Selected points - 1st week

EuroSummer School

Observation and data reduction with the Very Large Telescope Interferometer

Goutelas, France
June 4-16, 2006

Guy Perrin
Observatoire de Paris
10 June 2006

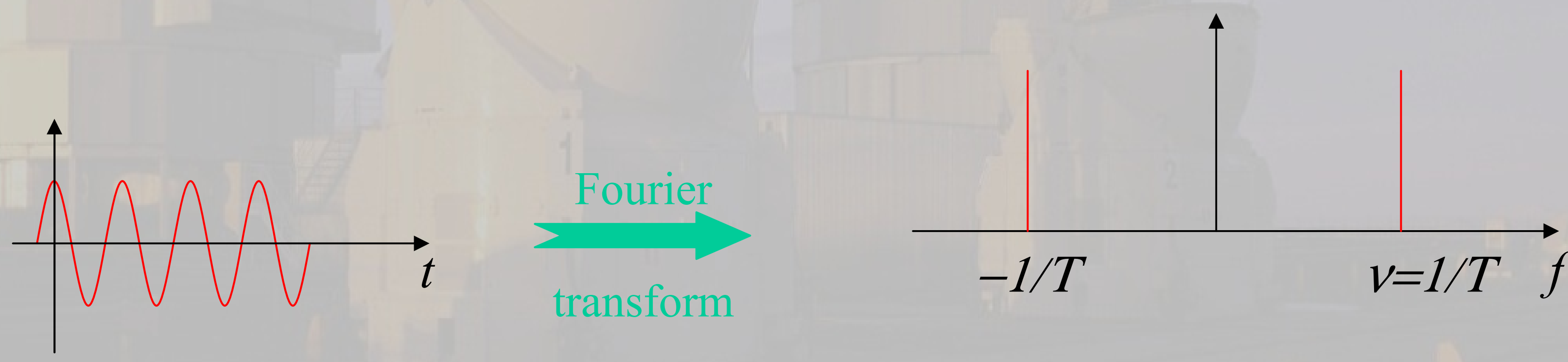
Disclaimer

This is not a good lecture !

Spatial frequency

$$\psi(f) = \int_{-\infty}^{+\infty} \Psi(t) e^{-2i\pi f t}\, dt$$

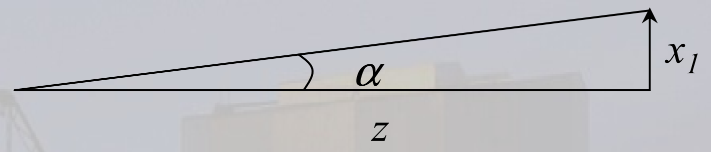
Frequency f and time t are ***conjugated variables*** by the Fourier transform.



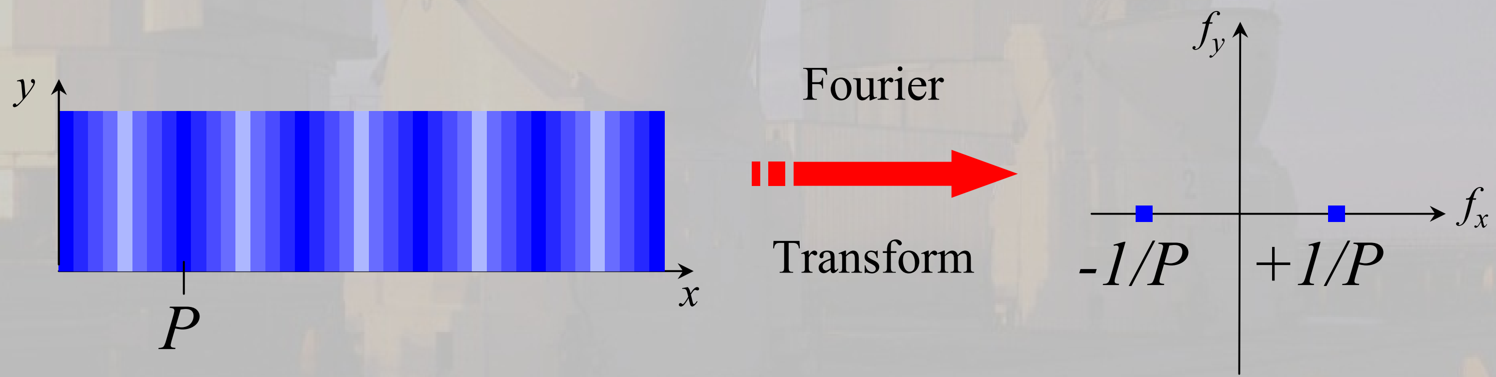
$$\Psi(t) = \cos(2\pi \frac{t}{T})$$

$$\psi(f) = \frac{1}{2}\left(\delta(f - \frac{1}{T}) + \delta(f + \frac{1}{T}) \right)$$

Angular coordinates are natural when the source is at a large distance from the diaphragm (or pupil). This is in particular true in astronomy for which objects are located at quasi infinite distances.



Spatial frequencies are not so obvious but this notion is not so difficult. A spatial frequency is the reciprocal of a characteristic scale of an object :

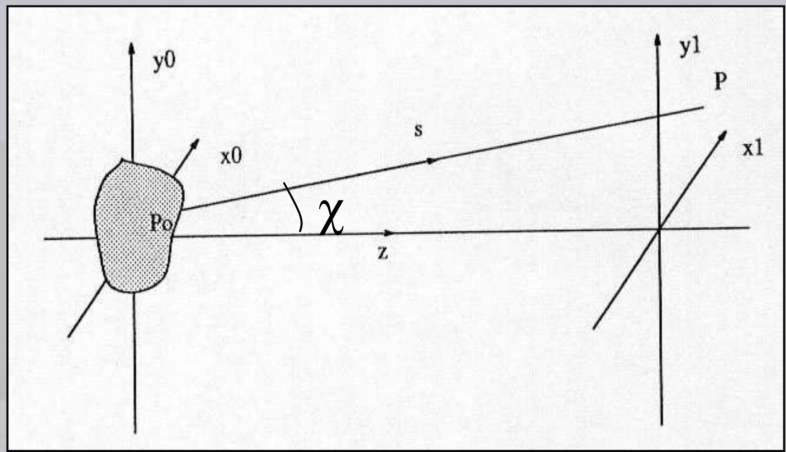


Spatial frequency coordinates have reciprocal dimensions with respect to coordinates in direct space (m ➔ m^{-1}).

Direct coordinates and Fourier space coordinates are ***conjugated*** by the Fourier transform.

Diffraction, imaging, Optical Transfer Function

Fraunhofer Diffraction



Hypotheses :

- 1. Small diffraction angles : $\cos\chi \approx 1$
- 2. Diaphragm and screen are small w.r.t. *z* : *1/s ≈ 1/z*
- 3. *z* much larger than the Rayleigh distance: $z_R = \frac{\left(x_0{}^2 + y_0{}^2\right)_{\max}}{\lambda}$

$$\psi_1(x_1,y_1) \approx \frac{e^{\frac{2i\pi}{\lambda}z}}{i\lambda z}.e^{\frac{i\pi}{\lambda z}\left[x_1{}^2+y_1{}^2\right]}\iint_{diaph}\Psi_0(x_0,y_0)e^{-\frac{2i\pi}{\lambda z}[x_0x_1+y_0y_1]}\,dx_0dy_0$$

Traditionally, variables are changed into :

Pupil plane $\begin{cases} u=\dfrac{x_0}{\lambda} \\ v=\dfrac{y_0}{\lambda} \end{cases}$ (rad^{-1} or arcsec^{-1})

Image plane $\begin{cases} \alpha=\dfrac{x_1}{z} \\ \beta=\dfrac{y_1}{z} \end{cases}$ (rad or arcsec)

$$\psi_1(\alpha,\beta) \approx \frac{e^{i\pi\frac{z}{\lambda}\left[2+\alpha^2+\beta^2\right]}}{i\dfrac{z}{\lambda}}\iint_{diaph}\Psi_0(\lambda u,\lambda v)e^{-2i\pi[\alpha u+\beta v]}\,dudv$$

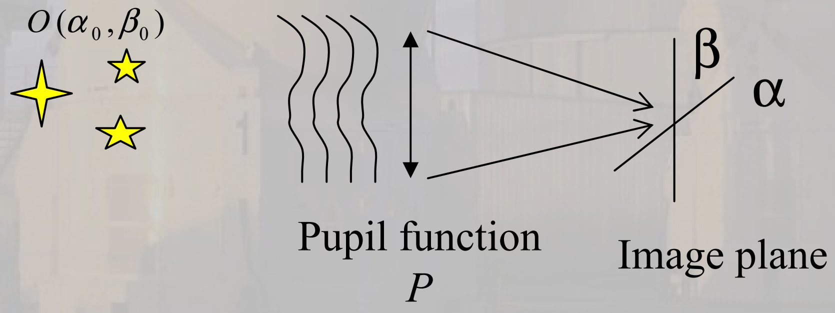
The diffracted field is proportional to the Fourier Transform of the field in the pupil plane :

$$\psi(\alpha,\beta) \propto TF\{\Psi(u,v)\}$$

Point source response:

$$I(\alpha,\beta) = \left\langle \left|\Psi(\alpha,\beta)\right|^2 \right\rangle_t \equiv \left|\Psi(\alpha,\beta)\right|^2$$

Extended object:



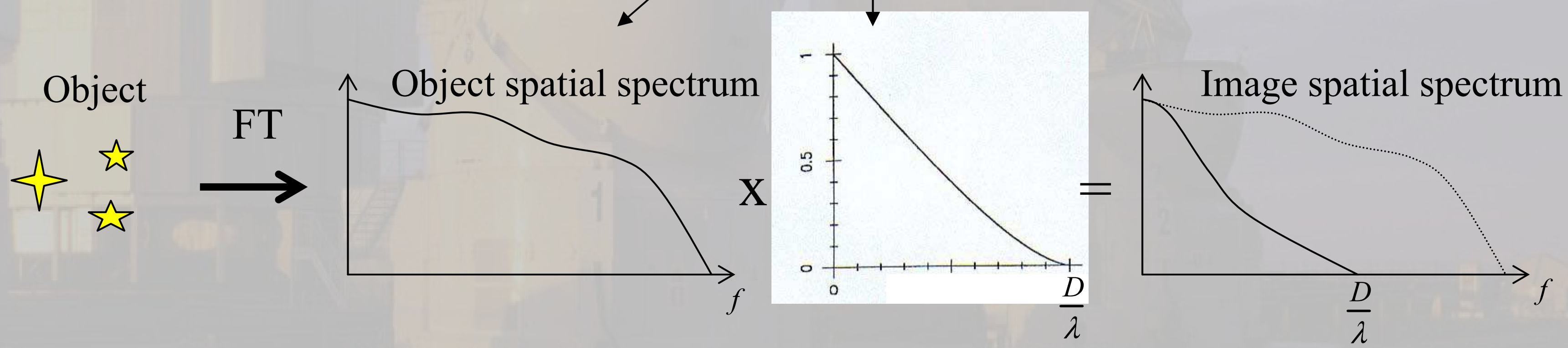
$$Im(\alpha,\beta) = \iint O(\alpha_0,\beta_0)\, I(\alpha-\alpha_0,\beta-\beta_0)\, d\alpha_0 d\beta_0 = O * I\,(\alpha,\beta)$$

The image is the convolution of the source spatial intensity distribution by the point spread function

Image frequency contents

Image spectrum :

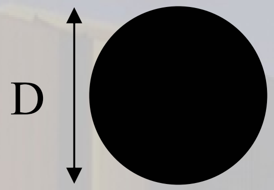
$$S(u,v) = FT\{Im(\alpha,\beta)\} = FT\{O(\alpha,\beta)\}.OTF(u,v)$$



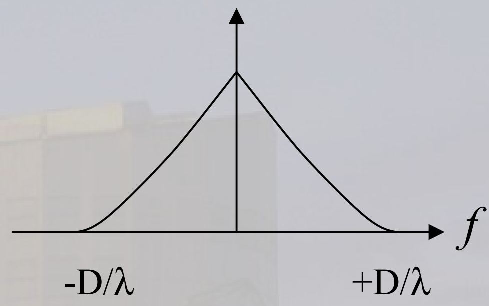
The optical system is a ***low-pass filter*** with a cut-off frequency ***D/λ***

The interferometer as a *band-pass filter*

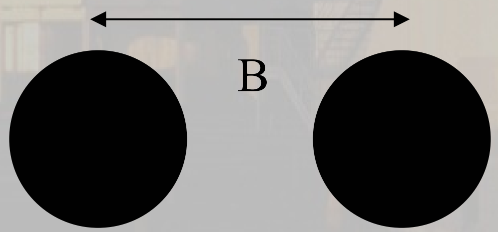
Pupil

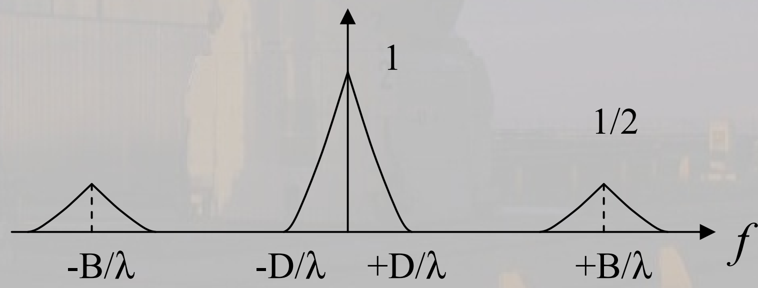


Optical transfer function

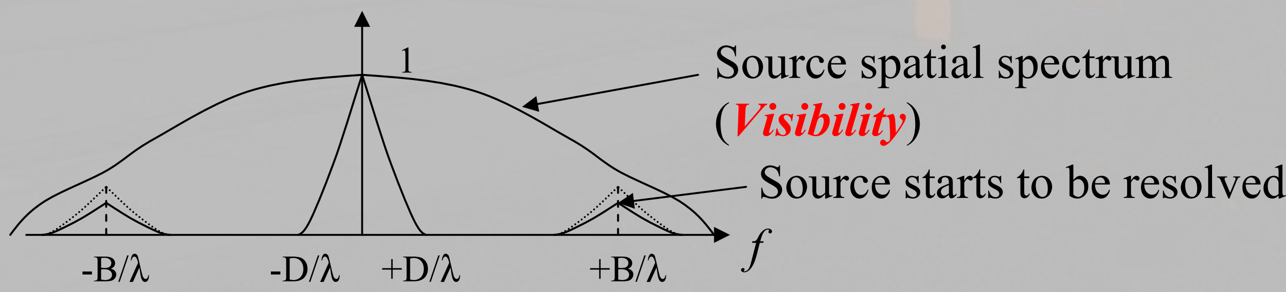


The same image formation theory applies to the interferometer

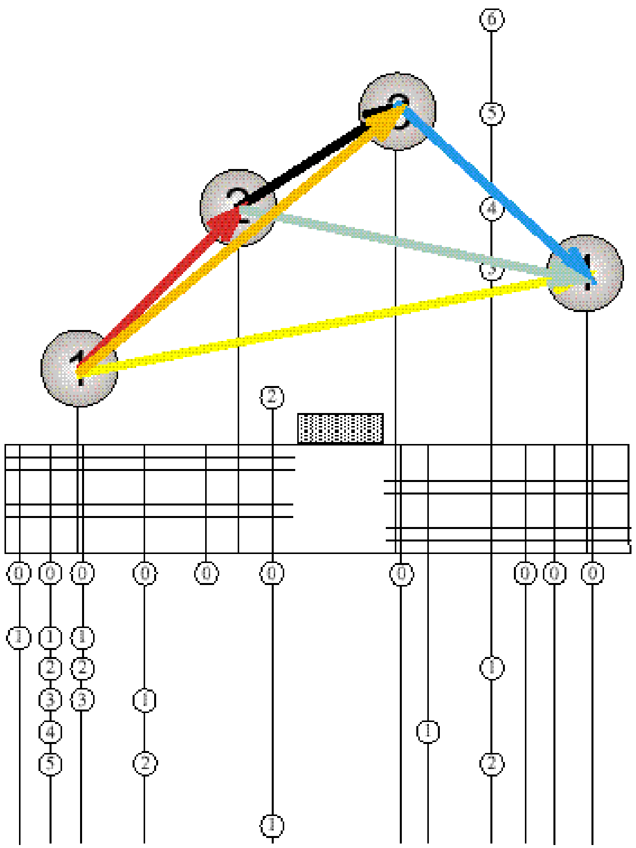




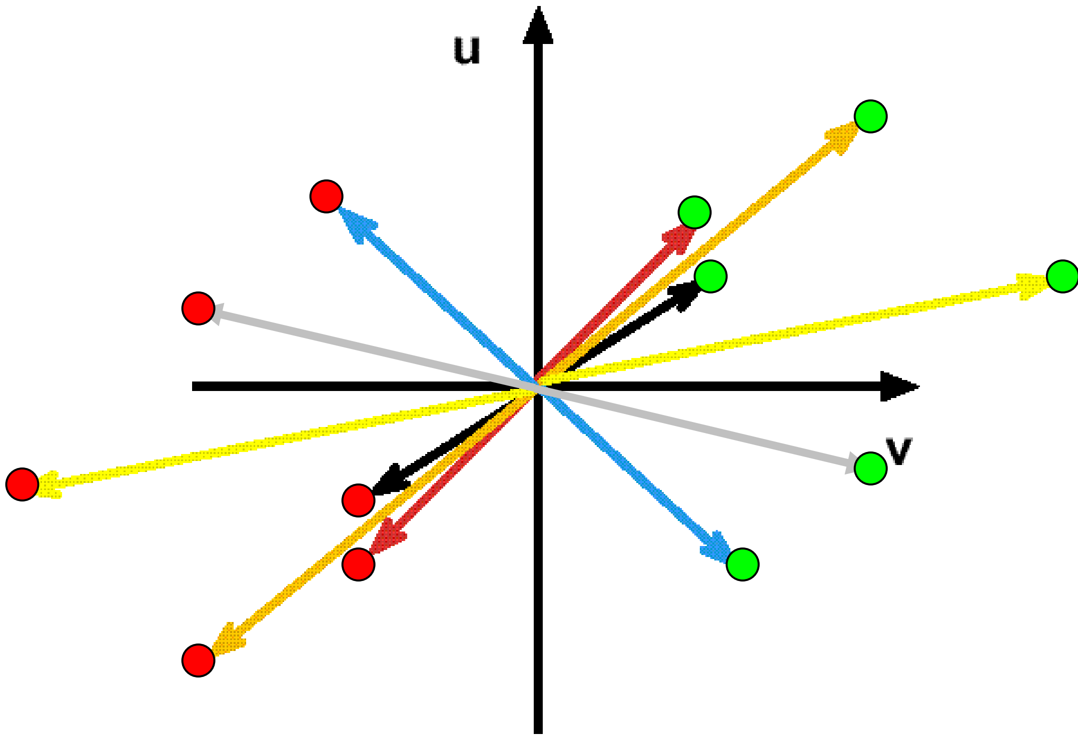
What happens for
an extended source ?



Fourier plane sampling



This is the uv-plane:



Note: This is the uv-plane for an object at zenith.
In general, the projected baselines have to be used.

(u,v) tracks

(u,v) tracks are ellipses whose center is on the v axis. (u ⌫ East, v ⌫ North)

General equation for (u,v) tracks :

$$u^2 + \left(\frac{v - (B_z/\lambda)\cos\delta}{\sin\delta} \right)^2 = \frac{B_x^2 + B_y^2}{\lambda^2}$$

B_x, B_y and B_z are the coordinates of the baseline vector projected onto the axes pointing towards East, North and the meridian, respectively

Particular cases :

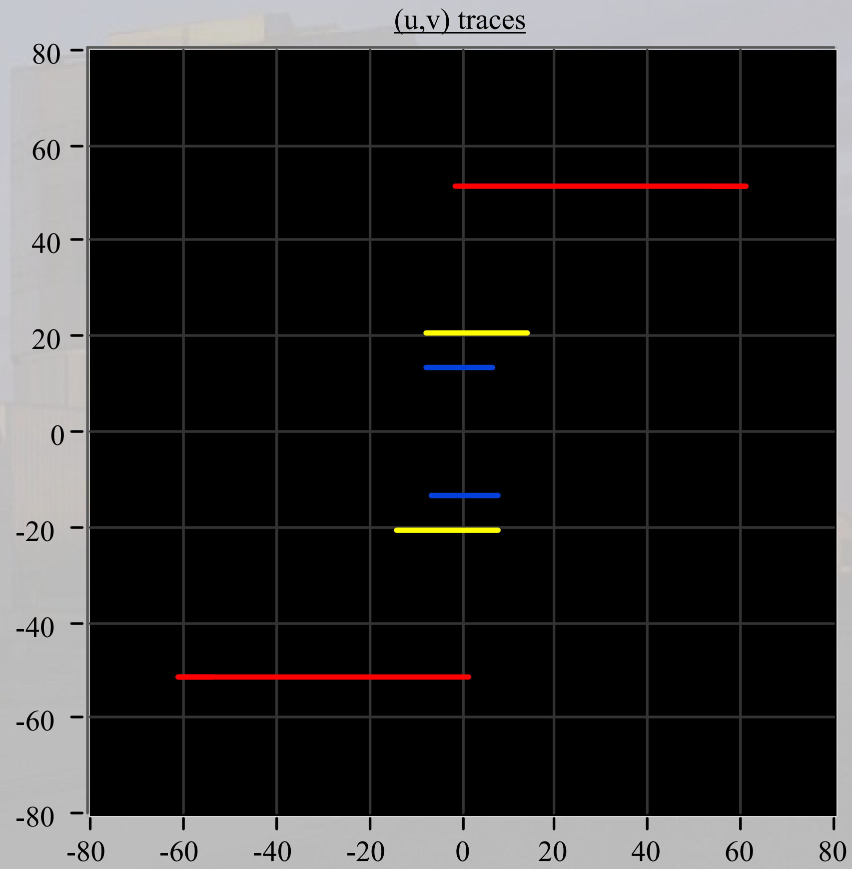
- $\delta = 0°$: (u,v) tracks are straight lines parallel to the u axis
- $\delta = 90°$: (u,v) tracks are circles centered on the origin

Examples of (*u*,*v*) coverages at IOTA (Arizona)

Three 45 cm relocatable siderostats

Lattitude = 31.4°

Hour angle range : -4h , +4h



$\delta = 0°$

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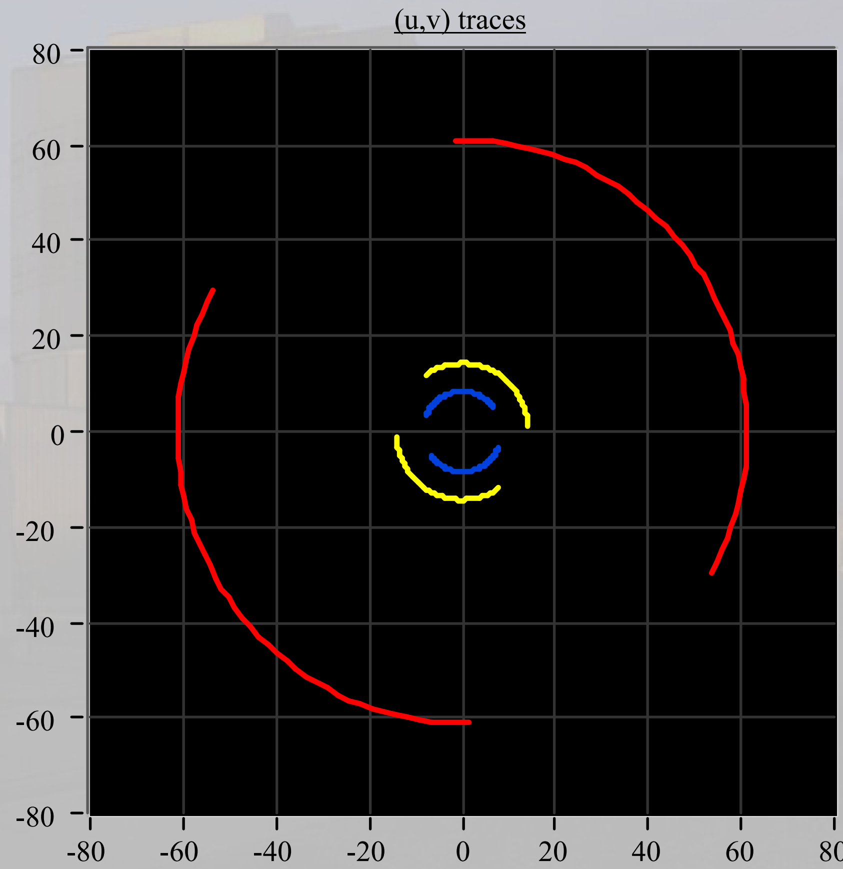

$\delta = 90°$

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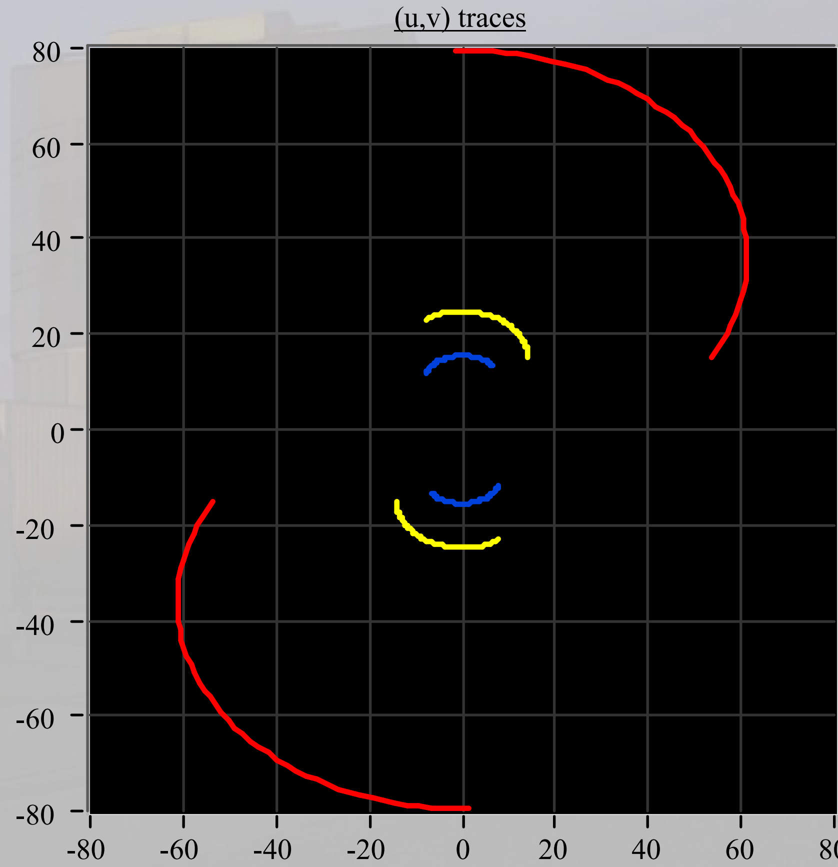


δ = +45°

Coherence

Coherence of light waves

Let $E(\vec{P},t)$ be a light wave.

As for random variables, a correlation can be defined between fields at different times or at different locations.

In the first case, this is called ***temporal coherence***:

$$Corr(E(\vec{P},t),E(\vec{P},t+\tau)) = \frac{\left\langle \left(E(\vec{P},t)-\left\langle E(\vec{P},t)\right\rangle\right)\left(E(\vec{P},t+\tau)-\left\langle E(\vec{P},t+\tau)\right\rangle\right)^* \right\rangle}{\sqrt{\left\langle \left|E(\vec{P},t)-\left\langle E(\vec{P},t)\right\rangle\right|^2\right\rangle \left\langle \left|E(\vec{P},t+\tau)-\left\langle E(\vec{P},t+\tau)\right\rangle\right|^2\right\rangle}} = \frac{\left\langle E(\vec{P},t).E^*(\vec{P},t+\tau)\right\rangle}{\sqrt{\left\langle \left|E(\vec{P},t)\right|^2\right\rangle \left\langle \left|E(\vec{P},t+\tau)\right|^2\right\rangle}}$$

In the second case, it is ***spatial coherence***:

$$Corr(E(\vec{P},t),E(\vec{P}+\Delta\vec{P},t)) = \frac{\left\langle E(\vec{P},t).E^*(\vec{P}+\Delta\vec{P},t)\right\rangle}{\sqrt{\left\langle \left|E(\vec{P},t)\right|^2\right\rangle \left\langle \left|E(\vec{P}+\Delta\vec{P},t)\right|^2\right\rangle}}$$

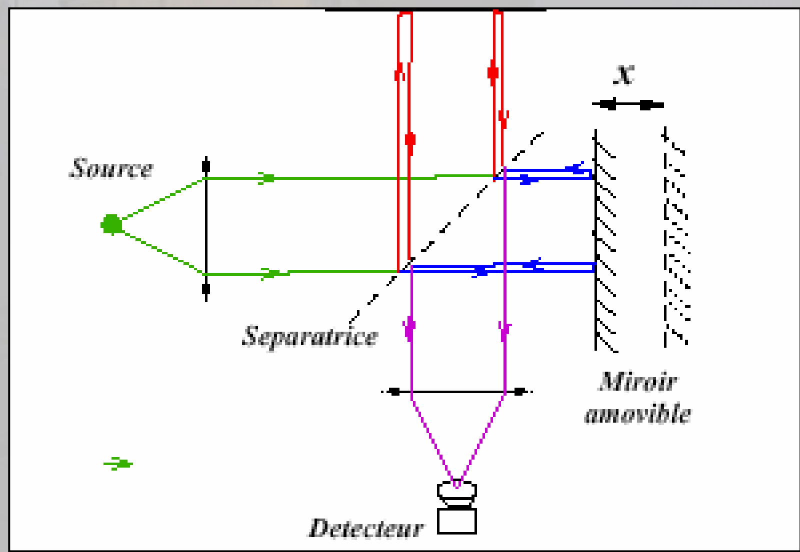
Temporal coherence

Measuring the temporal coherence of the electric field means measuring:

$$\left\langle E(\vec{P},t).E^{*}(\vec{P},t+\tau)\right\rangle_{\Delta t>>1/\nu}$$

How shall we do?

At optical wavelengths?



The Michelson interferometer

The Michelson interferometer (polychromatic case)

Fields at different wavelengths are not coherent.

The interferogram measured with the Michelson interferometer is therefore the sum of mochromatic interferograms in the band:

$$I_{tot}(\tau) = \int_{\Delta\lambda} I_{tot}(\lambda,\tau)\,d\lambda = \int_{\Delta\sigma} I_{tot}(\sigma,\tau)\,d\sigma$$

(σ = 1/λ is the wavenumber)

$$= 2I + 2\,\mathrm{Re}\left[\int_{\Delta\sigma} I(\sigma)\exp(2i\pi\sigma.c\tau)\,d\sigma\right]$$

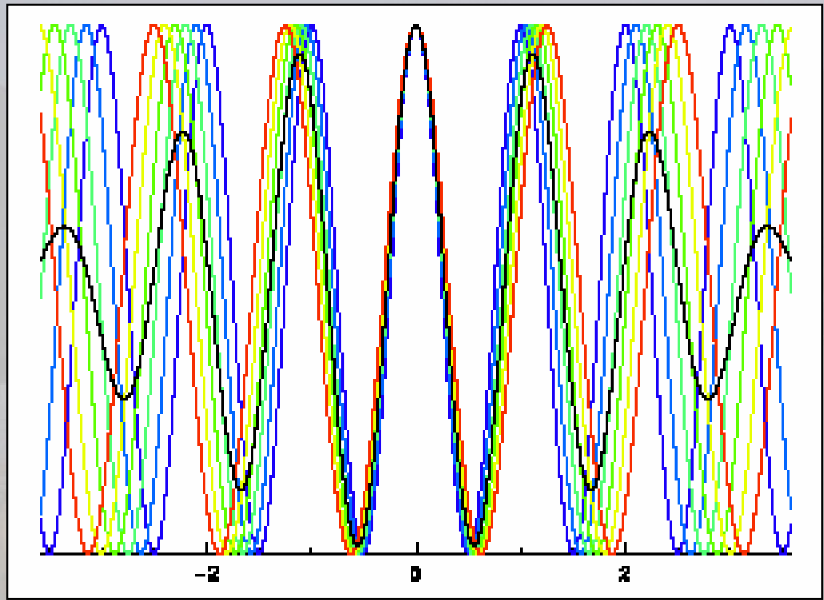
The complex degree of coherence in the polychromatic case is therefore:

(proof: Wiener-Khintchine theorem):

$$\gamma(\tau) = \frac{\int_{\Delta\sigma} I(\sigma)\exp(-2i\pi\sigma.c\tau)\,d\sigma}{\int_{\Delta\sigma} I(\sigma)\,d\sigma}$$

The interferogram of the Michelson set-up therefore allows to measure the spectrum of the source ➔ principle of the ***Fourier transform spectrometer***

Coherence length



Opd in units of λ

$$l_c = \frac{1}{\Delta\sigma} = \frac{\lambda^2}{\Delta\lambda}$$

Number of fringes $= \frac{l_c}{\lambda} = \frac{\lambda}{\Delta\lambda}$

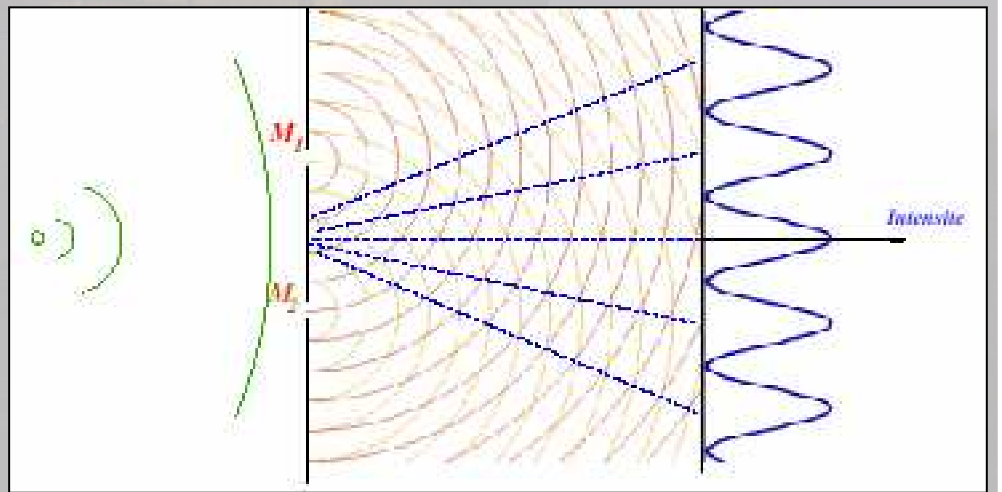
Spatial coherence

Measuring the spatial coherence of the field means measuring:

$$\left\langle E(\vec{P},t).E^{*}(\vec{P}+\Delta\vec{P},t)\right\rangle_{\Delta t>>1/\nu}$$

How shall we do?

At optical wavelengths?



The Young slit experiment

The spatial interferometer (extended source)

The source is extended and uncoherent (i.e. the fields emitted by individual points are not correlated).

The intensity of the field in the focal plane is:

$$I_{tot}(\vec{B},\tau) = \int_{source} \left\langle \left| E(P,\vec{S},t) + E(P+\vec{B},\vec{S},t+\tau) \right|^2 \right\rangle d^2\vec{S}$$

$$= 2 \int_{source} I(\vec{S},\lambda) d^2\vec{S} + 2\,\mathrm{Re}\left(\int_{source} I(\vec{S},\lambda) \exp\left[i\left(\omega\tau - 2\pi\vec{S}.\frac{\vec{B}}{\lambda} \right)\right] d^2\vec{S} \right)$$

$$= 2 \int_{source} I(\vec{S},\lambda) d^2\vec{S} + 2\left(\int_{source} I(\vec{S},\lambda) d^2\vec{S} \right) \mathrm{Re}\left(\mu_{12}(\vec{B})\exp(i\omega\tau)\right)$$

Which yields for the complex coherence factor:

$$\mu_{12}(\vec{B}) = \frac{\int_{source} I(\vec{S},\lambda)\exp\left[-2i\pi\vec{S}.\frac{\vec{B}}{\lambda}\right] d^2\vec{S}}{\int_{source} I(\vec{S},\lambda) d^2\vec{S}}$$

• This result is known as the ***Zernike-Van Cittert theorem*** : ***the complex coherence factor is equal to the Fourier transform of the spatial intensity distribution of the source.***

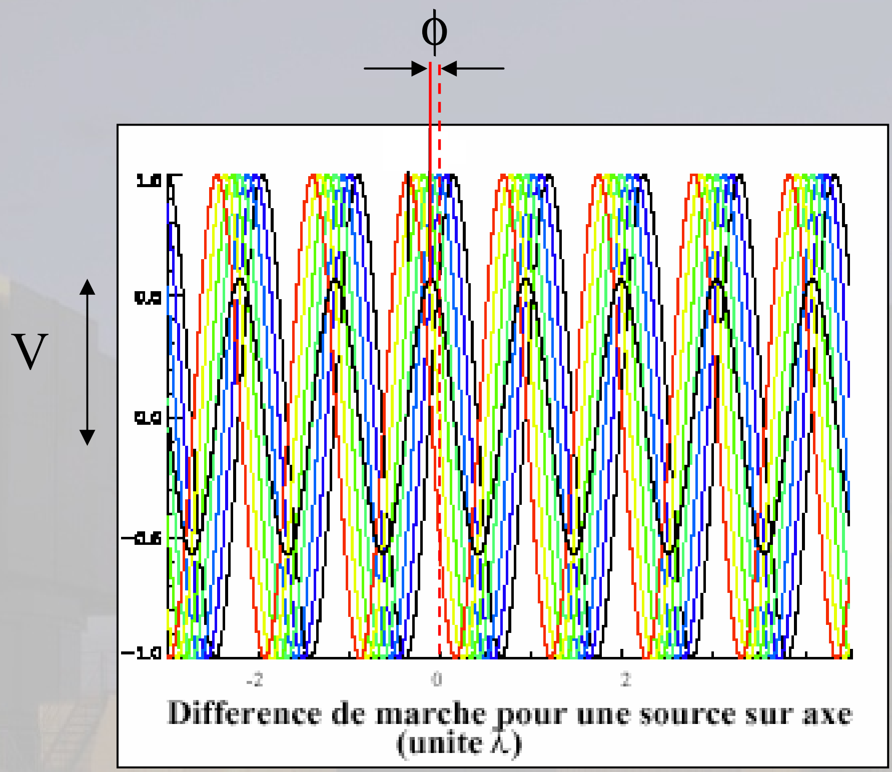
• The conjugated coordinates are the angular direction ***S*** and the spatial frequency ***B***/λ.

• Measuring a value of the complex coherence factor yields a value of the spatial spectrum of the source.

• The ***interferometer*** is therefore a ***band-pass filter*** (as opposed to a single pupil which is a low-pass filter) giving access to the very high spatial frequencies of the source.

• Measuring the complex coherence factor at several spatial frequencies allows to restore the spatial intensity distribution of the source.

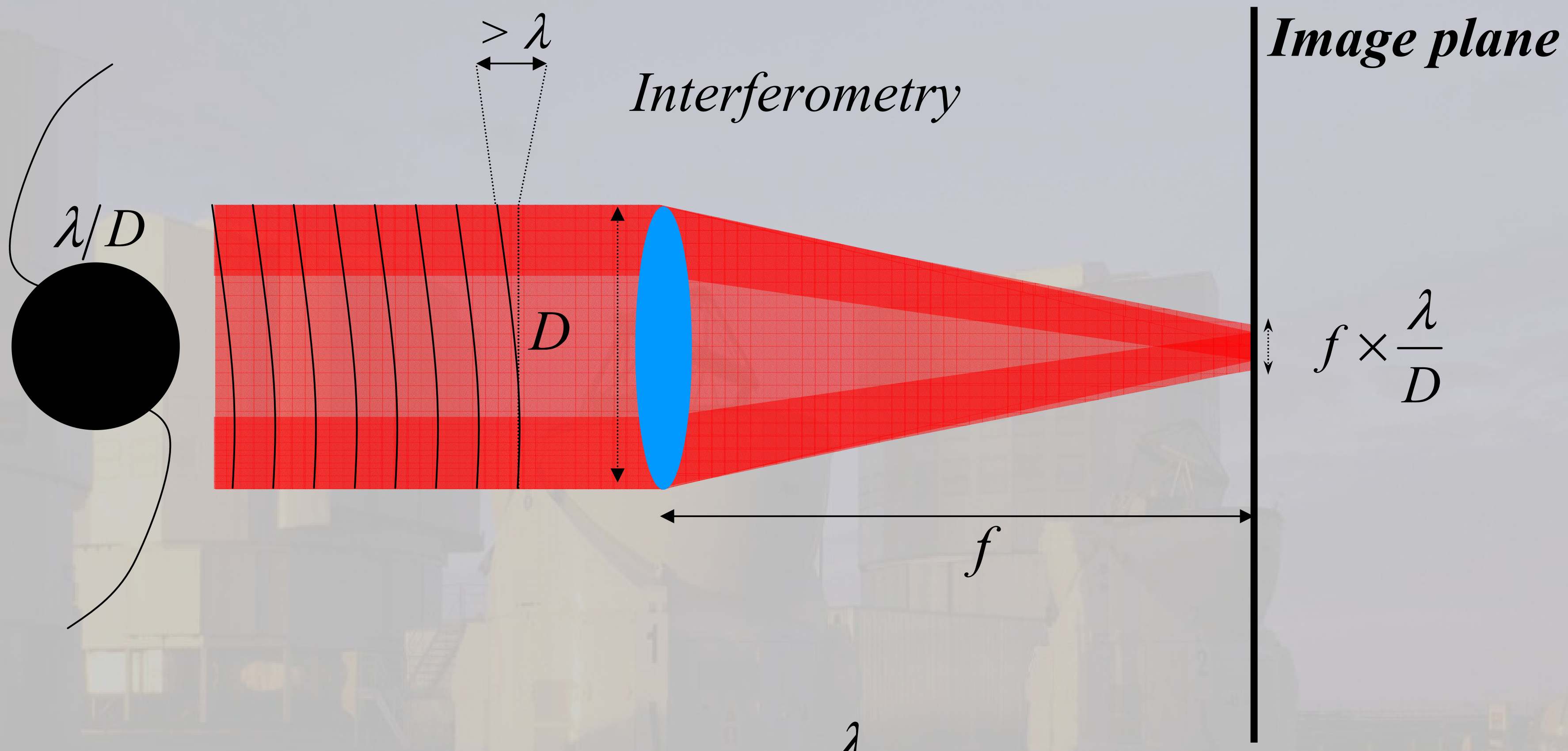
-> ***aperture synthesis technique***



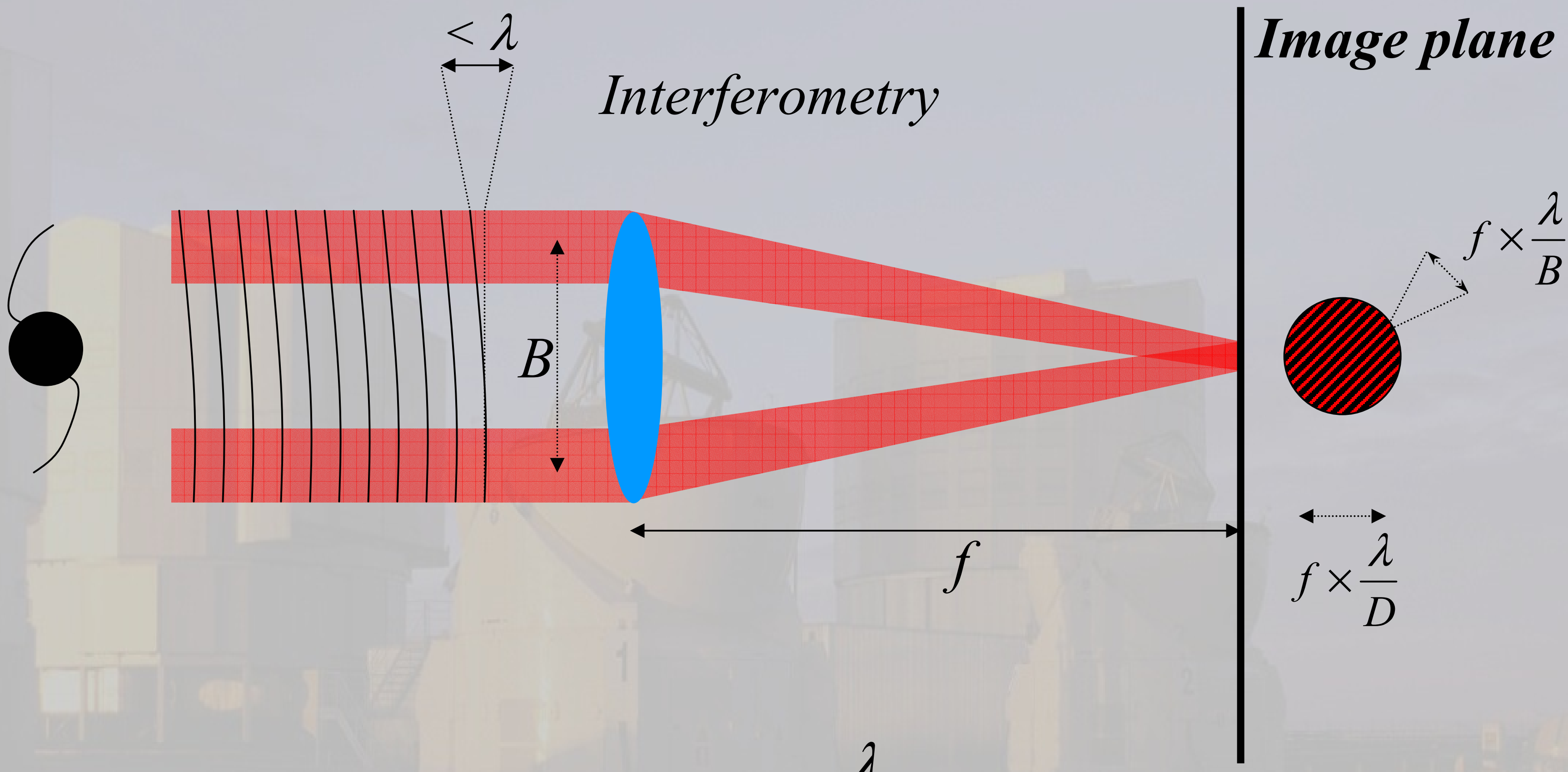
The *complex coherence factor* is usually called the *complex visibility*:

- the modulus is the fringe contrast of the interference pattern
- the phase is derived from the position of the central fringe with respect to the zero optical path difference (zero opd or zopd):

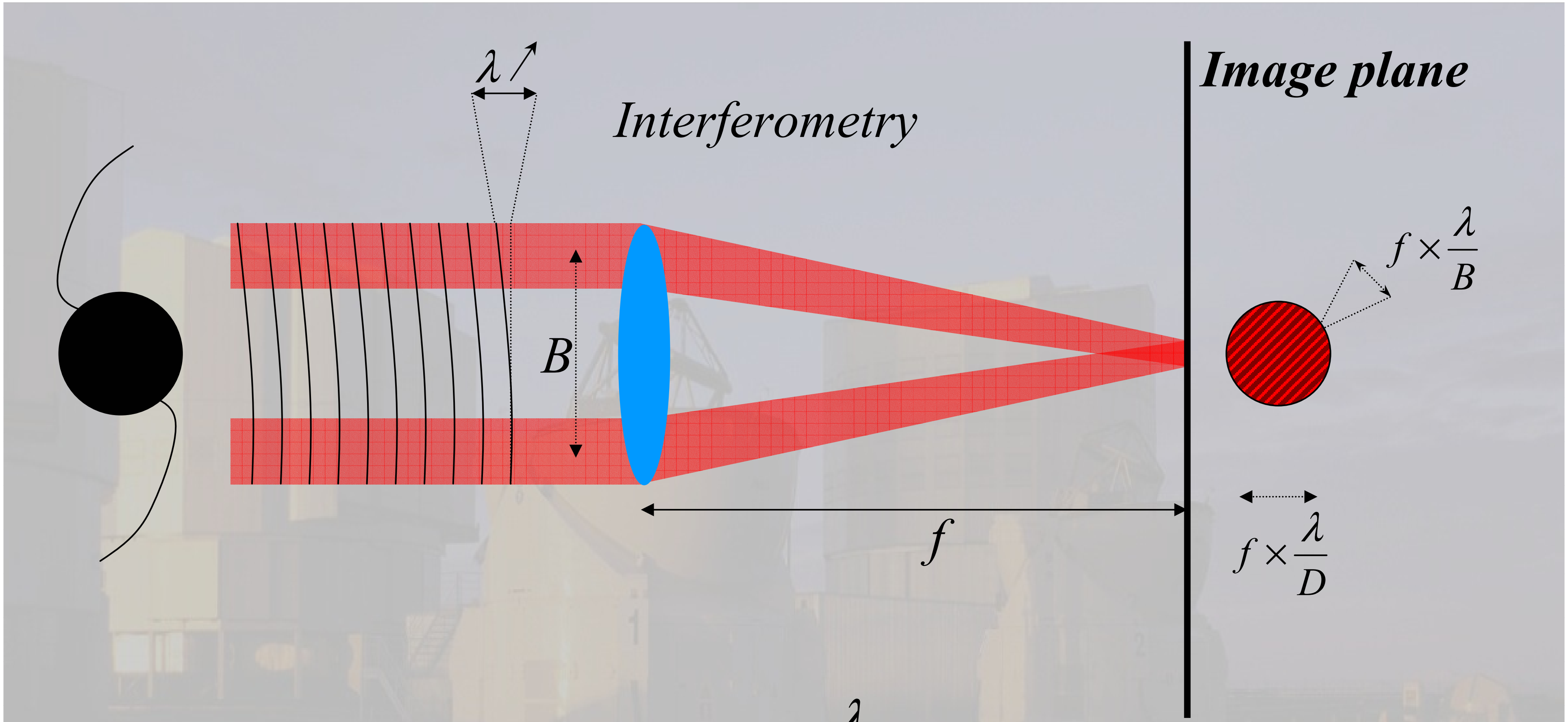
$$\varphi = \frac{2\pi\delta}{\lambda}$$



Angular resolution : $\frac{\lambda}{D}$

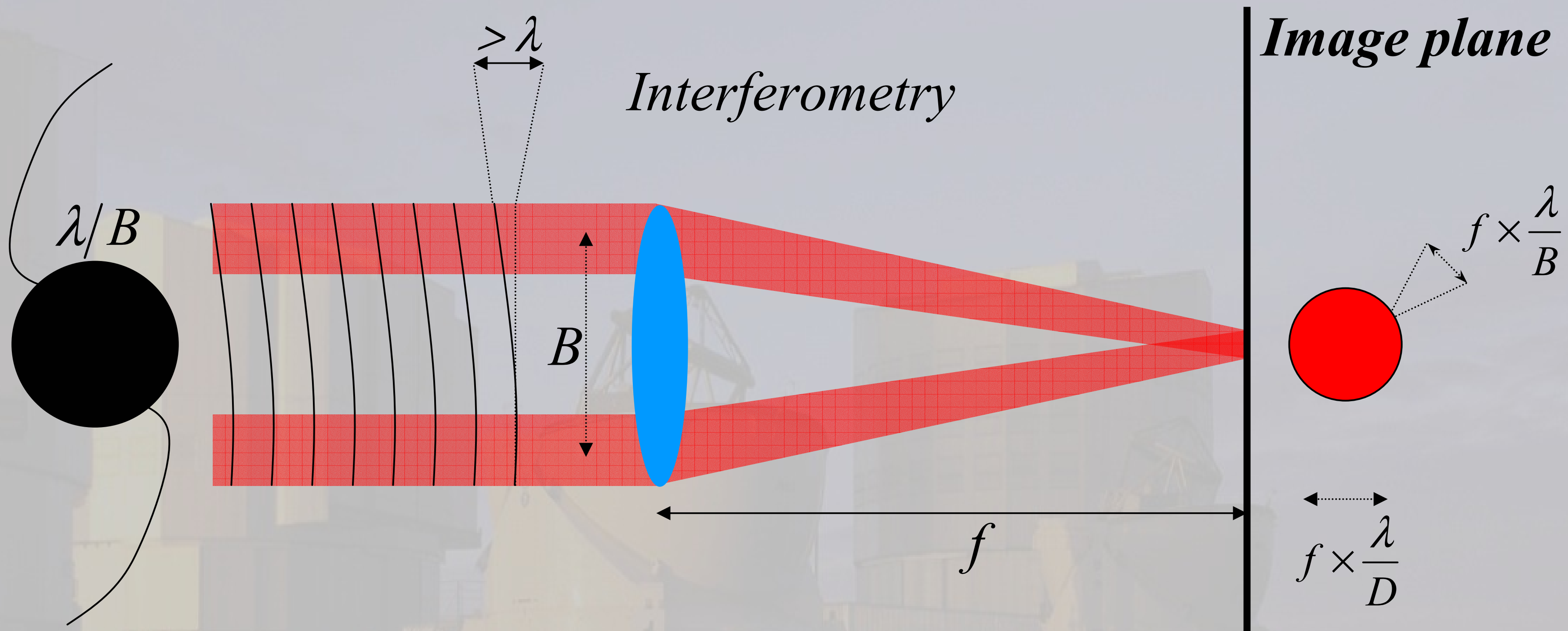


Angular resolution : $\frac{\lambda}{B}$

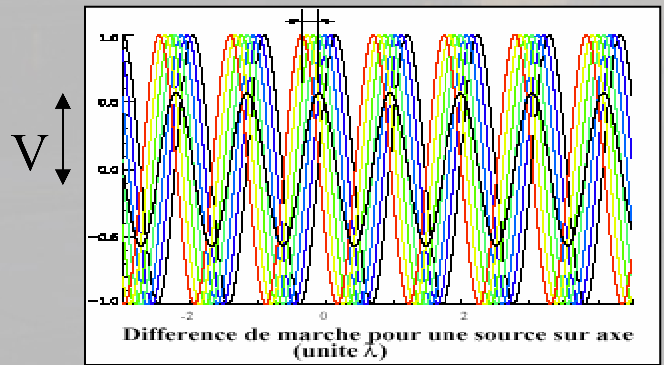


Angular resolution : $\frac{\lambda}{B}$

Interferometry



Angular resolution : $\frac{\lambda}{B}$

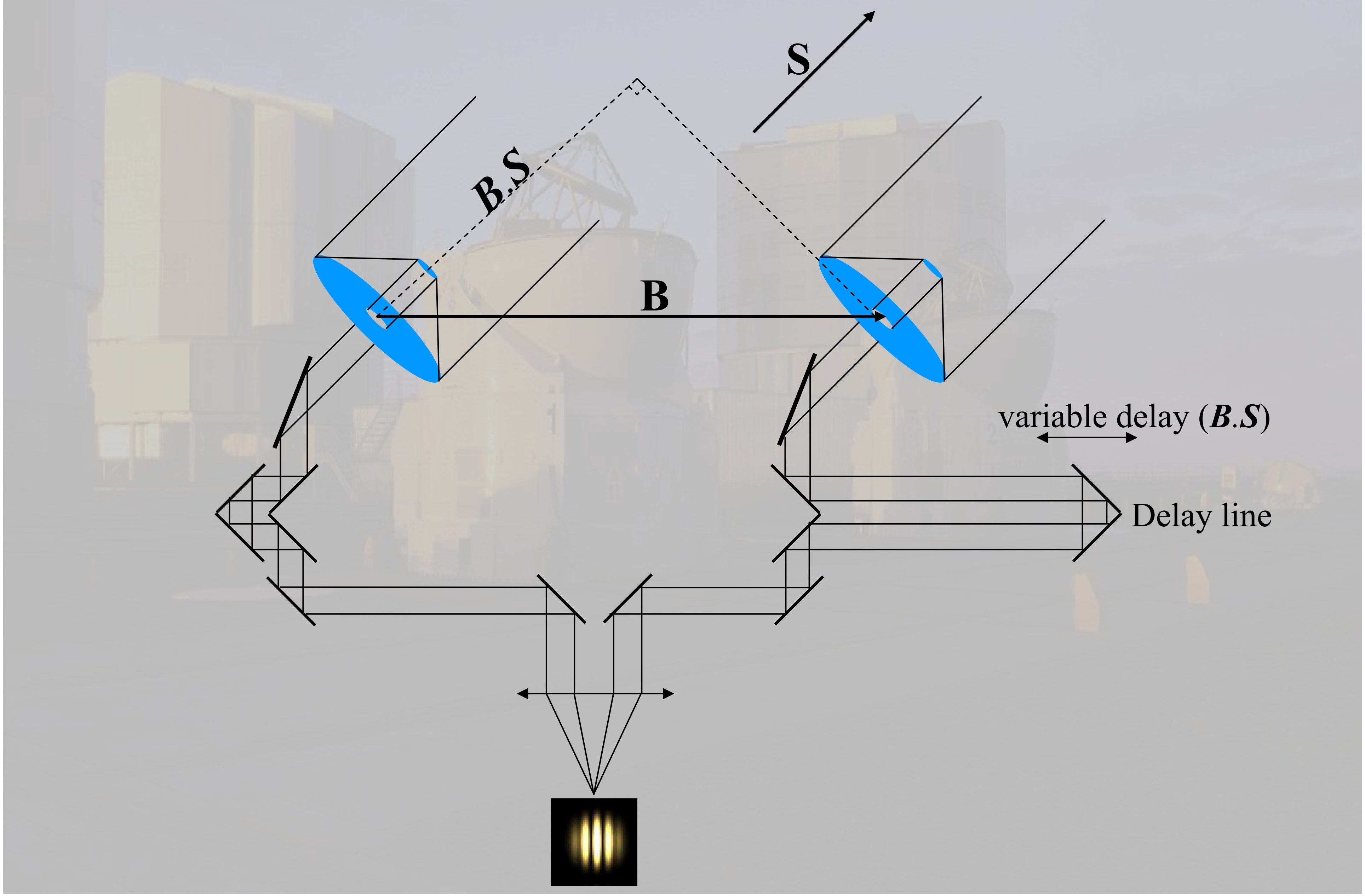


$$\int_{-\varphi}^{+\varphi} \cos x \, dx = 0 \quad \text{when} \quad \varphi = \pi$$

$$\text{hence} \quad \varphi = \pi = \frac{2\pi}{\lambda} \times \frac{B\Delta\alpha}{2} \Rightarrow \Delta\alpha = \frac{\lambda}{B}$$

The optical interferometer

Astronomical interferometry in practice

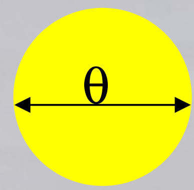


Model fitting and visibility models

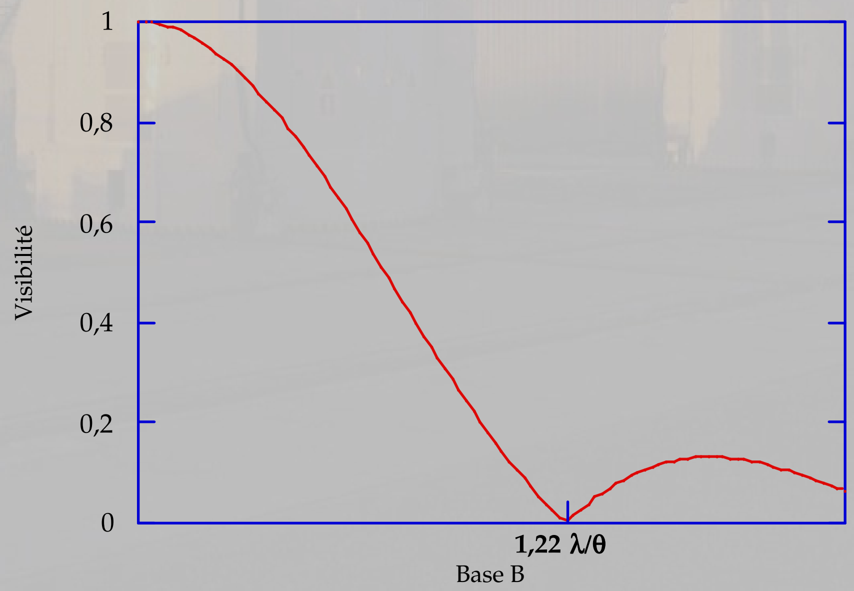
Two examples of visibility functions

Uniform disk: $I(\vec{S}) = \Pi(\frac{S}{\theta})$ with θ the angular diameter.

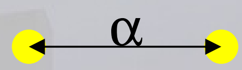
The visibility function is equal to: $V(\vec{B}) = \frac{2J_1\left(\frac{\pi\theta B}{\lambda}\right)}{\frac{\pi\theta B}{\lambda}}$



Visibility function (modulus) :

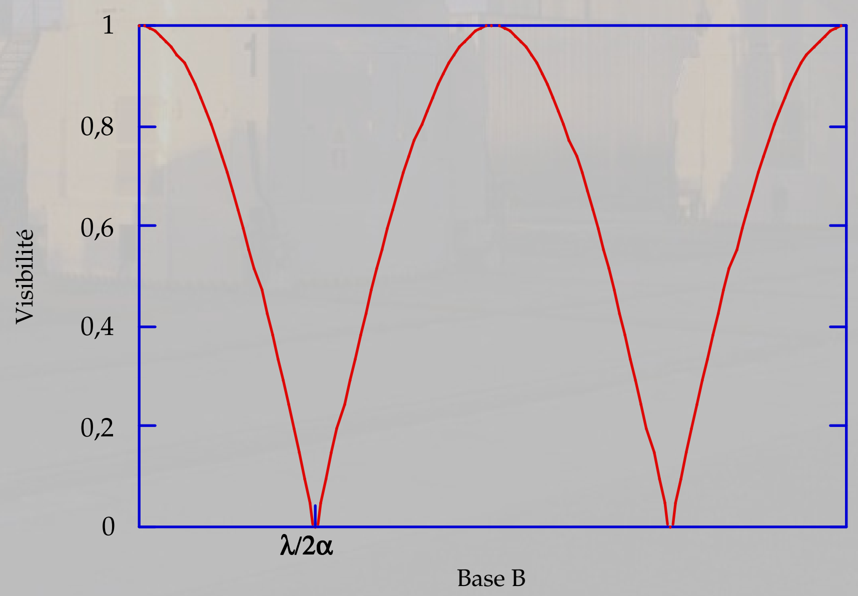


Binary star: $I(\vec{S}) = \delta\left(\vec{S} - \frac{1}{2}\vec{\alpha}\right) + \delta\left(\vec{S} + \frac{1}{2}\vec{\alpha}\right)$



The visibility function equals: $V(\vec{B}) = \cos\left(\frac{\pi\vec{\alpha}.\vec{B}}{\lambda}\right)$

Visibility function (modulus) for $\boldsymbol{B} // \boldsymbol{\alpha}$:



Quiz

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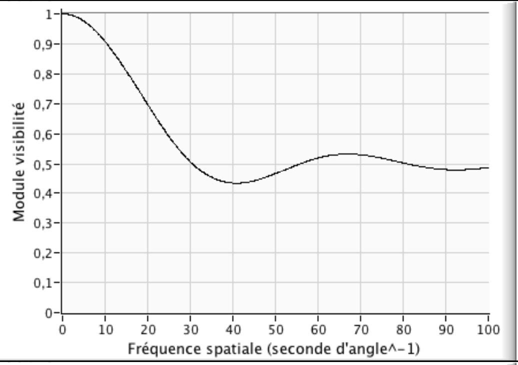
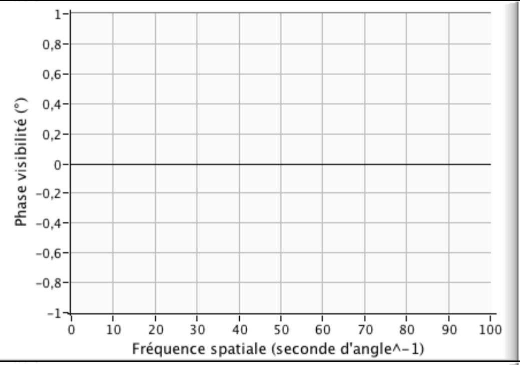
<tr><td>Object 1</td><td>
</td><td>
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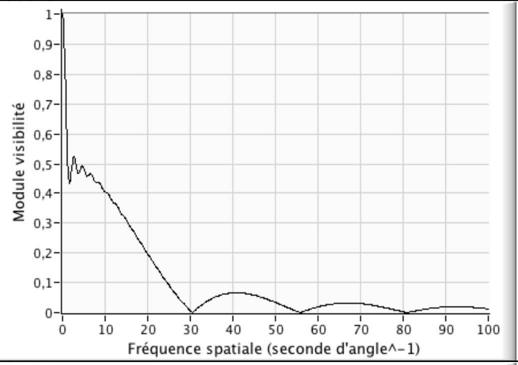
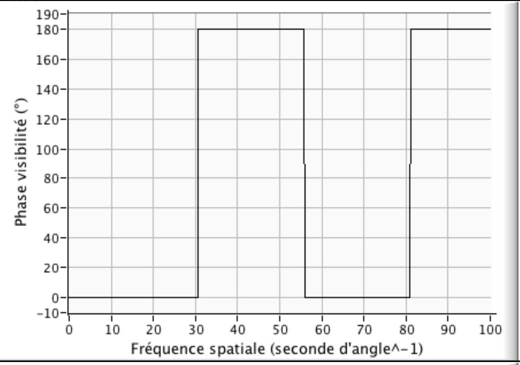
<tr><td>Object 2</td><td>
</td><td>
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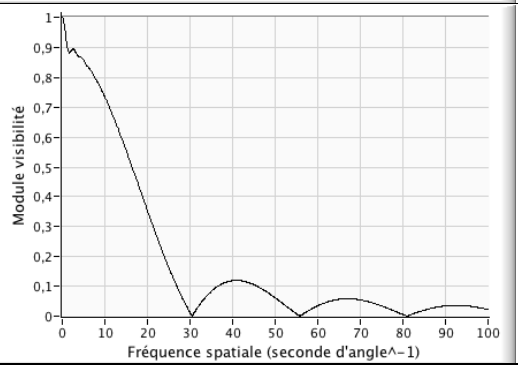
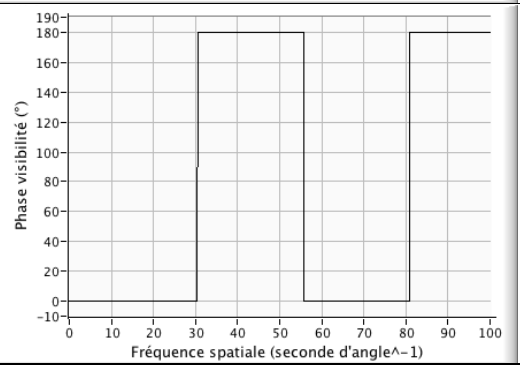
<tr><td>Object 3</td><td>
</td><td>
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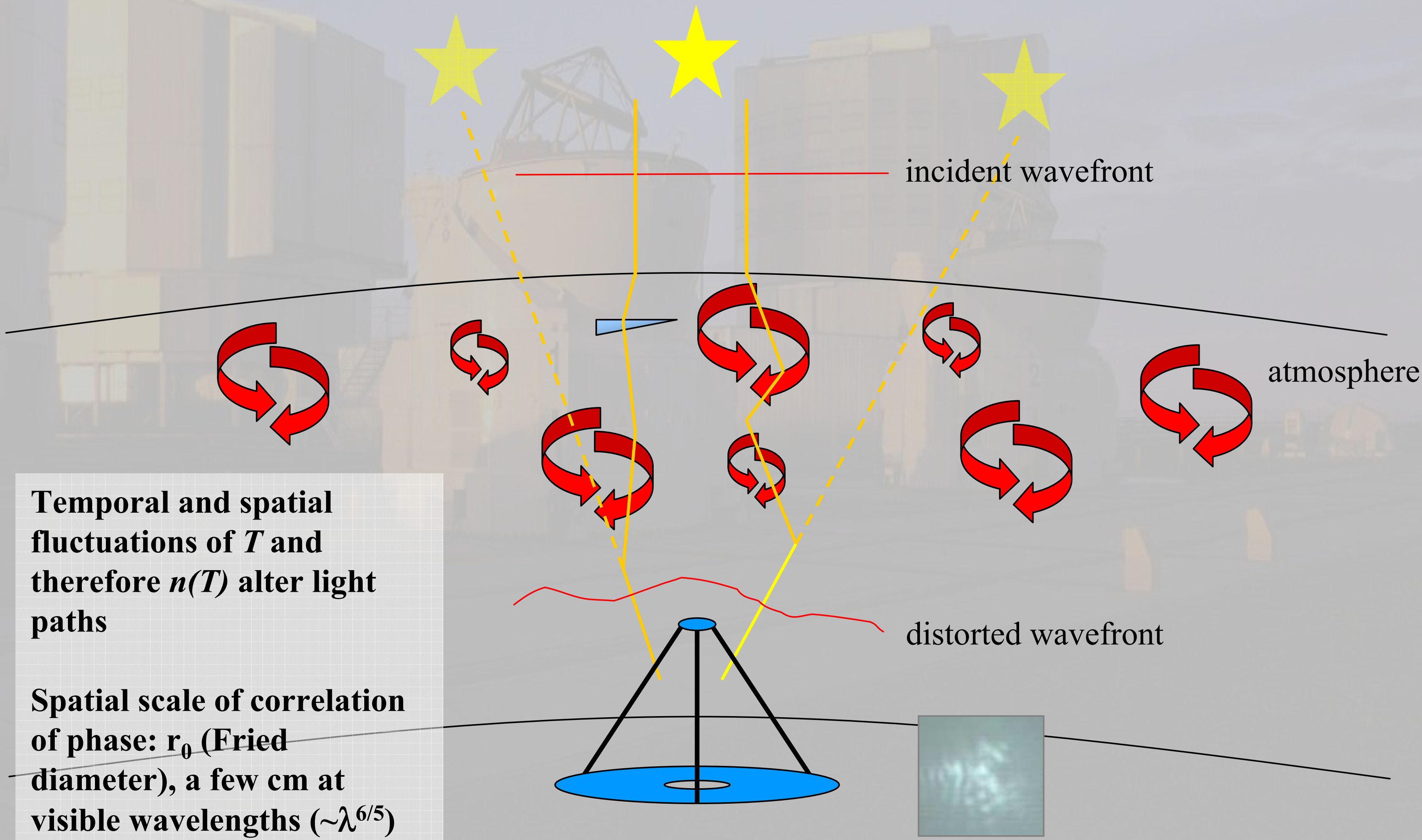
Atmospheric turbulence

Atmospheric turbulence

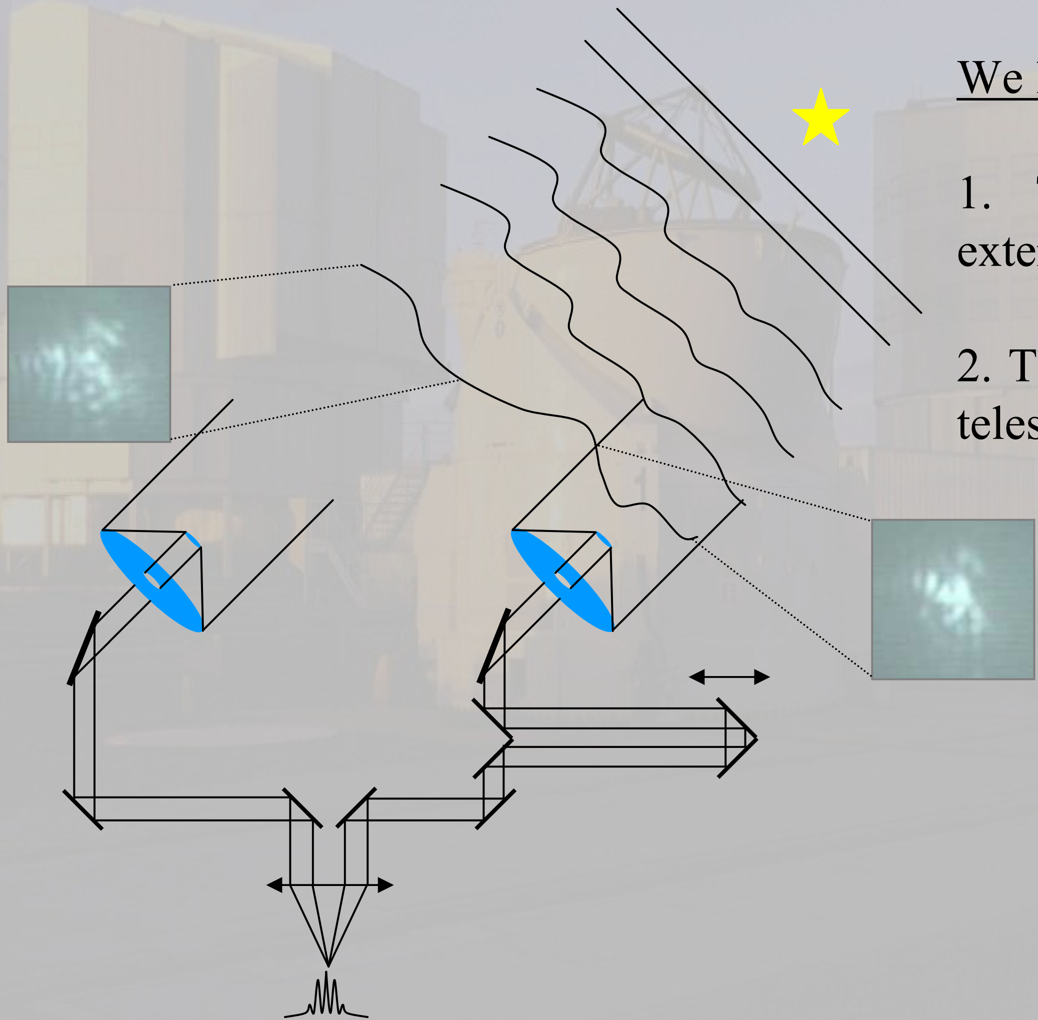


Temporal and spatial fluctuations of *T* and therefore *n(T)* alter light paths

Spatial scale of correlation of phase: r_0 (Fried diameter), a few cm at visible wavelengths ($\sim\lambda^{6/5}$)

The issue of atmospheric turbulence

Spatial coherence loss



We lose twice:

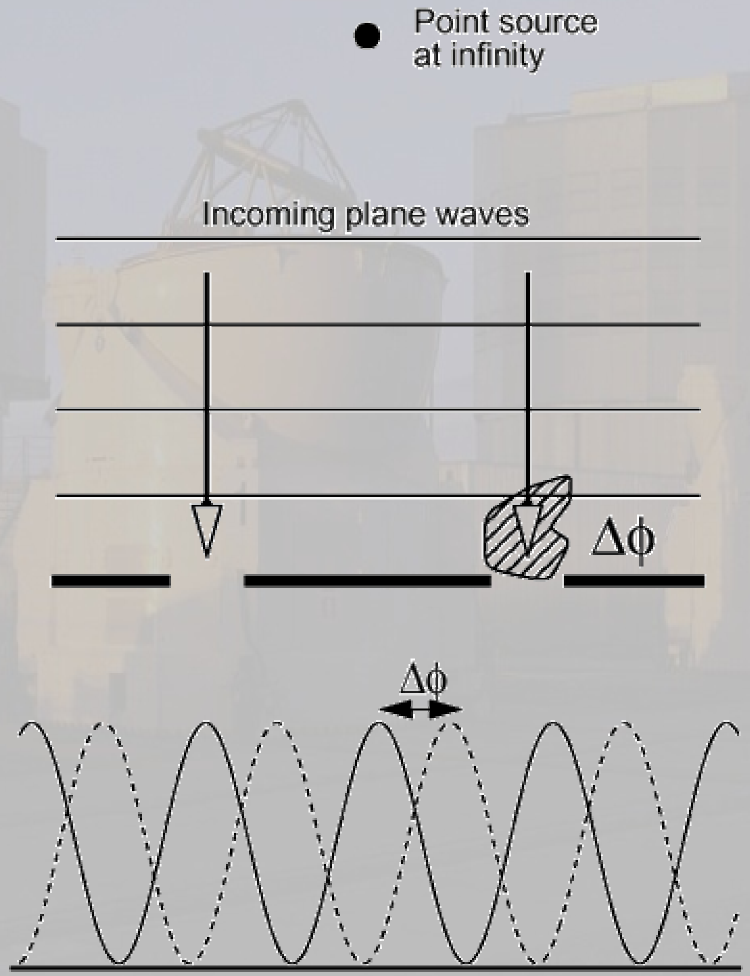
1. The object is apparently more extended hence less spatially coherent.

2. The object looks different in the two telescopes ➲ poorer correlation.

AO corrected image

Phase: closure, differential

Atmosphere Corrupts the Phase



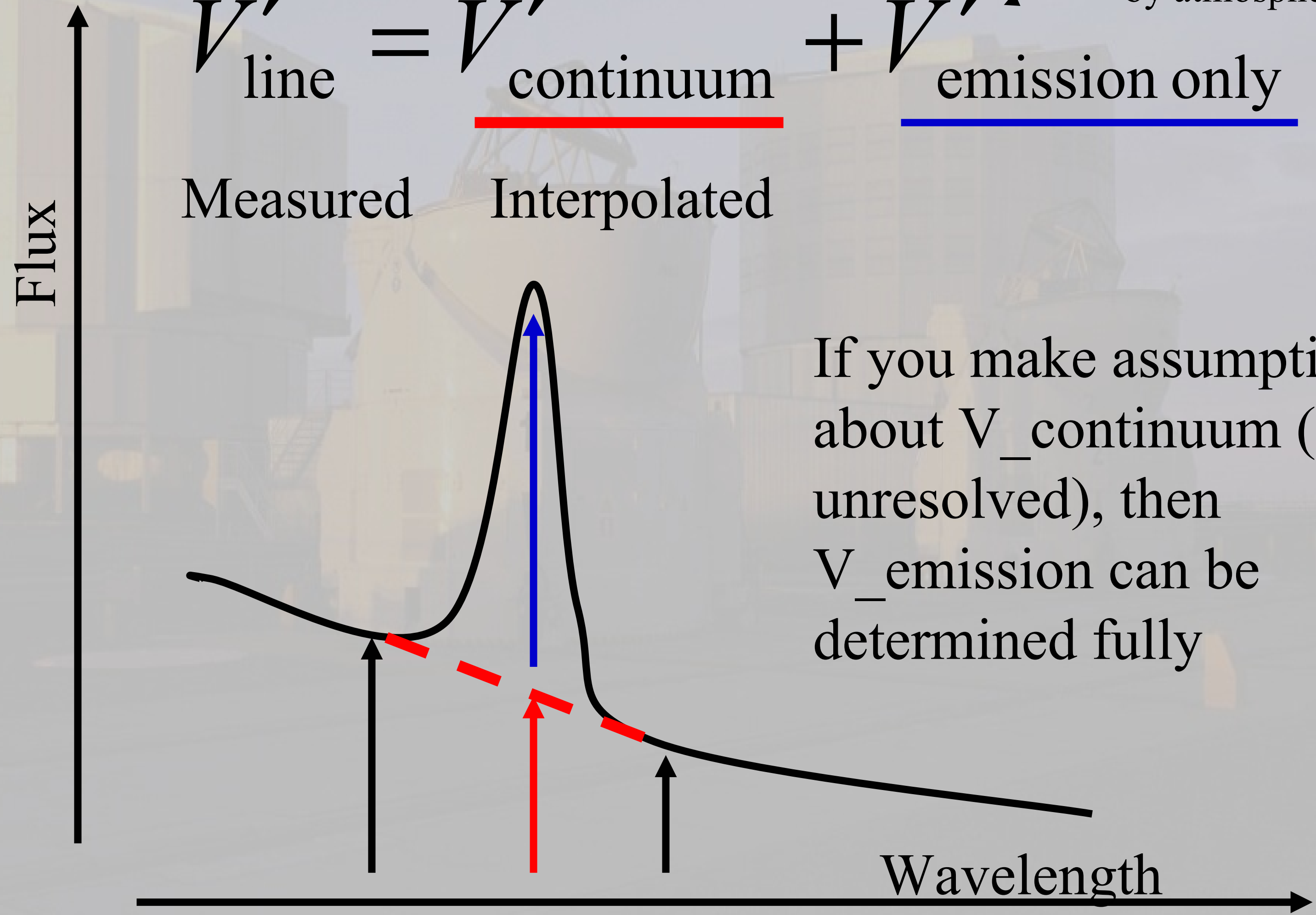
The absolute phase is ill-defined

Differential Phase

$$\tilde{V}'_{\text{line}} = \tilde{V}'_{\text{continuum}} + \tilde{V}'_{\text{emission only}}$$

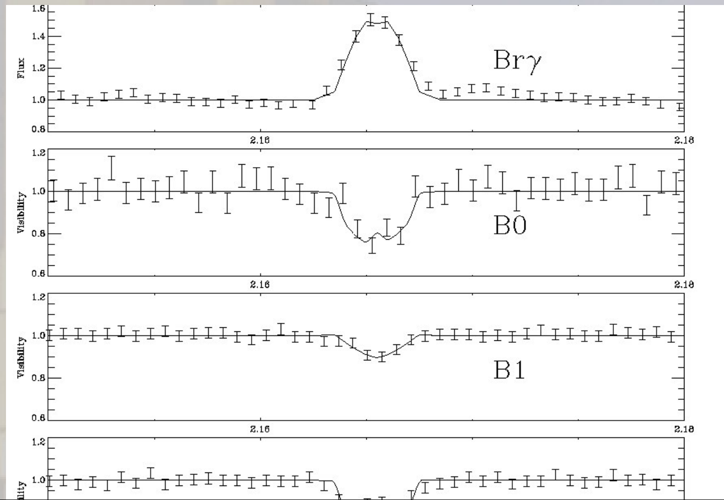
Prime indicates corrupted by atmospheric piston

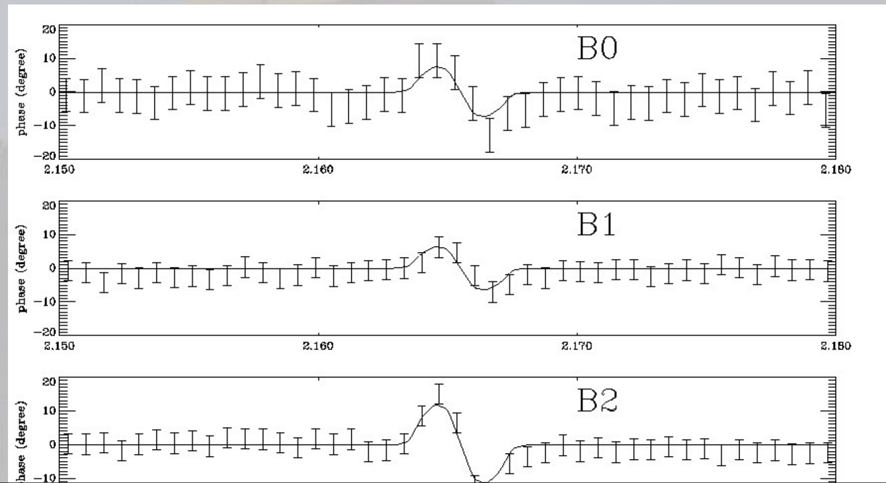
Measured Interpolated

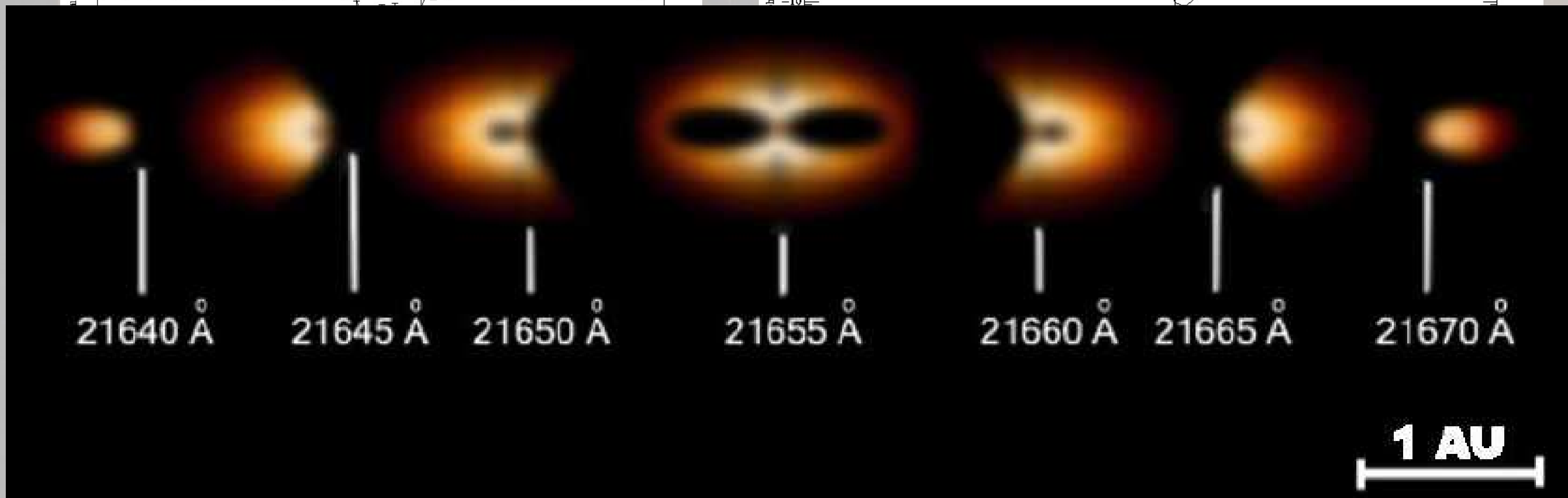


If you make assumption about V_continuum (e.g., unresolved), then V_emission can be determined fully

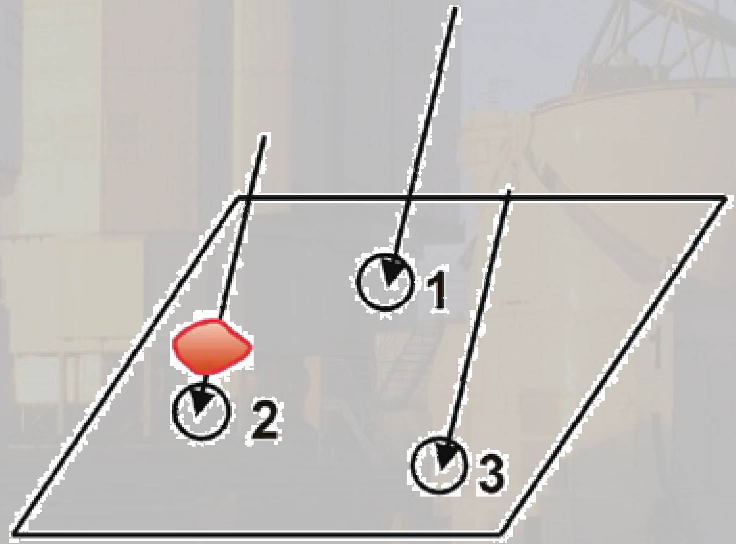
Differential Phases with VLTI-AMBER: What might this be?







The “Closure Phase” Is Not Corrupted



Observed		Intrinsic		Atmosphere
$\Phi(1\text{-}2)$	=	$\Phi_o(1\text{-}2)$	+	$[\phi(2)-\phi(1)]$
$\Phi(2\text{-}3)$	=	$\Phi_o(2\text{-}3)$	+	$[\phi(3)-\phi(2)]$
$\Phi(3\text{-}1)$	=	$\Phi_o(3\text{-}1)$	+	$[\phi(1)-\phi(3)]$

Closure Phase (1-2-3) $= \Phi_o(1\text{-}2)+\Phi_o(2\text{-}3)+\Phi_o(3\text{-}1)$

How Much Phase Information?

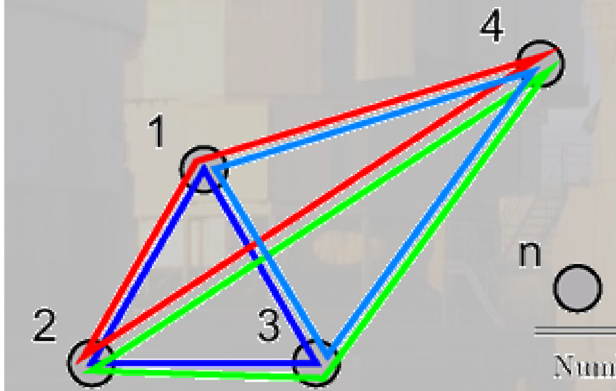
Closure Phases are not all independent from each other.

Number of Closure Phases

$$\binom{N}{3} = \frac{(N)(N-1)(N-2)}{(3)(2)},$$

Number of Fourier Phases

$$\binom{N}{2} = \frac{(N)(N-1)}{2}$$



Number of Independent Closure Phases

$$\binom{N-1}{2} = \frac{(N-1)(N-2)}{2}$$

Number of Telescopes	Number of Fourier Phases	Number of Closing Triangles	Number of Independent Closure Phases	Percentage of Phase Information
3	3	1	1	33%
7	21	35	15	71%
21	210	1330	190	90%
27	351	2925	325	93%
50	1225	19600	1176	96%

Summary of a few Important Points

The closure phases are independent of all **telescope-specific** phase errors.

The closure phases are all 0 or 180 degrees for sources with **point symmetry**.

Object must be resolved (~> half fringe spacing B/λ) to have non-zero CP

-- CP $\propto$ (baseline)^3

-- Phase $\propto$ (baseline)

Based on J.D. Monnier

Calibration

Calibrator Sources

Formally, *anything* can be a calibration source.

However, assessing our measurement *accuracies* we must account for uncertainties in our ability to predict the properties of the calibration source:

$$V_{trg}^2 = (V_{model-cal}^2 / V_{meas-cal}^2) V_{meas-trg}^2$$

$$\delta V_{trg}^2 \propto (\partial V_{model-cal}^2 / \partial model)\, \sigma_{model}$$

Traditionally (realistically) this has meant that we choose calibrators whose properties are as simple as possible – <u>single</u> stars!

$$\delta V_{trg}^2 \propto (\partial V_{model-cal}^2 / \partial \Theta)\, \sigma_{\Theta}$$

Choose star diameter as small as possible as then the derivative is 0

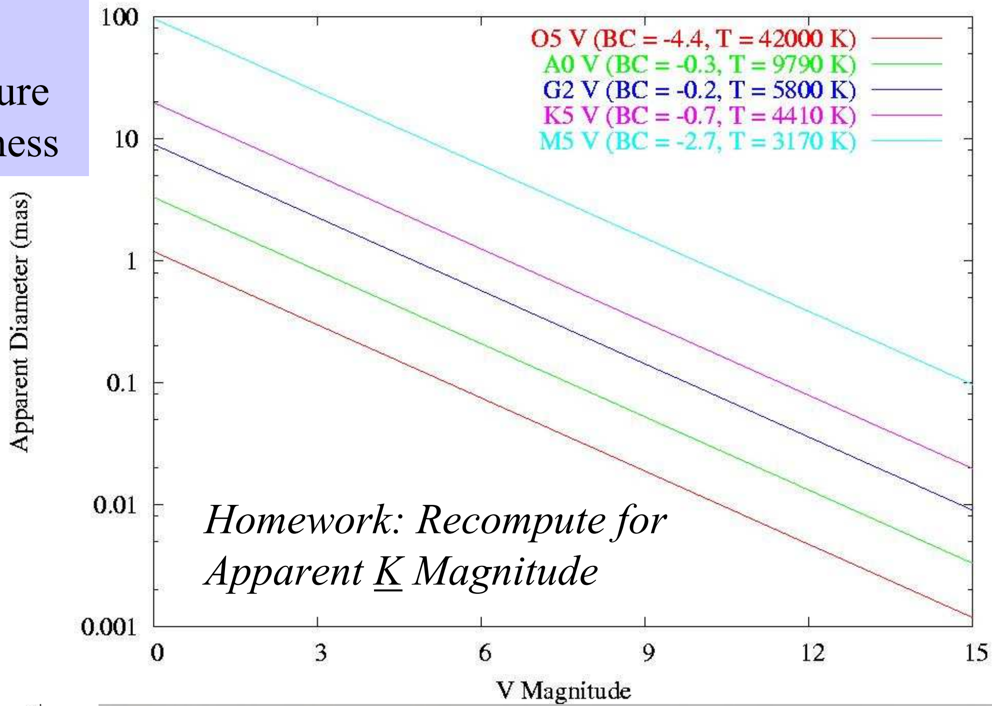
Based on A. Boden

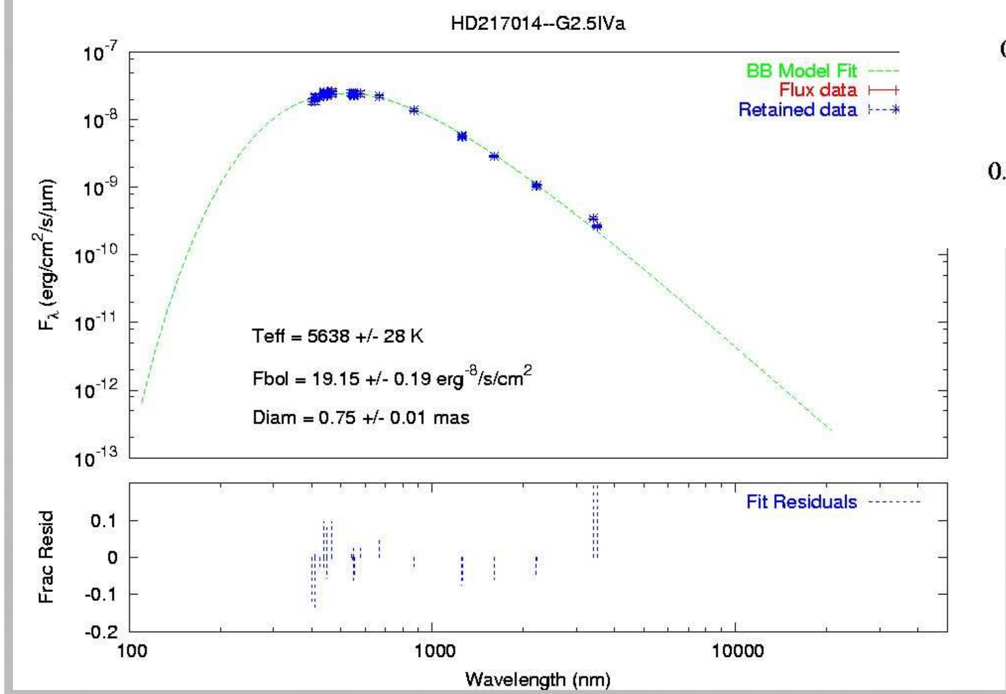
Estimating Angular Sizes of Stars (3)

$$\Theta = \sqrt{\frac{4 F_{Bol}}{\sigma T_{eff}^4}}$$

$$= 8.17 mas \bullet 10^{-0.2(V+BC)} \bullet \left[\frac{T_{eff}}{5800 K}\right]^{-2}$$

- So estimating apparent size is *easy*: all you need is *bolometric flux* and *effective temperature*!
- In this sense effective temperature is just a proxy for surface brightness





- Because stellar size is proportional to (sqrt of) brightness, bright & "point-like" does not formally exist
- Source resolution is a question of instrument resolution and sensitivity
- For given apparent brightness, "hotter" stars appear smaller