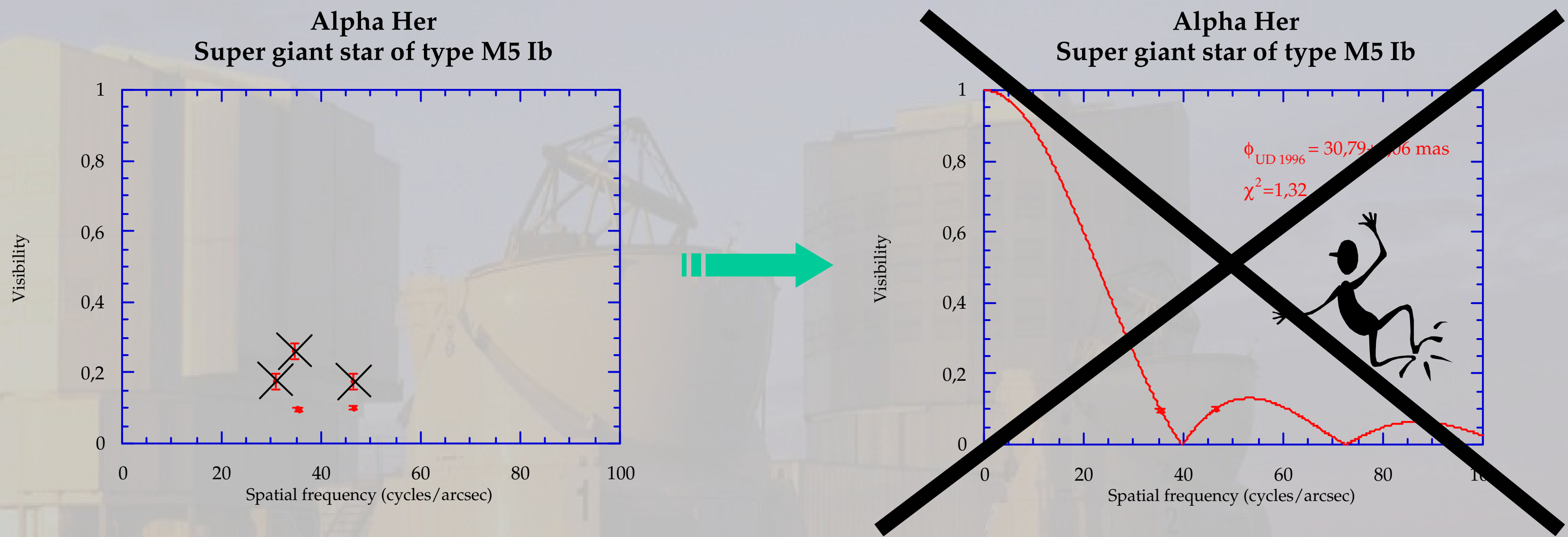
✓ Definitions

✓ Sources of biases

✓ Miscalibrations and biases

4. Single-mode interferometers

Rejecting bad data

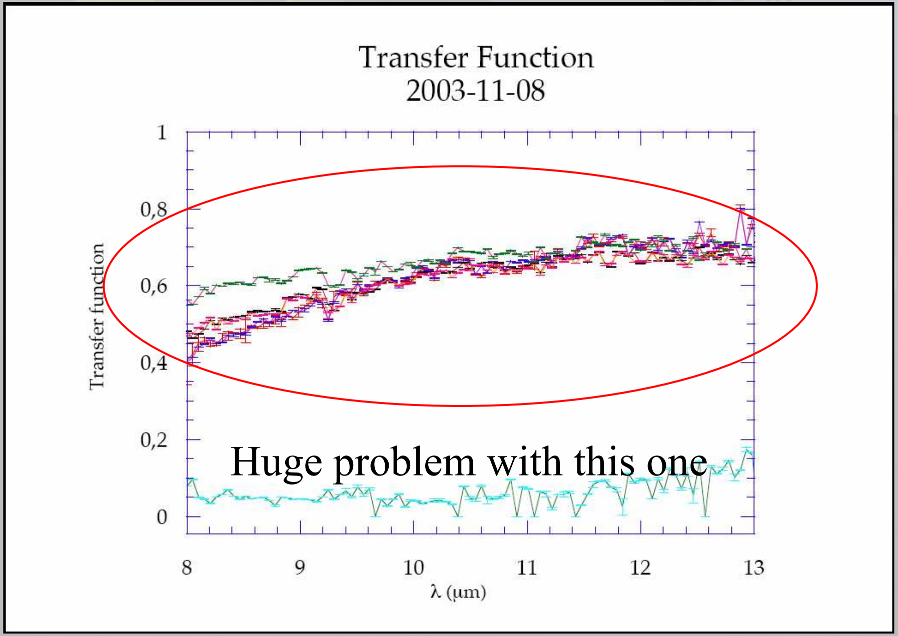


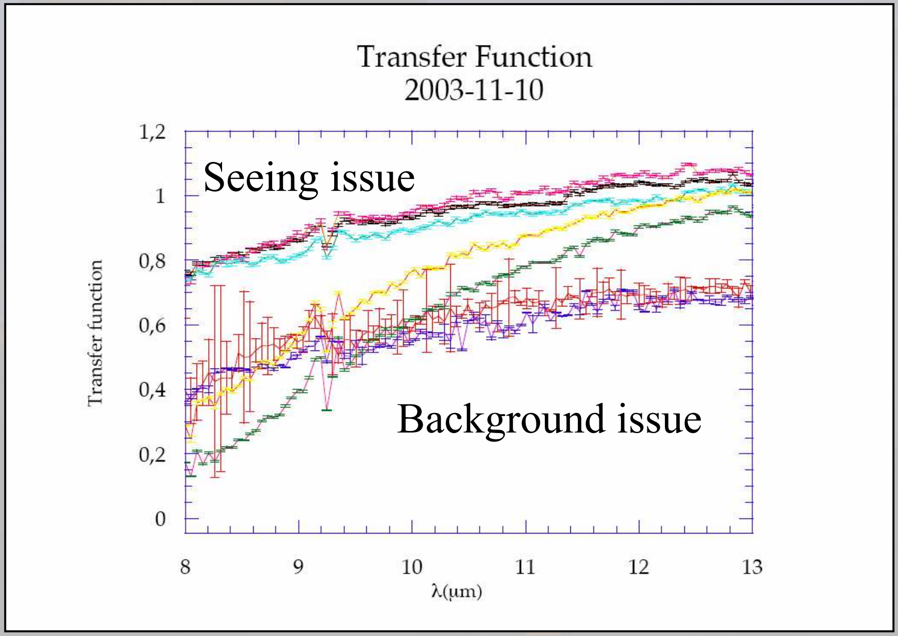
Examples of selection criteria:

- reject data for which the instrument was not stable (varying transfer function)
- (probably) reject data for which statistical distributions of μ^2 are not gaussian

Examples will be shown in L11

Example of MIDI data: Betelgeuse





Same selection applied to the star data

Assessing more realistic error bars

Error bars are first estimated for each series of scan (histogram method and propagation of errors).

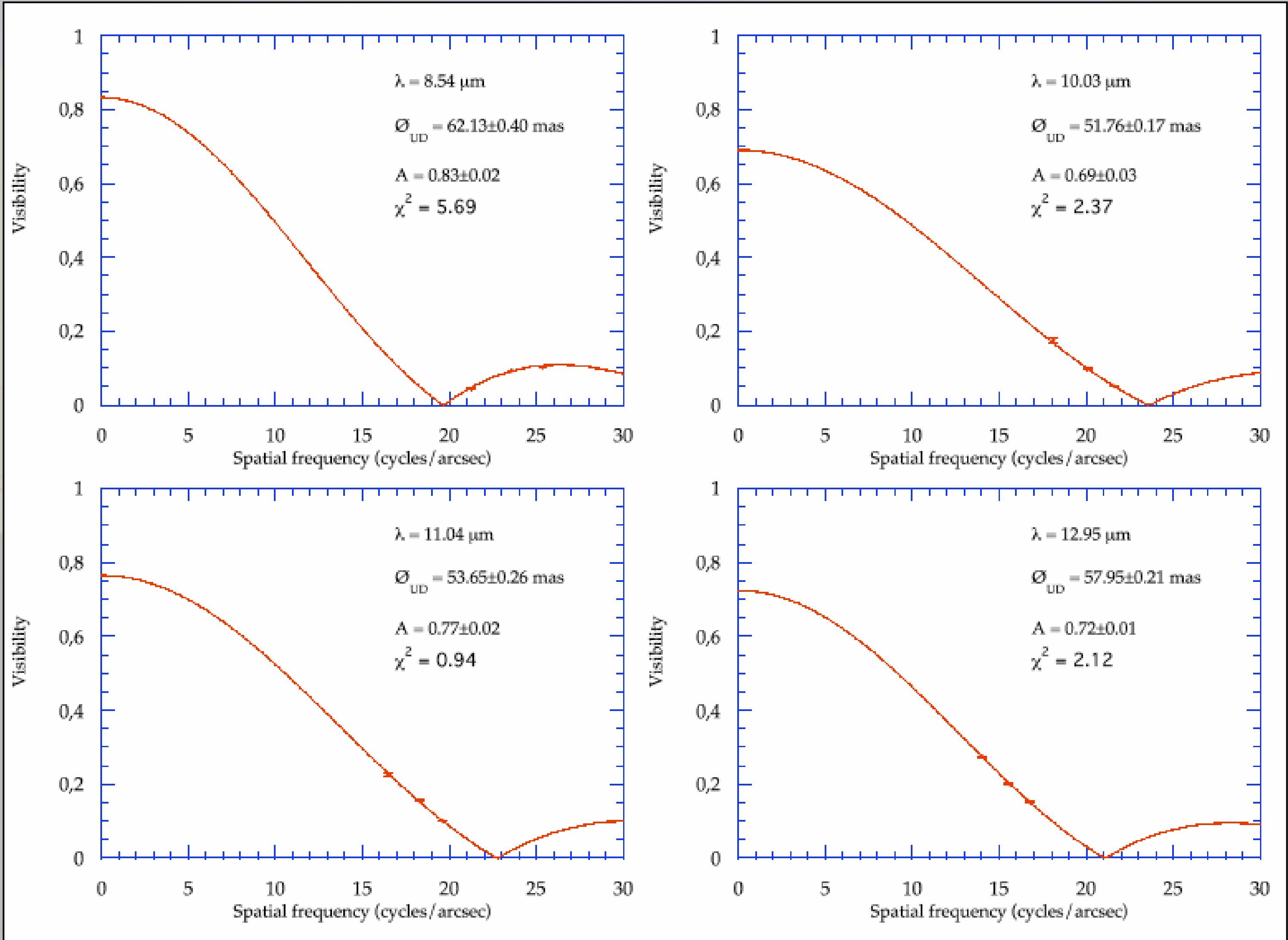
Visibilities are then binned by spatial frequencies -> several visibility estimates per bin.

The consistency of visibility sets per bin is checked:

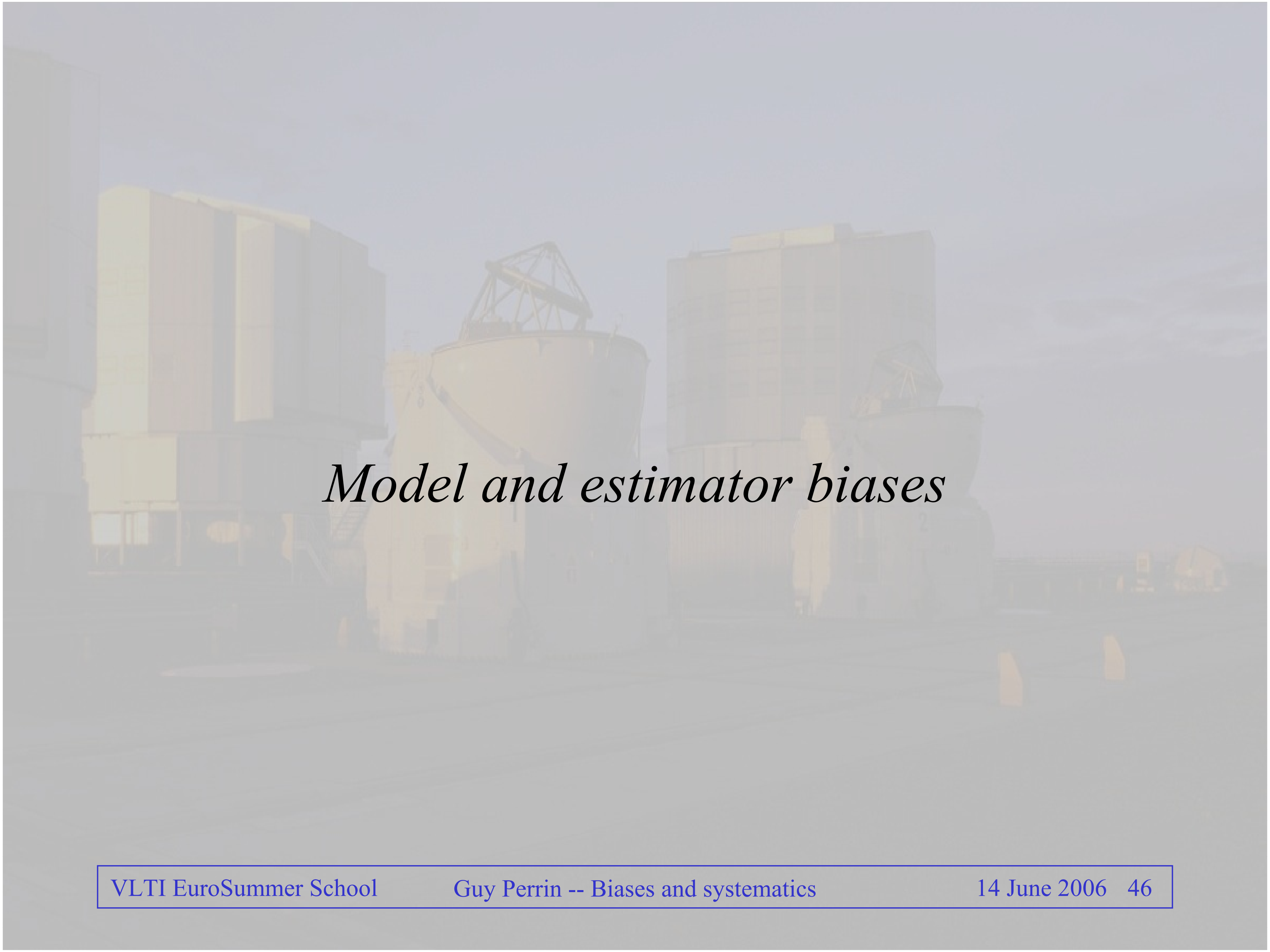
$$\chi^2 = \frac{1}{N-1}\sum_{i=0}^{N-1}\frac{(M-X_i)^2}{\sigma_i^2}$$

If $\chi^2>1$ then the variance of the estimated average is multiplied by χ^2 to make the scattered visibility estimates consistent.

Assessing error bars



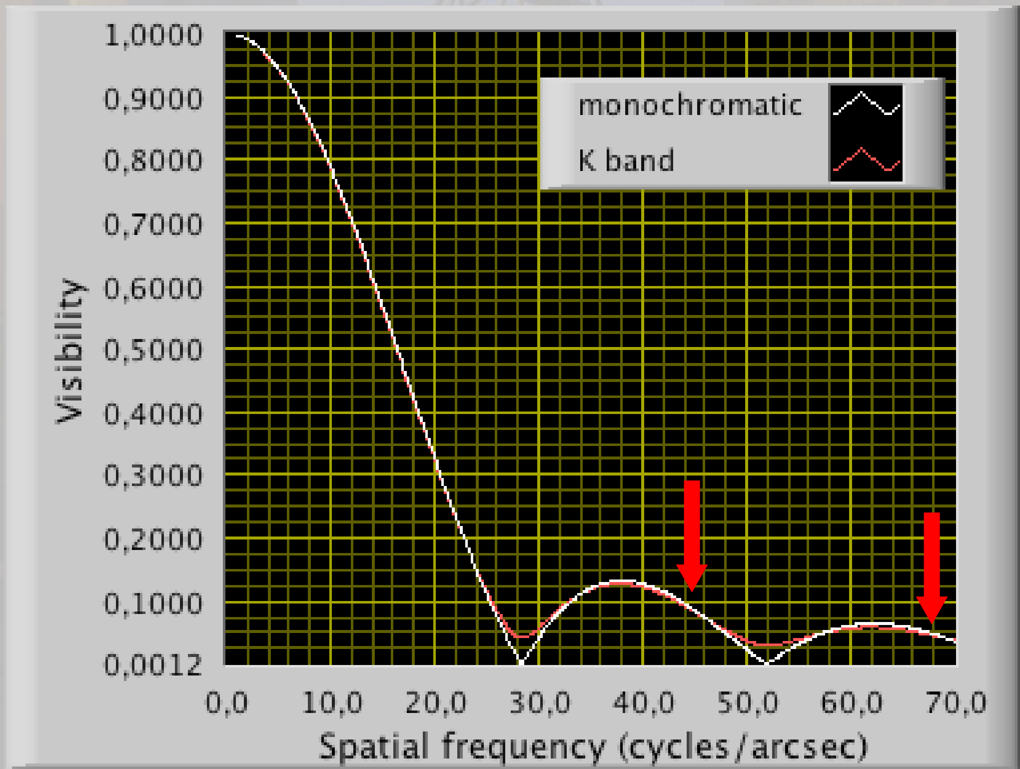
Perrin et al. (2006)



Model and estimator biases

Errors and biases on fringe contrasts measurements

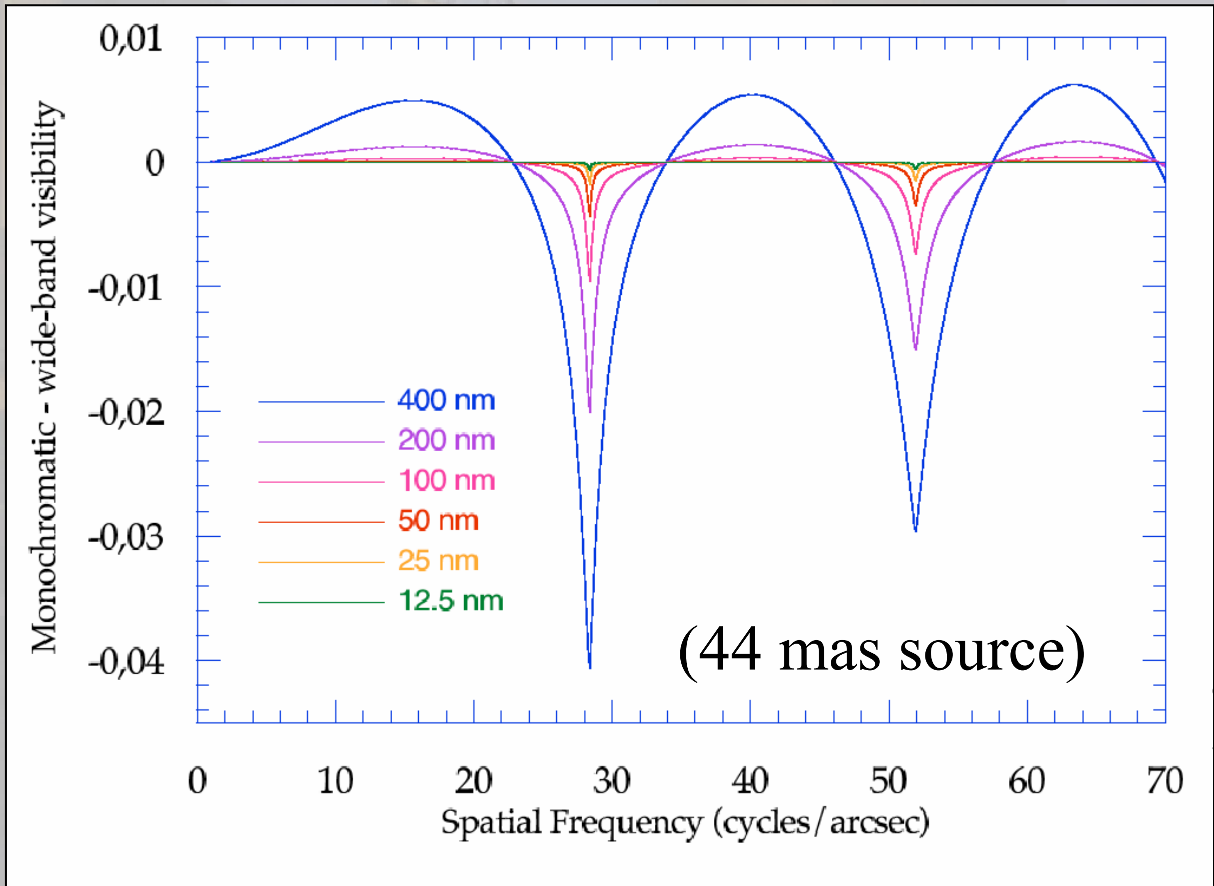
Wide band vs. Monochromatic estimator



$$\tilde{\mu}^2 \propto \int_{band} \mu^2(\sigma) B^2(\sigma) d\sigma$$

Errors and biases on fringe contrasts measurements

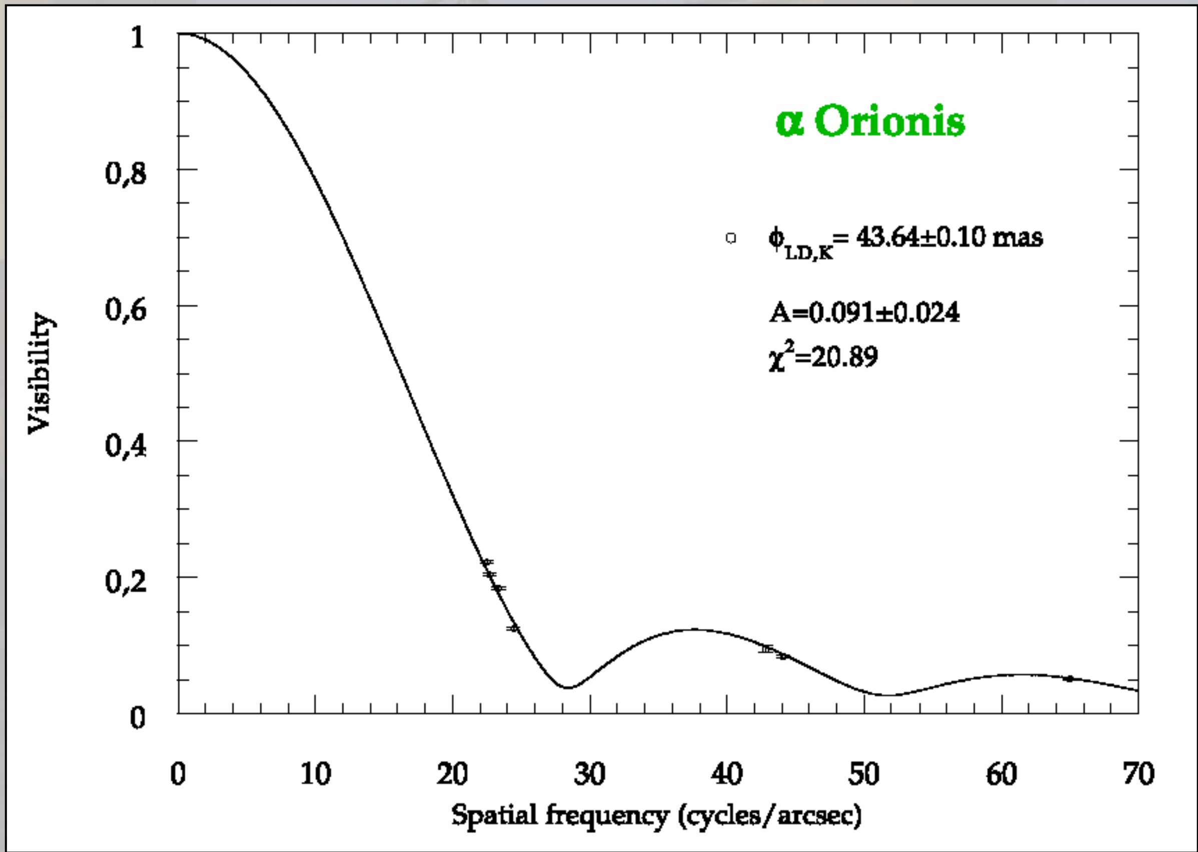
Wide band vs. Monochromatic estimator



Perrin & Ridgway (2005)

Errors and biases on fringe contrasts measurements

Wide band vs. Monochromatic estimator



Perrin et al. (2004)

Outline

- ✓ Definitions
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Correlated noise and relative interferometry

If different sets of visibilities have ***calibrators in common*** then different measurements have ***errors in common****

When fitting data, measurements cannot be assumed independent

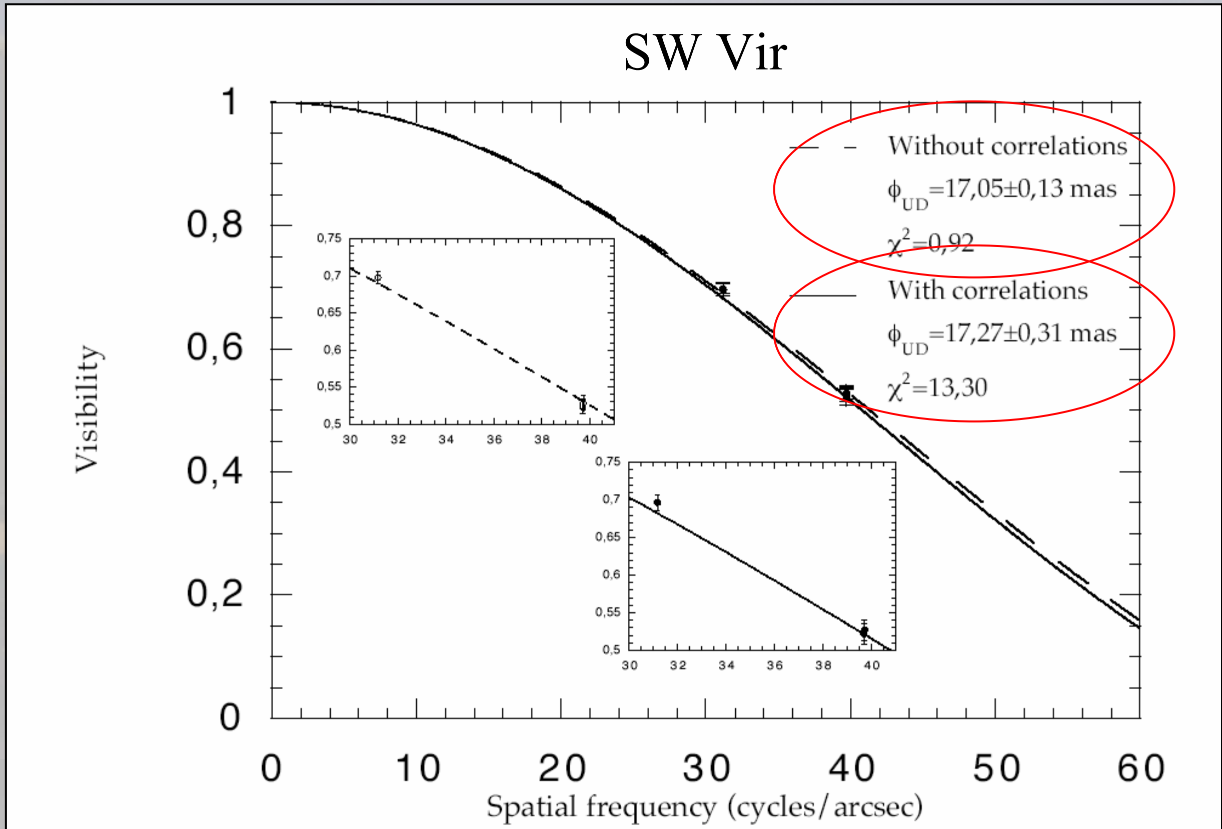
➲ Lower accuracy on fitted parameters (correlated errors do not average down to zero)

However, systematic errors can be disentangled from statistical errors to improve accuracy on parameters

* may be true for data acquired in different spectral channels (AMBER)

- common pixels
- same piston noise

Correlated noise



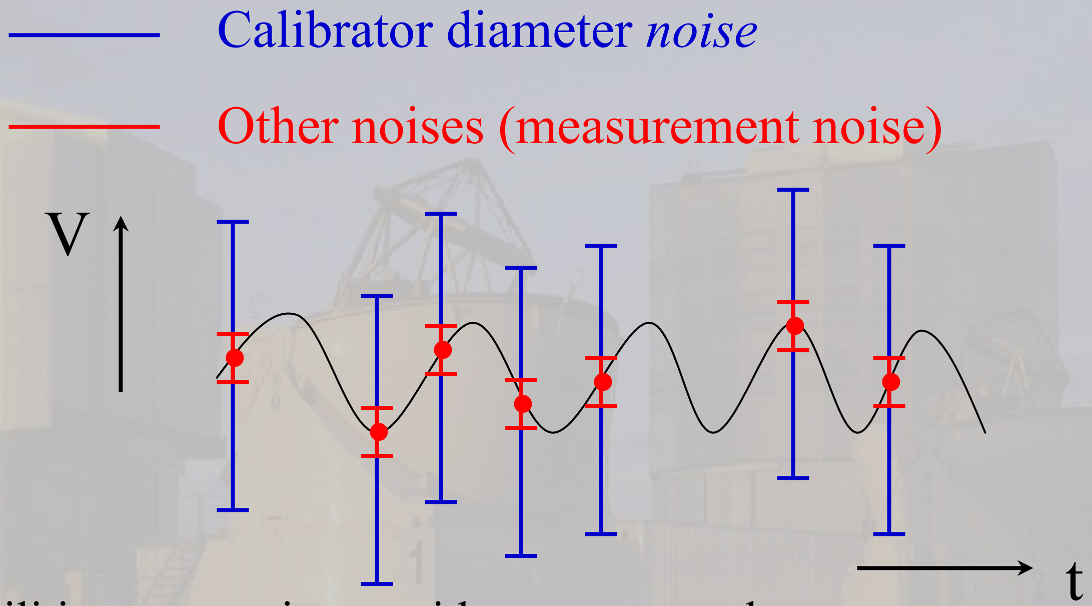
Perrin et al. (2003)

$$C = \begin{bmatrix} 1 & 0.96 & 0.96 \\ 0.96 & 1 & 0.97 \\ 0.96 & 0.97 & 1 \end{bmatrix}.$$

A single calibrator was used

Only 4% of the noise is uncorrelated

Correlated noise and relative interferometry



Absolute visibilities are consistent with a constant value:

- absolute diameter (e.g.) can be determined whose accuracy is limited by that of calibrator(s)

The periodic modulation is compatible with relative visibilities

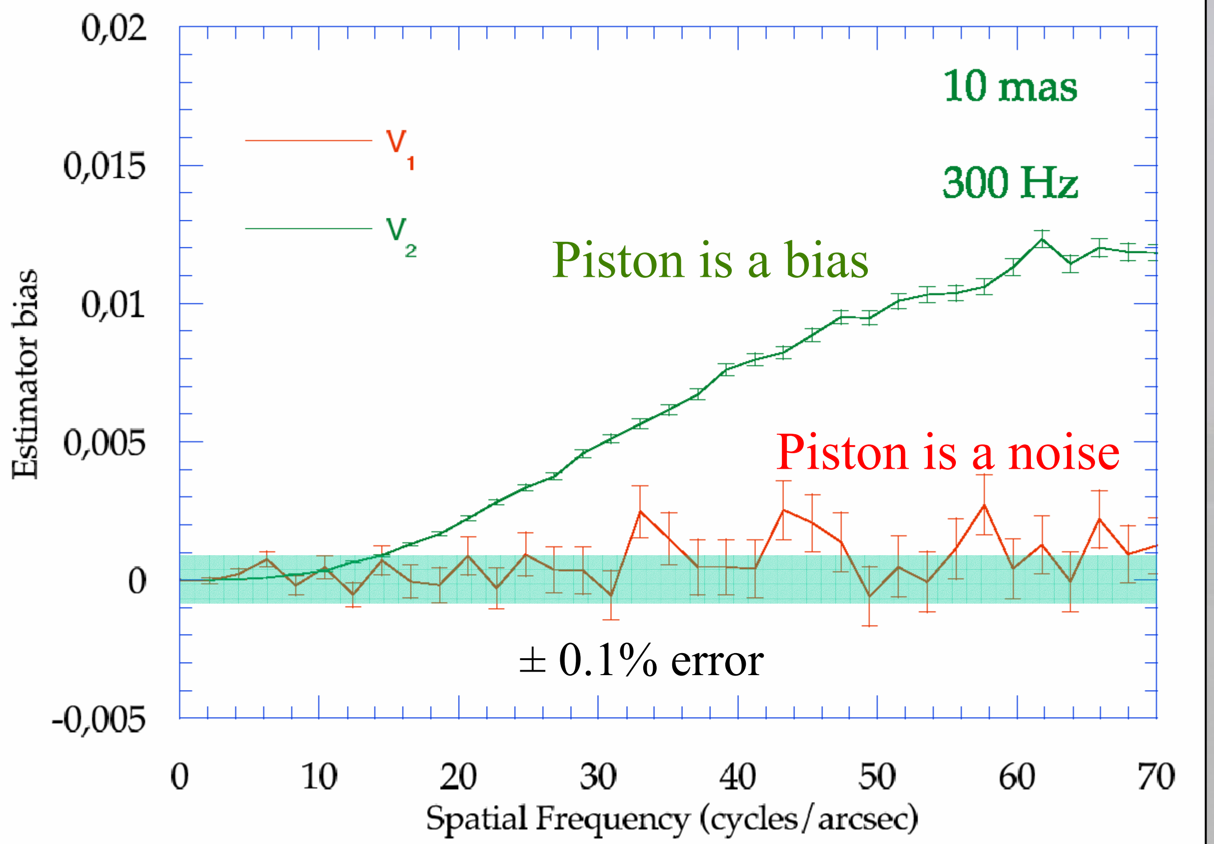
- relative diameter (e.g.) variation can be determined

Rather than using several calibrators, use of a single stable calibrator may be a good strategy to detect tiny variations

The end

Errors and biases on fringe contrasts measurements

Effect of atmospheric piston (if not corrected)



Perrin & Ridgway (2005)